Combined beam checks

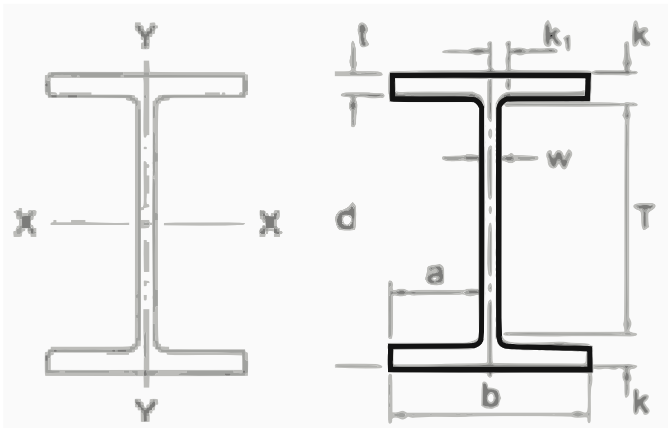
Process a comprehensive design check for a W-section beam.

Calculation ID: Blank calculation id

Verification status: Work in progress

Assumptions:

- Calc is applicable for Class 1, 2 and 3 W sections only
- Tension resistance is checked based on yielding only
- Compression force is taken as a positive value



W-Beam Axis

Inputs

Compression force in member	$C_f = 50\,kN$
Tension force in member	$T_f = 50\,kN$
Bending moment on member about x-axis	$M_{fx} = 50\,kN$
Bending moment on member about y-axis	$M_{fy} = 50\,kN$
Shear force on member in x-axis direction	$V_{fx} = 50\,kN$
Shear force on member in y-axis direction	$V_{fy} = 50\,kN$
Overall depth of section	$d = 257\,mm$
Overall flange width	$b = 204\,mm$
Web thickness	$w = 8.9\,mm$
Flange thickness	$t = 15.7\,mm$
Full unreduced cross-sectional area of member	$A = 8.58{\times}10^3\,mm^2$
Radius of gyration about centroidal principal x axis	$r_x = 110\,mm$
Radius of gyration about centroidal principal y axis	$r_y = 51\,mm$
Elastic section modulus about x-axis	$S_x = 8.06{\times}10^5\,mm^3$
Elastic section modulus about y-axis	$S_y = 2.18{\times}10^5\,mm^3$
Moment of inertia major axis	$I_x = 1.04{\times}10^8\,mm^4$
Moment of inertia minor axis	$I_y = 2.22{\times}10^7\,mm^4$
Plastic section modulus about x-axis	$Z_x = 9.01{\times}10^5\,mm^3$
Plastic section modulus about x-axis	$Z_y = 3.32{\times}10^5\,mm^3$
Saint-Venant torsion constant of cross-section	$J = 6.25{\times}10^5\,mm^4$
Torsional warping constant of cross-section	$C_w = 3.24{\times}10^{11}\,mm^6$

Yield strength	$F_y = 350\,MPa$
Tensile strength	$F_u = 410\,MPa$
Modulus of elasticity of steel	$E = 2{\times}10^5\,MPa$
Shear modulus of steel	$G = 7.7{\times}10^4\,MPa$
Effective length factor for buckling about x-axis	$K_x = 1$
Unbraced length or span of member about principal x-axis direction	$L_x = 2{\times}10^3\,mm$
Effective length factor for buckling about y-axis	$K_y = 1$
Unbraced length or span of member about principal y-axis direction	$L_y = 2{\times}10^3\,mm$
Unbraced length or span of member for torsional buckling	$L_z = 2{\times}10^3\,mm$
Distance from centroid of shear center in principal x-axis direction	$x_0 = 0\,mm$
Distance from centroid of shear center in principal y-axis direction	$y_0 = 0\,mm$
Lateral support condition for a member	$Condition = \text{Laterally\_Unsupported}$
Moment resistance coefficient accounting for moment gradient	$\omega_2 = 1$
Coefficient to determine equivalent uniform bending effect in beam-columns for the principal x-axis	$\omega_{1x} = 1$
Coefficient to determine equivalent uniform bending effect in beam-columns for the principal y-axis	$\omega_{1y} = 1$

Calculation

Classification of W section for axial compression

Width of stiffened or unstiffened compression elements

$$b_{el} = \frac{b}{2} = \frac{204\,mm}{2} = 102\,mm$$

S16-14 11.2 Table 2 — Width to thickness ratio

$$\frac{b_{el}}{t} = \frac{b_{el}}{t} = \frac{102\,mm}{15.7\,mm} = 6.5$$

$\frac{b_{el}}{t} = 6.5$

S16-14 11.2 Table 1 — Width to thickness limit for elements in axial compression

$$Limit = \frac{200}{\sqrt{F_y}} = \frac{200}{\sqrt{350\,MPa}} = 10.7$$

Check for maximum width to thickness ratio for flange

$$\frac{b_{el}}{t} \leqslant \frac{200}{\sqrt{F_y}}$$
$$6.5 \leqslant \frac{200}{\sqrt{350\,MPa}}$$
$$6.5 \leqslant 10.7$$

Flange does not exceed maximum permissible width to thickness ratio.

$Check = \text{Flange is not Class 4}$

Clear depth of web between flanges

$$h = d - 2 \times t$$
$$= 257\,mm - 2 \times 15.7\,mm$$
$$= 226\,mm$$

S16-14 11.3.2 b)

Width to thickness ratio

$$\frac{h}{w} = \frac{h}{w}$$
$$= \frac{226\,mm}{8.9\,mm}$$
$$= 25.3$$

$$\frac{h}{w} = 25.3$$

S16-14 11.2 Table 1

Maximum width to thickness ratio for webs supported on both edges

$$Limit = \frac{670}{\sqrt{F_y}}$$
$$= \frac{670}{\sqrt{350\,MPa}}$$
$$= 35.8$$

Check for maximum width to thickness ratio for web

$$\frac{h}{w} \leqslant \frac{670}{\sqrt{F_y}}$$
$$25.3 \leqslant \frac{670}{\sqrt{350\,MPa}}$$
$$25.3 \leqslant 35.8$$

Web does not exceed maximum permissible width to thickness ratio!

$$Check = \text{Web is not Class 4}$$

Classification of W section for flexural compression

Width of stiffened or unstiffened compression elements

$$b_{el} = \frac{b}{2}$$
$$= \frac{204\,mm}{2}$$
$$= 102\,mm$$

S16-14 11.2 Table 2

Width to thickness ratio

$$\frac{b_{el}}{t} = \frac{b_{el}}{t}$$
$$= \frac{102\,mm}{15.7\,mm}$$
$$= 6.5$$

$$\frac{b_{el}}{t} = 6.5$$

S16-14 11.2 Table 2

Section classification limit for Class 1 sections

$$Class\ 1\ Limit = \frac{145}{\sqrt{F_y}}$$
$$= \frac{145}{\sqrt{350\,MPa}}$$
$$= 7.75$$

S16-14 11.2 Table 2

Section classification limit for Class 2 sections

$$Class\ 2\ Limit = \frac{170}{\sqrt{F_y}}$$
$$= \frac{170}{\sqrt{350\,MPa}}$$
$$= 9.09$$

S16-14 11.2 Table 2

Section classification limit for Class 3 sections

$$Class\ 3\ Limit = \frac{200}{\sqrt{F_y}}$$
$$= \frac{200}{\sqrt{350\,MPa}}$$
$$= 10.7$$

$$Class = \text{Class\_1}$$

Clear depth of web between flanges

$$h = d - 2 \times t$$

$$= 257\,mm - 2 \times 15.7\,mm$$
$$= 226\,mm$$

S16-14 11.3.2 b) | Width to thickness ratio

$$\frac{h}{w} = \frac{h}{w}$$
$$= \frac{226\,mm}{8.9\,mm}$$
$$= 25.3$$

$$\frac{h}{w} = 25.3$$

Axial compressive load at yield stress

$$C_y = F_y \times A \times 10^{-3}$$
$$= 350\,MPa \times 8.58{\times}10^{3}\,mm^2 \times 10^{-3}$$
$$= 3{\times}10^{3}\,kN$$

S16-14 11.2 Table 2 | Section classification limit for Class 1 sections

$$Class\ 1\ Limit = \frac{1.1{\times}10^{3}}{\sqrt{F_y}} \times \left(1 - 0.39 \times \frac{C_f}{\phi \times C_y}\right)$$
$$= \frac{1.1{\times}10^{3}}{\sqrt{350\,MPa}} \times \left(1 - 0.39 \times \frac{50\,kN}{0.9\ \times 3{\times}10^{3}\,kN}\right)$$
$$= 58.4$$

S16-14 11.2 Table 2 | Section classification limit for Class 2 sections

$$Class\ 2\ Limit = \frac{1.7{\times}10^{3}}{\sqrt{F_y}} \times \left(1 - 0.61 \times \frac{C_f}{\phi \times C_y}\right)$$
$$= \frac{1.7{\times}10^{3}}{\sqrt{350\,MPa}} \times \left(1 - 0.61 \times \frac{50\,kN}{0.9\ \times 3{\times}10^{3}\,kN}\right)$$
$$= 89.8$$

S16-14 11.2 Table 2 | Section classification limit for Class 3 sections

$$Class\ 3\ Limit = \frac{1.9{\times}10^{3}}{\sqrt{F_y}} \times \left(1 - 0.65 \times \frac{C_f}{\phi \times C_y}\right)$$
$$= \frac{1.9{\times}10^{3}}{\sqrt{350\,MPa}} \times \left(1 - 0.65 \times \frac{50\,kN}{0.9\ \times 3{\times}10^{3}\,kN}\right)$$
$$= 100$$

$$Class = \text{Class\_1}$$

Shear capacity

Shear in the x-axis direction

Clear depth of web between flanges

$$h = d - 2 \times t$$
$$= 257\,mm - 2 \times 15.7\,mm$$
$$= 226\,mm$$

Shear area

$$A_w = d \times w$$
$$= 257\,mm \times 8.9\,mm$$
$$= 2.29{\times}10^{3}\,mm^2$$

$$A_w = 2.29{\times}10^{3}\,mm^2$$

Ultimate shear stress

$$F_s = 0.66 \times F_y$$
$$= 0.66 \times 350\,MPa$$
$$= 231\,MPa$$

$$F_s = 231\,MPa$$

S16-14 13.4.1.1 a) i) | Factored shear resistance of a member or component

$$V_r = \phi \times A_w \times F_s \times 10^{-3}$$
$$= 0.9\ \times 2.29{\times}10^{3}\,mm^2 \times 231\,MPa \times 10^{-3}$$
$$= 476\,kN$$

$$V_r = 476\,kN$$

$$\boxed{V_{rx} = 476\,kN}$$

Shear in the y-axis direction

Overall depth of section for weak axis shear (depth of 2 flanges)

$$d = 2 \times b = 2 \times 204\,mm = 408\,mm$$

Clear depth of web between flanges for weak axis shear (width of unstiffened compression elements)

$$h = \frac{b}{2} = \frac{204\,mm}{2} = 102\,mm$$

Web thickness for weak axis shear (flange thickness)

$$w = 15.7\,mm$$

Shear area

$$A_w = d \times w = 408\,mm \times 15.7\,mm = 6.41{\times}10^3\,mm^2$$

$$A_w = 6.41{\times}10^3\,mm^2$$

Ultimate shear stress

$$F_s = 0.66 \times F_y = 0.66 \times 350\,MPa = 231\,MPa$$

$$F_s = 231\,MPa$$

S16-14 13.4.1.1 a) i) — Factored shear resistance of a member or component

$$V_r = \phi \times A_w \times F_s \times 10^{-3} = 0.9 \times 6.41{\times}10^3\,mm^2 \times 231\,MPa \times 10^{-3} = 1.33{\times}10^3\,kN$$

$$V_r = 1.33{\times}10^3\,kN$$

$$\boxed{V_{ry} = 1.33{\times}10^3\,kN}$$

Slenderness ratio check

Slenderness ratio about x-axis

$$\frac{K_x L_x}{r_x} = \frac{K_x \times L_x}{r_x} = \frac{1 \times 2{\times}10^3\,mm}{110\,mm} = 18.2$$

Slenderness ratio about y-axis

$$\frac{K_y L_y}{r_y} = \frac{K_y \times L_y}{r_y} = \frac{1 \times 2{\times}10^3\,mm}{51\,mm} = 39.2$$

S16-14 10.4.1 — Slenderness ratio of member

$$\frac{KL}{r} = \max\left(\frac{K_x L_x}{r_x}, \frac{K_y L_y}{r_y}\right) = \max(18.2, 39.2) = 39.2$$

$$\frac{KL}{r} = 39.2$$

$$\frac{KL}{r} \leqslant 200$$

$$39.2 \leqslant 200$$

$$39.2 \leqslant 200$$

Slenderness ratio does not exceed maximum permissible slenderness ratio.

$$Check = \text{Slenderness ratio ok!}$$

Axial compression capacity

Factored axial compressive resistance based on flexural buckling (about x-axis and y-axis)

S16-14 13.3.1 — Euler buckling stress about x-axis

$$F_{ex} = \frac{\pi^2 \times E}{\left(\frac{K_x \times L_x}{r_x}\right)^2} = \frac{(\pi)^2 \times 2{\times}10^5\,MPa}{\left(\frac{1 \times 2{\times}10^3\,mm}{110\,mm}\right)^2} = 5.97{\times}10^3\,MPa$$

S16-14 13.3.1 — Euler buckling stress about y-axis

$$F_{ey} = \frac{\pi^2 \times E}{\left(\frac{K_y \times L_y}{r_y}\right)^2} = \frac{(\pi)^2 \times 2{\times}10^5\,MPa}{\left(\frac{1 \times 2{\times}10^3\,mm}{51\,mm}\right)^2} = 1.28{\times}10^3\,MPa$$

Euler buckling stress

$$F_e = \min\left(F_{ex}, F_{ey}\right) = \min\left(5.97{\times}10^3\,MPa, 1.28{\times}10^3\,MPa\right) = 1.28{\times}10^3\,MPa$$

S16-14 13.3.1 — Non-dimensional slenderness parameter

$$\lambda = \sqrt{\frac{F_y}{F_e}} = \sqrt{\frac{350\,MPa}{1.28{\times}10^3\,MPa}} = 0.522$$

$$\lambda = 0.522$$

S16-14 13.3.1 — Factored compressive resistance of a member or component

$$C_r = \frac{\phi \times A \times F_y}{\left(1 + \lambda^{2\times n}\right)^{\frac{1}{n}}} \times 10^{-3} = \frac{0.9 \times 8.58{\times}10^3\,mm^2 \times 350\,MPa}{\left(1 + 0.522^{\,2\times 1.34}\right)^{\frac{1}{1.34}}} \times 10^{-3} = 2.4{\times}10^3\,kN$$

$$C_r = 2.4{\times}10^3\,kN$$

Factored axial compressive resistance based on torsional buckling

Effective length factor for torsional buckling

$$K_z = 1$$

S16-14 13.3.2 — Polar radius of gyration about shear centre

$$\overline{r}_o^{\,2} = x_0^{\,2} + y_0^{\,2} + r_x^{\,2} + r_y^{\,2} = (0\,mm)^2 + (0\,mm)^2 + (110\,mm)^2 + (51\,mm)^2 = 1.47{\times}10^4\,mm^2$$

S16-14 13.3.1 — Euler buckling stress

$$F_e = \left(\frac{\pi^2 \times E \times C_w}{(K_z \times L)^2} + G \times J\right) \times \frac{1}{A \times \overline{r}_o^{\,2}} = \left(\frac{(\pi)^2 \times 2{\times}10^5\,MPa \times 3.24{\times}10^{11}\,mm^6}{(1 \times 2{\times}10^3\,mm)^2} + 7.7{\times}10^4\,MPa \times 6.25{\times}10^5\,mm^4\right) \times \frac{1}{8.58{\times}10^3\,mm^2 \times 1.47{\times}10^4\,mm^2} = 1.65{\times}10^3\,MPa$$

S16-14 13.3.1 — Non-dimensional slenderness parameter

$$\lambda = \sqrt{\frac{F_y}{F_e}} = \sqrt{\frac{350\,MPa}{1.65{\times}10^3\,MPa}} = 0.461$$

$$\lambda = 0.461$$

Parameter for compressive resistance $n = 1.34$

S16-14 13.3.1 Factored compressive resistance of a member or component

$$C_r = \frac{\phi \times A \times F_y}{(1+\lambda^{2\times n})^{\frac{1}{n}}} \times 10^{-3}$$
$$= \frac{0.9 \times 8.58{\times}10^3\,mm^2 \times 350\,MPa}{\left(1+0.461^{\,2\times 1.34}\right)^{\frac{1}{1.34}}} \times 10^{-3}$$
$$= 2.47{\times}10^3\,kN$$

$$C_r = 2.47{\times}10^3\,kN$$

Overall axial compressive resistance

Non-dimensional slenderness parameter $\lambda = 0.522$

$$\lambda = 0.522$$

Factored compressive resistance of member based on flexural buckling $C_{r,flexural} = 2.4{\times}10^3\,kN$

Factored compressive resistance of member based on torsional buckling $C_{r,torsional} = 2.47{\times}10^3\,kN$

S16-14 13.3 Factored compressive resistance of a member or component

$$C_r = \min\left(C_{r,flexural}, C_{r,torsional}\right)$$
$$= \min\left(2.4{\times}10^3\,kN, 2.47{\times}10^3\,kN\right)$$
$$= 2.4{\times}10^3\,kN$$

$$C_r = 2.4{\times}10^3\,kN$$

$$\boxed{C_r = 2.4{\times}10^3\,kN}$$

Moment capacity (laterally unsupported member)

Strong axis bending

Plastic moment resistance

$$M_p = Z \times F_y \times 10^{-6}$$
$$= 9.01{\times}10^5\,mm^3 \times 350\,MPa \times 10^{-6}$$
$$= 315\,kNm$$

$$M_p = 315\,kNm$$

Critical elastic moment of a laterally unbraced beam

$$M_u = \frac{\omega_2 \times \pi}{L} \times \sqrt{E \times I_y \times G \times J + \left(\frac{\pi \times E}{L}\right)^2 \times I_y \times C_w} \times 10^{-6}$$
$$= \frac{1 \times \pi}{2{\times}10^3\,mm} \times \sqrt{2{\times}10^5\,MPa \times 2.22{\times}10^7\,mm^4 \times 7.7{\times}10^4\,MPa \times 6.25{\times}10^5\,mm^4 + \left(\frac{\pi \times 2{\times}10^5\,MPa}{2{\times}10^3\,mm}\right)^2 \times 2.22{\times}10^7\,mm^4 \times 3.24{\times}10^{11}\,mr}$$
$$10^{-6}$$
$$= 1.51{\times}10^3\,kNm$$

$$M_u = 1.51{\times}10^3\,kNm$$

S16-14 13.6 a) i) Factored moment resistance of a member or component

$$M_r = \min\left(1.15 \times \phi \times M_p \times \left(1 - \frac{0.28 \times M_p}{M_u}\right), \phi \times M_p\right)$$
$$= \min\left(1.15 \times 0.9 \times 315\,kNm \times \left(1 - \frac{0.28 \times 315\,kNm}{1.51{\times}10^3\,kNm}\right), 0.9 \times 315\,kNm\right)$$
$$= 284\,kNm$$

$$M_r = 284\,kNm$$

$\boxed{M_{rx} = 284\,kN}$

Weak axis bending

S16-14 13.5 a)

Factored moment resistance of a member or component

$$\begin{aligned} M_r &= \phi \times F_y \times Z \times 10^{-6} \\ &= 0.9\ \times 350\,MPa \times 3.32{\times}10^{5}\,mm^3 \times 10^{-6} \\ &= 105\,kNm \end{aligned}$$

$M_r\ = 105\,kNm$

$\boxed{M_{ry} = 105\,kN}$

Axial tension capacity

S16-14 13.2 a) i)

Factored tensile resistance of a member or component

$$\begin{aligned} T_r &= \phi \times \frac{A_g \times F_y}{1{\times}10^3} \\ &= 0.9\ \times \frac{8.58{\times}10^3\,mm^2\ \times 350\,MPa}{1{\times}10^3} \\ &= 2.7{\times}10^3\,kN \end{aligned}$$

$T_r\ = 2.7{\times}10^3\,kN$

$\boxed{T_r = 2.7{\times}10^3\,kN}$

Capacity check summary

Summary table of demands, capacities and utilisation ratios for axial forces, moments and shear forces:

Summary	**Compression, Cf/Cr**	**Tension, Tf/Tr**	**Strong Axis Moment, Mfx/Mrx**	**Weak Axis Moment, Mfy/Mry**	**Shear in X-Axis Direction, Vfx/Vrx**	**Shear in Y-Axis Direction, Vfy/Vry**
Demand (kN)	50	50	50	50	50	50
Capacity (kN)	2395.786	2702.7	283.815	104.58	475.53	1331.724
Utilisation	0.021	0.019	0.176	0.478	0.105	0.038

Bibliography

S16-14 CSA S16 (2014) Canadian Standards Association: Design of steel structures. Canada.