Week 0 Solutions

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Julia Lab Exercises

Exercise 1

We're now ready for the first edX Exercise - the result of which you need to enter into the edX grading forms. Consider $|b_0\rangle = \sqrt{\frac{1}{3}}\,|0\rangle + \sqrt{\frac{2}{3}}\,|1\rangle$. What real-valued vectors are orthogonal to this vector? In other words given $|b_0\rangle$ which $|b_1\rangle$ satisfy $\langle b_1|b_0\rangle = 0$?.

Your task is to find parameters $\alpha_1 \geq 0$ and β_1 such that $|b_1\rangle = \begin{pmatrix}\alpha_1\\ \beta_1\end{pmatrix}$ is orthogonal to $|b_0\rangle$.

Take a guess, use Julia to find the vector for you, or compute it by hand! By clicking in the code box just below you can fill in values for α_1 and β_1 and calculate $\langle b_1|b_0\rangle$ to see how you are doing.

To start you off we have set α_1 and β_1 to $\sqrt{\frac{1}{2}}$ and $\sqrt{\frac{1}{2}}$ respectively.

Homework 0

1. In this question we will investigate the simplest of quantum communication tasks: sending a classical bit using a qubit. So let's start by imagining our two favourite protagonists: Alice and Bob. Alice, who we imagine to be a PhD student at CalTech, wants to send some information to Bob, who is a post-doc at TU Delft. However Bob only accepts messages coming through their shared quantum communicator, which can prepare, send, receive and measure qubits. Now imagine Alice wants to send a very simple message, namely a bit (either a 1 or a 0).

   In order to do this she encodes her binary value by preparing a qubit in the standard basis, so

   $$0 \rightarrow |0\rangle$$

   $$1 \rightarrow |1\rangle$$

   Let's imagine she wants to send a 0, so she prepares the state $|0\rangle$ and sends it to Bob. Now let's imagine that Bob knows Alice sent a qubit encoded in the standard basis (They agreed on this while they were at a conference the month before) and hence when he receives Alice's qubit he measures it in the standard basis. This means he will get either a 1 with probability p_1, or a 0 with probability p_0 as a classical outcome from the machine. What would now be the correct probabilities assigned to these measurement outcomes?