Elevator

In a five-story office building, there are n elevators transporting people to higher or lower levels. Unfortunately, the elevators have not been programmed in a very intelligent way, as each one of them works according to the following policy:

- An elevator is never idle. That is, it is either moving, or halted at a level to allow people to enter or exit.
- An elevator's direction (up/down) can only be changed at the bottom and top level. This means that the elevator repeatedly operates in the following cycle:

$$0 \to 1 \to 2 \to 3 \to 4 \to 3 \to 2 \to 1 \to 0 \to ...,$$

  regardless of the event that no one is waiting on the top or bottom level.
- If an elevator reaches a certain level (say i), it halts and opens the doors if either one of the persons in the cabin have destination level i, or there is at least one person queueing at level i to go into the current direction of the elevator.

Each elevator has a limited capacity of 10 persons. About the people using the elevator, we make the following assumptions:

- People arrive at level i according to a Poisson process with rates λ_i, for $i = 0, 1, 2, 3, 4$. The values for λ_i (arrivals per minute) are given below:

$$\lambda_0 = 13.1, \lambda_1 = 3.4, \lambda_2 = 2.1, \lambda_3 = 9.2, \lambda_4 = 8.8.$$

- Each person arriving at level i takes the elevator to level j with probability p_{ij}. These probabilities, in matrix form, are given below:

$$p_{ij} = \begin{pmatrix} 0 & 0.1 & 0.3 & 0.4 & 0.2 \\ 0.7 & 0 & 0.1 & 0.1 & 0.1 \\ 0.6 & 0.2 & 0 & 0.1 & 0.1 \\ 0.6 & 0.2 & 0.1 & 0 & 0.1 \\ 0.5 & 0.2 & 0.2 & 0.1 & 0 \end{pmatrix}$$

- Persons who want to go to a higher (lower) level only enter the elevator when it is moving upward (downward).
- If the elevator stops at level i, and the number of people queueing (to go into the current direction of the elevator) plus the number of people currently inside the elevator exceeds the capacity, the space is filled up to full capacity (according to the first come first served (FCFS) policy), while the remainder of the queue waits for the next elevator to arrive at level i.

Furthermore, we assume that the time it takes the elevator to move one level up or down is deterministic and equal to 6 seconds. The total time it takes to open the doors is exponentially distributed with mean equal to 3 seconds. Additionally, it takes one second per person to enter or leave the cabin. The time to close the doors is *again* exponentially distributed with mean 3. So if an elevator stops at a given floor and three people leave the cabin and 2 people enter the cabin, the total time the elevator spends at that floor is equal to $X_1 + 3 + 2 + X_2$ where X_1 and X_2 are exponentially distributed with mean 3. By convention, we say that the waiting time of a person ends at the moment they enter the elevator. People leaving the cabin always have priority over those entering the cabin, meaning that when an elevator stops at a certain floor, first all people wanting to leave at that level will exit the cabin. Only after the last person exits the cabin, new people are allowed to enter the cabin. We also assume that elevators move independently of each other. If there are two or more elevators present at the same floor, all of which are allowing people to enter, the queue of waiting people will still be served according to the FCFS policy. This means that you may get a cyclic pattern of customers successively entering elevators (for example $1, 2, 3, 1, 2, 3, 1, \ldots$) until the next elevator is full and leaves.

Write a simulation that resembles this scenario to answer the following questions, while you vary the number of elevators.

1. What are the 95% confidence intervals for the mean waiting time for a person waiting for the elevator at level $i = 0, 1, 2, 3$ and 4?
2. What is the expected number of people present in the elevator?
3. Starting at level i, what is the probability not being able to enter the cabin due to the capacity limitation?
4. How many elevators would you recommend? The criterion you use should be based on the fraction of people that have to wait longer than five minutes.

For this assignment you should also consider the following two model variations.

5. Suppose that the people at levels 1 and 2 collectively decide to be more active and take the stairs down. That is, $p_{10} = p_{20} = p_{21} = 0$. How does this affect the expected waiting time at all levels?
6. Now, instead of assuming that people from levels 1 and 2 take the stairs, we assume that impatient people take the stairs. You can construct your own (realistic) impatience policy, assuming that the critical waiting time before people stop waiting for the elevator increases as the number of stairs that they have to take increases. How does this affect the expected waiting time at all levels?

Write a detailed report, addressing the questions posed above and the components of your discrete-event simulation. Make sure that your report is well-written, well-structured, and that you interpret all of your results. The page limit is 8 pages (excluding appendices).
Upload your report to Canvas before the deadline and include your source code in the appendix *and* in a ZIP file.