2022-2023 SMML
MEET 2 – No Calculators
ROUND 1

1. The number of elements in set *A* is 20, the number of elements in set *B* is 40. The number of elements in set *C* is 60. If the number of elements in $A \cap B$ is 15, then number of elements in $B \cap C$ is 25, and the number of elements in $A \cap C$ is 20, then how many elements are in set *B* and NOT in sets *A* or *C*?

2. If $S = \{x \in \mathbb{Z} \mid 7 \leq x \leq 777\}$, *M* is the set of integers that are multiples of 7, and M' denotes the complement of *M*, then how many elements are in $S \cap M'$?

3. If $A = LCM(200, 2022)$ and $B = LCM(75, 2022)$, then find the $GCF(A, B)$.

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MEET 2 – No Calculators
ROUND 1

1. Note that all of the elements of A are in $A \cap C$. So, $A \cap B \cap C$ has 15 elements, $A \cap B \cap C'$ has 0 elements, and $A' \cap B \cap C$ has 10 elements. This leaves 15 elements in $A' \cap B \cap C'$.

The answer is $\boxed{15}$

2. The question is asking how many integers are their from 7 to 777 (inclusively) that are not multiples of 7. There are $777/7 = 111$ integers that are multiples of 7 from 1 to 777. So, there are 666 integers that are not. Subtracting off the 6 integers from 1 to 6, we get 660.

The answer is $\boxed{660}$

3. First, $2022 = 2 \cdot 1011$ and $200 = 2 \cdot 100$. Since 1011 and 100 clearly share no factors, we see that $LCM(200, 2022) = 2 \cdot 100 \cdot 1011 = 100 \cdot 2022$.

Similarly, we can see that 75 and 2022 share only a factor of 3, so $LCM(75, 2022) = 25 \cdot 2022$.

So, the $GCF(100 \cdot 2022, 25 \cdot 2022) = 25 \cdot 2022 = \dfrac{100 \cdot 2022}{4} = \dfrac{202200}{4} = 50550$

The answer is $\boxed{50550}$

2022-2023 SMML
MEET 2 – No Calculators
ROUND 2

1. How many integer values satisfy $2x-1<4x-9<3x+23$?

2. Find the sum of all integer solutions to $|1+|1+x||<6$.

3. If C is the set of all values that satisfy the system of inequalities $4x+1\le -4$ and $-4x-3\ge 1$ and D is the set of all values that satisfy the system of inequalities $-3x-2\le 4$ and $-4x-4\ge -2$, then what percent of the values in set D are also values in set C?

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MEET 2 – No Calculators
ROUND 2

1. Separating the inequality, we get

$8 < 2x$ and $x < 32$

$4 < x < 32$

So, there are $31 - 4 = 27$ values that work.

The answer is $\boxed{27}$

2. This becomes

$-6 < 1 + |1 + x| < 6$

$-7 < |1 + x| < 5$

Since the absolute value can't be negative the constraint on the left is redundant, and we get

$|1 + x| < 5$

$-5 < 1 + x < 5$

$-6 < x < 4$

The only integer solutions which don't cancel with another integer solution when added are -5 and -4, whose sum is -9.

The answer is $\boxed{-9}$

3. First, solve the two systems:

$4x \le -5$ and $-4x \ge 4$

$x \le -\frac{5}{4}$ and $x \le -1$

$x \le -\frac{5}{4}$

$-3x \le 6$ and $-4x \ge 2$

$x \ge -2$ and $x \le -\frac{1}{2}$

$-2 \le x \le -\frac{1}{2}$

Since $-\frac{5}{4}$ is the midpoint of the interval for set D, the percent of solutions that are in D that are also in C is 50.

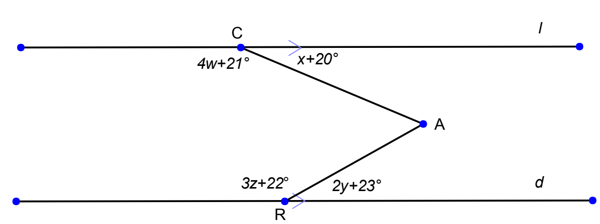
The answer is $\boxed{\text{50 (\% sign optional)}}$

2022-2023 SMML
MEET 2 – No Calculators
ROUND 3

1. In increasing length order, the sides of a trapezoid have lengths 12, 18, 18, and x. What is the longest possible integral length of the median?

2. Line l passes through the points $(2, c)$ and $(5, 3c+6)$. Line m passes through the points $(2c+3, 2)$ and $(4c+7, 10)$. What non-zero value of c makes lines l and m parallel?

3. In the following diagram, the lines l and d are parallel, and the measures of four angles are given. If w, x, y, and z are all positive integers, then what is the smallest possible value for $m\angle CAR$



2022-2023 SMML
MEET 2 – No Calculators
ROUND 3

1. We know that the x must be less than the sum of all the other sides, so x must be less than 48. Although the two sides of length 18 can not be the two bases (for then it would be a parallelogram), one of the length 18 sides CAN be a base. So, we know the median length must be strictly less than $\frac{48+18}{2}=33$. So, the largest possible integral value is 32.

The answer is $\boxed{32}$

2. Setting the slopes of the two lines equal to each other and solving we get…

$$\frac{3c+6-c}{5-2}=\frac{10-2}{4c+7-2c-3}$$
$$\frac{2c+6}{3}=\frac{8}{2c+4}=\frac{4}{c+2}$$
$$2c^2+4c+6c+12=12$$
$$2c^2+10c=0$$
$$2c(c+5)=0$$
$$c=0 \text{ or } c=-5$$

However, we are looking for non-zero values.

The answer is $\boxed{-5}$

3. We know that $4w+21^\circ+x+20^\circ=180^\circ$, so $x+4w=139^\circ$.
The smallest we can make x to ensure that w is a positive integer is 3.
We also know that $2y+23^\circ+3z+22^\circ=180^\circ$, so $2y+3z=135^\circ$.
The smallest we can make y to ensure that z is a positive integer is 3.
$m\angle CAR=x+20^\circ+2y+23^\circ=52^\circ$ when x and y are 3.

The answer is $\boxed{52^\circ}$

2022-2023 SMML
MEET 2 – No Calculators
ROUND 4

1. How many integer solutions satisfy the inequality $(x+5)(x+1)(x-2)^2(x-4)(x-5)\le 0$

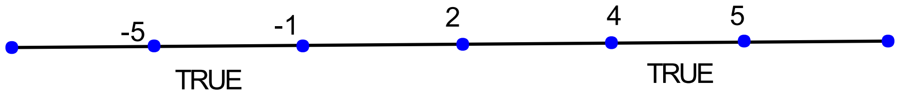
2. Find the sum of all solutions, x, that satisfy $2(f(x))^2-3f(x)-2=0$ given the table of values:

x	1	2	4	8	16	32
$f(x)$	1/2	-1/2	2	-2	2	0

3. Find the smallest positive value for k such that the equation $2x^2-12x+k=0$ has 2 distinct integer solutions.

2022-2023 SMML
MEET 2 – No Calculators
ROUND 4

1. Setting up a number line it is clear that the inequality is false for values beyond 5. Alternating every interval, except crossing 2 (because it is a double root), we get the following result.



So, we get -5, -4, -3, -2, -1, 2, 4, and 5 as integer solutions.

The answer is $\boxed{8}$

2. Let $u = f(x)$ and the equation becomes $2u^2 - 3u - 2 = 0$. Solving, we get $(2u+1)(u-2) = 0$, so $u = -\frac{1}{2}$ or $u = 2$.

So, $f(x) = -\frac{1}{2}$ or $f(x) = 2$.

These values occur when x is 2, 4, or 16. The sum of these is 22.

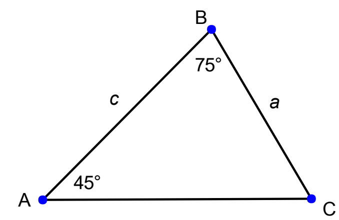
The answer is $\boxed{22}$

3. Plugging into the quadratic formula, we get $x = \frac{-12 \pm \sqrt{144-8k}}{4}$. While the minimum value for k to get a rational solution is $\frac{23}{8}$, we need $144-8k$ to be a multiple of 16, so that, taken outside the square root, we have a multiple of 4 to cancel with the denominator to ensure that we have an integer solution. The smallest number that is a multiple of 16 and is also a perfect square is 64. Solving $144-8k = 64$, we get $k = 10$.

The answer is $\boxed{10}$

2022-2023 SMML
MEET 2 – No Calculators
ROUND 5

1. Find the length of side c for the depicted triangle given that $a = 4\sqrt{6}$



2. Let A be the set of all trigonometric functions that when the function is applied for some real-valued angle, the answer is $\frac{\sqrt{3}}{2}$.

Let B be the set of all trigonometric functions that when the function is applied for some real-valued angle, the answer is 1.

Let C be the set of all trigonometric functions that when the function is applied for some real-valued angle, the answer is $\sqrt{3}$.

Which trig functions are in $(A \cap B)' \cup (B \cup C)'$

3. If $\sin A = \frac{\sqrt{3}}{4}$ and $\tan B = \frac{\sqrt{5}}{3}$, then find the value of $32\cos^2 A + 42\cos^2 B$.

2022-2023 SMML
MEET 2 – No Calculators
ROUND 5

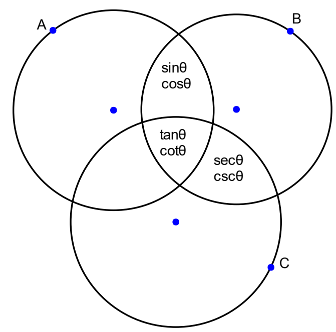
1. First, we determine that $m\angle C = 60^\circ$. Then we apply the law of sines:

$$\frac{c}{\sin 60^\circ} = \frac{4\sqrt{6}}{\sin 45^\circ}$$

$$c = \frac{4\sqrt{6}\cdot 2\cdot\sqrt{3}}{\sqrt{2}\cdot 2} = 12$$

The answer is $\boxed{12}$

2. Making a Venn diagram, we get the following:



We see that $(A\cap B)'\cup(B\cup C)' = \{\sec\theta, \csc\theta\}\cup\varnothing$

The answer is $\boxed{\sec\theta, \csc\theta \text{ (in either order)}}$

3. Note that there is no need to solve for values of a single triangle, because it is not stated that A and B are in the same triangle.

Since $\sin A = \frac{\sqrt{3}}{4}$, $\cos^2 A = 1-\sin^2 A = 1-\frac{3}{16} = \frac{13}{16}$

Since $\tan B = \frac{\sqrt{5}}{3}$, $\sec^2 B = 1+\tan^2 B = 1+\frac{5}{9} = \frac{14}{9}$, so $\cos^2 B = \frac{9}{14}$

$$32\cos^2 A + 42\cos^2 B = 26 + 27 = 53$$

The answer is $\boxed{53}$

2022-2023 SMML
MEET 2 – No Calculators
TEAM ROUND

1. What is the product of all values b that make $(|2b|, b)$ the fourth coordinate of a kite with other coordinates $(-1,4)$, $(-1,7)$, and $(2,7)$.

2. Find the area of the triangle formed by the equations $4y = 3x - 20$, $8y = -3x + 176$, and the y-axis.

3. Find the area of quadrilateral $ACBD$ with vertices $A(-9,-3)$, $B(12,-3)$, $C(-4,9)$, and $D(6,-11)$.

2022-2023 SMML
MEET 2 – No Calculators
TEAM ROUND

1. If we label the points $A(-1,4)$, $B(-1,7)$, $C(2,7)$, and $D(|2b|,b)$, then it is clear that AB and BC are one pair of congruent sides. Since the slope between A and C is 1, then slope between B and D must be -1. We can find the equation of a line that point D must lie on: $y-7=-(x+1)$. If b is positive, we get $b-7=-(2b+1)$, which yields $b=2$. If b is negative we get $b-7=-(-2b+1)$, which yields $b=-6$. The product of these is -12.

The answer is $\boxed{-12}$

2. First, find the y-intercept of each line, which are -5 and 22. Thus, the length of the base is 27.

Secondly, find the x-value where they intersect. This will be the height.

$$3x+8y=176$$
$$6x-8y=40$$

$$9x=216$$
$$x=24$$
$$A=\frac{1}{2}(27)(24)=(27)(12)=270+54=324$$

The answer is $\boxed{324}$

3. Although one can chop the triangle up into pieces, a simpler method is found in Heron's area formula.

Find the distance between all the points:

$AB=21$, $BC=\sqrt{16^2+12^2}=20$ (3-4-5 right triangle), and $AC=\sqrt{5^2+12^2}=13$

Note that ABC is not a right triangle itself.

The perimeter is 54, so the semi-perimeter is 27.

$$A=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{27\cdot6\cdot7\cdot14}=\sqrt{9\cdot9\cdot4\cdot49}=9\cdot2\cdot7=126$$

Similarly, $BD=10$ and $DA=\sqrt{15^2+8^2}=17$

So, perimeter is 48, so the semi-perimeter is 24.

$$A=\sqrt{24\cdot3\cdot14\cdot7}=\sqrt{7^2\cdot2^4\cdot3^2}=84$$

So, the total area is 210.

The answer is $\boxed{210}$

2022-2023 SMML
MEET 2 – No Calculators
ANSWERS

Round 1

1.) 15

2.) 550

3.) 50550

Round 2

1.) 27

2.) -9

3.) 50 (% sign optional)

Round 3

1.) 32

2.) -5

3.) 52°

Round 4

1.) 8

2.) 22

3.) 10

Round 5

1.) 12

2.) $\sec\theta, \csc\theta$ (in either order)

3.) 53

Team Round

1.) -12

2.) 324

3.) 210