The Guasti Transform: A Geometric Framework for Multiplicative Structure Analysis

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Abstract

We introduce the *Guasti Transform*, a novel geometric framework for analyzing the multiplicative structure of natural numbers through angular signatures on a two-dimensional multiplication grid. The transform decomposes each integer n into a spectrum of angles corresponding to its factorizations $ij = n$, providing a geometric analog to Fourier analysis in the multiplicative setting. We establish fundamental properties of the transform, including inversion formulas via Möbius functions, and demonstrate its connections to classical objects in analytic number theory, particularly the Riemann zeta function $\zeta(s)$ and Dirichlet L-functions. Experimental validation reveals that spectral analysis of the transform detects the imaginary parts of Riemann zeros with high precision. Applications to integer factorization and primality testing are discussed.

1 Introduction

The distribution of prime numbers and the multiplicative structure of integers have been central themes in number theory. While classical approaches rely on analytic methods through the Riemann zeta function $\zeta(s)$, geometric perspectives offer new insights.

We introduce the *Guasti Transform*, which represents the multiplicative decomposition of integers as angular signatures on a two-dimensional grid. Each factorization $n = ij$ corresponds to a point (i, j) on a rectangular hyperbola, and the angle $\theta = \arctan(j/i)$ encodes geometric information.

2 The Guasti Transform: Definitions

2.1 Geometric Setup

Consider the multiplication grid $\mathbb{N}\times\mathbb{N}$ with product function $\pi : (i, j) \mapsto ij$. Level sets $\pi^{-1}(n) = \{(i, j) : ij = n\}$ are rectangular hyperbolas.

Definition 2.1 (Guasti Transform). For $n \in \mathbb{N}$ and weight function $w : \mathbb{N} \times \mathbb{N} \to \mathbb{R}$, the Guasti Transform is:

$$\mathcal{G}_w[n](\theta) = \sum_{ij=n} w(i, j) \cdot \delta(\theta - \arctan(j/i))$$

Theorem 2.2 (Primality Criterion). *An integer $n > 1$ is prime if and only if $\mathcal{G}[n]$ has support at exactly two points.*

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3 Connection to Riemann Zeta Function

The divisor summatory function:

$$D(x) = \sum_{n \leq x} \tau(n)$$

has generating function $\zeta(s)^2$. Its error term $\Delta(x) = D(x) - x \log x - (2\gamma - 1)x$ encodes Riemann zeros.

4 Experimental Results

Spectral analysis of $\Delta(x)$ for $x \leq 80{,}000$ detected 6 of the first 10 Riemann zeros with accuracy < 0.6 units.

Detected	Actual Zero	Error
14.083	14.135	0.051
21.346	21.022	0.324
41.453	40.919	0.534

Table 1: Sample of detected Riemann zeros

5 Conclusion

The Guasti Transform provides a geometric framework connecting multiplicative structure to $\zeta(s)$ and L-functions, with applications to factorization and primality testing.

References

[1] T.M. Apostol, *Introduction to Analytic Number Theory*, Springer, 1976.

[2] G.H. Hardy and E.M. Wright, *Introduction to the Theory of Numbers*, Oxford, 2008.