2021-2022 SMML
MEET 3 – No Calculators
ROUND 1

1. How many ways can the letters in the word CALCULUS be rearranged?

2. A fair 6-sided die (with numbers 1 thru 6), a fair 10-sided die (with numbers 1 thru 10), and a fair 4-sided die (with numbers 7 thru 10) are rolled. What is the probability that two of the same number are rolled?

3. The probability that Mia makes a free throw is 40%. What is the probability that Mia makes **exactly** 3 of her next 5 shots.

2021-2022 SMML
MEET 3 – No Calculators
ROUND 1

1. There are 8 letters in CALCULUS, with three letters being duplicated, so the number of possible rearrangements is $\frac{8!}{2!2!2!} = 7! = 5040$

The answer is $\boxed{5040}$

2. There is no chance that the 6-sided die and 4-sided die roll the same number. The probability that the 10-sided die rolls a number from 1 to 6 is 6/10. The probability that it rolls a number from 7 to 10 is 4/10. If it rolls a number from 1 to 6, the probability that the 6-sided die matches it is 1/6. If it rolls a number from 7 to 10, the probability that the 4-sided die matches it is ¼. Thus the overall probability of getting a pair is $\frac{6}{10} \cdot \frac{1}{6} + \frac{4}{10} \cdot \frac{1}{4} = \frac{2}{10} = \frac{1}{5}$

The answer is $\boxed{\frac{1}{5}}$

3. The probability of making a shot is 2/5, and the probability of missing a shot is 3/5. The probability of making 3 shots and then missing 2 shots is $\left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^2$. But we can arrange the three shots that are made in $\binom{5}{3}$ ways. So the total probability is

$$\left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^2 \binom{5}{3} = \frac{2^3 \cdot 3^2 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{5^5 \cdot 3 \cdot 2 \cdot 2} = \frac{2^4 \cdot 3^2}{5^4} = \frac{144}{625}$$

The answer is $\boxed{\frac{144}{625}}$

2021-2022 SMML
MEET 3 – No Calculators
ROUND 2

1. If $\dfrac{x^3+4x^2+9x+16}{x+5} = P(x) + \dfrac{R}{x+5}$ where $P(x)$ is a polynomial and R is a constant, then find the value of R.

2. Simplify completely: $1-\dfrac{1-\dfrac{2}{1-x}}{x+\dfrac{3+x}{1-x}}$

3. Simplify completely, leaving your final answer in fully factored form:
$\dfrac{x^3b^2+x^3c^2-a^3b^2-a^3c^2}{xb^4-xc^4-ab^4+ac^4}$

2021-2022 SMML
MEET 3 – No Calculators
ROUND 2

1. Either synthetic or long division can be used here. Synthetic division would give

$$\begin{array}{r|rrrr} & 1 & 4 & 9 & 16 \\ -5 & & -5 & 5 & -70 \\ \hline & 1 & -1 & 14 & -54 \end{array}$$

So $R=-54$

The answer is $\boxed{-54}$

2. Simplifying, we get

$$1-\frac{1-\frac{2}{1-x}}{x+\frac{3+x}{1-x}}=1-\frac{1-x-2}{x-x^2+3+x}=1-\frac{-x-1}{-x^2+2x+3}=1-\frac{x+1}{x^2-2x-3}=1-\frac{x+1}{(x+1)(x-3)}=1-\frac{1}{x-3}$$

$$=\frac{x-3-1}{x-3}=\frac{x-4}{x-3}$$

The answer is $\boxed{\dfrac{x-4}{x-3}}$

3. Factoring by grouping, we get

$$\frac{x^3b^2+x^3c^2-a^3b^2-a^3c^2}{xb^4-xc^4-ab^4+ac^4}=\frac{x^3\left(b^2+c^2\right)-a^3\left(b^2+c^2\right)}{x\left(b^4-c^4\right)-a\left(b^4-c^4\right)}=\frac{\left(x^3-a^3\right)\left(b^2+c^2\right)}{(x-a)\left(b^4-c^4\right)}$$

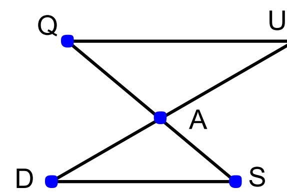
Factoring the difference of cubes and squares we get

$$=\frac{(x-a)\left(x^2+ax+a^2\right)\left(b^2+c^2\right)}{(x-a)\left(b^2+c^2\right)\left(b^2-c^2\right)}=\frac{x^2+ax+a^2}{b^2-c^2}=\frac{x^2+ax+a^2}{(b+c)(b-c)}$$

The answer is $\boxed{\dfrac{x^2+ax+a^2}{(b+c)(b-c)}}$

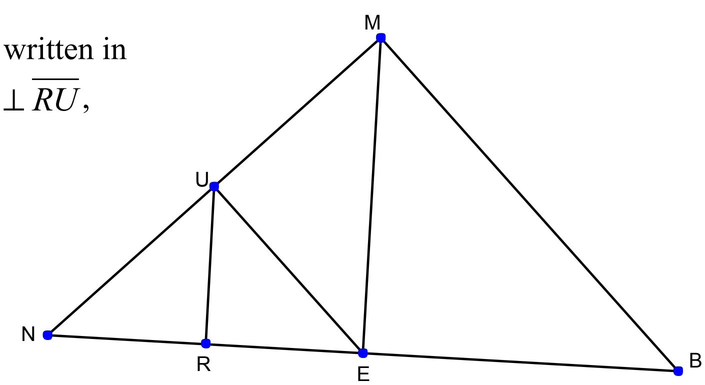
2021-2022 SMML
MEET 3 – No Calculators
ROUND 3

1. In the diagram to the right, parallel lines $\overleftrightarrow{QU}$ and $\overleftrightarrow{DS}$ are intersected by perpendicular transversals $\overleftrightarrow{QS}$ and $\overleftrightarrow{UD}$. If $QA = 6$, $AD = 5$, $DS = 13$, then find the area of ΔAQU.



2. One leg of a right triangle with perimeter 70 is 20. What is the length of the other leg?

3. The perimeter of ΔNMB can be written in the form $a+b\sqrt{c}$. Find $a+b+c$ if $\overline{NB} \perp \overline{RU}$, $\overline{NM} \perp \overline{UE}$, $\overline{NB} \perp \overline{EM}$, $\overline{NM} \perp \overline{MB}$, $\overline{EU}$ bisects $\overline{NM}$, and $NU = 3$.



2021-2022 SMML
MEET 3 – No Calculators
ROUND 3

1. Because ΔDAS is a right triangle, we get that $AS = 12$. Because the lines are parallel, we know that ΔDAS and ΔUAQ are similar. Since $\frac{QA}{AS} = \frac{6}{12} = \frac{1}{2}$, we know that the ratio of the areas will be $\frac{1}{4}$. Since ΔDAS is a right triangle, its area is $\frac{1}{2} \cdot 5 \cdot 12 = 30$, so the area of ΔUAQ is $\frac{15}{2}$.

The answer is $\boxed{\frac{15}{2}}$

2. Label the missing sides *b* and *c*. Then we know that $b + c = 50$ and that $20^2 + b^2 = c^2$. Solving this system, we get

$400 + b^2 = (50 - b)^2$

$400 = (50 - b)^2 - b^2$

$400 = (50 - b + b)(50 - b - b)$

$400 = 50(50 - 2b)$

$8 = 50 - 2b$

$2b = 42$

$b = 21$

The answer is $\boxed{21}$

3. Because *U* is the midpoint of the hypotenuse $\overline{NM}$, $UE = EM = NU = 3$. The Pythagorean theorem gives us $NE = 3\sqrt{2}$. Using the Pythagorean theorem on triangle NME we get that $ME = 3\sqrt{2}$. Because ME is the altitude from the right angle of ΔNMB we get that

$NM^2 = NE(NE + EB)$

$36 = 3\sqrt{2}\left(3\sqrt{2} + EB\right)$

$6\sqrt{2} = 3\sqrt{2} + EB$

$EB = 3\sqrt{2}$

Using the Pythagorean theorem one last time, we get that $MB = 6$.

So, the perimeter is $12 + 6\sqrt{2}$, so $a + b + c = 20$

The answer is $\boxed{20}$

2021-2022 SMML
MEET 3 – No Calculators
ROUND 4

1. Solve for x: $\left(\sqrt[x]{\sqrt[4]{81}}\right)^{2/3} = \sqrt{27}$

2. Find the product of all solutions to $2^{2x} - 2^2 \cdot 3^2 \cdot 2^x + 2^7 = 0$

3. Find the product abc if $2^{3a-4b} \cdot 3^{4a-6b} \cdot 25^{-2c/b} = 324000$

2021-2022 SMML
MEET 3 – No Calculators
ROUND 4

1. We can rewrite the initial equation as $\left(3^{4/4x}\right)^{2/3} = 3^{3/2}$, which then becomes $3^{2/3x} = 3^{3/2}$, so $\frac{2}{3x} = \frac{3}{2}$, so $x = \frac{4}{9}$

The answer is $\boxed{\frac{4}{9}}$

2. We can rewrite this as $2^{2x} - 36 \cdot 2^x + 128 = 0$, which we can factor into $\left(2^x - 4\right)\left(2^x - 32\right) = 0$. This gives us $2^x = 2^2$ and $2^x = 2^5$, so $x = 2$ or 5, and the product of these solutions is 10.

The answer is $\boxed{10}$

3. We can rewrite this as $2^{3a-4b} \cdot 3^{4a-6b} \cdot 5^{-4c/b} = 4 \cdot 81 \cdot 1000 = 2^2 \cdot 3^4 \cdot 10^3 = 2^5 \cdot 3^4 \cdot 5^3$. This gives us $\begin{matrix} 3a - 4b = 5 \\ 4a - 6b = 4 \end{matrix}$, which becomes $\begin{matrix} +12a - 16b = 20 \\ -12a + 18b = -12 \end{matrix}$ which gives $2b = 8$, so $b = 4$ and plugging back in gives us $a = 7$. Using $5^{-4c/4} = 5^3$, we get $c = -3$. So the product $abc = -84$

The answer is $\boxed{-84}$

2021-2022 SMML
MEET 3 – No Calculators
ROUND 5

1. Find the exact value of $\sin 15^{\circ} \cos 15^{\circ}$

2. Find the exact value of $\tan 75^{\circ}$

3. Find the sum of all solutions of $\cos^4 x - \sin^4 x = -\frac{1}{2}$ given that $0 \le x < 2\pi$

2021-2022 SMML
MEET 3 – No Calculators
ROUND 5

1. $\sin 15^\circ \cos 15^\circ = \frac{1}{2}\left(2\sin 15^\circ \cos 15^\circ\right) = \frac{1}{2}\sin 30^\circ = \frac{1}{4}$

The answer is $\boxed{\frac{1}{4}}$

2. Using the half angle formula for tangent, we get:

$$\tan 75^\circ = \frac{\sin 150^\circ}{1+\cos 150^\circ} = \frac{\frac{1}{2}}{1-\frac{\sqrt{3}}{2}} = \frac{1}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$$

The answer is $\boxed{2+\sqrt{3}}$

3. Factoring, we get $\left(\cos^2 x+\sin^2 x\right)\left(\cos^2 x-\sin^2 x\right) = -\frac{1}{2}$. The Pythagorean and double angle identities give us $(1)(\cos 2x) = -\frac{1}{2}$. Solving this we get $2x = \left\{\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}\right\}$ on the interval $[0, 4\pi]$, which gives us $x = \left\{\frac{2\pi}{6}, \frac{4\pi}{6}, \frac{8\pi}{6}, \frac{10\pi}{6}\right\}$. Adding these together we get $\frac{24\pi}{6} = 4\pi$

The answer is $\boxed{4\pi}$

2021-2022 SMML
MEET 3 – No Calculators
TEAM ROUND

1. Solve for x: $(x-1)(x-1)^2(x^2-1)(x-1)^3(x^3-1)(x-1)^4(x^4-1)<0$

2. Solve for x: $\det\left(\begin{bmatrix} 1 & -1 \\ 2 & -2 \\ 3 & -3 \end{bmatrix}\begin{bmatrix} 1 & x & 5 \\ 2 & 4 & 6 \end{bmatrix}\right)=0$

3. Find the sum of all of the integer solutions to $\begin{vmatrix} x & 20 \\ 21 & x+1 \end{vmatrix}+\begin{vmatrix} x+2 & 20 \\ 22 & x-3 \end{vmatrix}\le\begin{vmatrix} x+1 & 10 \\ 86 & x+3 \end{vmatrix}+3$

2021-2022 SMML
MEET 3 – No Calculators
TEAM ROUND

1. Grouping the $(x-1)$ terms we start with and factoring, we get $(x-1)^{10}(x+1)(x-1)(x-1)(x^2+x+1)(x-1)(x+1)(x^2+1)<0$. This yields $(x-1)^{13}(x+1)^2(x^2+x+1)(x^2+1)<0$. The last two terms do not have any real roots, so we are left with $(x-1)^{13}(x+1)^2<0$ which gives us the solution $(-\infty,-1)\cup(-1,1)$.

The answer is $\boxed{(-\infty,-1)\cup(-1,1) \text{ (OR } -\infty<x<-1 \text{ or } -1<x<1\text{)}}$

2. $$\begin{bmatrix} 1 & -1 \\ 2 & -2 \\ 3 & -3 \end{bmatrix}\begin{bmatrix} 1 & x & 5 \\ 2 & 4 & 6 \end{bmatrix}=\begin{bmatrix} -1 & x-4 & -1 \\ -2 & 2x-8 & -2 \\ -3 & 3x-12 & -3 \end{bmatrix}$$

Taking the determinant of this matrix we get $(6x-24+6x-24+6x-24)-(6x-24+6x-24+6x-24)=0$. Since $0=0$ and there are no domain constraints, the answer is ° .

The answer is $\boxed{° \text{ (OR } (-\infty,\infty)\text{)}}$

3. The determinants simplify to

$x^2+x-20(21)+x^2-x-6-20(22)\le x^2+4x+3+3-10(86)$

$2x^2-20(43)-6\le x^2+4x+6-20(43)$

$x^2-4x-12\le 0$

$(x+2)(x-6)\le 0$

This gives us the solution set $[-2,6]$. The sum of the integer solutions is $3+4+5+6=18$

The answer is $\boxed{18}$

2021-2022 SMML
MEET 3 – No Calculators
ANSWERS

Round 1

1.) 5040

2.) $\frac{1}{5}$

3.) $\frac{144}{625}$

Round 2

1.) -54

2.) $\frac{x-4}{x-3}$

3.) $\frac{x^2+ax+a^2}{(b+c)(b-c)}$

Round 3

1.) $\frac{15}{2}$

2.) 21

3.) 20

Round 4

1.) $\frac{4}{9}$

2.) 10

3.) -84

Round 5

1.) $\frac{1}{4}$

2.) $2+\sqrt{3}$

3.) 4π

Team Round

1.) $(-\infty,-1)\cup(-1,1)$
(OR $-\infty < x < -1$ or $-1 < x < 1$)

2.) ° (OR $(-\infty,\infty)$)

3.) 18