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1 Propositional Logic

P: Paolo has a Job M: Mary has a Job R: Roberto has a J

1. $\neg(P \to M)$ φ_1

2. $(P \wedge M) \to \neg R$ φ_2

3. $(M \wedge R) \to (P \vee \neg M)$ φ_3

For a set of formulas to be consistent there should be at least an assignment that satisfies all of them

$$\neg(P \to M) \wedge ((P \wedge M) \to \neg R) \wedge ((M \wedge R) \to (P \vee \neg M))$$

1. $CNF(\varphi_1)$: $\neg(\neg P \vee M)$ or $(P \wedge \neg M)$

2. $CNF(\varphi_2)$: $\neg(P \wedge M) \vee \neg R$ or $(\neg P \vee \neg M \vee \neg R)$

3. $CNF(\varphi_3)$: $\neg(M \wedge R) \vee (P \vee \neg M)$ or $(\neg M \vee \neg R \vee P \vee \cancel{\neg M})$

$\downarrow$

$$P \wedge \neg M \wedge (\neg P \vee \neg M \vee \neg R) \wedge (\neg M \vee \neg R \vee P)$$

branch on P

$\mu(P) = T$

$\{\neg M\}; \{\neg M, \neg R\};$

branch on M

$\mu(M) = \bot$

SAT

There is $\mu(P) = T$, $\mu(M) = \bot$ that satisfies all of them
They are consistent (i can put $\mu(R) = T$ or $\bot$)

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2. First order logic

$F(x,y)$: x is a friend of y

1. $\forall x. \neg F(x,x)$ φ_1

2. $\forall x. \forall y. (F(x,y) \rightarrow F(y,x))$ φ_2

3. $\forall x. \forall y. \forall z. ((F(x,y) \wedge F(y,z)) \rightarrow F(x,z))$ φ_3

1) For a set of sentences to be consistent, there should be at least an interpretation that satisfies all of them

Consider an $I: \langle D, g \rangle$ s.t. $D = \{c\}$ and $g(F) = \emptyset$ (*)

1. $\models_I \forall x. \neg F(x,x)$ TRUE because $F(c,c)$ is false

2. $\models_I \forall x. \forall y. (F(x,y) \rightarrow F(y,x))$ TRUE because $F(c,c)$ is false

3. $\models_I \forall x. \forall y. \forall z. ((F(x,y) \wedge F(y,z)) \rightarrow F(x,z))$ TRUE because $F(c,c)$ is false

2) $D = \{Mary, Susan, Juliet\}$

$g(F) = \{(Mary, Susan), (Susan, Juliet)\}$

1. $\models_I \varphi_1$ TRUE

2. $\models_I \varphi_2$ FALSE, since Mary is a friend of Susan, but not the contrary.

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3. $\vDash_I \varphi_3$ FALSE, since Mary is a friend of Susan, Susan is a friend of Juliet, but Mary is not a friend of Juliet

↓ All these considering the given interpretation

3) $\exists x. \forall y. (F(x,y))$ φ_4

So that $(\varphi_1, \varphi_2, \varphi_3) \vDash \varphi_4$ all models that satisfies $\varphi_1, \varphi_2, \varphi_3$ must satisfy φ_4

↳ (*) Using the interpretation used for the first question we can simply notice that it satisfies $\varphi_1, \varphi_2, \varphi_3$ and not φ_4 since $F(c,c)$ is FALSE

So much so φ_4 is not a logical consequence of $\varphi_1, \varphi_2, \varphi_3$