Quantum Mechanics, Physics 732
Final Assignment, due May 14, 2015

This assignment is based on papers

- "Quantum control and process tomography of a semiconductor quantum dot hybrid qubit" by Dohun Kim *et al.*, Nature **511**, pp. 70 - 74 (2014), also available as arXiv:1401.4416v1;
- "Fast coherent manipulation of three-electron states in a double quantum dot" by Zhan Shi *et al.*, Nat. Commun. **5**, p. 3020 (2014), arXiv:1308.0588v1.

Consider a system described by the Hamiltonian

$$H = \begin{pmatrix} \varepsilon/2 & 0 & \Delta_1 & -\Delta_2 \\ 0 & \varepsilon/2 + \Delta_L & -\Delta_3 & \Delta_4 \\ \Delta_1 & -\Delta_3 & -\varepsilon/2 & 0 \\ -\Delta_2 & \Delta_4 & 0 & -\varepsilon/2 + \Delta_R \end{pmatrix}, \tag{1}$$

where Δ_i are fixed parameters and provided in Nat. Commun. paper, while ε can be changed quickly by applying voltage pulses to metallic gates of a double quantum dot.

1. Calculate the energy spectrum of the system as a function of bias ε and find level crossing or minimal energy gaps.
2. At initial moment of time, a large negative bias is applied, $-\varepsilon_{\text{in}} \gg \max_i \Delta_i$, and the system is in the ground state. Then, the bias changes linearly in time as $\varepsilon = \varepsilon_{\text{in}} + vt$. Calculate the probabilities of occupation of energy states at $t \to \infty$. Can you find a reasonable approximation for the ground state probability using the Landau-Zener formula?
3. Now, consider X and Z rotation pulses, discussed in Nature paper. For X rotations, the system is initiated in the ground state at large negative ε and then quickly brought to the configuration with the minimal level separation between the ground state and the first excited state. The system stays in this configuration over time t_X. Calculate the probabilities of occupation of energy states at $t = t_X$. What is the best choice of t_X to realize $\pi/2$ rotation about X axis?
4. Model the Ramsey experiment, see Fig. 1 in Nature paper.

Write a summary of your calculations, provide corresponding plots and numerical values. Submit your program that you wrote to perform numerical calculations. You can choose any software packages and programming languages, the recommended software is Mathematica, Matlab or python.

updated April 24, 2015