Queueing for lunch

Photo: Cursor TU/e.

In this assignment, you will consider a queueing system you are probably familiar with, namely the MetaForum's food outlet (canteen) during lunch time. A (simplified) layout of the food outlet is depicted in Figure 1. Arriving customers grab their food and then join one of the three queues and wait patiently until they can pay for their purchases. In this assignment, we are interested in the waiting times of the customers. Although this might not always be true in practice, we assume that there are always *three* available servers (one per queue) and that you can pay with cash or by bank card. Analyzing the

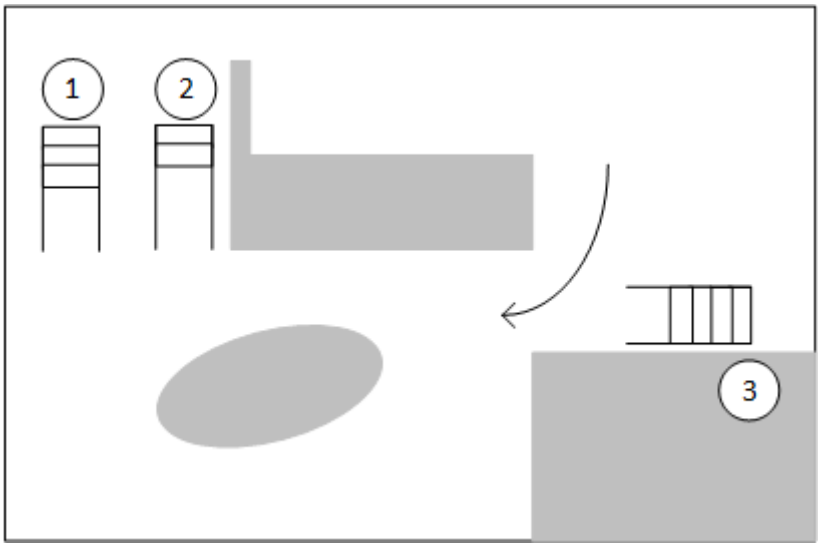

Figure 1: Map of the MetaForum canteen.

complex reality of the situation is of course nearly impossible. Therefore, we abstract from the real setting and study the system under the following modeling assumptions:

- Hungry customers arrive to the canteen in groups. The group size N is geometri-

cally distributed with mean 3. Hence,

$$\mathbb{P}(N = k) = \frac{1}{3} \cdot \left(\frac{2}{3}\right)^{k-1}, \qquad k = 1, 2, ...$$

- After arriving to the entrance, every customer independently grabs their food and beverages of choice, which takes an exponentially distributed amount of time with mean of 80 seconds, before joining the shortest queue. Ties are broken at random. Once customers have chosen a queue, they do not switch, even if one of the other queues becomes shorter or even empty.
- We assume that all cashiers work equally hard, but the service time of a customer depends on his method of payment: service times are exponentially distributed with means 20 and 12 seconds for payment by cash and card, respectively. It has been observed that 40 % of customers pay cash.
- We are only interested in the system dynamics within the time frame 12.00h-13.00h. During this period, the groups of customers arrive to the canteen according to a Poisson process with rate λ (per minute). You may assume that the system is empty at 12.00h.
  Hint: this means that you should only simulate the system for this hour (not longer).
- Separate queues form in front of each cash register in use. There is always a cashier available, at each of the three registers.

Given this model description, build a discrete-event simulation *with event scheduling* (as taught in the lectures) to answer the following questions for multiple values of λ (see below):

1. What is the expected sojourn time S of an arbitrary customer, that is the time between them entering (possibly with some collegues) and leaving the canteen? What is the expected time they spend waiting in one of the queues? What is the expected number of customers inside the canteen? Also retrieve the standard deviation (sd) of these performance metrics.
2. What is the sojourn time of an arriving *group* in total, denoted by S_G, that is, the time between the group arriving at the canteen, and the last person completing service at a cashier station?
3. Give the distribution of the number of customers present in the canteen: provide the average number of customers, the standard deviation, and plot a histogram.
4. Give approximate 95% confidence intervals for the mean sojourn time of an arbitrary customer and of an arbitrary group.

Be sure to include a table like this in your report, filled with simulation results, obviously:

λ	$\mathbb{E}[W_1]$	sd$[W_1]$	$\mathbb{E}[W_2]$	sd$[W_2]$	$\mathbb{E}[W_3]$	sd$[W_3]$	$\mathbb{E}[S]$	sd$[S]$	$\mathbb{E}[S_G]$	sd$[S_G]$
1										
2										
3										
4										

Model extensions

Select at least three stars **[***]** of the following model extensions to make the model more realistic and answer the corresponding question(s):

1. **[*]** The cashier at station 1 has only just started as of today. Therefore, she works at a different speed than her colleagues. Each service time is 25% longer in her case. What is the expected waiting time of customers joining her queue, as opposed to the waiting time in the other two queues?
2. **[*]** To reduce the congestion in the main canteen during lunch hours, a 'mobile store' is placed in de Markthal of the MetaForum. It is expected that 15% of the groups will visit this new food stand instead of the main canteen. How does this affect the expected sojourn and waiting time of customers still visiting the main canteen?
3. **[*]** The canteen switches to bank card payment only, not allowing for cash payments anymore. Assume that this does not affect the arrival rate. How does this affect the expected sojourn and waiting time of customers?
4. **[**]** Customers may switch queues, because another queue is shorter. How does this impact the waiting times of the customers and the occupation rate of the servers?
5. **[?]** You are encouraged to come up with any other extension to make the model more realistic or interesting, but please ask one of the lecturers (Peter or Marko) for confirmation and to indicate how many stars you can earn.

Please make sure that you implement each of the extensions *separately*, so do *not* create one model with multiple extensions simultaneously.

Grading and knock-out criteria

Be sure to interpret all of your results! This assignment will be 25% of your final grade. The assignment is made in groups of three students. Each group should hand in a well-written report and the source code of their simulation programs in Canvas. The **page limit** of the report is **6 pages** excluding references and appendix. Ensure that **each** class is saved in its own **class file** and is named appropriately so that we immediately recognize the structure of your simulation program. Make sure to check Canvas for more information like deadlines or the rubric.

Please upload your report as a PDF file and upload the source code *in a separate zip file*.