Simulation et Applications Interactives

Modeling and Geometric Deformation Techniques

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Course Outline

1. Modeling Techniques
    - Revolve
    - Extrude
    - Loft
2. Geometric Deformation Techniques
    - Key shape interpolation
    - Warping
    - Global deformations
    - FFD (Free Form Deformations)

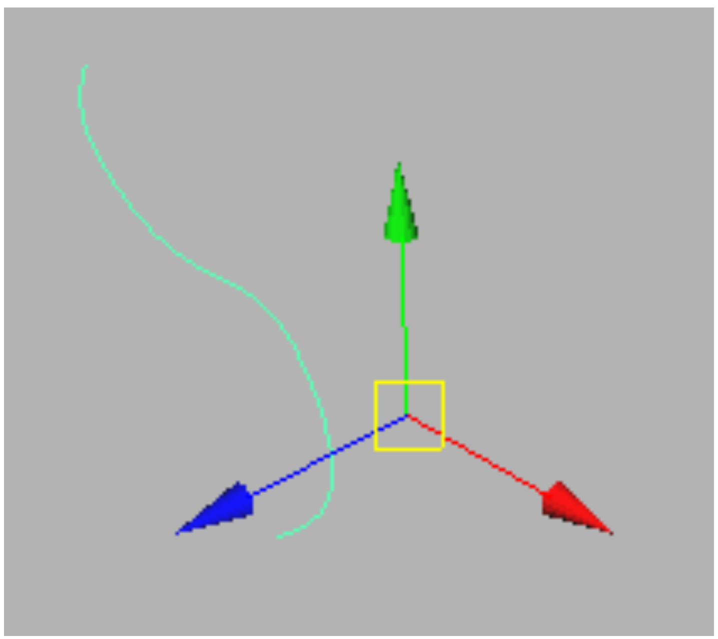
1. Modeling Techniques

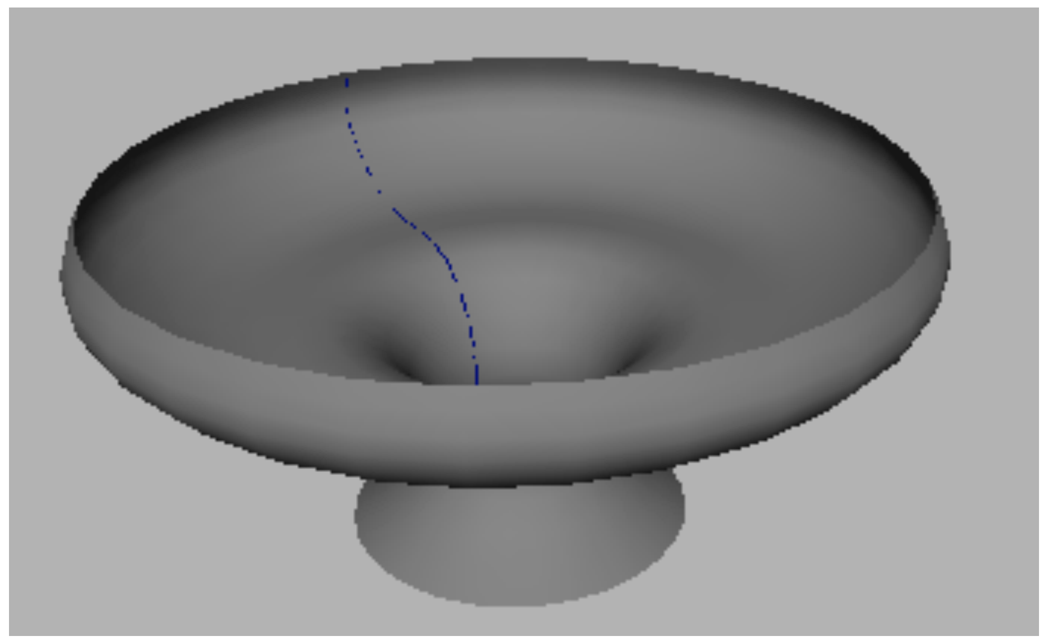
Free Form Primitives

- Free Form means:
  - Any type of geometry
  - Any type of topology (sphere, donut)
- Free Form primitives are:
  - Revolve
  - Loft
  - Extrude / Sweep (general extrude)

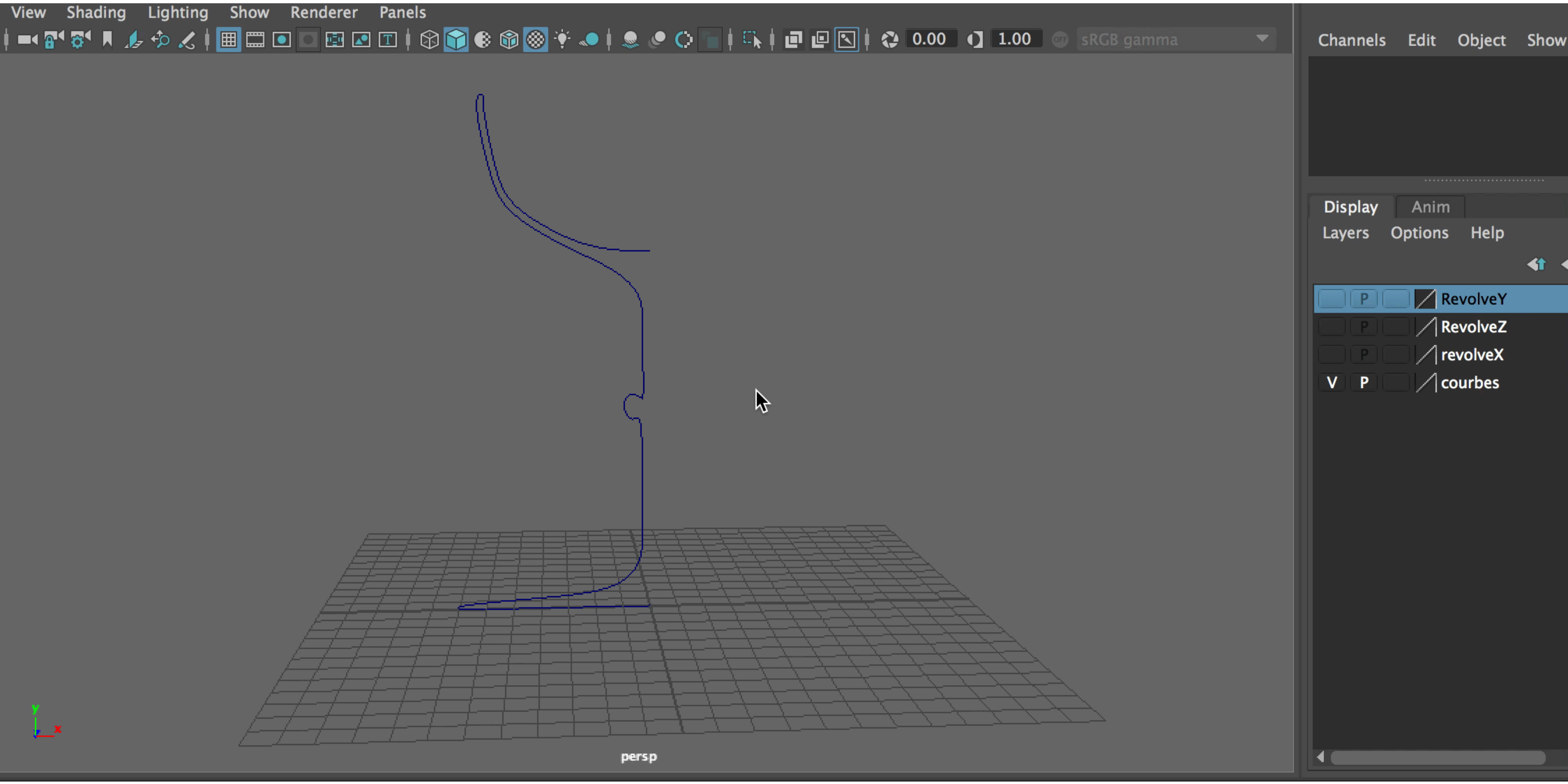
Revolve

- Rotate a profile curve around an axis





Revolve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RevolveY
RevolveZ
revolveX
V P courbes

Revolve

Around z-axis

- General parameterisation of the surface

$$x(\theta, t) = C_x(t) \ \cos\theta - C_y(t) \ \sin\theta$$
$$y(\theta, t) = C_x(t) \ \sin\theta + C_y(t) \ \cos\theta$$
$$z(\theta, t) = C_z(t)$$

with t the curve parameter that varies from 0 to 1

$C(t)$ the profile curve,

θ the rotation angle around the axis

Revolve

Around z-axis

- **If profile curve in the x-z-plane**

$$x(\theta, t) = C_x(t)\ \cos\theta$$
$$y(\theta, t) = C_x(t)\ \sin\theta$$
$$z(\theta, t) = C_z(t)$$

- **If profile curve in the y-z plane**

$$x(\theta, t) = -\ C_y(t)\ \sin\theta$$
$$y(\theta, t) = C_y(t)\ \cos\theta$$
$$z(\theta, t) = C_z(t)$$

Revolve

Around z-axis

- **Example 1**

$$C_x(t) = 0$$

$$C_y(t) = 5$$

$$C_z(t) = 10\ t$$

Revolve

Around z-axis

- **Example 2**

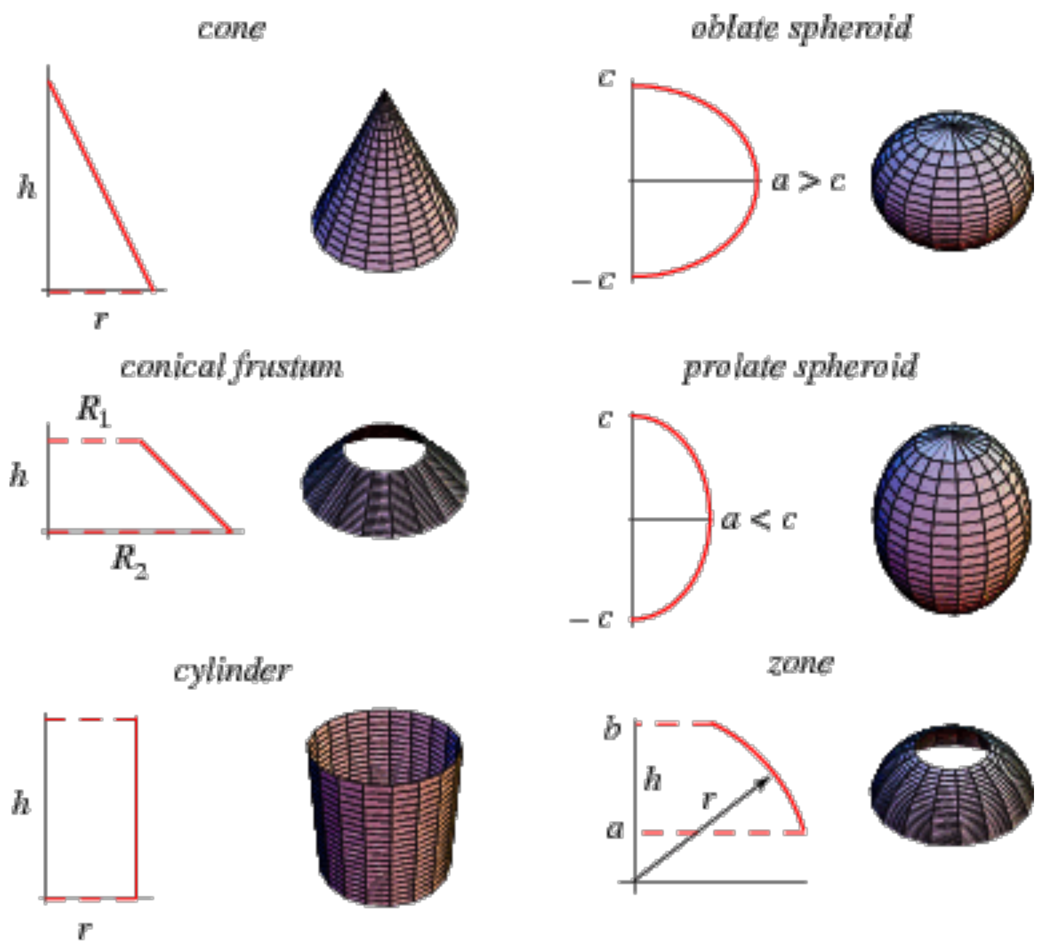
$$C_x(t) = 0$$

$$C_y(t) = 2 - t$$

$$C_z(t) = t$$

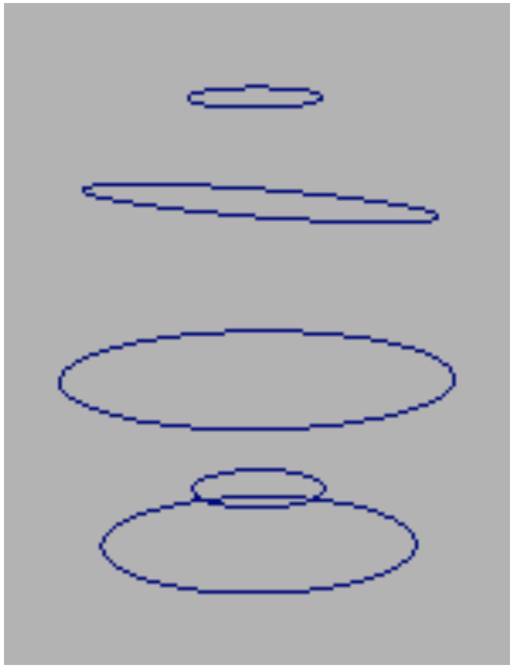
Revolve

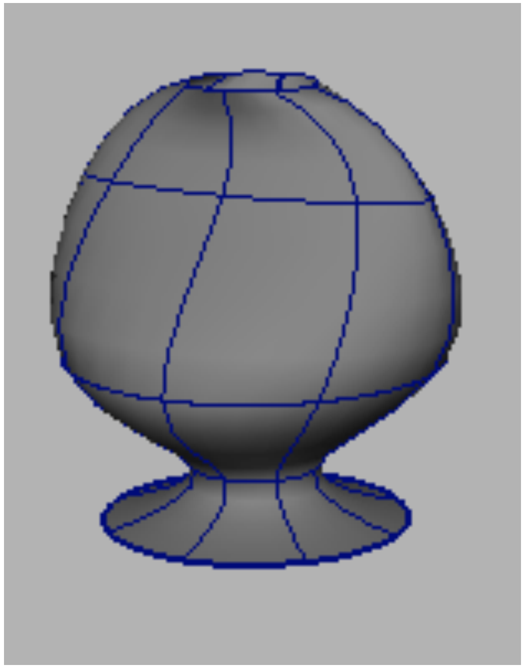
Standard Surfaces



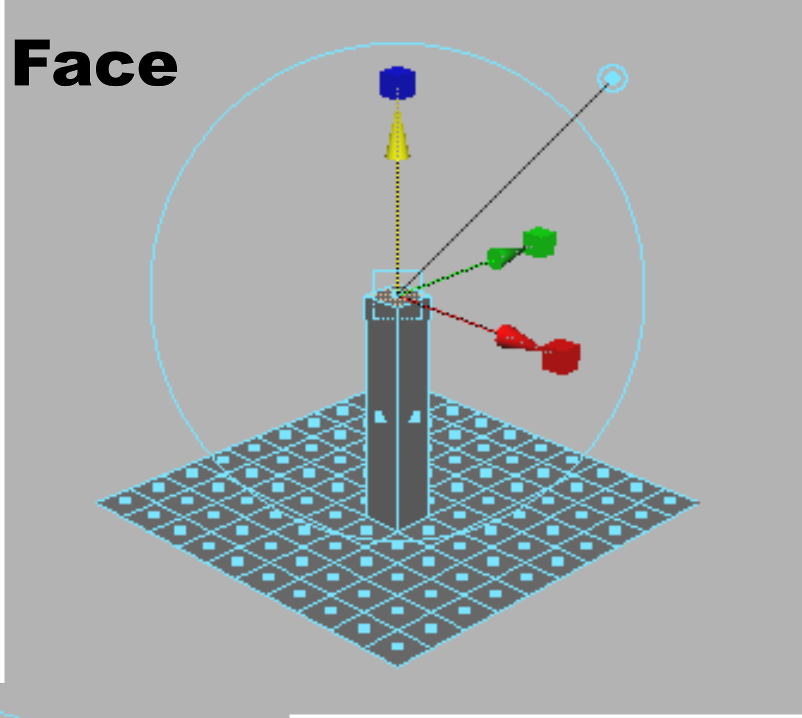
Loft

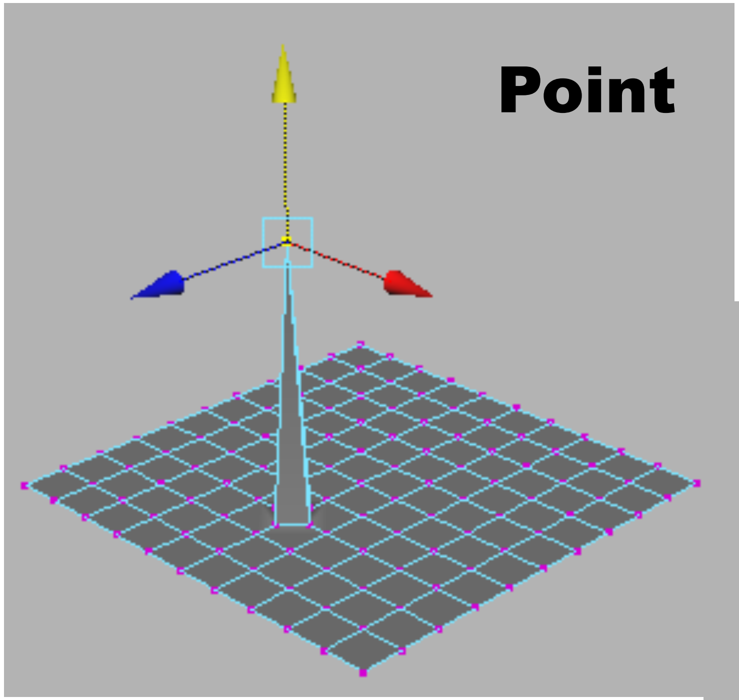
- Creates a surface between two (or more) curves
- Need to have the same number of CPs on each curve

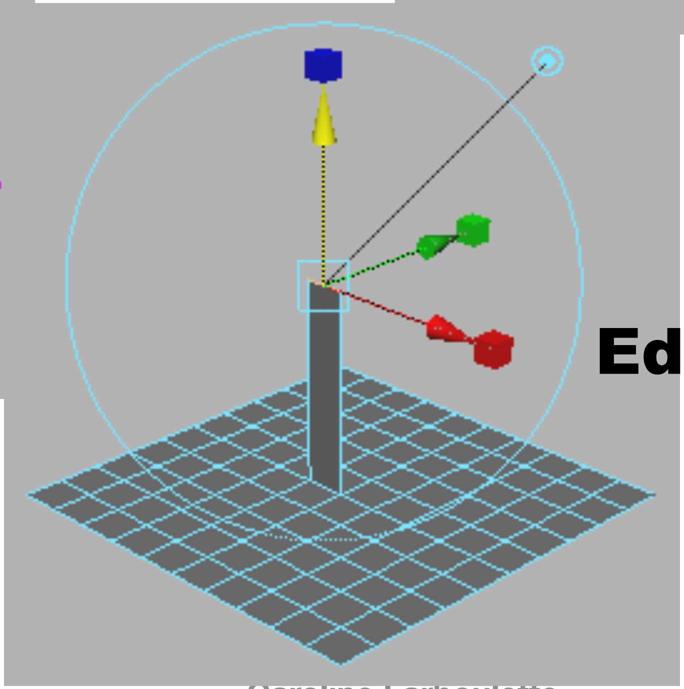




Extrude







Extrude

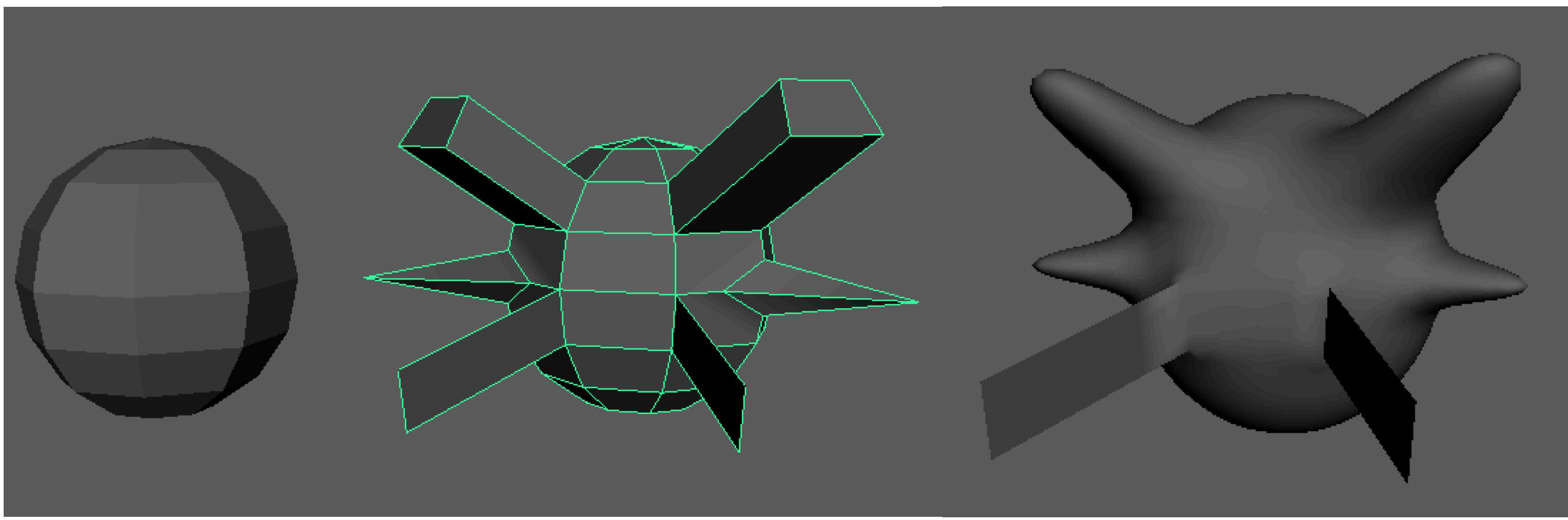
Equation

- Points displaced along direction $\overrightarrow{n}$ for a given distance l

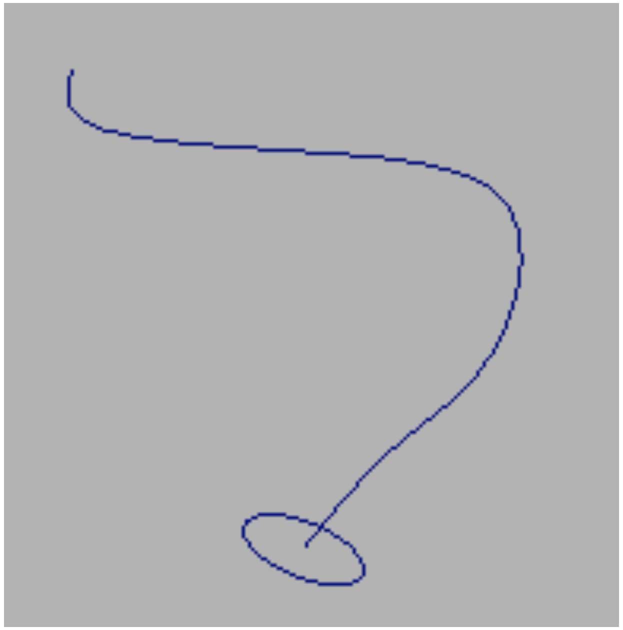
$$P_{new} = P + \vec{n} * l$$

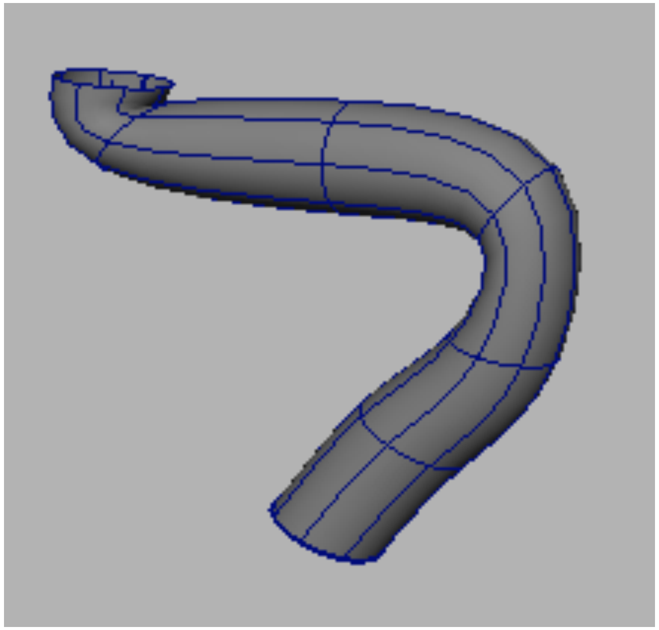
Extrude

Examples



Extrude Along Curve

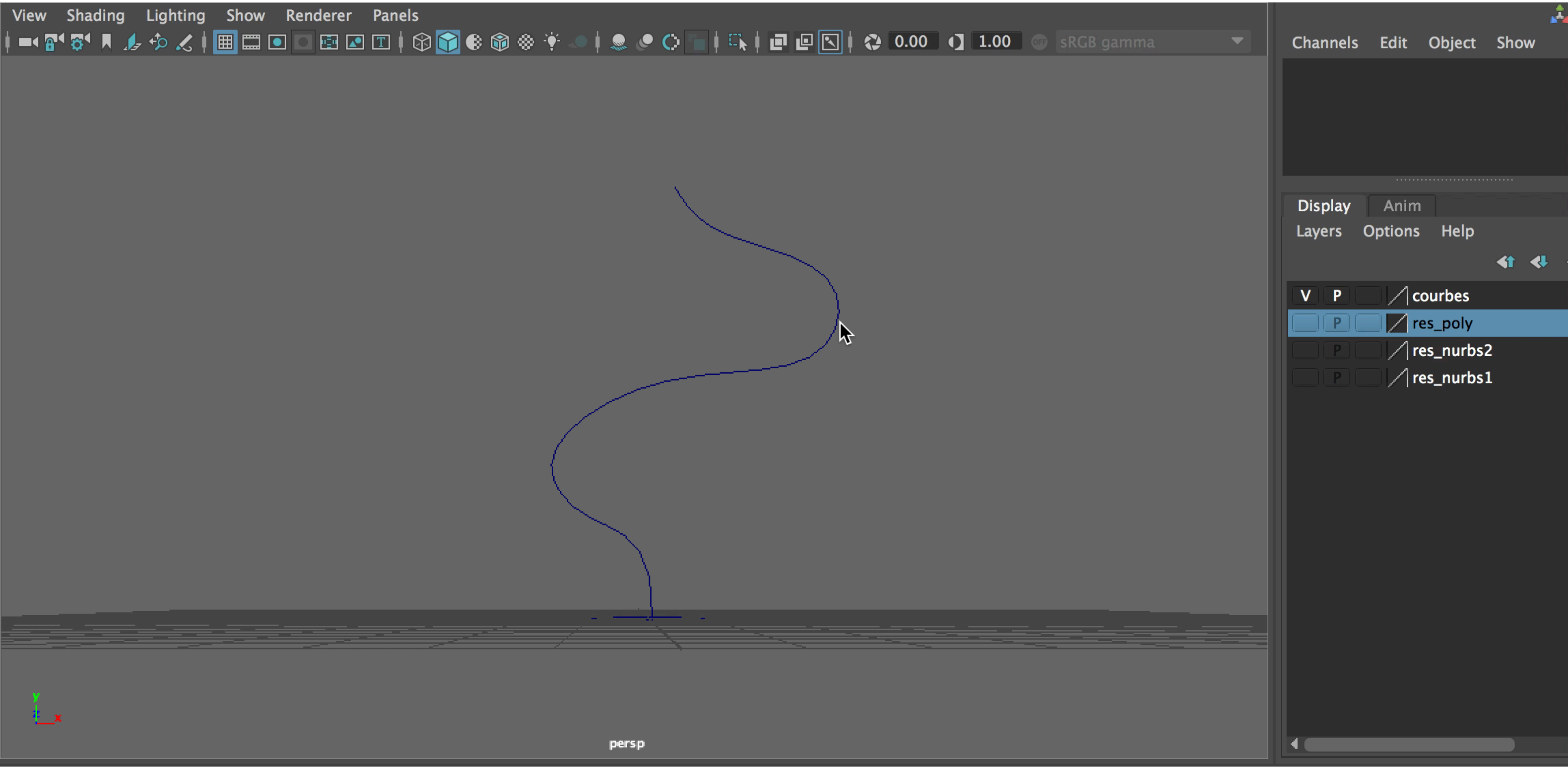




Sweep

- Profile curve extruded along the path curve
- Profile curve undergoes transformations of rotation, scale...

Extrude / Sweep



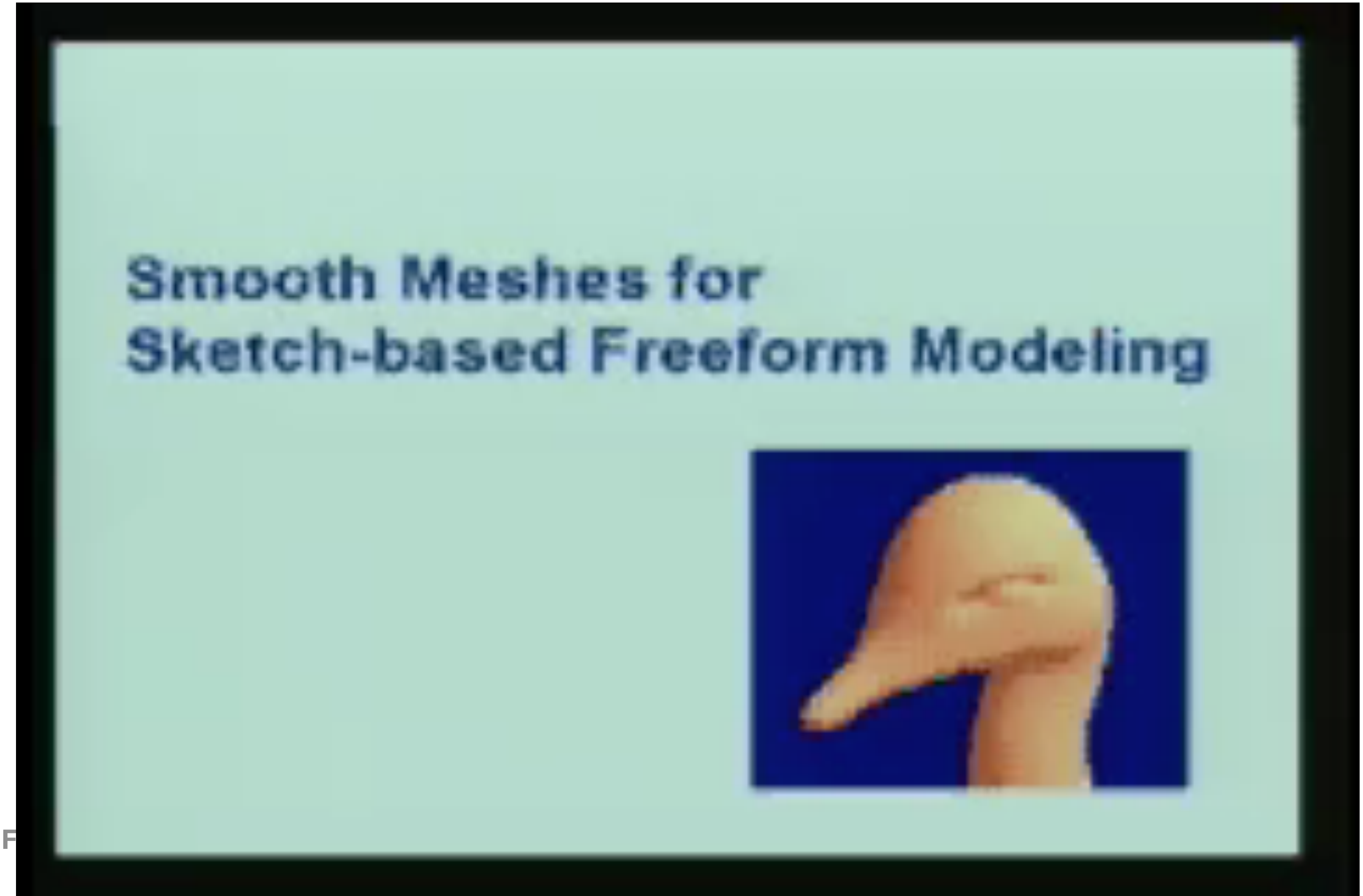
Teddy (1999)

Takeo Igarashi

http://www-ui.is.s.u-tokyo.ac.jp/~takeo/java/smoothteddy/index.html

Smooth Teddy (2003)

Takeo Igarashi



2. Geometric Deformation Techniques

Keyshape Interpolation

- Linear interpolation between 2 keyshapes

$$v_i^{interpolated}(t) = (1-t) \cdot v_i^{neutral} + t \cdot v_i^{smile}$$

with $0 \leq t \leq 1$ and for all vertices v_i of the mesh

neutral face

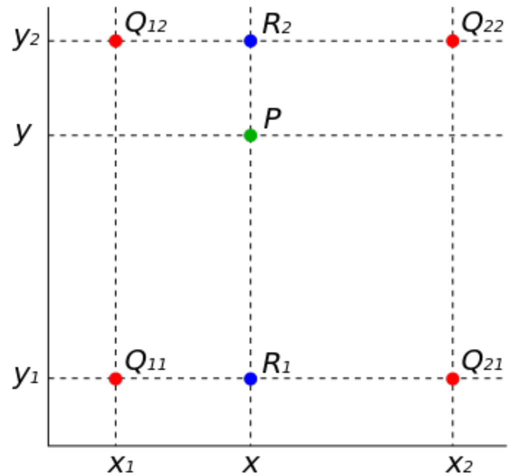
interpolated image

smiling face

Keyshape Interpolation

- Cosine interpolation
  - Creates acceleration / deceleration effects at beginning and end of the interpolation
- Spline interpolation

- Bilinear interpolation to interpolate between 4 keyshapes

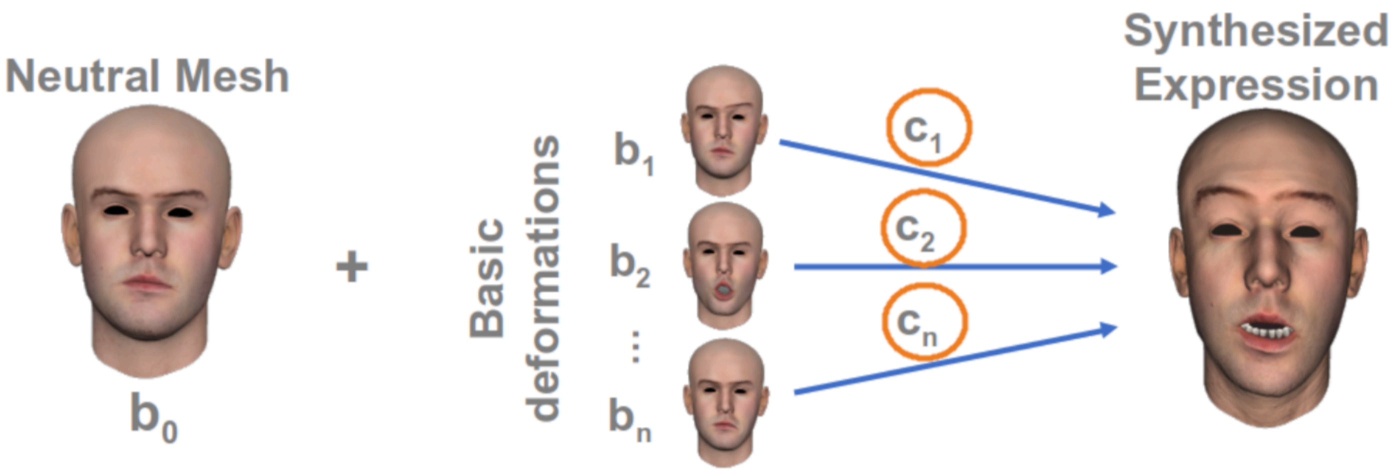


Blendshapes

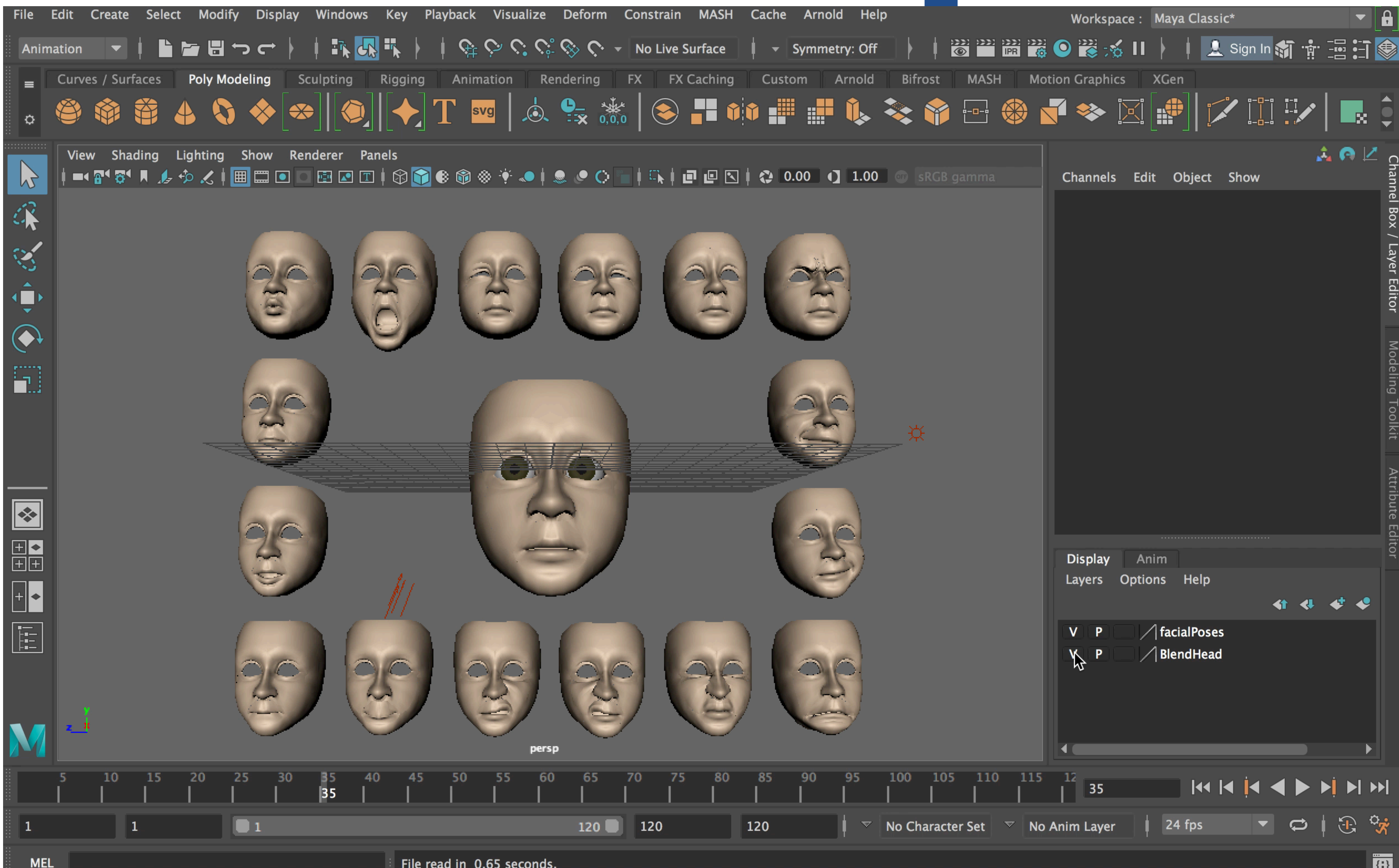
- **Sum of interpolations**

$$Shape(t) = Shape_0 + \sum_k w_k(t) \cdot Shape_k$$

$w_k(t)$ **varies from 0 to 1**



Blendshapes



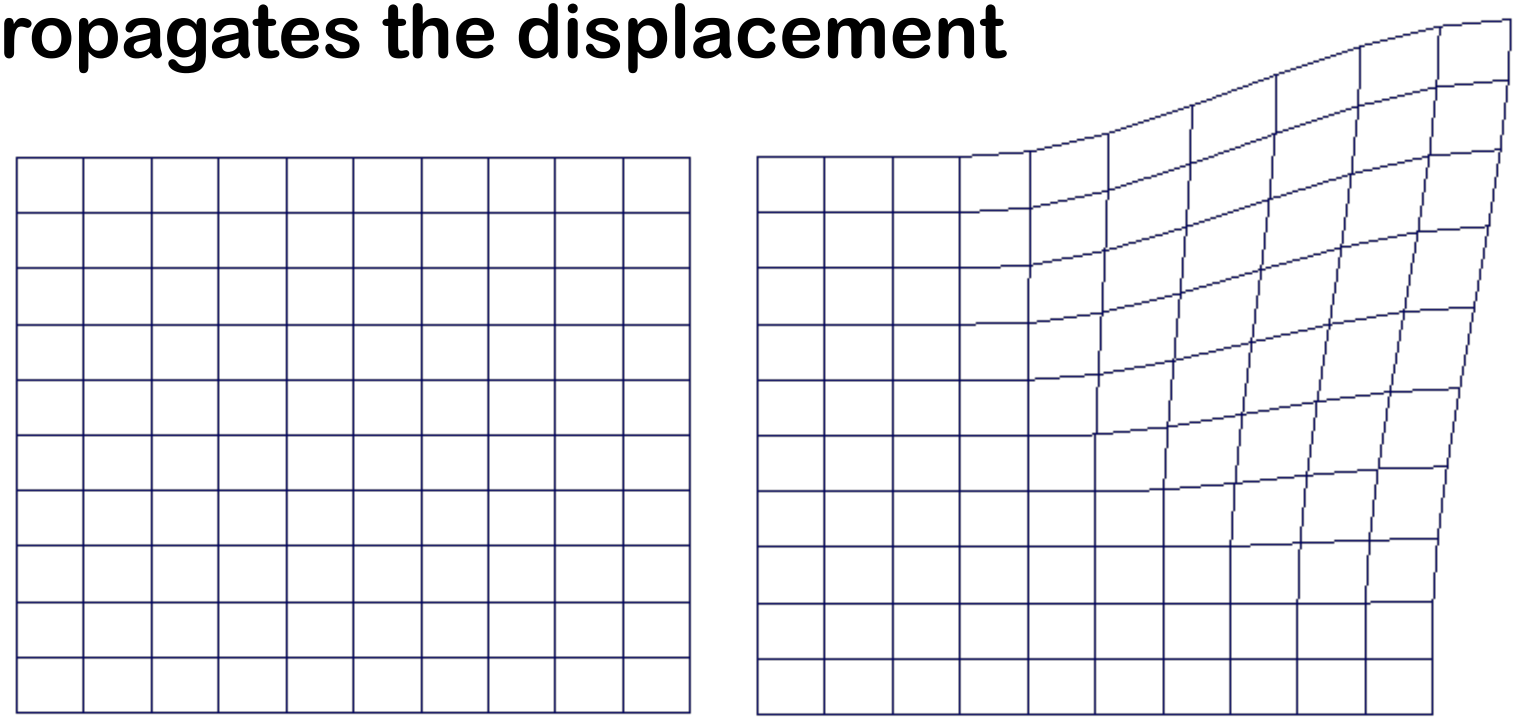
Blendshape Animation

Keyshape / Blendshape Interpolation

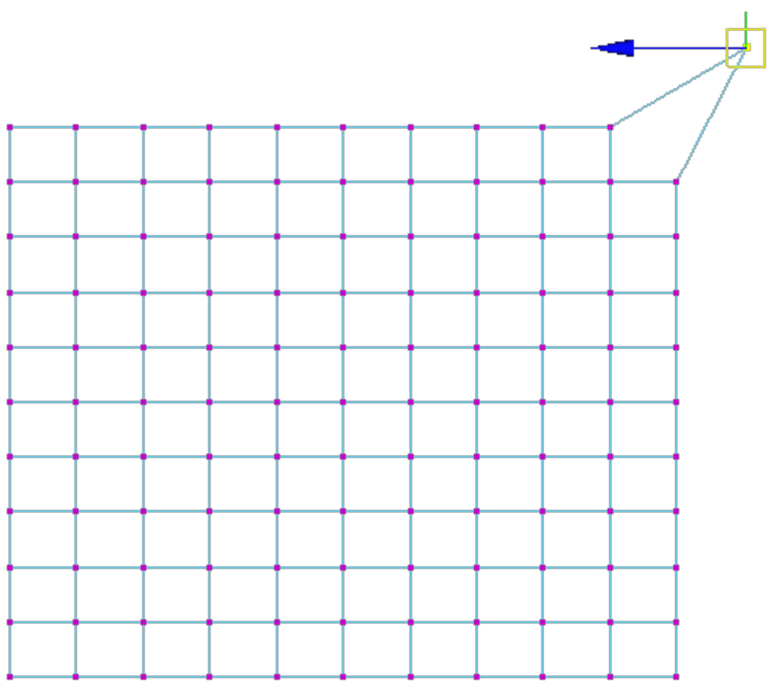
- Done on vertices, control points ...
- Memory consumption
- Artist dependant
  - Use of scanned or captured data
    - Noise and reconstruction problems
    - Results more accurate
- All degrees of liberty have to be covered
- Deformation must not be in opposition

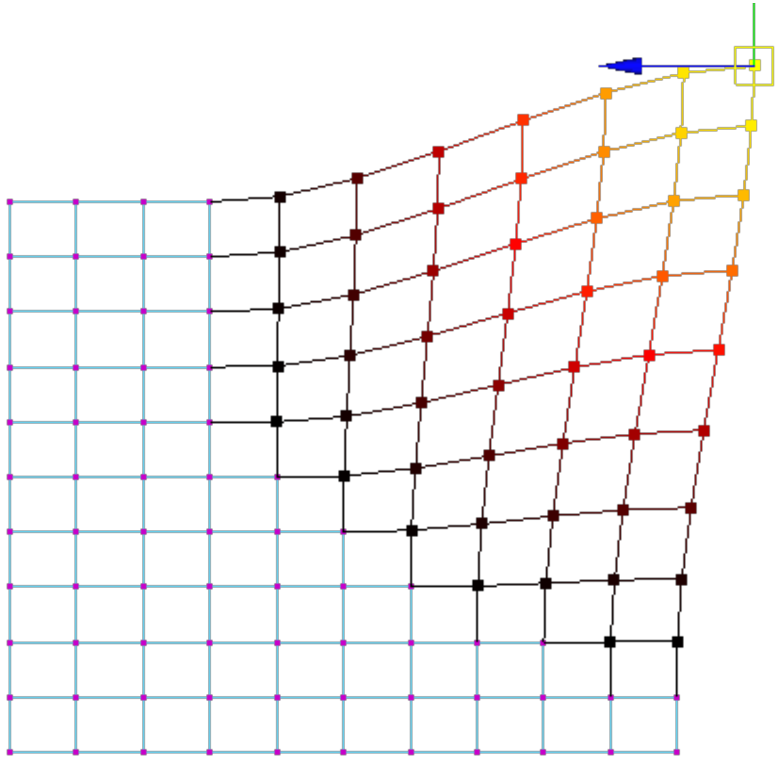
Warping

- Displace one vertex
- Propagates the displacement



Without / With Warping





Warping

- Displace one vertex
- Propagates the displacement
- Attenuation is function of a distance metric
  - Euclidean Distance
  - Geodesic Distance
  - Number of edges
  - ...

Warping

Example

$$S(i) = 1 - \left(\frac{i}{n+1}\right)^{k+1} \text{ for } k \geq 0$$

$$S(i) = 1 - \left(\frac{i}{n+1}\right)^{-k+1} \text{ for } k < 0$$

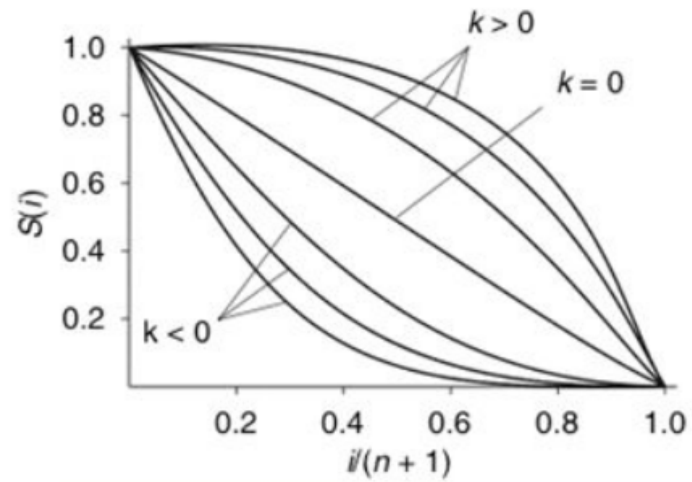
- $S(i)$ = scaling of the transformation applied
- n = distance (number of edges)
- k = some coefficient

Warping

Example

$$S(i) = 1 - (\frac{i}{n+1})^{k+1} \text{ for } k \geq 0$$

$$S(i) = 1 - (\frac{i}{n+1})^{-k+1} \text{ for } k < 0$$



- $k = 0$: linear deformation
- $k > 0$: more elastic deformation
- $k < 0$: more rigid deformation

Global Deformations

Global and local deformations of solid primitives, Barr, SIGGRAPH ‘84

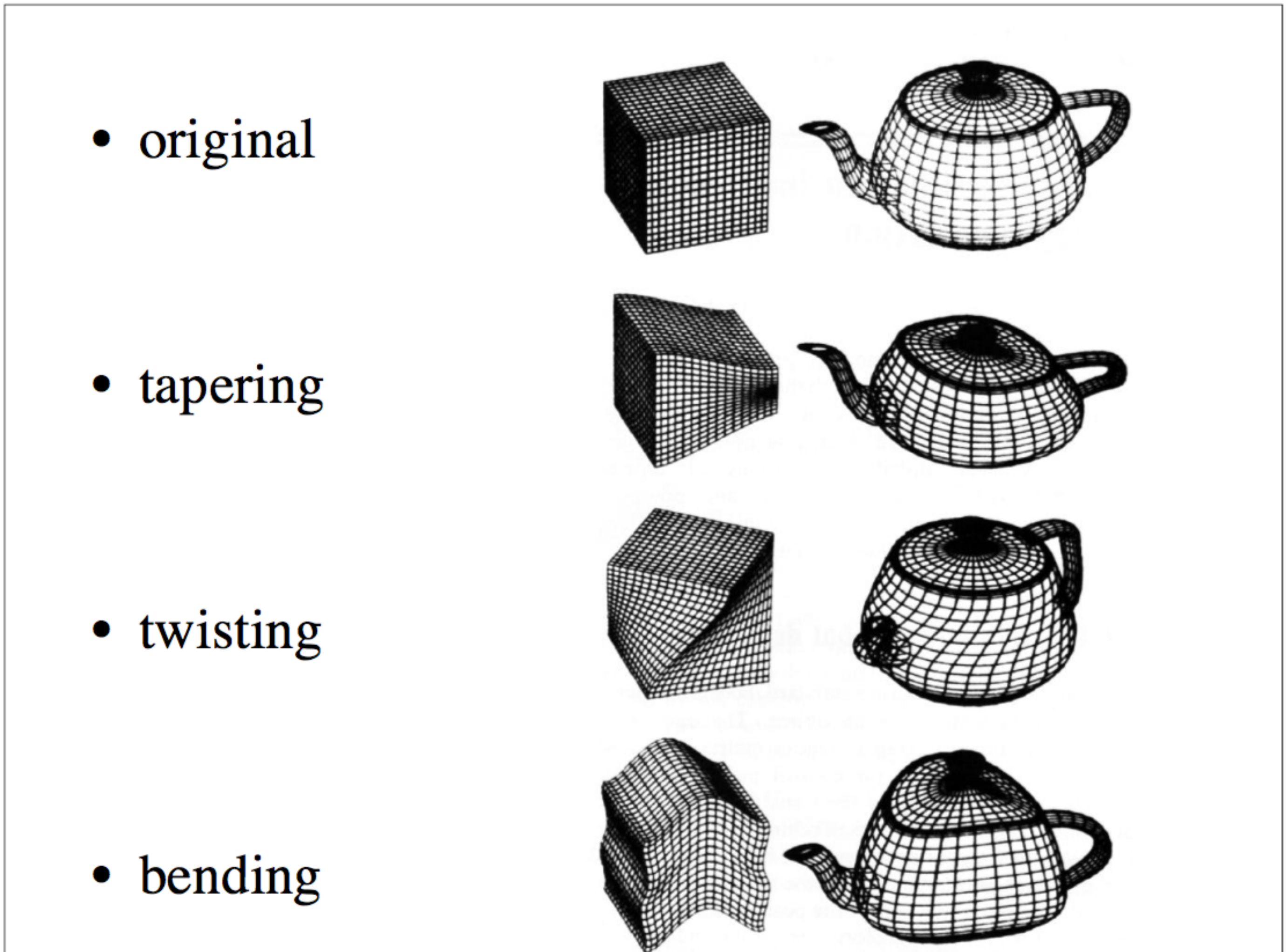
- Traditional affine transformations

(= linear transformations + translation)

  - Rotation, Translation, Scale

- Global deformations
  - Taper
  - Twist
  - Bend

Global Deformations



Definition

$$X = F(x)$$

- x : undeformed points
- X : deformed points
- F : transformation matrix

Property

- Tangents and normals can also be transformed
- For tangents, you multiply by the Jacobian J

$$\frac{\partial X}{\partial u} = J \frac{\partial x}{\partial u}$$

- For normals, the transformation matrix is the inverse transpose of the Jacobian matrix J multiplied by det J

$$n^X = det J J^{-1T} n^x$$

Reminder

- Jacobian:

$$J_{ij} = \frac{\partial X_i}{\partial x_j}$$

- det:

$$det \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = (a_1 b_2 c_3 - a_1 b_3 c_2) + (b_1 c_2 a_3 - b_1 a_2 c_3) + (c_1 a_2 b_3 - c_1 b_2 a_3)$$

Rotation and Translation

$$R_x T(\theta, t) = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & \cos(\theta) & -\sin(\theta) & t_y \\ 0 & \sin(\theta) & \cos(\theta) & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scaling

$$X = a_1 x \qquad Y = a_2 y \qquad Z = a_3 z$$

$$J = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}$$

- Det J = volume ratio between the two shapes

$$\det J = a_1 a_2 a_3$$

Scaling
Normal Transformation Matrix

$$J^{-1T} = \begin{bmatrix} 1/a_1 & 0 & 0 \\ 0 & 1/a_2 & 0 \\ 0 & 0 & 1/a_3 \end{bmatrix}$$

$$det J J^{-1T} = \begin{bmatrix} a_2 a_3 & 0 & 0 \\ 0 & a_1 a_3 & 0 \\ 0 & 0 & a_1 a_2 \end{bmatrix}$$

Tapering
Along z-axis

- **Vertices transformation**

$$X = f(z)x \qquad Y = f(z)y \qquad Z = z$$

$$Tz = \begin{bmatrix} f(z) & 0 & 0 \\ 0 & f(z) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Tapering

Normal Transformation Matrix

$$J = \begin{bmatrix} f(z) & 0 & f'(z)x \\ 0 & f(z) & f'(z)y \\ 0 & 0 & 1 \end{bmatrix}$$

$$det J J^{-1T} = \begin{bmatrix} f(z) & 0 & 0 \\ 0 & f(z) & 0 \\ -f(z)f'(z)x & -f(z)f'(z)y & f(z)^2 \end{bmatrix}$$

- $\det J = f(z)^2$: **volumetric rate of expansion**

Tapering

Example

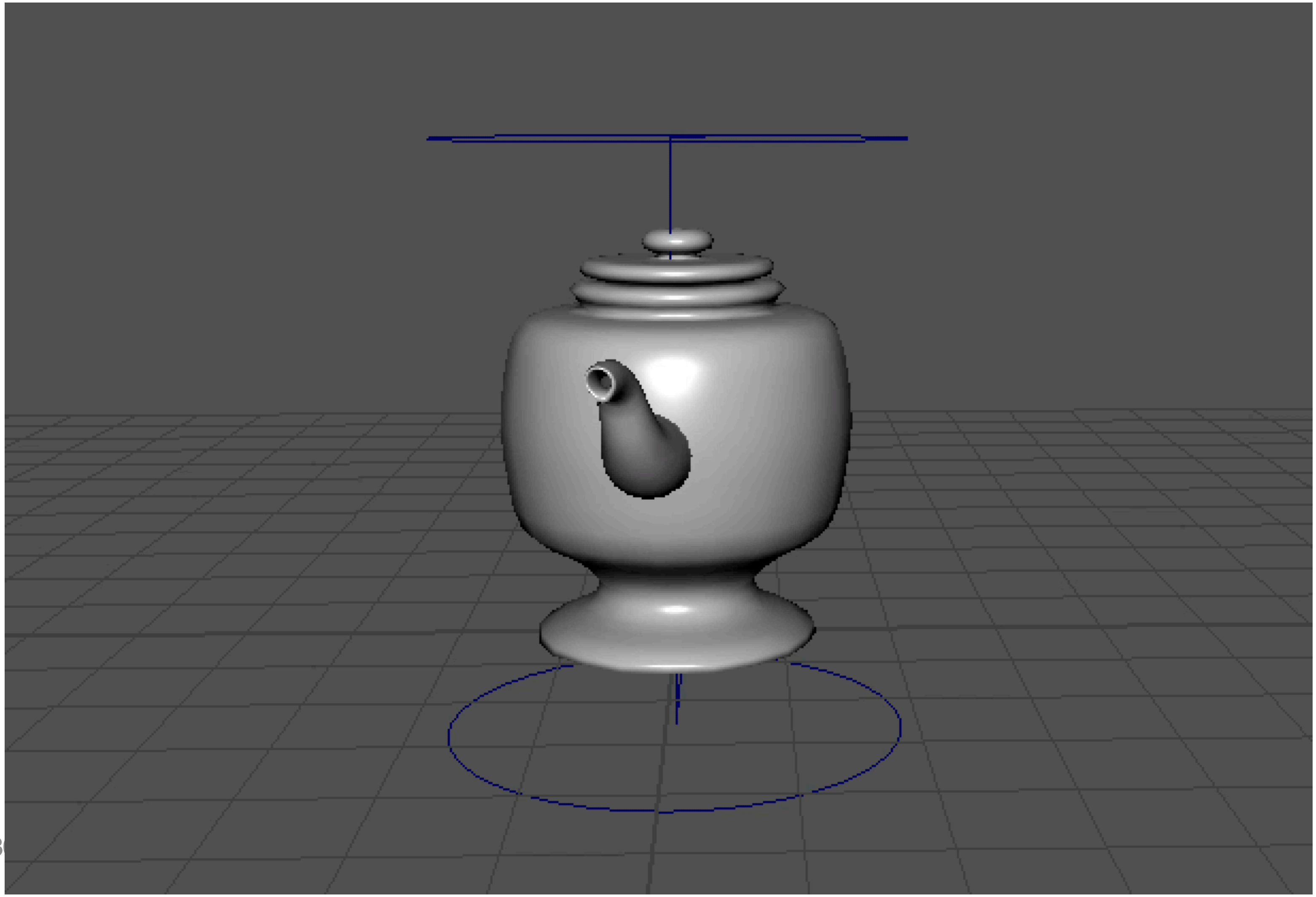
$$Tz = \begin{bmatrix} f(z) & 0 & 0 \\ 0 & f(z) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Cube centered over z-axis
- z varying from 0 to 1

$$f(z) = z$$

Twisting

Examples



Twisting

Around z-axis

- Let $$\theta = f(z)$$

$$\begin{aligned} X &= x\cos(\theta) - y\sin(\theta) \\ Y &= x\sin(\theta) + y\cos(\theta) \\ Z &= z \end{aligned}$$

$$Tz = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Twisting

Normal Transformation Matrix

$$J = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & -x\sin(\theta)f'(z) - y\cos(\theta)f'(z) \\ \sin(\theta) & \cos(\theta) & x\cos(\theta)f'(z) - y\sin(\theta)f'(z) \\ 0 & 0 & 1 \end{bmatrix}$$

Twisting

Normal Transformation Matrix

$$J = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & -x\sin(\theta)f'(z) - y\cos(\theta)f'(z) \\ \sin(\theta) & \cos(\theta) & x\cos(\theta)f'(z) - y\sin(\theta)f'(z) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\det J = \cos^2(\theta) + \sin^2(\theta) = 1$$

Twisting

Normal Transformation Matrix

$$J = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & -x\sin(\theta)f'(z) - y\cos(\theta)f'(z) \\ \sin(\theta) & \cos(\theta) & x\cos(\theta)f'(z) - y\sin(\theta)f'(z) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\det J = \cos^2(\theta) + \sin^2(\theta) = 1$$

Volume does not change !

Twisting

Normal Transformation Matrix

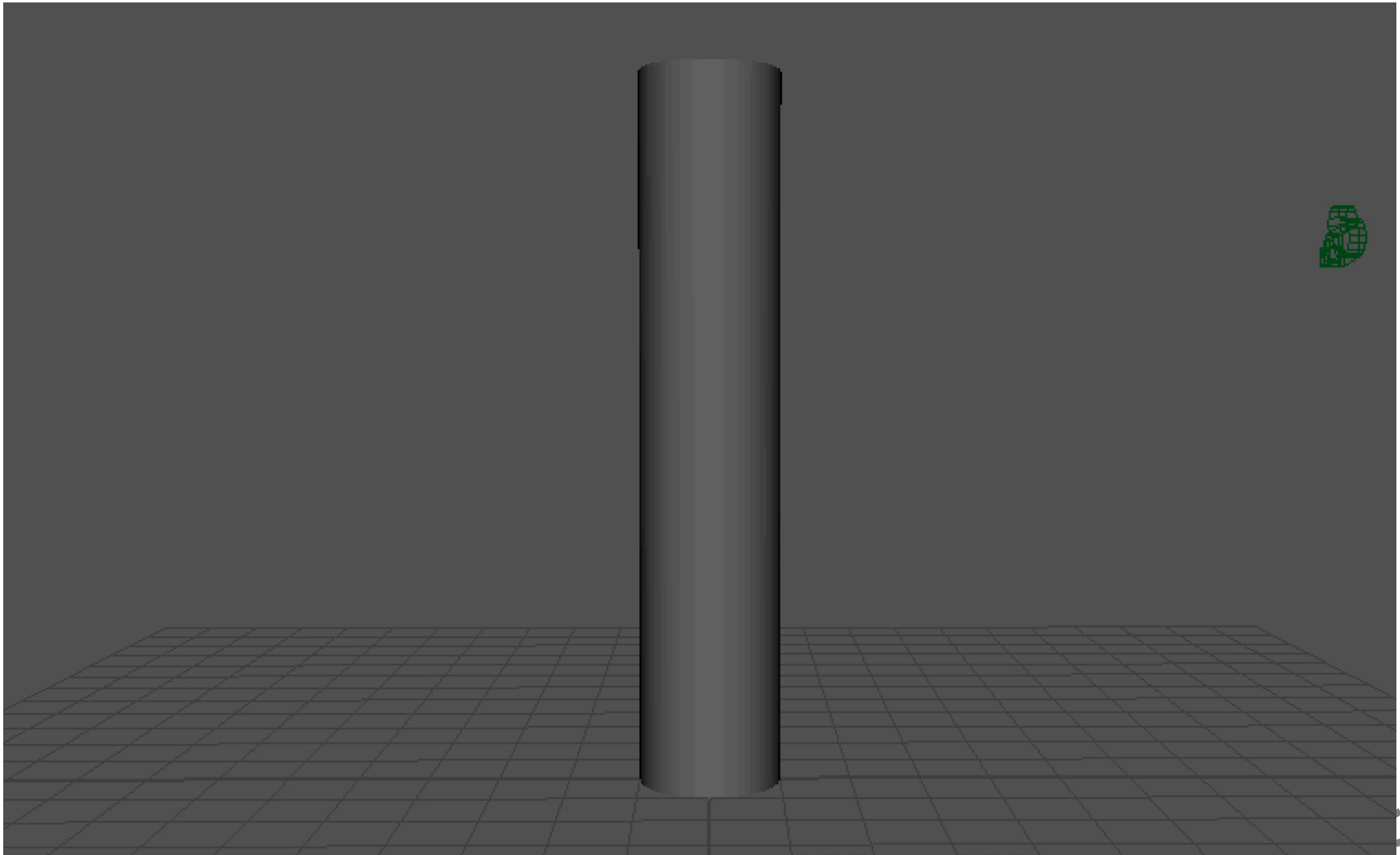
$$J = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & -x\sin(\theta)f'(z) - y\cos(\theta)f'(z) \\ \sin(\theta) & \cos(\theta) & x\cos(\theta)f'(z) - y\sin(\theta)f'(z) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\det J = \cos^2(\theta) + \sin^2(\theta) = 1$$

- **Tnormals :** $det J J^{-1T} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ yf'(z) & -xf'(z) & 1 \end{bmatrix}$

Bending

Examples



Bending

Along y-axis

Bending angle: $\theta = k(y - y_0)$

$$X = x$$
$$Y = y_0 - \sin(\theta)(z - 1/k)$$
$$Z = 1/k + \cos(\theta)(z - 1/k)$$

FFD - Principle

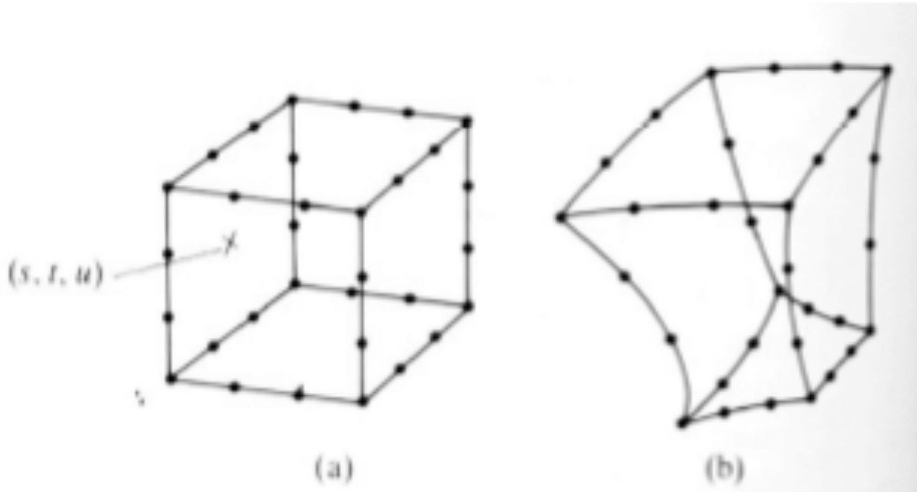
Free Form Deformation of Solid Geometric Models, Sederberg & Parry, 1986

- FFD = Free Form Deformations
- Local coordinate grid super-imposed over an object
- For each vertex, coordinates relative to this local grid are computed and registered
- The grid is then manipulated by the user
- Each vertex is mapped back into the modified grid -> modified global position

FFD

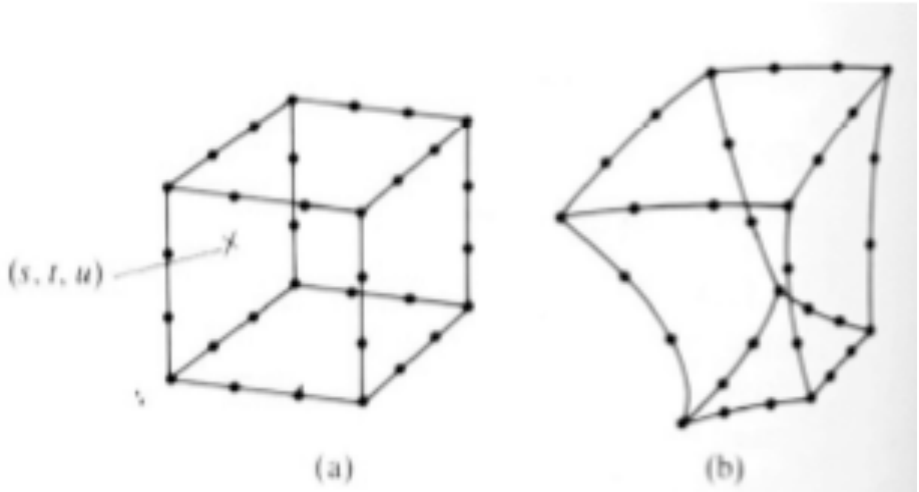
$$\forall u \in [0,1], \forall v \in [0,1], \forall w \in [0,1]$$

$$Q(u,v,w) = \sum_{i=1}^{l} \sum_{j=1}^{m} \sum_{k=1}^{n} F_i(u) F_j(v) F_k(w) P_{ijk}$$



Applying FFD

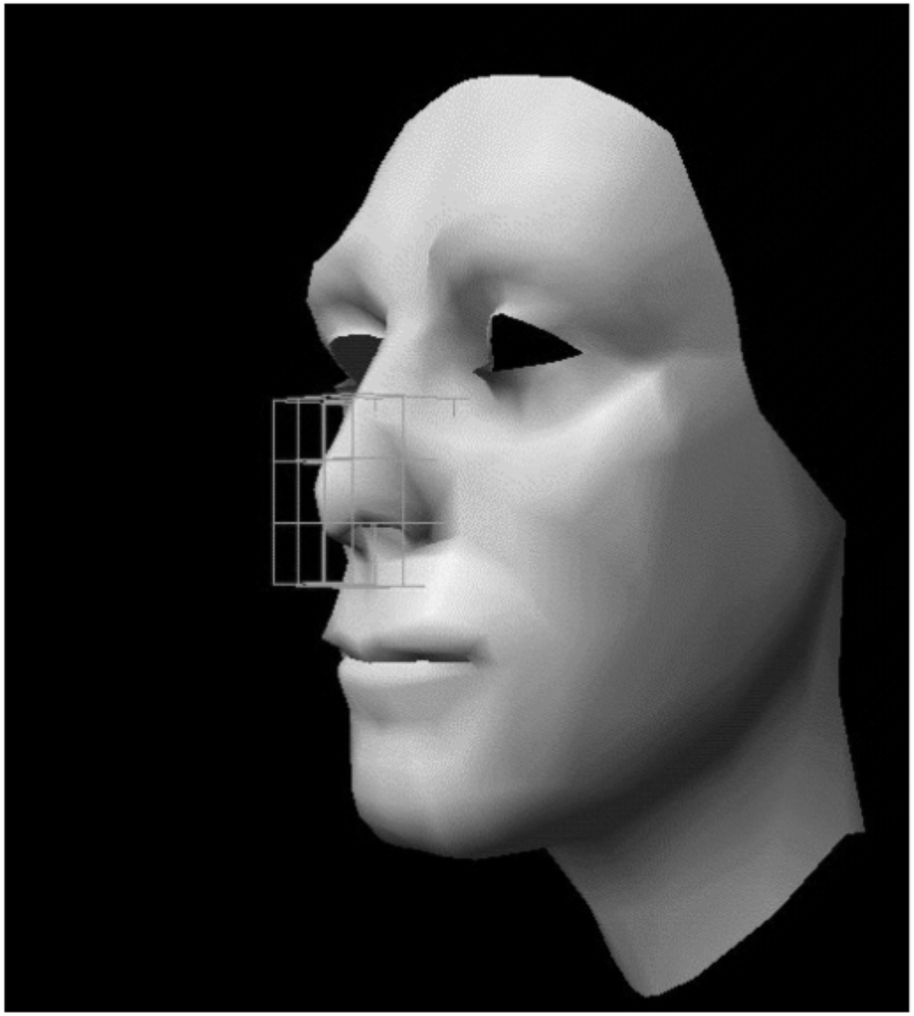
- For each point, compute its relative position in the cube (s,t,u)
- Move the control points Pijk
- Compute the new position using the formula

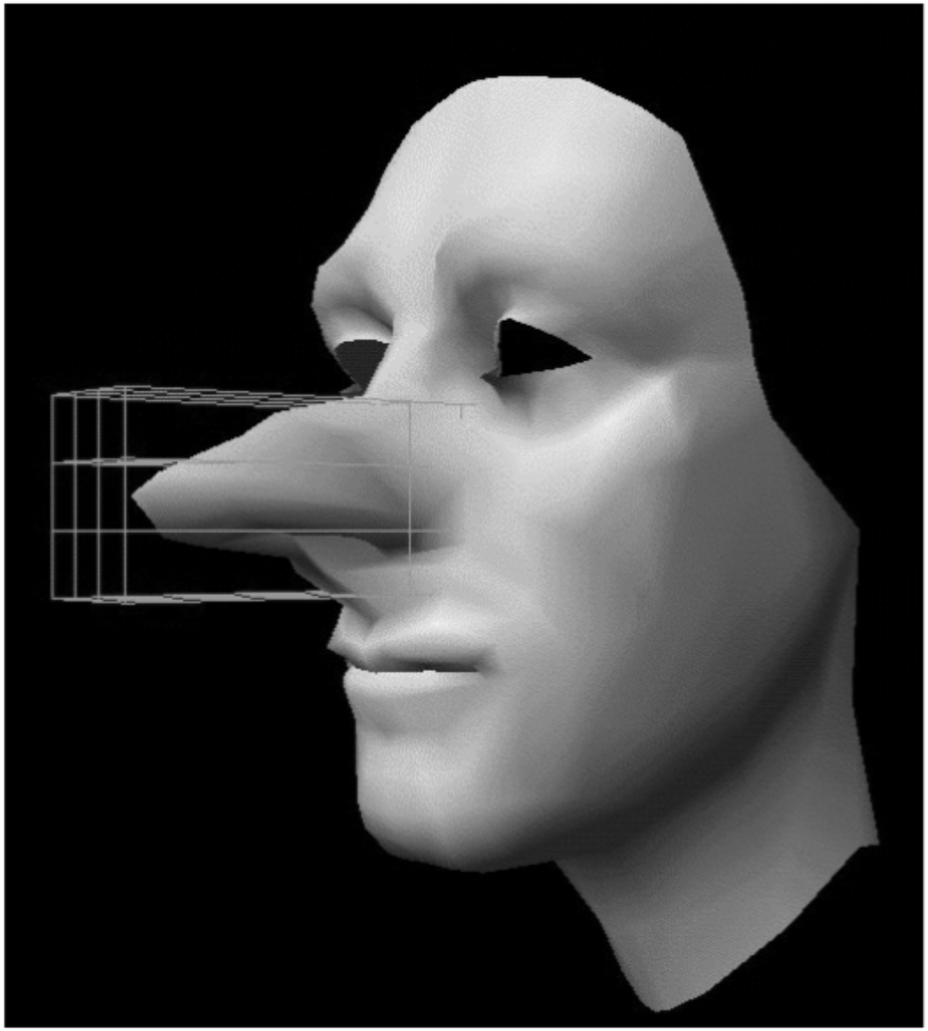


FFD

- High level control
- Ensures continuity of the deformed surface
- Deformation not always easy to control
  - Surfacic FFDs
    - Still indirect deformation
    - Closer to the manipulated surface
    - Very dependant of the control mesh and resolution

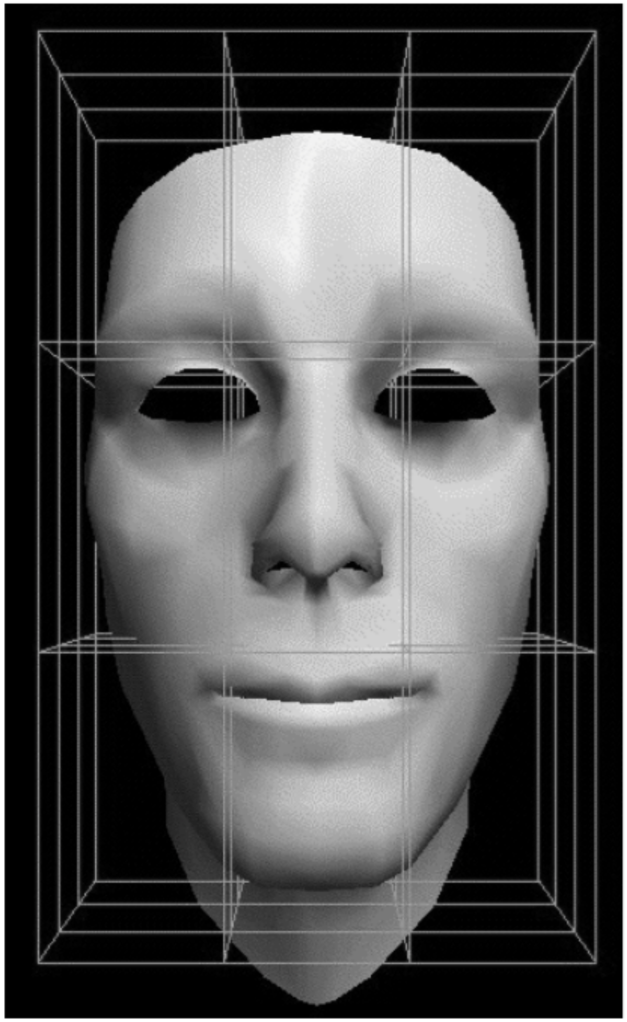
FFD Example

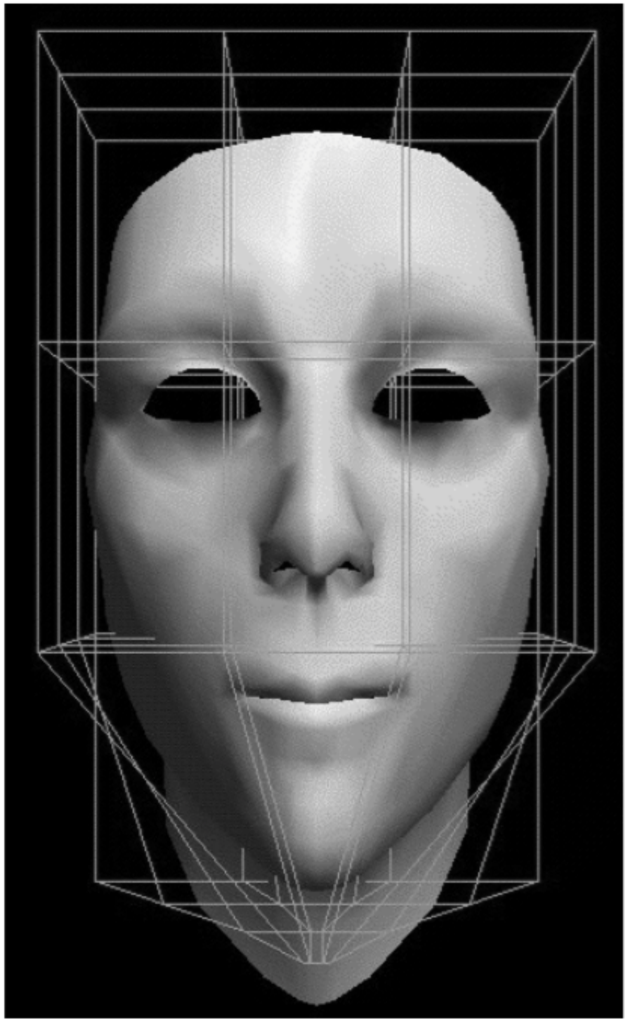




FFD Example

FFD Example





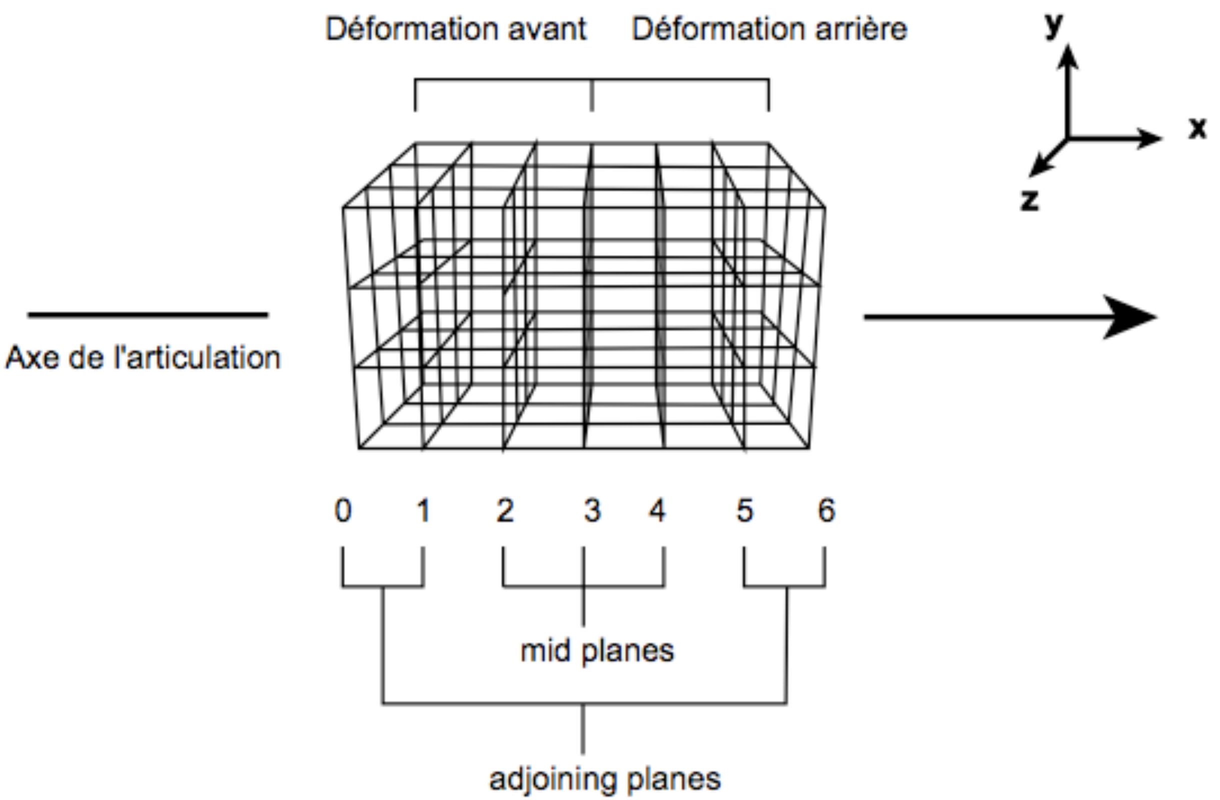
FFD Example

- Video

Muscles - FFDs

Layered construction for deformable animated characters, Chadwick et al., SIGGRAPH' 89)

- M



Muscles - FFDs

Deformation of the belly

Muscle contraction (use contractibility eq)

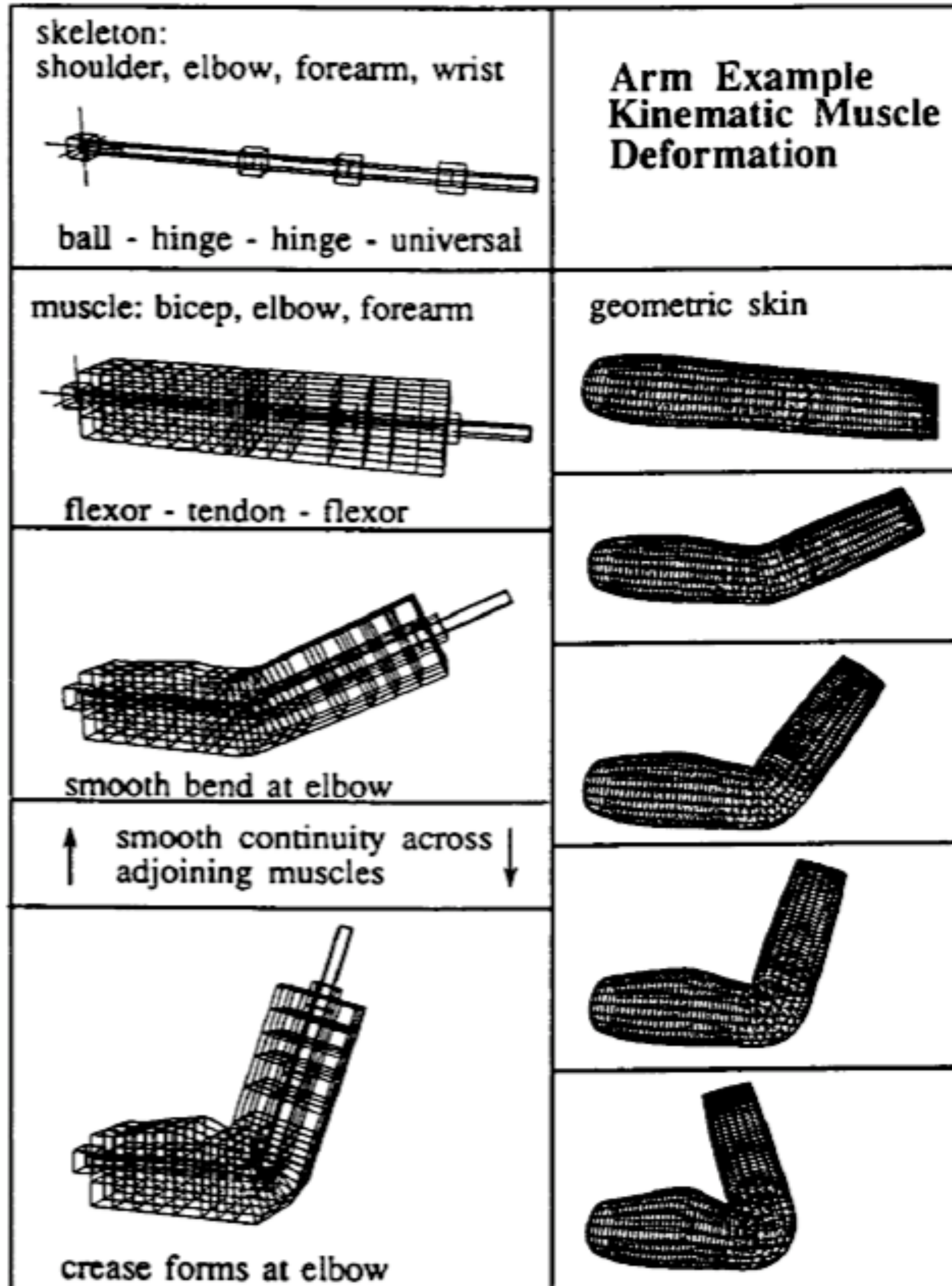
1. **Computation of the shortening R (depends on θ)**
2. **Propagation of the shortening to the** ***mid-planes*** **(if x↘, y↗ & z↗)**
3. **Deformation of the** ***adjoining planes*** **to ensure continuity**

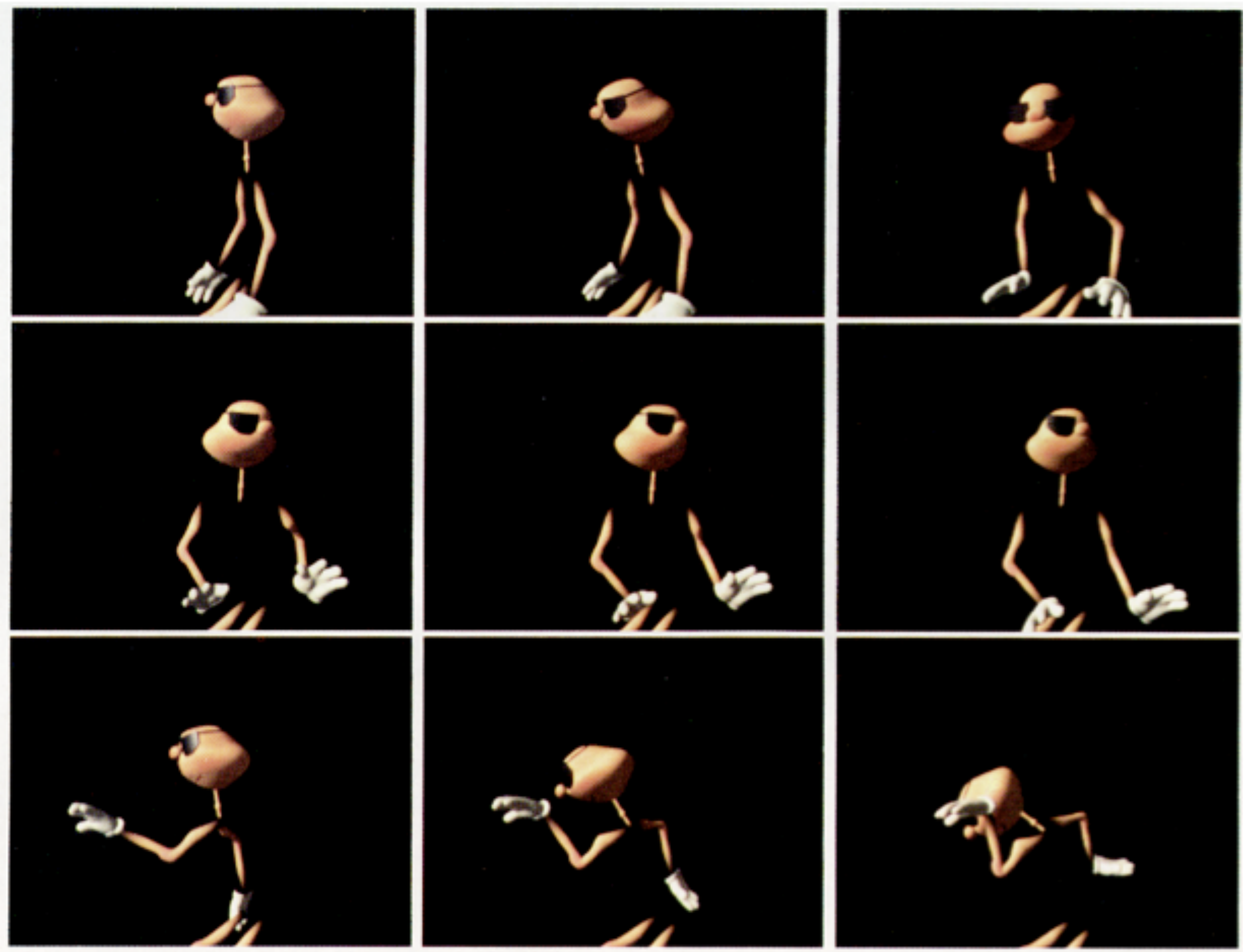
Muscles - FFDs

Deformation of the tendons

1. Compute *angle* = rotation angle / 2
2. Move mid-planes 2 and 3 by *angle* around z and mid-plane 4 by *-angle*
3. Slide adjoining planes away from mid-pl.
4. Rotate trailing deform by joint angle
5. Check for intersections and continuity C1
6. Scale if needed to maintain volume

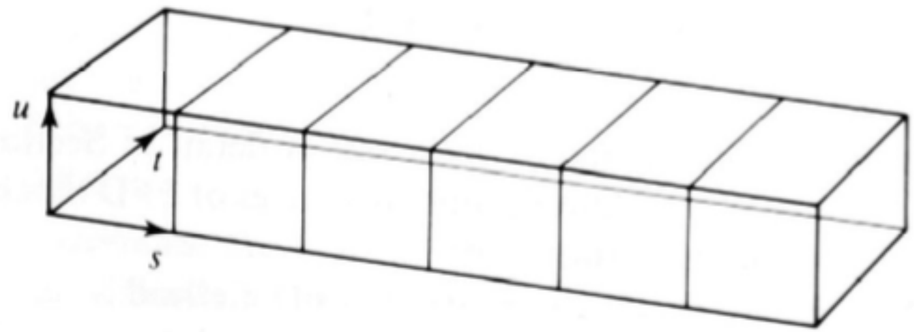
Resul


skeleton:
shoulder, elbow, forearm, wrist
ball - hinge - hinge - universal
Arm Example
Kinematic Muscle
Deformation
muscle: bicep, elbow, forearm
flexor - tendon - flexor
geometric skin
smooth bend at elbow
smooth continuity across
adjoining muscles
crease forms at elbow

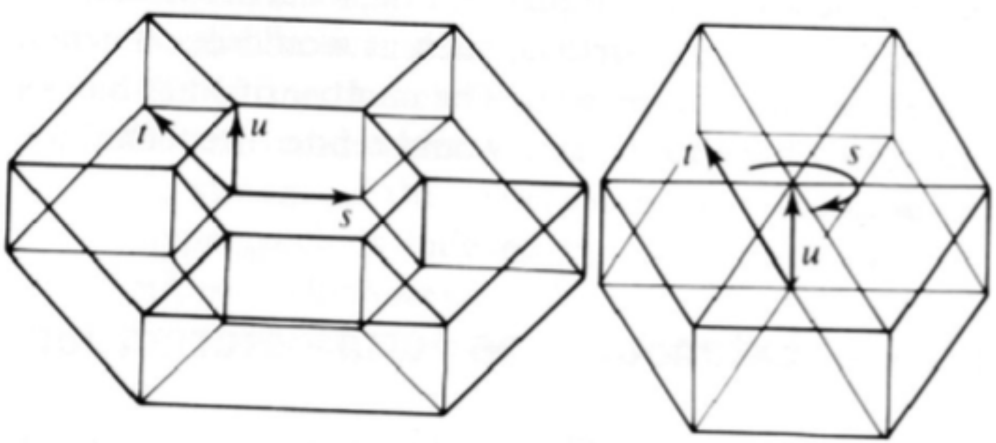


Extended FFD

Extended Free-From Deformation: A Sculpturing Tool for 3D Geometric Modeling, Coquillart, 1990



(a)



(b) (c)

Ext