Sets

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Section 1.1 exercises - Answers

1. Which of the following are sets?
   (a) is not a set.
   (b) is not a set.
   (c) is not a set.
   (d) is a set.
   (e) is a set.

2. Let $S = \{-2, -1, 0, 1, 2, 3\}$. Describe each of the following sets as $\{x \in S : p(x)\}$, where $p(x)$ is some condition on x.
   (a) $A = \{x \in S : x > 0\}$
   (b) $B = \{x \in S : x \geq 0\}$
   (c) $C = \{x \in S : x < 0\}$
   (d) $D = \{x \in S : |x| = 2 \vee x = max(S)\}$

3. Determine the cardinality of each of the following sets:
   (a) $|A| = 5$
   (b) $|B| = 11$
   (c) $|C| = 51$
   (d) $|D| = 2$
   (e) $|E| = 1$
   (f) $|F| = 2$

4. Write each of the following sets by listing its elements within braces.
   (a) $A = \{-3, -2, -1, 0, 1, 2, 3, 4\}$

(b) $B = \{-2, -1, 0, 1, 2\}$
(c) $C = \{1, 2, 3, 4\}$
(d) $D = \{0, 1\}$
(e) $E = \{\}$

5. Write each of the following sets in the form $\{x \in \mathbb{Z} : p(x)\}$, where $p(x)$ is a property concerning x.
   (a) $A = \{x \in \mathbb{Z} : x < 0\}$
   (b) $B = \{x \in \mathbb{Z} : |x| \leq 3\}$
   (c) $C = \{x \in \mathbb{Z} : |x| \leq 2 \wedge x \neq 0\}$

6. The set $E = \{2x : x \in \mathbb{Z}\}$ can be described by listing its elements, namely $E = \{..., -4, -2, 0, 2, 4, ...\}$. List the elements of the following sets in a similar manner.
   (a) $A = \{..., -3, -1, 1, 3, 5, ...\}$
   (b) $B = \{..., -8, -4, 0, 4, 8, ...\}$
   (c) $C = \{..., -5, -2, 1, 4, 7, ...\}$

7. The set $E = \{..., -4, -2, 0, 2, 4, ...\}$ of even integers can be described by means of a defining condition by $E = \{y = 2x : x \in \mathbb{Z}\} = \{2x : x \in \mathbb{Z}\}$. Describe the following sets in a similar manner.
   (a) $A = \{y = 3x + 2 : x \in \mathbb{Z}\} = \{3x + 2 : x \in \mathbb{Z}\}$
   (b) $B = \{y = 5x : x \in \mathbb{Z}\} = \{5x : x \in \mathbb{Z}\}$
   (c) $C = \{y = x^3 : x \in \mathbb{N}\} = \{x^3 : x \in \mathbb{N}\}$

8. (a) Describe the set A by listing its elements.
   $A = \{-3, -2, 2, 3\}$
   (b) Give an example of three elements that belong to B but do not belong to A.
   $\frac{5}{2}$, $\frac{7}{2}$, 4
   (c) Describe the set C by listing its elements.
   $C = \{\sqrt{2}, 2\}$
   (d) Describe the set D in another manner.

$D = \{x \in \mathbb{Q} : x = 2\}$

(e) Determine the cardinality of each of the sets A,C and D

$|A| = 4$

$|C| = 2$

$|D| = 1$

9. Determine C.

$C = \{10, 13\}$