

BT2201 Finance Project - G38

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Portfolio Optimization

Modern portfolio theory (MPT) tells us that investors are risk averse and given a level of risk, they will choose the portfolios that offer the most return. This is where portfolio optimization comes into play.

To perform the optimization we will need

- To read asset price data
- Calculate the mean returns for the time period (10/2001 - Present)
- Assign random weights to the assets and then use those to build an efficient frontier through rounds of trialing

Disclaimer: For following data, expense ratio defined “per annum”, tracking difference defined “over 1 year”

In order to construct an even more comprehensive and diversified portfolio, we suggest including 2 additional ETFs, alongside the 16 ETFs provided to us.

Initial set of 16 suggested ETFs:

AGG: iShares Core U.S. Aggregate Bond ETF
SCHP: Schwab U.S. TIPS ETF
VCSH: Vanguard Short-Term Corporate Bond ETF
BNDX: Vanguard Total International Bond ETF
EMB: iShares J.P. Morgan USD Emerging Markets Bond ETF

1306: Nomura TOPIX Exchange Traded Fund ETF
VOO: Vanguard S&P 500 ETF
VO: Vanguard Mid-Cap ETF
VSS: Vanguard FTSE All-World ex-US Small-Cap ETF
VGK: Vanguard FTSE Europe ETF
VWO: Vanguard FTSE Emerging Markets ETF

1343: Nomura NEXT FUNDS Tokyo Stock Exchange REIT Index ETF
VNQ: Vanguard Real Estate ETF
VNQI: Vanguard Global ex-U.S. Real Estate ETF

IAU: iShares Gold Trust ETF
BCI: ETFS Bloomberg All Commodity Strategy K-1 Free ETF

Our suggested ETFs:

2561: iShares Core Japan Government Bond ETF

<https://www.blackrock.com/jp/individual-en/en/products/313081/ishares-core-japan-government-bond-etf>

Expense Ratio: 0.07% (low)

Tracking difference: -0.26%

Benchmark: FTSE Japanese Government Bond Index - JPY

The 16 ETFs listed above, although well-diversified across asset classes and region, does not include JPY Fixed-Income, hence we believe including the 2561 ETF will provide exposure to a broad range of fixed-rate, local-currency Japanese government bonds. Also, it is a cost effective way to diversify our portfolio using fixed income.

SCHA: Schwab U.S. Small-Cap ETF

<https://www.schwabassetmanagement.com/resource/scha-fact-sheet>

Expense Ratio: 0.04% (low)

Tracking difference: 0.02%

Benchmark: Dow Jones U.S Small-Cap Total Stock Market Total Return Index - USD

Although the 16 ETF listed above comprised of a few ETF that covers the emerging market, it is more proportionately offerings from China, Taiwan, India, Canada and Japan. It would be beneficial to further diversify into the US emerging market consisting of small-cap stocks, in which companies have much resources to innovate. Small-cap stocks tend to have more room to move than large-caps, so they exhibit greater volatility. More importantly, small-caps are more risky than more stable, large-cap stocks. As an added bonus, small caps aren't perfectly correlated with large caps, offering a diversification benefit.

Currency Conversion

For the assets which are tracking USD indexes, we had to use the time-series matrix of USDJPY daily rates to convert the ETF with currency in USD to JPY, which was every ETF except 1306 and 1343.

This was done using the formula $(1+\text{return}) \times (1+\text{USDJPY}) - 1$. This is to ensure that the values we are tracking are on a comparable basis.

Mean Annual Returns of Market, Risk-free Asset, ETFs

	Mean Annual Returns
mkt	0.0737013
rf	0.0054607
AGG	0.0401157
SCHP	0.0514788
VCSH	0.0378906
BNDX	0.0397449
VWOB	0.0776141
1306	0.0671742
VOO	0.1152926
VO	0.1327294
VSS	0.1032819
VGK	0.0911835
VWO	0.1360688
1343	0.0944193
VNQ	0.1457270
VNQI	0.0956096
IAU	0.1016365
BCI	0.0410220

Covariance Matrix between 16 ETFs

	AGG	SCHP	VCSH	BNDX	VWOB	1306	VOO	VO	VSS	VGK	VWO	1343	VNQ	VNQI	IAU	BCI
AGG	0.0087	0.0077	0.0078	0.0077	0.0093	0.0037	0.0125	0.0128	0.0096	0.0118	0.0138	0.0022	0.0129	0.0097	0.0051	0.0085
SCHP	0.0077	0.0088	0.0075	0.0073	0.0088	0.0035	0.0110	0.0113	0.0084	0.0100	0.0120	0.0019	0.0117	0.0086	0.0053	0.0086
VCSH	0.0078	0.0075	0.0084	0.0079	0.0094	0.0036	0.0137	0.0140	0.0103	0.0128	0.0147	0.0025	0.0143	0.0103	0.0049	0.0092
BNDX	0.0077	0.0073	0.0079	0.0084	0.0091	0.0026	0.0136	0.0137	0.0094	0.0121	0.0142	0.0013	0.0142	0.0092	0.0049	0.0088
VWOB	0.0093	0.0088	0.0094	0.0091	0.0151	0.0066	0.0205	0.0214	0.0175	0.0209	0.0241	0.0046	0.0216	0.0170	0.0074	0.0134
1306	0.0037	0.0035	0.0036	0.0026	0.0066	0.0423	0.0088	0.0100	0.0182	0.0111	0.0139	0.0191	0.0058	0.0201	0.0041	0.0081
VOO	0.0125	0.0110	0.0137	0.0136	0.0205	0.0088	0.0569	0.0585	0.0379	0.0551	0.0595	0.0050	0.0600	0.0327	0.0112	0.0241
VO	0.0128	0.0113	0.0140	0.0137	0.0214	0.0100	0.0585	0.0635	0.0411	0.0580	0.0631	0.0061	0.0651	0.0354	0.0124	0.0265
VSS	0.0096	0.0084	0.0103	0.0094	0.0175	0.0182	0.0379	0.0411	0.0438	0.0467	0.0492	0.0112	0.0387	0.0378	0.0132	0.0243
VGK	0.0118	0.0100	0.0128	0.0121	0.0209	0.0111	0.0551	0.0580	0.0467	0.0704	0.0679	0.0068	0.0605	0.0393	0.0144	0.0281
VWO	0.0138	0.0120	0.0147	0.0142	0.0241	0.0139	0.0595	0.0631	0.0492	0.0679	0.0887	0.0078	0.0672	0.0434	0.0183	0.0324
1343	0.0022	0.0019	0.0025	0.0013	0.0046	0.0191	0.0050	0.0061	0.0112	0.0068	0.0078	0.0424	0.0046	0.0169	0.0042	0.0051
VNQ	0.0129	0.0117	0.0143	0.0142	0.0216	0.0058	0.0600	0.0651	0.0387	0.0605	0.0672	0.0046	0.0992	0.0332	0.0125	0.0240
VNQI	0.0097	0.0086	0.0103	0.0092	0.0170	0.0201	0.0327	0.0354	0.0378	0.0393	0.0434	0.0169	0.0332	0.0423	0.0108	0.0213
IAU	0.0051	0.0053	0.0049	0.0049	0.0074	0.0041	0.0112	0.0124	0.0132	0.0144	0.0183	0.0042	0.0125	0.0108	0.0301	0.0164
BCI	0.0085	0.0086	0.0092	0.0088	0.0134	0.0081	0.0241	0.0265	0.0243	0.0281	0.0324	0.0051	0.0240	0.0213	0.0164	0.0371

CAPM Regression

According to the CAPM formula, we will first get the beta of each stock by regressions; then further calculate the expected excess return of each stock. The beta is a measure of an asset's risk relative to the overall market benchmark.

We conducted regression on the returns of each stock against the return of market (mkt). An example of one of our regression is shown below for the AGG ETF, where the Beta of AGG is the coefficient of the 'mkt' variable (4.721e-01) and Variance of residual standard error is the squared value of Residual Standard Error (0.003906)^2.

Using the CAPM formula: $E(R_{etf}) - R_f = \beta(ETF) * (E(R_{mkt}) - R_f)$, where R_f is the risk-free rate of 2.166938e-05 which is calculated from taking the mean of the 'rf' in the dataset. The formula is equivalent to: Expected excess return of ETF = $\beta(ETF) * \text{Expected excess return of MKT}$. We used the mean of the market calculated from the dataset to get the Expected excess MKT return = 0.0002924657 - risk-free = 0.0002707963.

We then ran an iteration of all 16 ETFs using their respective Betas calculated from the regressions to calculate the Expected Excess Returns of each ETF in the portfolio shown in the table below.

The values are also known as the Market Risk Premium which represents the additional return over the risk-free rate, which is required to compensate investors for investing in a riskier asset class compared to risk-free assets.

```
##
## Call:
## lm(formula = AGG ~ mkt, data = returns)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.051019 -0.001939  0.000005  0.001903  0.033917
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 2.112e-05  5.347e-05   0.395   0.693
## mkt         4.721e-01  5.758e-03  81.991 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.003906 on 5338 degrees of freedom
## Multiple R-squared:  0.5574, Adjusted R-squared:  0.5573
## F-statistic: 6723 on 1 and 5338 DF, p-value: < 2.2e-16
```

Table 1: Expected Excess Returns using CAPM

AGG	SCHP	VCSH	BNDX	VWOB	ETF_1306	VOO	VO	VSS	VGK	VWO	ETF_1343	VNQ	VNQI	IAU	BCI
0.0001278	0.0001159	0.0001346	0.0001323	0.0001883	0.0001289	0.0004057	0.0004201	0.0003117	0.0004115	0.000446	7.01e-05	0.0004132	0.0002881	0.0001022	0.0002141

Next, we considered portfolios that matter the most to investors...

- Minimum Variance portfolio
- Market/Tangency portfolio

Firstly, we assigned random weights to 16 ETFs and calculated the annualized portfolio returns and portfolio risk. To find the optimal portfolio with the minimum variance and maximum Sharpe ratio, we looped this process 5000 times.

The minimum variance portfolio is catered to the most risk-adverse investors. It aims to minimize the price volatility of the overall portfolio, as well as the maximum drawdown that the portfolio might potentially face.

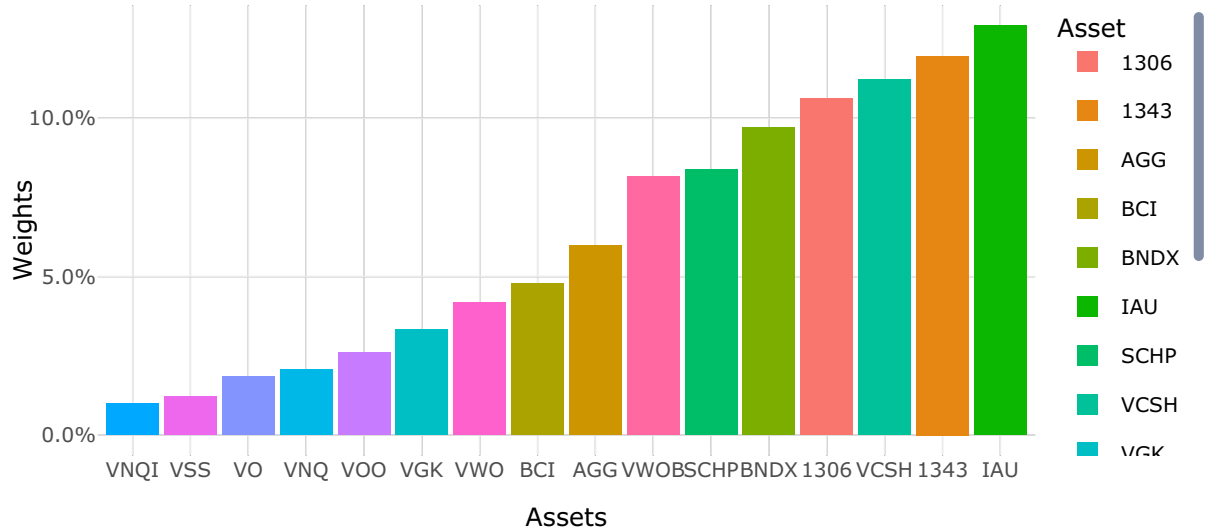
The tangency portfolio is the portfolio of ETFs that has the highest Sharpe ratio. It has the best risk-adjusted performance.

Individual ETF Weights (Minimum Variance)

Table 2: Individual Weights to minimise Risk

Return	Risk	SharpeRatio	Asset	Weights
0.0768999	0.1061137	0.7246931	AGG	0.0598644
0.0768999	0.1061137	0.7246931	SCHP	0.0838557
0.0768999	0.1061137	0.7246931	VCSH	0.1121908
0.0768999	0.1061137	0.7246931	BNDX	0.0970344
0.0768999	0.1061137	0.7246931	VWOB	0.0815981
0.0768999	0.1061137	0.7246931	1306	0.1062897
0.0768999	0.1061137	0.7246931	VOO	0.0260619
0.0768999	0.1061137	0.7246931	VO	0.0183799
0.0768999	0.1061137	0.7246931	VSS	0.0120919
0.0768999	0.1061137	0.7246931	VGK	0.0333594
0.0768999	0.1061137	0.7246931	VWO	0.0419537
0.0768999	0.1061137	0.7246931	1343	0.1196414
0.0768999	0.1061137	0.7246931	VNQ	0.0206384
0.0768999	0.1061137	0.7246931	VNQI	0.0100607
0.0768999	0.1061137	0.7246931	IAU	0.1291566
0.0768999	0.1061137	0.7246931	BCI	0.0478229

Minimum Variance Portfolio Weights

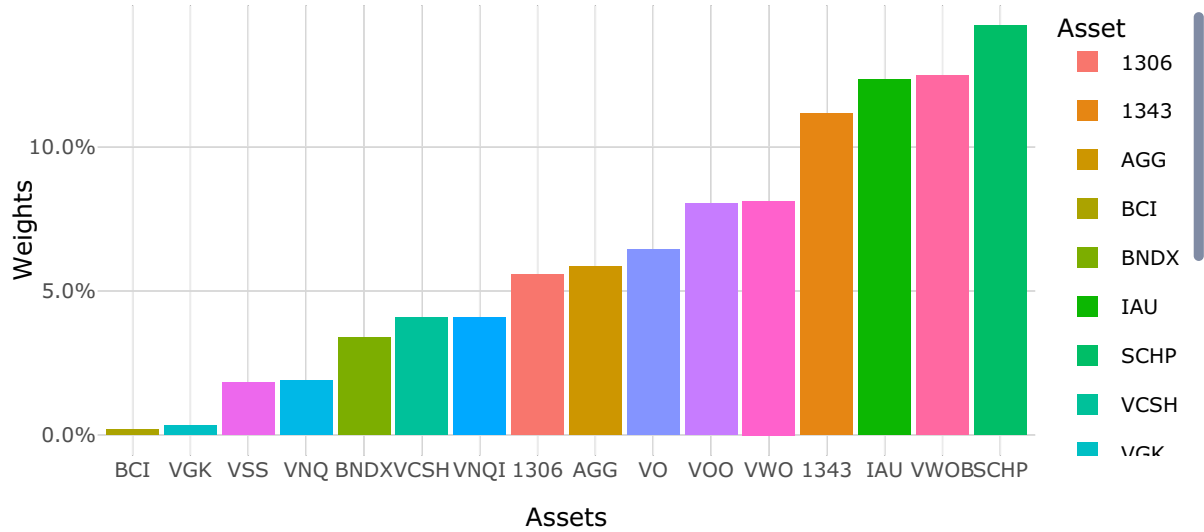


Individual ETF Weights (Maximise Sharpe Ratio)

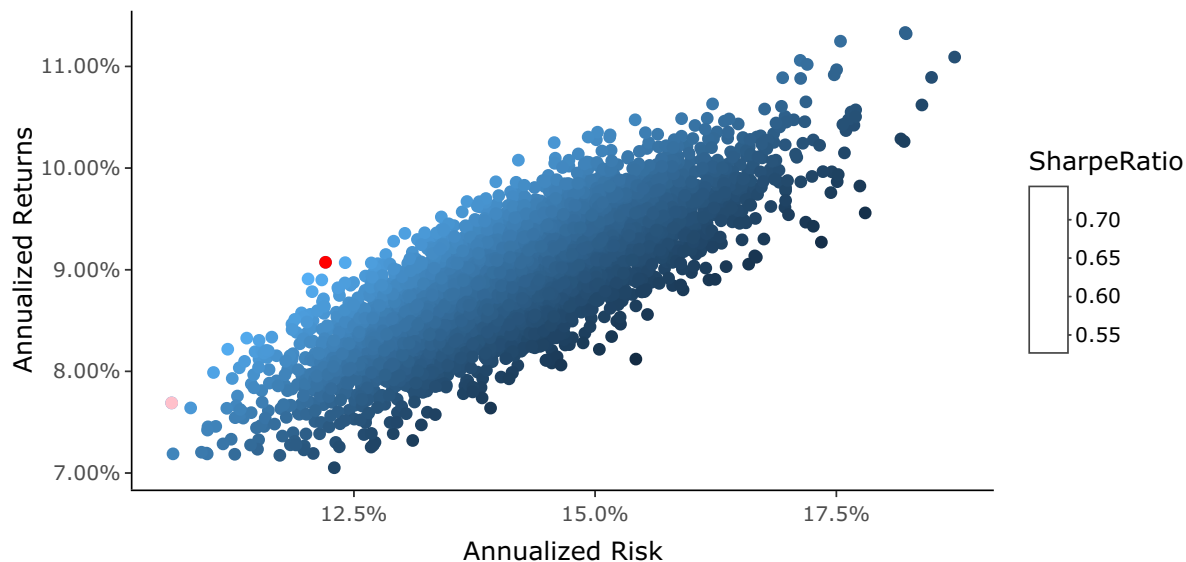
Table 3: Individual Weights to maximise Sharpe Ratio

Return	Risk	SharpeRatio	Asset	Weights
0.0907295	0.122062	0.7433063	AGG	0.0585435
0.0907295	0.122062	0.7433063	SCHP	0.1420371
0.0907295	0.122062	0.7433063	VCSH	0.0406949
0.0907295	0.122062	0.7433063	BNDX	0.0338981
0.0907295	0.122062	0.7433063	VWOB	0.1247386
0.0907295	0.122062	0.7433063	1306	0.0556980
0.0907295	0.122062	0.7433063	VOO	0.0804569
0.0907295	0.122062	0.7433063	VO	0.0643796
0.0907295	0.122062	0.7433063	VSS	0.0184011
0.0907295	0.122062	0.7433063	VGK	0.0032096
0.0907295	0.122062	0.7433063	VWO	0.0812482
0.0907295	0.122062	0.7433063	1343	0.1116260
0.0907295	0.122062	0.7433063	VNQ	0.0190124
0.0907295	0.122062	0.7433063	VNQI	0.0407267
0.0907295	0.122062	0.7433063	IAU	0.1233046
0.0907295	0.122062	0.7433063	BCI	0.0020246

Tangency Portfolio Weights



Plotting the Efficient Frontier



By observation of the efficient frontier, we chose the following 3 portfolios as our model portfolios:

- Low risk: **Pink** point (GMVP)
- Medium risk: **Red** point (Tangency Portfolio)
- High risk: A point upwards and rightwards of the Tangency Portfolio (1/N Portfolio)

Backtesting

For in-sample testing, we benchmarked our portfolio against the following 60/40 stocks-bonds portfolio. The benchmark portfolio consists of 60% weight in VTI, and 40% weight in BND, as follows.

	Weight	Ticker	ETF name	Investment Themes
A	60%	VTI	Vanguard Total Stock Market ETF	Equity, U.S, Large Cap
B	40%	BND	Vanguard Total Bond Market ETF	Bond, U.S, All-Term

Portfolios vs 60/40 Benchmark Returns

We compared returns over the past ~20 years to observe how our 3 model portfolio performs against the 60/40 benchmark set.

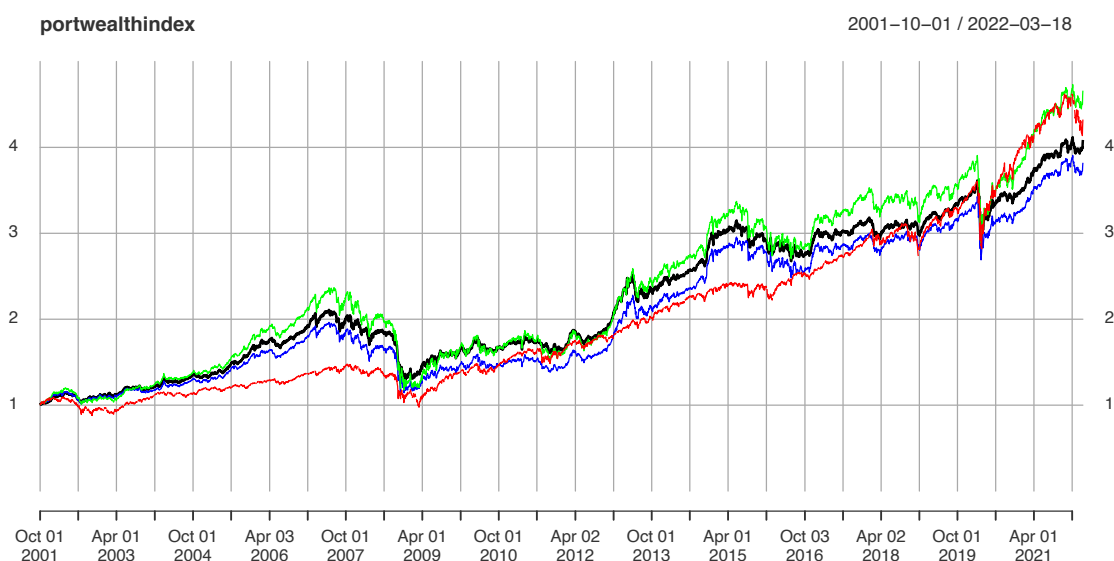
Legend

Black Line: Tangency Portfolio

Blue Line: Global Mean-Variance Portfolio

Green Line: 1/N Portfolio

Red Line: 60/40 Benchmark



Portfolios vs 60/40 Benchmark Summary Statistics

	Total_Return	Return_pa	Volatility_pa	SharpeRatio_pa	Max Drawdown
Tangency Portfolio	3.086458	0.0686859	0.1083816	0.0449856	0.4027040
Global Min Variance Portfolio	2.820746	0.0653006	0.1072094	0.0405589	0.4207909
1/N Weighted Portfolio	3.660164	0.0753319	0.1361315	0.0379181	0.5035114
60/40 Benchmark	3.216394	0.0726291	0.1110595	0.0432751	0.3398451

The statistics calculated are in line with our expectations.

The **Global Minimum Variance Portfolio (GMVP)** has the lowest volatility per annum, as well as the lowest return per annum compared to the other 3 portfolios. This portfolio will be marketed to our more **risk-averse** investors.

The **Tangency Portfolio** has the highest Sharpe Ratio per annum compared to the other 3 portfolios. It fares better than the GMVP returns-wise, but it also comes with a higher risk, as shown by the volatility per annum. This portfolio will be marketed and catered to our **medium-risk** investors.

Finally, the **1/N Weighted Portfolio** has the best in-sample performance out of the 4 portfolios. However, it also comes with a much higher risk as expected. This portfolio will thus be marketed to our more **risk-tolerant** investors.