The ladder operators are defined as

$$L_+ = L_x + \imath L_y$$

$$L_- = L_x - \imath L_y$$

$$\begin{aligned}
L_+L_z &= L_zL_+ + [L_+, L_z] \\
&= L_zL_+ + [L_x, L_z] + \imath[L_y, L_z] \\
&= L_zL_+ + (-\imath\hbar L_y) + \imath(\imath\hbar L_x) \\
&= L_zL_+ - \hbar(L_x + \imath L_y) \\
&= L_zL_+ - \hbar L_+ \\
[L_i, L_j] &= \imath\hbar\epsilon_{ijk}L_k
\end{aligned}$$

Therefore,

$$\begin{aligned}
L_+L_z|l,m\rangle &= L_zL_+|l,m\rangle - \hbar L_+|l,m\rangle \\
\hbar m L_+|l,m\rangle + \hbar L_+|l,m\rangle &= L_zL_+|l,m\rangle
\end{aligned}$$

From the orthonormality of the vectors in the set $\bigcup_i \{|l,m_i\rangle\}$, we can conclude that

$$L_+|l,m\rangle \propto |l,m+1\rangle$$

and similarly,

$$L_+|l,m\rangle \propto |l,m-1\rangle$$

Let,

$$L_+|l,m\rangle = C^+_{l,m}|l,m+1\rangle$$

and

$$L_-|l,m\rangle = C^-_{l,m}|l,m-1\rangle$$

From this, we conclude that

$$\langle l,m|L_-L_+|l,m\rangle = C^-_{l,m+1}C^+_{l,m}$$

$$\langle l,m|L_+L_-|l,m\rangle = C^+_{l,m-1}C^-_{l,m}$$

Also,

$$\begin{aligned}
L_-L_+ &= (L_x - \imath L_y)(L_x + \imath L_y) \\
&= L_x^2 - \imath\left(L_yL_x - L_xL_y\right)) + L_y^2 \\
&= L_x^2 + L_y^2 + L_z^2 - L_z^2 - \hbar L_z \\
&= L^2 - L_z^2 - \hbar L_z
\end{aligned}$$

Similarly,

$$L_+L_- = L^2 - L_z^2 + \hbar L_z$$

This implies,

$$\langle l,m|L_-L_+|l,m\rangle = \hbar^2(l^2+l-m^2-m)$$
$$\langle l,m|L_+L_-|l,m\rangle = \hbar^2(l^2+l-m^2+m)$$

Since, $L_+^\dagger = L_-$,

$$\begin{aligned} L_+|l,m\rangle \leftrightarrow & \langle l,m|L_+^\dagger \\ & \langle l,m|L_- \end{aligned}$$

Hence,

$$\langle l,m|L_-L_+|l,m\rangle = C^-_{l,m+1}C^+_{l,m} = C^{+*}_{l,m}C^+_{l,m}$$
$$\langle l,m|L_+L_-|l,m\rangle = C^+_{l,m-1}C^-_{l,m} = C^{-*}_{l,m}C^-_{l,m}$$

Using the above relations,

$$C^+_{l,m} = e^{\imath\phi^+_{l,m}}\hbar\sqrt{l^2+l-m^2-m}$$

$$C^-_{l,m} = e^{\imath\phi^-_{l,m}}\hbar\sqrt{l^2+l-m^2+m}$$

Since, the phase carries no actual physical signficance, we set $\phi^+ = \phi^- = 0$.

$$\langle l,m'|L_+|l,m\rangle = \hbar\delta_{m+1,m'}\sqrt{l^2+l-m^2-m}$$

$$\langle l,m'|L_-|l,m\rangle = \hbar\delta_{m-1,m'}\sqrt{l^2+l-m^2+m}$$

It is also clear, from the derivation that m ranges from $-l$ to l in steps of 1. Dimensionality of these matrices is therefore: $(2l+1)^2$ Using the formulae for the matrix elements for L_+ and L_- in matrix form, for
$l=\frac{1}{2}$:

$$L_+ = \hbar\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$L_- = \hbar\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

and for $l=1$:

$$L_+ = \hbar\begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}$$

$$L_- = \hbar\begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}$$

From the definitions for L_+ and L_-,

$$L_x = \frac{1}{2}\left(L_+ + L_-\right)$$

$$L_y = \frac{1}{2\imath}\left(L_+ - L_-\right)$$

Therefore, for $l = 1/2$:

$$L_x = \hbar \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$$

$$L_y = \hbar \begin{pmatrix} 0 & -\frac{i}{2} \\ \frac{i}{2} & 0 \end{pmatrix}$$

and for $l = 1$:

$$L_x = \hbar \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

$$L_y = \hbar \begin{pmatrix} 0 & -\frac{i}{\sqrt{2}} & 0 \\ \frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} \\ 0 & \frac{i}{\sqrt{2}} & 0 \end{pmatrix}$$

L_z can be found by

$$L_z = -\frac{\imath}{\hbar}\left[L_x, L_y\right]$$

For $l = 1$,

$$L_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$