



AwesomeMath Test C

March 16 – March 28, 2024

PLEASE READ CAREFULLY and THOROUGHLY

DEADLINE: Thursday, March 28, 2024, 5:00 PM CT

INSTRUCTIONS

- Students who wish to take Level 1 & Level 2 classes must submit solutions to Part I only.
- Students who wish to take Level 3 & Level 4 classes must submit solutions to Part II only.
- **Only one test and one part (Part I or Part II) will be graded per student.** Do NOT submit solutions to Part I and then additional solutions to Part II as a separate submission.
- Students accepted for Level 1 and Level 2 will not be able to take Level 3 and 4 classes, however students accepted for Level 3 and 4 are allowed to switch to lower-level classes if it turns out that their initial classes are too difficult.
- **This test is not timed, therefore, do not rush.** Show us your best work. Your solutions will be evaluated against mathematical experience and any math competition test scores provided on your application.
- We want you to have a positive and successful experience at our camps, therefore **your work should be yours and yours alone.** The purpose of the test is to ensure that you are placed in classes that fit your level and skills. If you consult outside resources (other people or online), then you are compromising your ability to succeed in our challenging program.
- **Do not be discouraged if you cannot solve all the questions.** We want to see the solutions you come up with no matter how many problems you solve. Some of the problems involve complex mathematical ideas, but all of them can be solved using only elementary techniques, admittedly combined in clever ways.
- **Include all significant steps in your reasoning and computation.** We are interested in your ability to present your work, so unsupported answers will receive less credit than well-reasoned progress towards a solution without a correct answer.
- In this document, you will find an answer sheet. **Print out or make several copies of the blank answer sheet.** Fill out the top of each answer sheet as you go. Start each problem on a new answer sheet.
- **You may handwrite or type your solutions.** If you type your solutions, your answer sheets should still include the same information as shown on the test packet answer sheets (9-digit UIN, First and Last Name, Problem #, etc.)

SUBMISSION REQUIREMENTS

- **Your solutions must be submitted as a single pdf document.** Do NOT submit test solutions as individual pages.
- **UPLOAD your solutions file** in accordance with the directions provided on your student dashboard.
- **DO NOT email us your solutions** as it will significantly delay receipt and grading of your solutions.



AwesomeMath Admission Test Cover Sheet

PLEASE PRINT LEGIBLY and DO NOT LEAVE ANY FIELDS BLANK

9-Digit UIN:

Last Name:

First Name:

Admission Test (check one): ☐ A ☐ B ☐ C ☐ D ☐ E

Part Completed (check one): ☐ Part I ☐ Part II

Email:

Important Reminders:

- **Make sure this COVER SHEET is the first page of your submission**, and that it is completely filled out.
- **UPLOAD your solutions file** in accordance with the directions provided on your student dashboard.
- **Your solutions must be submitted as a single pdf document.** Do NOT submit test solutions as individual pages.
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AwesomeMath Admission Test Answer Sheet

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Last Name:

Problem Number:

Write neatly! All work should be inside the box. Do NOT write on the back of the page!

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Test C

March 16 – 28, 2024

Part I

Levels 1 & 2

1. Find all 4-tuples (p, q, r, s) of primes such that

$$p^2 + q^2 + 4r^2 + 6s^2 = 2024.$$

2. Find the least positive integer n for which $\frac{n^4}{4} - 2024n^2 + 1$ is a product of four primes, not necessarily distinct.
3. Two lotteries each draw balls marked 1 through n . The first draws 6 out of the n balls and the second 5 out of n . Playing the same number of tickets at one drawing, the odds of hitting the jackpot at the second lottery is 10 times bigger than at the first. Find n .

4. Write $2 \cdot 2024^3$ as

$$\overline{ab} \cdot \overline{bc} \cdot \overline{cd} \cdot \overline{de} \cdot \overline{ef} \cdot \overline{fg} \cdot \overline{gh}$$

and $3 \cdot 2024^3$ as

$$\overline{pq} \cdot \overline{qr} \cdot \overline{rs} \cdot \overline{st} \cdot \overline{tu} \cdot \overline{uv} \cdot \overline{vw},$$

where \overline{xy} is the two-digit number whose digits are x and y .

5. For some positive integer n , in the equality

$$\prod_{k=2}^{10^n} \left(k - 2 + \frac{2}{k^2 - k + 1} \right) = \frac{(10^n + 1)!}{100 \dots 01},$$

the denominator of the fraction in the right-hand side has 2024 zeros. Find n .

6. Let n be a positive integer such that $6n^2$ has precisely 2024 positive divisors and $4n^2$ has precisely 2025 positive divisors. Find n .
7. Let ABC be a triangle with $AB = AC$ and area 2024. Point P lies on base BC and Q and R are the orthogonal projections of P onto AB and AC , respectively. Given that $PQ + PR = 22$, evaluate BC .
8. Solve in integers the equation

$$x^2 - 15 \min(x, y) + y^2 = 2024.$$

9. Let ABC be a triangle with $AB = AC = 2024$ for which there is a point D on side BC such that $AD = 1981$ and $|BD - CD| = 1846$. Evaluate $\angle BAC$.

10. Solve the equation

$$(16x - \{x\})[x] = 2024,$$

where $[x]$ and $\{x\}$ are the greatest integer less than or equal to x and the fractional part of x , respectively.

Part II

Levels 3 & 4

1. Solve in prime numbers the equation

$$p^3 + 2q + r^3 = 2024.$$

2. Let $a_k = k^2 - 78k + 2028$, $k = 1, 2, 3, \dots$. Evaluate

$$\frac{1^3}{a_1} + \frac{2^3}{a_2} + \dots + \frac{77^3}{a_{77}}.$$

3. Solve in integers the equation

$$(x^2 + 2yz)(y^2 + 2zx)(z^2 + 2xy) \min(x, y, z) = 2024.$$

4. Find the least positive integer n such that $S(n-1)S(n+1) = 2024$, where $S(N)$ is the sum of the digits of N .

5. Let a and b be real numbers for which there is an integer n such that $a^3 + nab + b^3 = 107$ and

$$9(a^2 - ab + b^2) + n(n + 3(a + b)) = \frac{2024}{n - 3(a + b)}.$$

Find n .

6. Let $a \in \left(0, \frac{\pi}{2}\right)$ such that

$$\sin^2 a \cos^2 a + \csc^2 2a = 2024.$$

Given that $\cos 4a = m\sqrt{n} - p$, where m, n, p are positive integers and n is square-free, find $m + n + p$.

7. Solve in real numbers the system of equations

$$xy + z + w = 50, \quad yz + w + x = 90, \quad zw + x + y = 184, \quad wx + y + z = 2024.$$

8. Quadrilateral $ABCD$ is inscribed in a circle with center O . Given that $AB = 13$, $BC = 23$, $CD = 35$, and $39AC = 25BD$, evaluate $\angle AOD$.

9. In the rectangular box $ABCD A' B' C' D'$, $AB = 2024$, $AA' + BD = 2074$, and $A'D = 102$. Find the volume of the rectangular box.

10. Solve the equation

$$45(2x - \{x\}^2) = [x]^2 + 2024,$$

where $[x]$ and $\{x\}$ denote the integer part and the fractional part of x , respectively.