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Generalized disjunctive programming model for the multi-period production planning optimization: An application in a polyurethane foam manufacturing plant

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ABSTRACT

A Generalized Disjunctive Programming (GDP) model for the optimal multi-period production planning and stock management is proposed in this work. The formulation is applied to a polyurethane foam manufacturing plant that comprises three stages: a first step that produces pieces with certain characteristics, a second process that involves the location of these pieces in a limited area and a third stage where pieces are stored in dedicated spaces. This article shows the GDP capabilities to provide a qualitative framework for representing the problem issues and their connections in a natural way, especially in a context where decisions integration is required. Due to the multi-period nature of the planning problem, a rolling horizon approach is suitable for solving it in reasonable computing time. It serves as a tool for analyzing the trade-offs among the different costs. Through the examples, the capabilities of the formulation and the proposed resolution method are highlighted.

1. Introduction

In the last years, there has been an interest for representing integer decisions and logical constraints through disjunctive programming. It symbolizes, in a straightforward manner, the logical conditions and relations in optimization problems (Balas, 1979). Generalized Disjunctive Programming (GDP) can be considered as an extension of disjunctive programming, which – besides involving algebraic constraints – also embeds disjunctions and logical propositions (Raman and Grossmann, 1994). In addition, GDP allows modeling nested decisions for the Boolean variables that are related.

GDP has been developed for modeling various applications of process systems ranging from supply chain (Rodriguez and Vecchietti, 2012), passing by plant design (Moreno et al., 2009; García-Ayala et al., 2012; Caballero et al., 2014), to scheduling problems (Knudsen and Foss, 2013; Castro and Marques, 2015). GDP models are formulated through continuous and Boolean variables, algebraic equations, disjunctions and logical clauses. The usual technique for solving GDPs consists in reformulating the problem as a mixed integer linear/nonlinear programming (MILP/MINLP) model. For this purpose, Big-M (Wolsey and Nemhauser, 1988) or Convex Hull (Lee and Grossmann, 2000) reformulations can be applied converting the Boolean decisions in binary variables and the logic relations into algebraic linear/nonlinear constraints. According to Grossmann and Lee (2003), Big-M reformulation generates a smaller MILP/MINLP in terms of equations and variables, while Convex Hull reformulation generates a tighter one. Recently, Trespalacios and Grossmann (2014) presented a review of MINLP and GDP methods for Process Systems Engineering (PSE), addressing the techniques for improving MINLP methods through GDP and their applications in process synthesis, planning and scheduling.

Different applications of GDP can be cited. In Moreno and Montagna (2012) a GDP model was formulated for solving the simultaneous design and production planning of a batch plant under uncertainty. They propose a two stages stochastic multi-period model where the design variables are modeled as here-and-now decisions which are made before the demand realization, while the production planning variables are delayed in a wait-and-see mode to optimize in the face of uncertainty. Capacity plant expansions are also considered through unit duplications

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Nomenclature

Indices
- i Block widths
- j Block densities
- k Block lengths
- h Rows in the curing area
- t Planning days

Sets
- *Blocks* Set of possible foam blocks of width i, density j, and length k
- *DP* Set of days t for which a detailed planning is performed

Positive variables
- θ_{it} Fixed cost per day for producing width i on day t
- mb_{ijkt} Number of missing blocks of width i, density j and length k in storage with respect to the safety stock on day t
- sf_{ijkt} Final stock of blocks of width i, density j and length k on day t
- sm_{ijkt} Intermediate stock of blocks of width i, density j and length k on day t

Boolean variables
- μ_{ijkt} Indicates if unsatisfied demand occurs for block of width i, density j and length k on day t
- v_{jt} Indicates if density j is produced on day t
- w_{jht} Indicates if density j is produced and assigned to be cured in ρ h on day t
- x_t Indicates that production is performed on day t
- y_{it} Indicates if the width i is selected to be produced on day t
- z_{jkht} Indicates if any block of density j and length k is placed on row h on day t

Binary variables
- U_{ijkt} It is one indicates unsatisfied demand occurs for block of density j and length k is placed on row h on day t, and otherwise if it is zero
- V_{jt} If it is one indicates density j is produced on day t, and otherwise if it is zero
- W_{jht} If it is one indicates density j is produced and assigned to be cured in row h on day t, and otherwise if it is zero
- X_t If it is one indicates production is performed on day t, and otherwise if it is zero
- Y_{it} If it is one indicates the width i is selected to be produced on day t, and otherwise if it is zero
- Z_{jkht} If it is one indicates if any block of density j and length k is placed on row h on day t, and otherwise if it is zero

Integer variables
- n_{ijkht} Number of blocks produced of width i, density j, and length k, placed in row h, on day t
- dn_{ijkt} Number of blocks of width i and density j and length k produced on day t

Parameters
- α_j Cost of changing density j
- β_{ijk} Unsatisfied demand cost for blocks of width i, density j and length k
- γ_{ijk} Cost of missing blocks of width i, density j and length k in storage with respect to safety stock
- δ Fixed cost of changing width from one day to another
- λ_{ijk} Safety factor for block of width i, density j and length k
- ρ_{ijk} Storage cost for block of width i, density j and length k
- σ_{ijkt} Standard deviation of daily demand of block of width i, density j and length k on day t
- σ^L_{ijk} Demand standard deviation over the lead time for block of width i, density j and length k
- τ_{ijk} Material handling cost for block of width i, density j and length k
- bw_i Width of blocks i
- cw_i Width of the curing area for block width i
- cl_i Length of the curing area for block width i
- d_{ijkt} Demand of blocks of width i, density j, and length k on day t
- dfc Daily fixed cost when production is performed
- fl_{ijk} Length of block of width i, density j, and length k occupied on the floor in the curing stage
- fs Minimal space that must be left between blocks in order to allow the air flow in the curing area
- L_{ijk} Replenishment time for block of width i, density j, and length k
- l_{ijk} Length of block of width i, density j, and length k
- l_min_j Minimal length to produce for density j
- ml Minimal length to be produced
- $Rows_i$ Number of available rows in the curing area when width i is selected
- s_{ijk} Number of blocks of width i, density j, and length k in stock at beginning of the day
- $smax_{ijk}$ Maximal stock level (stock capacity) for block of width i, density j and length k
- $smin_{ijk}$ Safety stock for block of width i, density j and length k
- TD_{ijkt} Target demand of block of width i, density j and length k on day t

both in series and in parallel. Recently, Drouven and Grossmann (2016) propose a large scale MINLP model involving GDP for the shale gas design and planning problem, considering strategic decisions. Rubio-Castro et al. (2012) present an MINLP model for the optimal retrofit of water networks of different plants in the same industrial area. They consider the in-plant and inter-plant integration between water sources and sinks, taking into account different operating conditions and costs through disjunctive programming.

Bowling et al. (2011) solve the facility location and production planning of a biorefinery supply chain (SC) using a GDP formulation. The optimal SC selection considering centralized and distributed plants is obtained. They apply the proposed framework to two examples in the biorefinery industry. Multiple process decisions are considered through a disjunctive model by Murillo-Alvarado et al. (2013). The authors propose the optimal pathways for biorefineries where the selection of feedstocks, products and processing stages are simultaneously taken into account.

Supplier and contract selections are optimized by Rodriguez and Vecchietti (2009)applying a hierarchical disjunctive formulation. Different contractual policies are considered in order to diminish the effect of uncertainty in the provision process. In 2010, these authors integrate inventory and delivery using a GDP approach (Rodriguez and Vecchietti, 2010). They propose two formulations considering several decision levels where different planning horizons are addressed. The detailed formulation gives rise to a non-linear disjunctive model. Several transformations are evaluated in order to generate a linear GDP reformulation which is solved using LogMIP (Vecchietti et al., 1999; Vecchietti and Grossmann, 1999).

Some scheduling approaches are also faced through GDP. Castro et al. (2014a) present a continuous-time formulation for scheduling a batch pulp plant integrating heating tasks. In other work, Castro et al. (2014b) formulate a continuous time model for a long-term scheduling of a gas engine power plant with parallel units.

Focusing on the industry to be considered in this work, despite that the mattress industry is developed in many countries and it involves challenging issues given the different involved processes (production, cutting, stock, distribution, among others), the published articles in the area are scarce. Lin et al. (2013) formulate a scheduling model for the foaming and manufacturing stages that minimizes the total job completion time. They propose a branch and bound algorithm in order to improve the computational performance of the model. Lanoë et al. (2013) evaluate the potential environmental impact of two bedding products, polyurethane (PU) foam mattress and pocket spring mattress through Life Cycle Assessment (LCA). They analyze different alternatives in order to reduce the environmental impact of the product's life cycle using an alternative End-of-Life scenario. Mogaji (2014) presents a simple linear programming (LP) model for process planning and control of PU foam production. The approach is developed to enhance production efficiency and presents seven modules that work together to support the decision-making task.

Taking into account the GDP capabilities, an industrial problem where requirements about decisions integration must be satisfied is explored in this work applying this modeling technique. The mattress industry presents a scenario where different connected and nested problems must be jointly represented. In particular, the multi-period production planning and stock management of foam blocks for a mattress industry is addressed. Basically, three stages are considered and simultaneously optimized: foaming, curing and storage. The model provides a detailed production program in an overall multi-period formulation. Several relations among these stages and the involved decisions are assessed. The used framework, GDP, allows a clear outline of the simultaneous optimization formulation and the relations among the problem elements. Thus, GDP can be considered as a powerful tool to address integrated problems of production planning.

In order to solve efficiently the multiperiod formulation, a rolling horizon approach is proposed, that simultaneously combines the detailed planning for the current production day and the aggregate planning for the rest of the days in the horizon time. This methodology has been used in many applications (Al-Ameri et al., 2008; Kopanos and Pistikopoulos, 2014; Zamarripa et al., 2016) and it consists on dividing the overall planning problem into sub-problems with shorter time horizons which are iteratively solved.

The proposed disjunctive model considers the production planning of a PU foam production plant. The approach simultaneously formulates a detailed program for determining the blocks to be foamed, the allocation of the foamed blocks in the curing area and the storage management taking into account production requirements. As an additional result, raw material procurement for the blocks production can be efficiently carried out. Therefore, this problem poses the integration of several decision levels and the simultaneous assessment of different trade-offs through a rolling horizon framework. Application of GDP has been shown as a suitable approach for treating such problems.

The manuscript is organized as follows: the problem description is addressed in section 2. Section 3 includes the disjunctive model formulation. Results are presented in the fourth section of the manuscript while conclusions are finally described in section five.

2. Problem statement

This work addresses a multi-period production planning problem for a process formed by three main stages: a first step that produces pieces with certain characteristics such as quality and dimensional features, a second process that involves the location of these pieces in a limited area during certain time and a third stage where pieces are stored in dedicated spaces. There are many industries that present this type of process stages such as the production of board boxes where in the first stage the paper layers are glued and corrugated and the sheets are cut, while in the following stage the board sheets are placed in a special area to dry and diminish temperature before the printing stage, and, finally, stored. Another case is found in the sawmill industry where after cutting the logs into smaller parts, they must be assigned to a specific place to eliminate the excess of humidity before they are moved to the storage area.

The first stage corresponds to PU foam blocks production. A single foaming machine is used for producing the PU blocks of different densities and dimensions. Blocks of only one width can be produced at a time since this is a feature fixed at the beginning of the foaming process. In addition, a long setup machine is required between width changes; therefore, one width can be selected per day. The involved set up cost, mainly labor cost, when a change of width is required from one day to the next is penalized in the objective function.

Each production day, different densities can be selected, but the order in which the densities must be produced is pre-established: from greater to lower densities. In addition, given that this is a continuous process, a material loss is produced in the transition of densities. A minimal length must be produced in order to diminish the scraps and the material cost involved is included in the objective function. The foaming machine has a guillotine to cut the foam according to different lengths forming the blocks. Taking into account the future use of the blocks, several sizes can be cut from each density according to products requirements. A scheme of the foaming process is shown in Fig. 1.

The second stage of the process corresponds to the location of pieces in the curing sector. The blocks are placed on a specific area and stay there from one day to the next, in order to obtain the required properties like diminishing temperature and increasing stability. This is a crucial stage due to the large dimensions involved.

After a day, the blocks are taken from the curing area and moved to the storage. This is the third stage of the process considered in the proposed formulation. In the inventory area, the blocks can be piled up and ordered by its size in a space with limited capacity. The pieces are stored until they are required in the next manufacturing process phase.

Fig. 2 shows the process main characteristics and decisions involved in this work.

Then, the problem for optimal multi-period production planning, curing and stock of foam blocks is stated as follows:

Given:

- the set of widths, lengths and densities of blocks
- the sequence of densities
- the minimum length per density per day
- the minimum length to be foamed per day
- the curing area size (width and length)

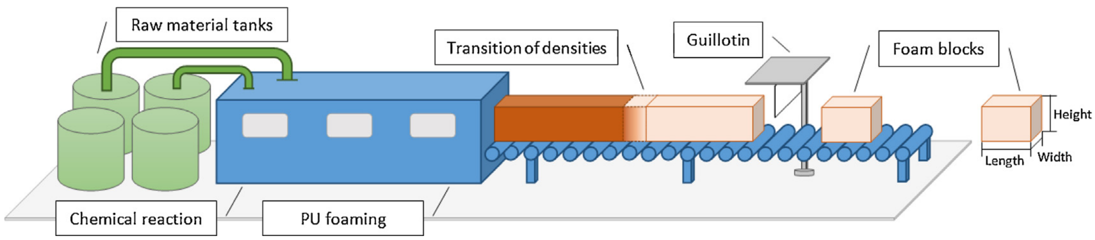

Fig. 1. Foaming process.

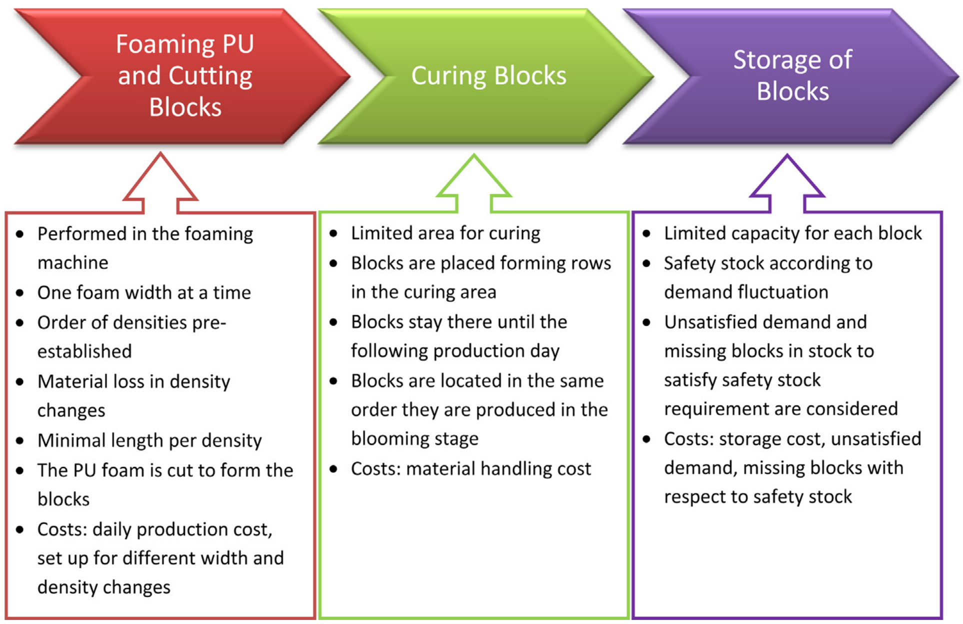

Fig. 2. Production process and main characteristics.

- the initial stock for each block in day 1
- the safety stock for each block
- the demand of blocks in each day during the planning horizon
- the number of days for detailed planning
- the number of days for production planning horizon
- the capacity in storage for each block type

Determine for each planning day:

- the width to be foamed
- the lengths and densities for the selected width
- the number of blocks of each type to be foamed
- the sequence in which the blocks are foamed
- the blocks arrangement in the curing area for a set of days (detailed planning days)
- the final stock of each block

with the objective of minimizing the total costs given by width set up, material loss due to density transition, storage, material handling in the curing area, unsatisfied demand and missing blocks in stock during a multi-period planning horizon.

3. Model formulation

3.1. Curing and stock parameters

As previously mentioned, after the blocks are produced, they are moved to the curing area using special carts. In this stage, the foam pieces are located on the floor in the same order they are produced, from left to right forming rows, as shown in Fig. 3. In order to favor the temperature decrease, a fixed space must be left between consecutive blocks. The number of rows is a given parameter for each width i, i.e. $Rows_i$. This parameter is calculated according to the curing area length and the row width as presented in Eq. (1). Note that the former is given by cl_i while the latter is given by the sum of the block width bw_i and the fixed space fs left between blocks. The function $floor\,(x)$ returns the greatest integer number less than or equal to x.

$$Rows_i = floor\left(\frac{cl_i}{bw_i + fs}\right) \quad \forall i \tag{1}$$

Even though a deterministic demand approach is considered, the plan takes into account a safety stock $smin_{ijk}$. assuming the company is willing to satisfy some extra demand level if a fluctuation occurs. This safety stock is defined according to the target demand and the standard deviation of the demand distribution as reported by You and Grossmann (2008). The target demand TD_{ijkt} for each block type of width i, density j and length k in planning day t can be calculated using Eq. (2). The set *Blocks* is introduced to define the feasible combinations of width, density and length considered in the process:

$$TD_{ijkt} = d_{ijkt} + smin_{ijk}, \ \forall t \ , \forall (i, j, k) \in Blocks \tag{2}$$

where d_{ijkt} is the mean daily demand for each block and $smin_{ijk}$ the safety stock for that block.

The safety stock is defined using parameter λ_{ijk} which is the safety factor, and represents the level of variability the company is willing to handle. Different safety factor can be assumed for each

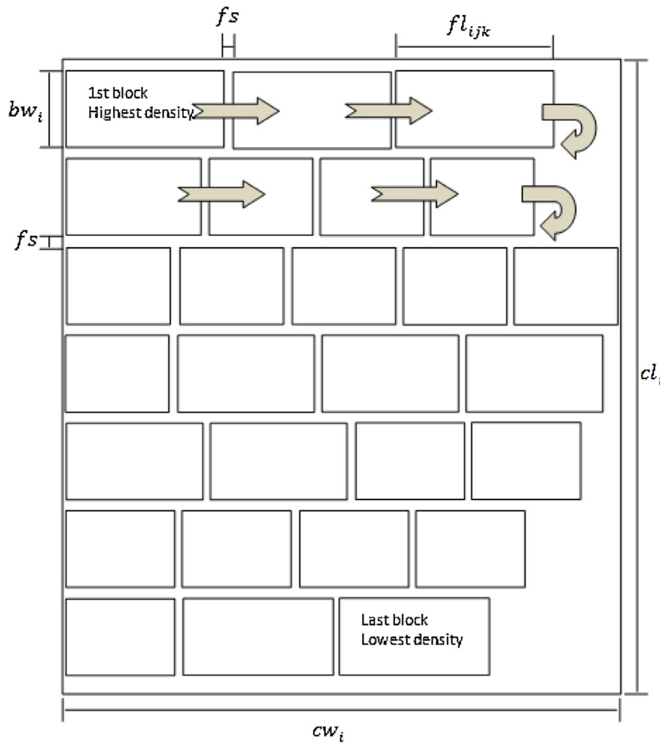

Fig. 3. Curing area and order of blocks.

product of width i, density j and length k. The larger the safety factor, the greater the safety stock. On the other hand, as safety stock increases, fewer back-orders or lost sales are expected. Safety factor parameter is multiplied by the demand standard deviation over the lead time, i.e. σ^L_{ijk}, to determine the safety stock as shown in Eq. (3).

$$smin_{ijk} = \lambda_{ijk} \cdot \sigma^L_{ijk} \quad \forall i, \forall j, \forall k \tag{3}$$

The lead time is called guaranteed service time, the required period for inventory review plus the replenishment time, i.e. L_{ijk}. Therefore, the standard deviation over the lead time is determined by Eq. (4).

$$\sigma^L_{ijk} = \sigma_{ijkt} \cdot \sqrt{L_{ijk}} \quad \forall i, \forall j, \forall k \tag{4}$$

where σ_{ijkt} represents the standard deviation of daily demand. If final stock at the end of each day is lower than the safety level, a cost is considered to take into account the missing blocks.

3.2. Equations and objective function

In this section, a generalized disjunctive programming approach is proposed for the multi-period planning problem applied to a foaming production plant.

The objective function in Eq. (5) is proposed in order to achieve a production plan that takes into account the involved trade-offs.

Min TC

where

$$TC = \sum_{j,t} \alpha_j \cdot v_{jt} + \sum_{\substack{i,j,k,t \\ \forall (i,j,k) \in Blocks}} \beta_{ijk} \cdot ud_{ijkt} + \sum_{\substack{i,j,k,t \\ \forall (i,j,k) \in Blocks}} \gamma_{ijk} \cdot mb_{ijkt} + \sum_t \delta \cdot x_t + \sum_{i,t} \theta_{it} + \sum_{\substack{i,j,k,t \\ \forall (i,j,k) \in Blocks}} \tau_{ijk} \cdot dn_{ijkt} + \sum_{\substack{i,j,k,t \\ \forall (i,j,k) \in Blocks}} \rho_{ijk} \cdot sf_{ijkt} \tag{5}$$

The first term in Eq. (5) represents the cost by density change. Let v_{jt} be a Boolean variable to account for the density j produced during period t. Each time the density is changed in the foaming machine, a loss of material is produced. The higher density the more expensive this loss is, which is given by parameter α_j.

In the second term, the unsatisfied demand of blocks is considered. It is determined by the non negative variable ud_{ijkt} which will be calculated in Eq. (15). It is assumed that some blocks are more important than others due to the customer requirements or the product prices related to them. Therefore, the parameter β_{ijk} is assigned to penalize each block that is not available for demand satisfaction.

Safety stock is established by the company in order to consider possible fluctuations in demand forecasts. In this sense, if the inventory level at the end of the working day is lower than the safety stock, the amount of missing blocks, mb_{ijkt}., is penalized in the third term of the objective function. Considering that some blocks are more important than others due to demand variations or customer requirements, a different cost parameter is assigned to each block, γ_{ijk}.

As it was mentioned, when the width is changed from one day to the next one, a setup cost must be taken into account which includes a fixed labor time. For this reason, the following term in the objective function considers the cost of setup when the width selected in a given day is different from that assigned to the following day, which is determined by Boolean variable x_t. In this case, a fixed cost, δ, is assumed.

Let introduce the non-negative variable θ_{it} meaning the fixed cost per day in case the width i is produced in the planning day t, which is determined in the first constraint of Eq. (13). Since only one width can be selected per day, at most one term of the sum of this variable over i can be positive.

In the last two terms material handling and storage costs are considered, respectively. The first one is given by the variable dn_{ijkt} that defines the number of produced blocks of width i, density j and length k in period t multiplied by the parameter unit material handling cost, τ_{ijk}. Storage cost is determined in the last term as the product of the non-negative variable sf_{ijkt} that represents the final stock of blocks of width i, density j and length k in period t by the unit storage cost, ρ_{ijk}.

Logic constraints

Logic relations between Boolean variables are modeled using propositional logic.

$$w_{j'h't} \rightarrow \bigvee_j w_{jht} \quad \forall j', \forall \left(h, h'\right) | h < h', \forall t \tag{6}$$

The curing area is organized in rows h in order to locate the foamed blocks that must be placed in this surface. Let w_{jht} be a Boolean variable that represents the location of density j to row h in period t. Therefore, Eq. (6) determines that a density must be assigned to all the previous rows than h' if $w_{j'h't}$ is true. The major purpose of this equation is to avoid empty spaces in the curing area when it is greater than the required surface to place the produced blocks. In addition, this avoids alternative solutions in the arrangement of blocks.

$$\neg w_{jht} \rightarrow \neg z_{jkht} \quad \forall j, \forall h, \forall k, \forall t \tag{7}$$

Let z_{jkht} be a Boolean variable that is true if any block of density j and length k are assigned to row h in period t. The logic relationship in Eq. (7) establishes that if w_{jht} is false, i.e. no block of density j is placed on row h in period t, then no block of length k of density j can be assigned to row h in the same period.

$$y_{it} \wedge y_{i't+1} \rightarrow x_t \quad \forall t, \forall \left(i, i'\right) | i \neq i' \tag{8}$$

Eq. (8) introduces a new Boolean variable x_t that is used in the objective function to penalize the change of widths in two consecutive days. If width i is selected in period t and width i' is chosen for t+1, then Boolean variable x_t must be true.

Inventory constraints

Once the blocks produced in the previous day are stored, the daily demand is taken from the inventory area. Let sf_{ijkt} be a continuous variable that defines the final stock of blocks of width i, density j and length k at the final of planning day t and let introduce the variable sm_{ijkt} for the intermediate stock of blocks of width i, density j and length k and the variable dn_{ijkt} for the number of blocks of width i, density j and length k produced in planning day t. These variables are non-negative. Three moments are considered in stock, the initial moment where the stock is equal to the final stock of the previous day sf_{ijkt-1} (considering the pieces produced t previous day, dn_{ijkt-1}), the intermediate moment where the stock sm_{ijkt} is the number of blocks in inventory after blocks are taken to the following downstream process (demand of blocks of the planning day) and the final stock sf_{ijkt} which is the inventory level after new blocks are produced and cured, dn_{ijkt}. Note that these new blocks will be available for the downstream process the next day because they have to be cured the day they are produced in order to obtain the required properties like diminishing temperature and increasing stability. In the intermediate moment, if the demand of blocks d_{ijkt} is greater than the available stock from the previous day sf_{ijkt-1}, the unsatisfied demand ud_{ijkt} is taken into account in the stock balance and penalized in the objective function. Fig. 4 shows these three moments. The intermediate stock of a block of width i, density j and length k is considered to estimate final stock and unsatisfied demand. Fig. 4a presents the case that the stock from the previous day is enough to satisfy demand while Fig. 4b shows the situation when unsatisfied demand occurs and the intermediate stock in this case is zero.

$$sf_{ijkt} = sm_{ijkt} + dn_{ijkt} \quad \forall (i, j, k) \in Blocks, \forall t \tag{9}$$

Eq. (9) determines the final stock for the day as the intermediate stock, sm_{ijkt}, plus the number of produced blocks in that period dn_{ijkt}. The intermediate stock is determined in disjunction (15).

$$sf_{ijkt} \leq smax_{ijk} \quad \forall (i, j, k) \in Blocks, \forall t \tag{10}$$

The final stock of blocks cannot exceed the stock capacity for each block, given by parameter $smax_{ijk}$ as shown in Eq. (10).

$$\sum_{h \leq Rows_i} n_{ijkht} = dn_{ijkt} \quad \forall (i, j, k) \in Blocks, \forall t \tag{11}$$

Let n_{ijkht} be the integer variable that defines the number of blocks of width i, density j and length k located in row h of the curing area in period t. Eq. (11) establishes that the sum of variable n_{ijkht} over the rows h must be equal to variabledn_{ijkt} that defines the number of produced blocks of width i, density j and length k in period t.

$$mb_{ijkt} \geq smin_{ijk} - sf_{ijkt} \quad \forall (i, j, k) \in Blocks, \forall t \tag{12}$$

Let mb_{ijkt} be a positive continuous variable that determines the number of blocks of width i, density j and length k in period t that are missing in stock with respect to the safety stock $smin_{ijk}$. This variable, calculated in Eq. (12), is introduced in the objective function in order to minimize the number of missing blocks in stock.

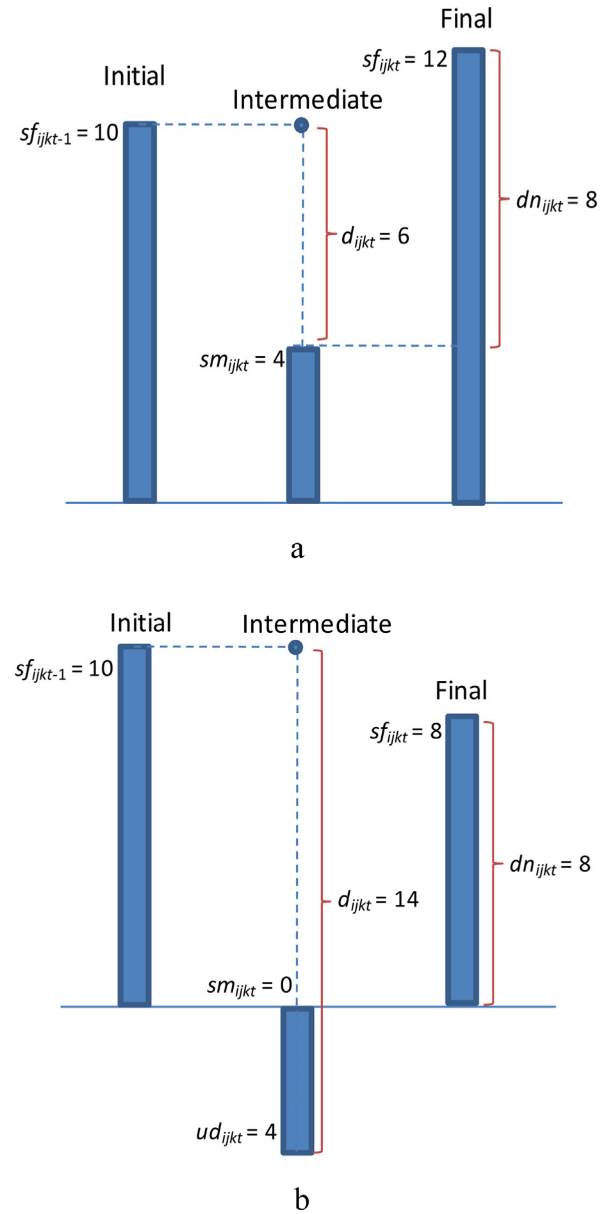

Fig. 4. a. Stock evolution when demand is satisfied. b. Stock evolution when unsatisfied demand occurs.

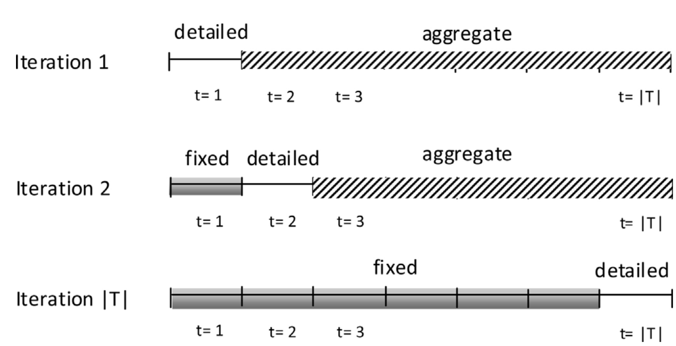

Fig. 5. Rolling horizon iterative scheme.

Disjunctive equations

$$\underset{i}{\vee}\begin{bmatrix} y_{it} \\ \theta_{it} = dfc \\ \sum_{\substack{i',j,k,h \\ \forall i' \neq i \\ \forall (i',j,k) \in Blocks \\ h \leq Rows_i}} n_{i'jkht} = 0 \\ \sum_{\substack{j,k \\ \forall (i,j,k) \in Blocks}} n_{ijkht} \cdot (fl_{ijk} + fs) \leq cw_i \quad \forall h \leq Rows_i \\ \sum_{\substack{j,k,h \\ \forall (i,j,k) \in Blocks \\ h \leq Rows_i}} n_{ijkht} \cdot l_{ijk} \geq ml \\ sf_{ijkt} - smin_{ijk} \geq 0 \quad \forall (i,j,k) \in Blocks \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} v_{jt} \\ \sum_{\substack{i,k,h \\ \forall (i,j,k) \in Blocks \\ h \leq Rows_i}} n_{ijkht} \cdot l_{ijk} \geq l\_min_j \\ \begin{bmatrix} \begin{bmatrix} w_{jht} \\ \sum_{\substack{i,k,h' \\ \forall (i,j',k) \in Blocks \\ h' < h}} n_{ij'kh't} = 0 \quad \forall j' > j \\ \sum_{\substack{i,k \\ \forall (i,j,k) \in Blocks \\ h \leq Rows_i}} n_{ijkht} \geq 1 \\ \begin{bmatrix} \begin{bmatrix} z_{jkht} \\ \sum_{\substack{i,k',h' \\ \forall (i,j,k') \in Blocks \\ k' < k \\ h' < h}} n_{ijk'h't} = 0 \\ \sum_{\substack{i \\ \forall (i,j,k) \in Blocks \\ h \leq Rows_i}} n_{ijkht} \geq 1 \end{bmatrix} \underline{\vee} \begin{bmatrix} \neg z_{jkht} \\ \sum_{\substack{i \\ \forall (i,j,k) \in Blocks \\ h \leq Rows_i}} n_{ijkht} = 0 \end{bmatrix} \forall k \end{bmatrix} \end{bmatrix} \underline{\vee} \begin{bmatrix} \neg w_{jht} \\ \sum_{\substack{i,k \\ \forall (i,j,k) \in Blocks \\ h \leq Rows_i}} n_{ijkht} = 0 \end{bmatrix} \forall h \end{bmatrix} \end{bmatrix} \underline{\vee} \begin{bmatrix} \neg v_{jt} \\ \sum_{\substack{i,k,h \\ \forall (i,j,k) \in Blocks \\ h \leq Rows_i}} n_{ijkht} = 0 \end{bmatrix}$$

$$\forall j, \forall t \quad (14) \tag{14}$$

Eq. (13) presents a disjunction where Boolean variable y_{it} is introduced to decide if the width i is produced in period t. If width i is selected in period t, a set of constraints must be considered.

If the Boolean variable is true, then a daily fixed cost must be considered. This is represented in the first equation of this disjunction, where the variable θ_{it}, introduced in the objective function, is equal to parameter dfc. In the following equation, no block of other width can be produced. In the next one, the curing area constraint is presented. As it was mentioned, the blocks are located in the curing sector forming rows. A fixed space fs must be left between blocks in order to guarantee the air stream. Each block occupies a length fl_{ijk} on the floor. Most blocks are cured lying down on the floor, thus fl_{ijk} is equal to the block length,l_{ijk}. However, some blocks of low density and short length are cured in standing position using a smaller surface. For those blocks, fl_{ijk} represents the block height. Then, the total width used of each row is given by the number of blocks of width i, density j and length k assigned to the row multiplied by the corresponding occupied length $\left(fl_{ijk} + fs\right)$. As shown in Fig. 3, this total length occupied in each row must be lower than or equal to the width of the curing area cw_i.

A minimal daily production length, ml, must be satisfied in the next equation. The total produced length is calculated multiplying the number of blocks produced n_{ijkht} by the block length l_{ijk}, and performing the sum over all densities j, lengths k and rows h.

The next condition states that for the selected width i, the safety stock must be satisfied. If width i is selected, the final stock of all blocks of that width that belong to *Blocks*, minus the safety stock $smin_{ijk}$, must be equal or greater than zero.

Eq. (14) presents a set of nested disjunctions. In the first decision level, the selection of densities j in period t is taken into account by Boolean variable v_{jt}. This is a yes-no disjunction. In the positive case, a minimal length must be satisfied for each selected density. The total produced length for density j must be at least l_min_j. In the negative case, no block of that density can be produced.

The following nested decision level is given by Boolean variable w_{jht} to define if a given density j is assigned to row h of the curing area in the day t. If this is not the case, then blocks of that density can be assigned to row h in period t as shown in the equation of the negative term of the disjunction, i.e. when $\neg w_{jht}$ is selected. Otherwise, if the density j is chosen for row h in the planning day t, the first constraint establishes that no block of greater density (j') can be assigned to any previous row (h'). The aim of this inequality is to guarantee that the blocks are produced and placed in the curing area in an increasing order of density. The second equation establishes that if this Boolean variable is true, at least one block must be produced of that density j and assigned to row h.

The last decision level involves the assignment of length k, density j and width i to row h in day t which is represented by Boolean variable z_{jkht} In the true case, the purpose of the first equation is to guarantee that longer blocks are placed before shorter ones for a given density j and width i, i.e. in decreasing order of length. This means that blocks of lengths k' cannot be assigned to rows h' given that $k' < k$ and $h' < h$. Similarly to the previous disjunction, if z_{jkht} is selected, at least one block of that density and length must be produced and placed on row h in planning day t. On the contrary, if this Boolean variable is false ($\neg z_{jkht}$), no block of that density and length can be assigned to row h in period t.

$$\underset{i}{\vee}\begin{bmatrix} y_{it} \\ \theta_{it} = dfc \\ \sum_{\substack{j,k \\ \forall (i,j,k) \in Blocks}} dn_{i'jkt} = 0 \quad \forall i' \neq i, \forall (i',j,k) \in Blocks \\ \sum_{\substack{j,k \\ \forall (i,j,k) \in Blocks}} dn_{ijkt} \cdot \left(fl_{ijk} + fs\right) \leq cw_i \cdot Rows_i \\ \sum_{\substack{j,k \\ \forall (i,j,k) \in Blocks}} dn_{ijkt} \cdot l_{ijk} \geq ml \quad \forall j, \forall k, \forall h \\ sf_{ijkt} - smin_{ijk} \geq 0 \quad \forall (i,j,k) \in Blocks \end{bmatrix} \quad \forall t \in NDP \tag{16}$$

$$\begin{bmatrix} \mu_{ijkt} \\ sm_{ijkt} = sf_{ijkt-1} - d_{ijkt} \\ ud_{ijkt} = 0 \end{bmatrix} \underline{\vee} \begin{bmatrix} \neg \mu_{ijkt} \\ ud_{ijkt} = d_{ijkt} - sf_{ijkt-1} \\ sm_{ijkt} = 0 \end{bmatrix}$$
$$\forall (i, j, k) \in Blocks, \forall t \tag{15}$$

Let μ_{ijkt} be a Boolean variable that is true if the demand of blocks from the downstream process is satisfied and false in the contrary case. Therefore, disjunction in Eq. (15) is introduced to define the intermediate stock sm_{ijkt} and the unsatisfied demand ud_{ijkt} of blocks of width i, density j and length k in period t. In the first term, the intermediate stock sm_{ijkt} is a non-negative continuous variable defined as the final stock of block (i,j,k) in the previous day sf_{ijkt-1} minus the demand of the block d_{ijkt} in the plan day. In this case, the unsatisfied demand of this block, ud_{ijkt}, is zero. In the second term, the final stock of the previous day sf_{ijkt-1}, is not enough to fulfill the demand of blocks d_{ijkt}. Thus, the unsatisfied demand of blocks ud_{ijkt} is a non-negative continuous variable given by the difference between the demand of block and the final stock of the previous day. In addition, the intermediate stock of blocks is zero.

Finally, the GDP model called P1 is given by Eqs. (5)–(15) and it is transformed into a Mixed Integer Linear Programming model using Big-M reformulation. This reformulation is presented in Appendix A as Supplementary Information.

3.3. Rolling horizon approach

Given the complexity of the strip packing constraints for the curing stage, the simultaneous planning problem is hard to solve in reasonable computing time. Therefore, a rolling horizon approach is proposed in order to improve the model performance that is compared with the original formulation. Besides, this methodology is suitable for multi-period formulations where aggregated information is available for long-term planning and real or more up-to-date information can be used for detailed (disaggregated) planning. The strategy is iteratively performed, updating the detailed and aggregate periods, while fixing the planning already solved (Lobos and Vera, 2016), as shown in Fig. 5.

In this approach, the planning horizon is divided into two periods, i.e. $T \equiv DP \cup NDP$. In the first one, i.e. $\forall t \in DP$, detailed decisions are considered, while in the second one, i.e. $\forall t \in NDP$, some of them are relaxed and the model is iteratively solved updating the sets DP and NDP until all detailed decisions are considered for the whole planning horizon. Regarding the curing stage, the detailed allocation of blocks in the area, the production sequence and the movements to locate them are taken into account $\forall t \in DP$. On the other hand, a space constraint over the curing area must be satisfied for the rest of the days in the planning horizon, $\forall t \in NDP$. Therefore, Eq. (16) replaces Eq. (13) for this second period while Eq. (13) remains for the detailed planning, i.e. $\forall t \in DP$.

Eq. (16) presents the disjunction where Boolean variable y_{it} selects the width i to be produced for the rest of the planning horizon where no detailed production plan is carried out, i.e. $\forall t \in NDP$. Once a width is selected, the first equation determines the fixed production cost while the following equation indicates that no block of other width can be produced. In the next one, the curing area constraint considers that the total used length must be lower than or equal to the available length, i.e. the number of rows multiplied by the curing area width ($cw_i \cdot Rows_i$.). The total used length is calculated summing up over densities j and lengths k, the product of the number of produced blocks dn_{ijkt} by the occupied length on the curing floor $\left(fl_{ijk} + fs\right)$.

As in Eq. (13), a minimal daily production length ml must be satisfied in the fourth equation. The total produced length is calculated multiplying the number of blocks produced dn_{ijkt} by the block length l_{ijk}, and performing the sum over all densities j and lengths k. This amount must be at least equal to the parameter ml.

If width i is selected to be produced, then the final stock of all blocks of that width that belong to $Blocks$, must fulfill the safety stock $smin_{ijk}$ as presented in the fifth equation.

$$\begin{bmatrix} \nu_{jt} \\ \sum_{\substack{i,k \\ \forall (i,j,k) \in Blocks}} dn_{ijkt} \cdot l_{ijk} \geq l\_min_j \end{bmatrix} \underline{\vee} \begin{bmatrix} \neg \nu_{jt} \\ \sum_{\substack{i,k \\ \forall (i,j,k) \in Blocks}} dn_{ijkh} = 0 \end{bmatrix} \forall j, \forall t \in NDP \tag{17}$$

Eq. (14) is applied for the detailed planning, i.e. $\forall t \in DP$, while Eq. (17) is considered for the rest of the planning horizon ($\forall t \in NDP$). The disjunction in Eq. (17) is used to choose the densities j in day t which is given by Boolean variable υ_{jt}. If a density j is produced in planning day t, a minimal length must be produced which is determined by the parameter l_min_j. In the negative case, no block of that density can be produced.

Finally, the rolling horizon approach, model P2, is solved applying Eqs. (5)–(17), considering the domain reformulation over T set for Eqs. (13) and (14) mentioned in this section. Please note that all disjunctions, from Eqs. (13)–(17) are exclusive since only one option is admitted.

4. Results

In this section, three cases are shown where the Big-M MILP reformulation of the GDP models is implemented. First, a base study case representing a typical operation of the plant is proposed. Example 1 presents a variation of this base case, incrementing the penalties for unsatisfied demands and stock-outs of all blocks. Finally, example 2 is presented where these costs and the safety stock are incremented only for a subset of blocks and they are analyzed in a separated way. For all cases, 3 different widths (i_1, i_2, i_3), 11 different densities (j_1, j_2, ..., j_{11}), and 10 blocks lengths (k_1, k_2, ..., k_{10}) are considered, and a total of 60 different types of blocks can be produced in the plant, i.e. the set*Blocks* has cardinality equal to 60. The safety stock parameter is computed according to Eq. (3), while minimum density length and minimum day length are fixed to 15 and 150 m, respectively. The curing area size is equal to 20.4 m width and 45 m long.

All the examples are implemented and solved in GAMS interface (Brooke et al., 2012), using CPLEX 12.5 solver, on an Intel Core i7, 3.4 GHz. The full space planning model given by model P1 comprises 198,444 constraints, 2528 continuous variables, and 31,780 discrete variables considering a planning horizon of 7 days (t_1, ..., t_7). When this model is solved for the base case, the best solution obtained after 3600 s of computation time is equal to \$ 21918 and the optimality gap is 22%, while applying the rolling horizon approach the objective function is \$ 17792 with a 0% optimality gap in 230 s. Therefore, the rolling horizon approach is performed in order to obtain a solution in a reasonable computing time.

Each iteration of the rolling horizon approach given by model P2 involves 32,688 constraints, 2528 continuous variables, and 4990 discrete variables. The resolution time for all the iterations is between 230 and 490 CPU sec for the different examples. In the following sections, the results applying the rolling horizon methodology are presented.

4.1. Base case

For this case, the considered cost parameters are: daily factory cost, dfc = 500; density change cost, α_j = 100; daily set-up cost, δ = 500; the material handling cost τ_{ijk} = 5; the storage cost ρ_{ijk} = 2; cost of unsatisfied demand of blocks, β_{ijk} = 60 for i_1 and i_2 and β_{ijk} = 120 for i_3; cost of stock-out of blocks γ_{ijk} = 30 for i_1 and i_2, while γ_{ijk} = 60 for i_3.

The optimal solution involves foaming blocks of width i_1 on t_3, i_2 on days t_1 and t_5, and i_3 on t_2. The unsatisfied demand of blocks over the planning horizon is equal to 41, while 70 blocks are missing regarding to the safety stock. Fig. 6 depicts the evolution of unsatisfied demand and stock-outs for each block width. It can be noted that on t_1 there is unsatisfied demand for blocks of width i_1 and i_2 while for i_3, the current initial stock is enough for covering its demand. Regarding the stock-out, there are only missing block for width i_1. This amount is increased on t_2 since i_1 is not foamed until t_3. On that day, due to the demand is taken at the beginning of the day, the unsatisfied demand for this width is also increased. After t_3, the missing blocks in stock and the unsatisfied demand is reduced for all the widths because the three widths are foamed at least once. Since width i_2 is again foamed on t_5, the unsatisfied demand and stock-outs for this width is reduced at the end of the planning horizon.

Regarding the curing stage, Fig. 7 shows the percentage of covered area in each day. It is worth to mention that since on days t_4, t_6 and t_7 no foam process is carried out, the curing area is empty.

Fig. 8 shows the level of inventory in the storage area. Stock capacity is enough during the whole planning horizon to satisfy the inventory needs. In addition, no relevant peaks and valleys are shown facilitating a balanced storage management. This is an appropriate result considering the consumption of other resources

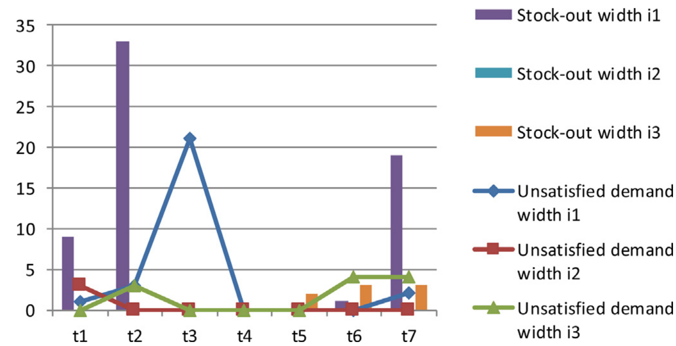

Fig. 6. Blocks management performance respect to satisfy safety stocks and production demands.

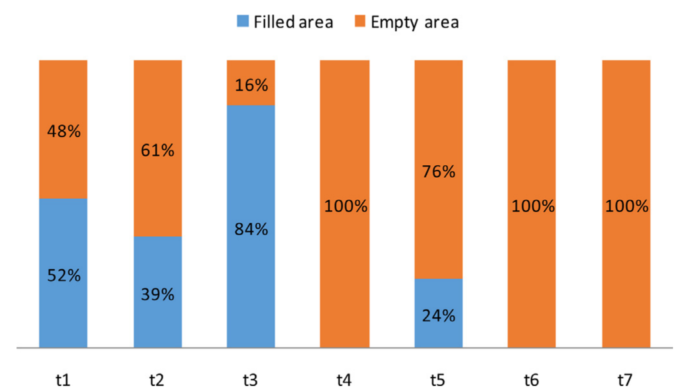

Fig. 7. Curing area covering along the planning horizon for the Base Case.

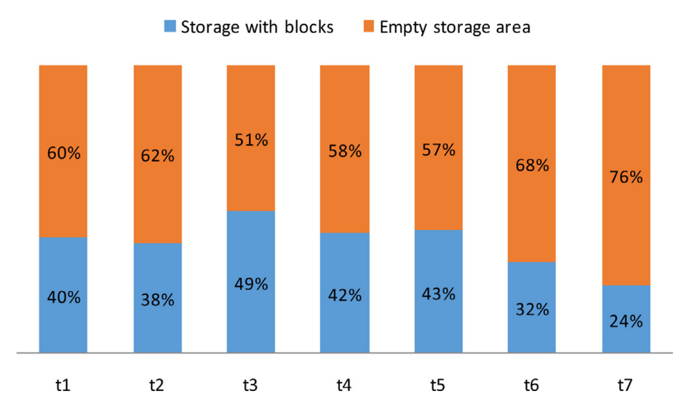

Fig. 8. Occupied and empty storage area along the planning horizon for the Base Case.

Table 1
Economical results for the Base Case.

Costs	$	% of the total cost
Production	2000	11.2
Width change (setup)	1000	5.6
Density change	2800	15.7
Material handling	2240	12.6
Storage	4292	24.1
Unsatisfied demand	3120	17.6
Stock-out	2340	13.2
Total	17792	

Table 2
Economical results for the Example 1.

Costs	$	% of the total cost
Production	2500	11.4
Width change (setup)	1500	6.8
Density change	3300	15
Material handling	2440	11.1
Storage	4356	19.9
Unsatisfied demand	6360	29
Stock-out	1500	6.8
Total	21956	

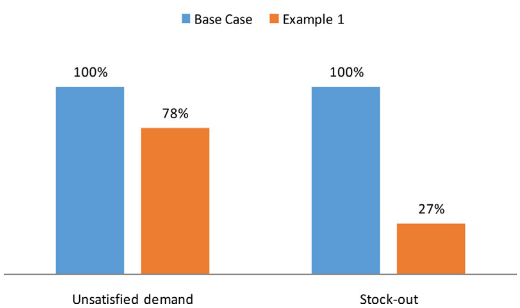

Fig. 9. Improvement of unsatisfied demand and stock-out results for example 1.

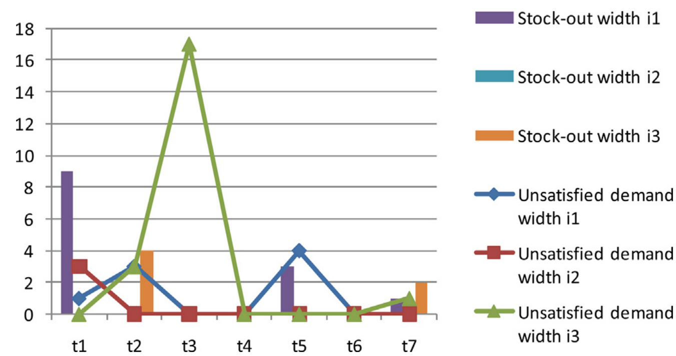

Fig. 10. Unsatisfied demand and stock-out for example 1.

such as labor. In fact, the maximum difference in the stock level, given between days t_3 and t_7, is 25% of the storage area.

In Table 1, the economical results of this example are shown. Through this table, some trade-offs can be analyzed. It can be seen that the storage cost is dominant, and therefore, the program tries to manage less amount of blocks in this sector, leading to higher unsatisfied demand and stock-outs. Although the unsatisfied demand represents the second dominant cost, in order to diminish this cost more blocks must be produced, and therefore, the costs related to production process (production, setups, and density change) also increase, which in sum represent 45.5%. Then, the system prefers to produce few days. But, due to the reduced number of production days, the missing stock and unsatisfied demand are increased.

4.2. Example 1: higher stock-out and unsatisfied demand cost

In this example, the impact of the cost of the missing blocks regarding to the safety stock and the cost of unsatisfied demand are increased. This can be associated with a company policy that pursues diminishing the unsatisfied demand as much as possible.

Suppose that parameters $\beta_{ijk} = 120$ for i_1 and i_2 and $\beta_{ijk} = 240$ for i_3 are adopted. Also, for the stock-out $\gamma_{ijk} = 60$ for i_1 and i_2, while $\gamma_{ijk} = 120$ for i_3 are considered, i.e. they are increased 100% from the Base Case example.

The optimal planning in this case involves five, instead of four, production days: in t_1 width i_2 is foamed, in t_2 the width i_1, in t_3 the width i_3, in t_5 the width i_2, and finally in t_6 the width i_1. The total number of unsatisfied demand blocks is equal to 32, while there are 19 blocks missed in stock. The unsatisfied demands and stock-outs decrease comparing with the Base Case as it is shown in Fig. 9. Even though the economical results shown in Table 2 are not comparable with the cost of the Base Case, they are displayed in order to analyze the trade-offs assessed in this case.

In this case, the dominant cost is the unsatisfied demand, and therefore, more production days are reached in order to satisfy the orders. The increment of the production days, and consequently the amount of produced blocks, increases processing, setup, density change, material handling and storage costs.

Fig. 10 shows the progress on unsatisfied and missed blocks in stock for the planning horizon. As it can be noticed, the stock-out of i_1 presented in the Base Case, is reduced in this case since i_1 is twice produced, as well as its unsatisfied demand on t_3. For the others widths, the unsatisfied demand and stock-outs are also reduced, except for i_3 on t_3 and t_2 respectively, because this width is produced in t_3 instead of t_2 as happen in the Base Case.

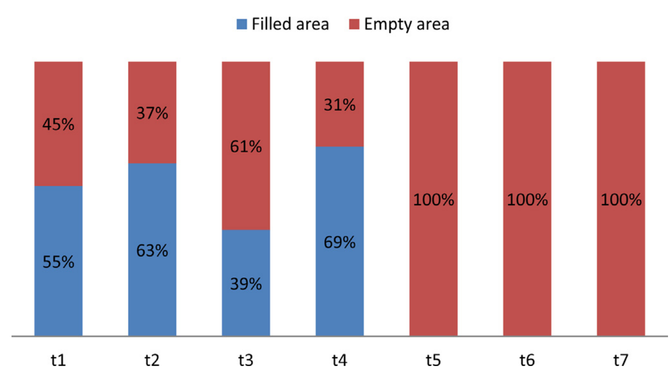

Fig. 11. Curing area covering along the planning horizon for Example 2.

Table 3
Safety stock for more demanded blocks.

	j_1				j_2		j_3			j_4	j_5	
	k_3	k_6	k_8	k_9	k_5	k_9	k_5	k_7	k_9	k_9	k_5	k_9
Base Case	10	2	1	1	0	3	5	0	8	1	1	10
Example 2	15	3	3	3	2	8	6	2	10	5	5	15

4.3. Example 2: increase safety stock of some blocks

In this example, the production manager detects that there is a set of critical blocks since they are used for manufacturing several types of mattresses of high demand. Therefore, the stock-out of these blocks must be reduced in order to respond on time to the demand. It is assumed that for width i_1, and densities j_1 to j_5 in all the defined lengths for these blocks (relations in set *Blocks*), their safety stock is increased according to the values displayed in Table 3

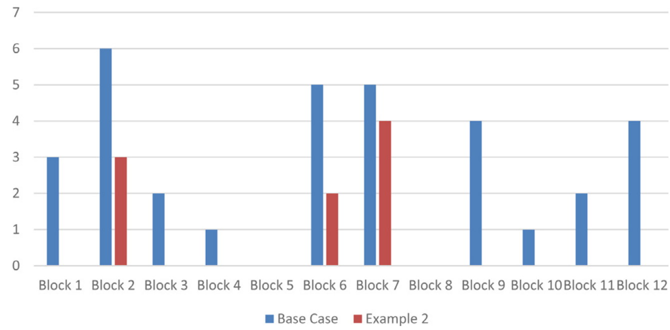

Fig. 12. Total stock-outs for more demanded blocks in the planning horizon.

respect to the Base Case. Also, the stock-out cost for these blocks is augmented from 30$/block to 120$/block. The rest of the model parameters are equal to those adopted for the Base Case.

The optimal solution in this case produces blocks on days t_1, t_2, t_3 and t_4. The foamed width in each day is i_1, i_2, i_3, and i_1 respectively. The curing area is covered along the planning horizon according to the percentages shown in Fig. 11. Since during t_5, t_6 and t_7 non production is carried out, the curing area is totally empty.

The unsatisfied demand of blocks over the planning horizon is equal to 58, while 106 blocks are missing regarding to the safety stock. Therefore, they are increased 41% and 51% respectively comparing with the Base Case. However, for the more demanded blocks, the stock-out is decreased from 33 to 9 blocks. Fig. 12 compares the total stock-outs between Base Case and Example 2 optimal solutions for each one of these blocks in the planning horizon, while Fig. 13 displays the number of missed blocks for each type of block day by day, for both Base Case (a) and Example 2 (b). In these two last figures, the more demanded blocks were numerated in order as they appear in Table 3 (block 1 represents the block of width i_1, density j_1 and length k_3, block 2 is of width i_1, density j_1 and length k_6, and so on).

It is worth to highlight that in this case the production planning is changed in order to avoid stock-out of the more demanded blocks. In this way, width i_1 is twice foamed along the planning horizon and, on t_1, i_1 is produced instead of i_2. Therefore, higher unsatisfied demand is carried out for blocks of width i_2 and i_3 because their production is carried out later in the planning horizon. But for more required blocks, the stock-outs is considerably improved (only 9 blocks of this type are missed in stock along the horizon time) such that the company can assure the production of the more required mattresses.

This example shows how the model can be used as a tool for solving different company scenarios, and respond with an appropriate solution.

5. Conclusions

In this article, the capability and easiness of GDP to formulate integrated models have been shown. A real problem where planning, storage and allocation decisions must be made was addressed. GDP was used as a tool capable of representing different relations and alternatives in order to determine an optimal operations management. The proposed GDP formulation allows a natural representation of the logic structure of the discrete decisions and a clear outline of the simultaneous optimization problem. The presented model, with a straightforward representation, can consider all the available options.

The MILP reformulation of the proposed GDP was applied to a multi-period production planning and stock management of foam blocks for the mattress industry. This is an appropriate problem taking into account the pursued objectives of this work. The production process comprises the foaming and curing stages as well as the storage of PU blocks, which are intermediate products in the mattress manufacturing. Given the large size of the multi-period formulation, a typical operation plan is very hard to solve considering the full space. Therefore, in order to improve the computational performance, a rolling horizon approach is addressed. Thus, the model is iteratively solved for each production day, providing a detailed production program for the current day and an aggregated program for the rest of the days in the time horizon. After all iterations are performed, a detailed plan is obtained for the complete planning horizon improving substantially the computation time in comparison to the full space formulation.

The optimal plan takes into account several costs such as the material loss cost when density is changed in the foaming machine, the unsatisfied demand of blocks cost, the stock-out cost when the inventory level at the end of the working day is lower than the safety stock, the set-up cost when a width is changed from one day to the following, handling material cost, storage cost, and the fixed cost per production day assuming that fixed resources (mainly labor) can be assigned to other productive activities in the company if no production is carried out.

The several trade-offs related to the costs are tested in the results section showing the impact of different scenarios on the final solution. Events such as increase in demand level and safety stock are tested. Also the sensitivity of the solution is analyzed when the involved costs fluctuate. From the examples, it can be concluded that the proposed formulation can effectively solve all the usual planning problems in the considered industry.

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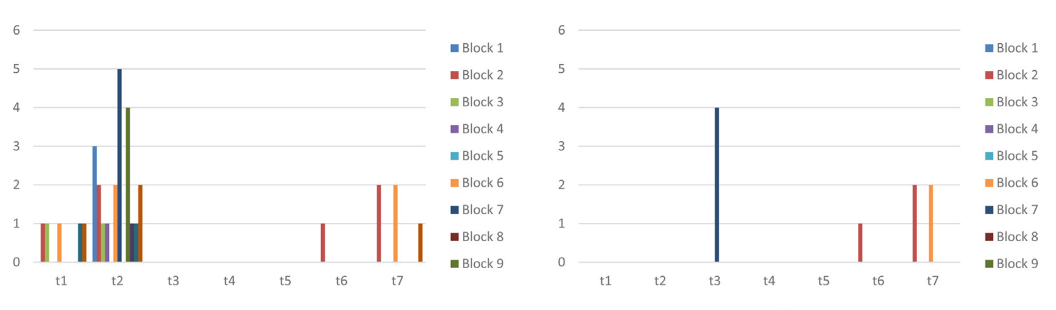

Fig. 13. a Stock-outs in each day for each type of the more demanded blocks for Base Case. b Stock-outs in each day for each type of the more demanded blocks for Example 2.

ities through their projects PIP2013-0682 and PICT-2012-2484, respectively.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.compchemeng.2017.03.006.

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