

# Formation of an Effective Non-Linear Seismic Analysis

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## Abstract

Structures in megacities are under serious threat because of unskilled design and construction of the structure. Sometimes building designers are more concerned in constructing different load-resistant members whereas neglecting its necessity and its performance. This rigid design leads to a variety of inaccurate predictions when evaluating structural response due to real-life environmental hazards, especially earthquakes, which is one of the root causes of building collapse. It is essential to form a comprehensive seismic analysis system that can accurately evaluate the earthquake response of a structure. Pushover analysis has been widely used in the seismic response analysis as its accessible concepts and convenient computations and can effectively evaluate the seismic performance of the selected structure. The fundamental theory and implementation of Pushover analysis are introduced. As a complementary approach with Pushover analysis, the Capacity Spectrum is also illustrated. The paper composes these two methods together to form a seismic analyzing modal which eliminates the potential drawback of Pushover analysis as well as accurately predicts where and when the structure fractures during an earthquake. To further verify the feasibility of the investigation, the paper is followed by a case study of a multi-story reinforced concrete building structure in Richmond BC. Using the seismic analyzing modal to predict the theoretical damages of the building during the earthquake

## Keywords

Pushover analysis, Capacity Spectrum, non-linear analysis, seismic performance.

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## 1. Introduction

The performance of a building can be undermined with time, and buildings accumulate damages during their life-time service due to external hazards such as hurricanes, earthquakes, etc. In recent decades, earthquake damages are still the root cause of house collapses and property damages worldwide. However, such a headache tends to relief as structural engineering has developed a comprehensive seismic analyzing system that can obtain more accurate damage assessment for structures.

To better understand the central concept of this paper, it is necessary to know the background knowledge of traditional seismic analysis. The seismic analyzing system can be categorized into two types (Fig 1) that enable the evaluation of performances of a building --- linear analysis and non-linear analysis. For the linear analysis, the force and the displacement are linearly related by specific formulas. The Response Reduction factor ( $R$ ) is one of the critical determinants for the linear approach. For instance, the value of  $R$  represents that only  $1/R$  of the seismic force is taken into the calculation of the structural deflection at the limit state. The drawback of such an assumption is that it inaccurately predicts further deflections in the ductile region. Because the structural response beyond the limit state is neither a linear extrapolation nor a perfectly ductile behavior with pre-

determined yielding capacity. Reasons for this complexity can be summarized as the p-delta effect, unstable seismic forces, and changing in stiffness of structural elements due to fracturing and yielding in the inelastic state. Although the linear approach gives an appropriate approximation of the elastic response and indicates where the maximum yielding capacity might first occur, this analysis cannot accurately predict the ductile behavior for the structure and the redistribution of loads during the yielding process. Thus, the non-linear analysis is more suitable for seismic evaluation of an existing structure as it considers both elastic and inelastic behavior of the structure. It predicts members which likely to reach critical states when subjected to the earthquake or, by tracking plastic hinge states, even highlights the potential cracks on the structure.

With the invention of powerful analyzing tools such as ETAB and SAP2000, seismic design is gradually transforming from a stage where a linear analysis for both elastic and inelastic structures, to a stage where a specially dedicated non-linear analysis that is mainly dealing with the inelastic state. In the non-linear analysis approach, Pushover analysis and Capacity Spectrum are fast becoming accepted methods for the current seismic analysis. Unlike the traditional Non-linear Response History analysis which is a time-consuming and may even be impractical for people who start to learn it, Pushover and Capacity Spectrum analysis are more acceptable methods due to their conceptual simplicity and ability to graphically describe a structure's response. Such techniques accelerate the speed of locating vulnerable points during the analyzing process and reduce the possibility of human errors. More specifically, Pushover analysis is the method that considering the sequential yielding of structural elements as the whole structure is laterally displaced beyond its elastic-plastic limits. The Capacity Spectrum Method mainly focuses on a graphical visualization on how well the building can perform when subjected to the seismic ground vibration. Therefore, these two analyses play a significant role in today's seismic risk analysis. However, the research finds out that there might be some potential shortcomings if using two methods independently.

In general, the essential goal of this paper is to investigate two seismic analyzing methods and find out how to compose these two branches into a whole comprehensive seismic evaluation system that can improve the reliability and accuracy of the prediction for seismic response at various earthquake intensities and structural conditions.

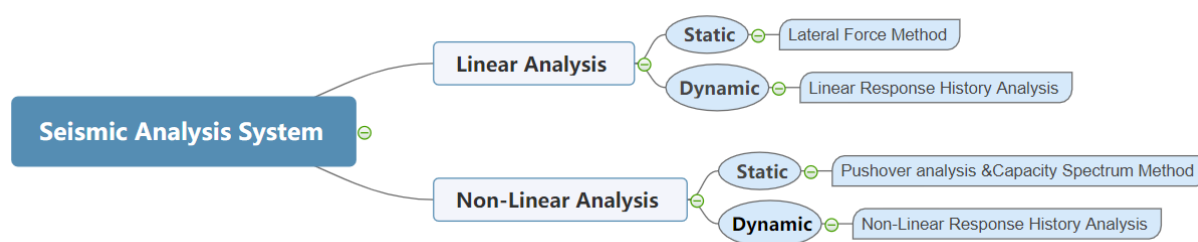


Figure 1: Composition of the seismic analysis system

## 2. Fundamental Theory of Pushover analysis

### 2.1 Introduction of the Pushover analysis

Pushover Analysis is a non-linear analysis procedure to estimate the strength capacity of a structure beyond its yielding point up to its ultimate fracture point in the inelastic range. During the analyzing process, Pushover analysis gives successive predictions of performance points on the structure by keeping track of sequent deflections of every element in the structure. Rely on the data of performance points, structural engineers can generally identify maximum forces and displacements that can be applied on structural elements. In order to apply the Pushover analysis in real-life structures, the methodology to test the design is: (1) generate a seismic analyzing modal based on all basic properties (types of frames, dimension for each frame, materials, etc.) of the structure; (2) insert the location of hinges and define the type of hinges for every structural element and finally (3) the Pushover analysis, followed by (4) adjustments and retrofitting of the primary design.

In this paper, SAP2000 is used as a powerful tool to accomplish the Pushover analysis. The primary mechanism of this test in SAP2000 is gently “push over” the structure by a monotonically increasing lateral load which is given by predetermined seismic data. The Pushover analysis for a specific seismic motion outputs results in two concepts. In the global level, a plot of the base shear ( $V_b$ ) vs. rooftop displacement ( $\Delta_{roof}$ , the displacement of a point on the roof from its original position). This plot describes what maximum lateral loads can be applied to the structure before its fracture, and what is the maximum displacement corresponding to this load. In the local level, the analysis outputs the detailed response for each plastic hinge on every structural element. Hinge states which are represented by multiple plots of ( $V_b$ ) vs. ( $\Delta_{roof}$ ) can help to decide on where the retrofiting is required and how much.

## 2.2 The importance of hinges

Three categories of hinges are shear hinges, flexural hinges, and axial hinges (Fig 2). Generally, hinges represent a relationship between localized forces and displacements for structural members under seismic loads. More specifically, the flexural hinge controls the rotational moment of a beam and the rest of two controls the displacement along two axes. The paper mainly focuses on force-displacement relation which is shown in Fig 3, A-B represents the linear elastic region from unloaded state A up to the yielding point B, followed by an inelastic region which the structure behaviors inelastically and its stiffness gradually reduces from B to C. C-D implies that the structure reaches its fracture point and a sudden reduction occurs in the load resistance. The inelastic region B-C is further divided into three non-linear states: Immediate Occupancy (IO), Life Safety (LS) and Collapse Prevention (CP). Three non-linear states give warnings where a certain hinge can be potentially fractured. The criteria for dividing the state significantly relies on the Design Code of the selected country. For example, one such specification is at 10%, 60% and 80% of the inelastic region B-C for IO, LS, and CP respectively. In the case study session, these three states play a significant role as indicators in defining vulnerable hinges (or members) in a building during an earthquake.

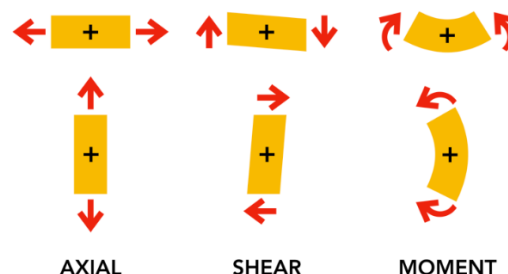


Figure 2: Hinge states categorization

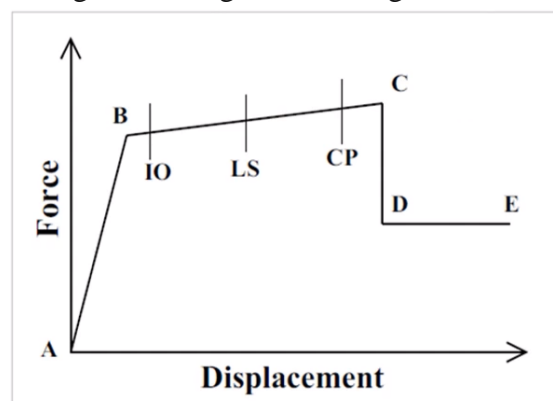


Figure 3: The elastic curve for a hinge

In SAP2000, those points could be identified by color bands to describe how plastic hinges form each stage Fig 3 where IO, LS, and CP represent Immediate Occupancy, Life Safety, and Collapse Prevention respectively. To better understand the meaning behind each category, Fig 4 is referred to as a graphic presentation for all damage levels. Before reaching the yielding point B (Fig 4a), the

building is capable of providing full resistances to seismic loads. If the building falls into the IO level (Fig 4b), it means the residential building is quickly able to provide shelter for people and continue functioning post-disaster. However, when the exacerbation continues to the LS level (Fig 4c), it indicates that the building is under severe damages, and only margins remain against collapses. Finally, the building is no longer capable of resisting any external forces after entering the CP level, it simply collapses.



Figure 4: Force-Deformation color bands for Pushover analysis



Figure 5: Levels of building performance: [a] Safe (B), [b] Immediate Occupancy (IO), [c] Life-safety (LS), [d] Collapse Prevention (CP) Sayin, B (2017)

### 2.3 Limitations for Pushover Analysis

The potential shortcoming for the concept of Pushover Analysis is that it only deals with the system with the first degree of freedom, which is hard to achieve in reality. It means the structure model is assumed to be equivalent to a single vertical frame fixed at the bottom with a single mass lumped at the top. This assumption conceptually ignores the fact that the building has unevenly distributed loads and plenty of joints with various damping ratios (related to the ductility and level of damages). In general, analyzing results would be more accurate if the drawback of Pushover analysis is mended by another method.

## 3. Fundamental Theory of Capacity Spectrum Method

### 3.1 Introduction of the Capacity Spectrum Method

Capacity Spectrum Method (CSM) is defined as the ratio of spectral displacement and spectral acceleration. Freeman originally introduced it in the 1970s (Freeman et al. 1975, Freeman 1978). This graphical presentation enables a visualization on how the structure behaves when subjected to the seismic ground motion. CSM tends to mend the shortcoming that exists in the Pushover analysis. Most current Pushover procedures use load patterns based on the 1st mode or code statics, but CSM doesn't only base on the 1st mode but higher modes which enable an evaluation of seismic response for multi-mode structure. Also, this method extends the range of seismic analysis as it can be adopted in the evaluation of various materials, such as steel and reinforced concretes. However, the accuracy of the CSM significantly relies on the data obtained from the Pushover analysis. For example, Capacity Spectrum is calculated from changing the base shear in x-axis and roof displacement in the y-axis of a non-linear static Pushover analysis to equivalent spectral accelerations ( $S_a$ ) and spectral displacements ( $S_d$ ). Furthermore, the performance point of the structure is obtained by a repetitive process of intersecting the elastic demand spectra with an effective damping ratio  $\xi$  (generally use 5%) and its capacity spectrum that given initially from the Pushover analysis (Fig 5). Thus, the accuracy of results from Pushover analysis is the determinant for the reliability of Capacity Spectrum.

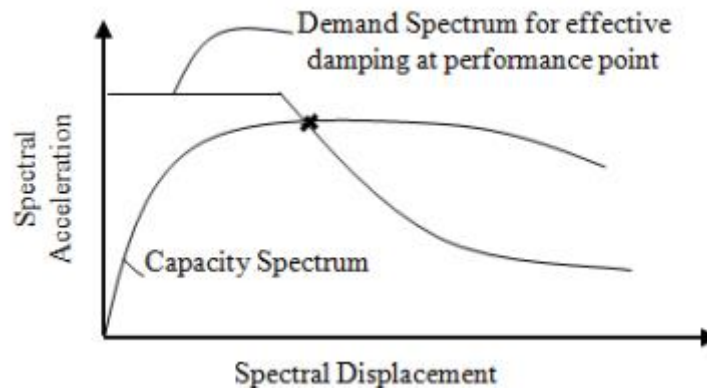


Figure 6: Capacity spectrum with an effective damping ratio.

### 3.2 Basic Formulas

The prerequisite for applying the Capacity Spectrum Method is identifying the dynamic characteristics of the structure and its dynamic response. Both data can be obtained from repetitively comparing the performance point (the intersection of the Demand curve and Capacity curve from Fig 5) of the structure. Thus, the first step of the CSM is to define these two curves. For the capacity curve, it can be calculated from the Pushover analysis by converting the base shear ( $V_b$ ) to spectral acceleration ( $S_a$ ) and roof displacement ( $\Delta_{roof}$ ) to spectral displacement ( $S_d$ ) within the formula (ACT-40, 1996) below:

$$S_a = \frac{\frac{V_b}{W}}{M_k/M} * g$$

$$S_d = \frac{\Delta_{roof}}{P_k * \phi_{roof}} * g$$

where

$M_k$  = modal mass

$M$  = total mass of the building

$P_k$  = mode participation factor

$W$  = total weight of the building

$\phi_{roof}$  = modal amplitude at the roof

$V_b$  = base shear for mode

$\Delta_{roof}$  = roof displacement for the mode

After obtaining the Capacity curve through the above formula. Demand curve must be calculated to find the exact location of the performance point. The value of the Demand curve is directly related to the value of effective damping ratio  $\zeta$ , which is expressed by the following formula.

$$\zeta_{eff} = \zeta_o + \zeta_D$$

The formula indicates that the effective damping ratio ( $\zeta_{eff}$ ) is the sum of the inherent damping ( $\zeta_o$ ) with the device damping ( $\zeta_D$ ). The first procedure of defining the demand curve is to build the elastic spectrum with damping equal to the effective damping of the structure. The spectral acceleration (y-axis) and the spectral displacement (x-axis) for the demand curve are obtained from the following two formulas:

$$S_a(T, \zeta_{eff}) = \frac{S_a(T, 5\%)}{B(T, \zeta_{eff})} \quad S_d(T, \zeta_{eff}) = \frac{S_d(T, 5\%)}{B(T, \zeta_{eff})}$$

Where  $S_a(T, 5\%)$  and  $S_d(T, 5\%)$ , is the spectral acceleration and the spectral displacement for the demand curve subjected to the 5% damping ratio,  $B(T, \zeta_{eff})$  is the damping reduction factor for damping ratio ( $\zeta_{eff}$ ). The general formula for calculating the Demand curve at 5% damping ratio  $S_a/S_d(T, 5\%)$  is:

$$S_d = g * \left( \frac{T}{2\pi} \right)^2 * S_a$$

After the calculation of capacity and demand curves, the exact position of the performance point can be located by intersecting two curves. The performance point is the most essential factor for the



computer to set the reference for determining hinge states and to visually display the structural response with different color bands (Fig 3). Such an illustration might seem to be a little abstract. The followed case study is a clue to understanding the central concept of the Capacity Spectrum Method. Moreover, the effect of damping ratios should be taken into consideration in the real-life seismic analysis, and it is essential in locating the performance point (Fig 7) as the demand curve varies dramatically when subjected to different damping ratios. For instance, the demand curve shifts outward with an increased damping ratio. The case study also considered the structural response with various damping ratios.

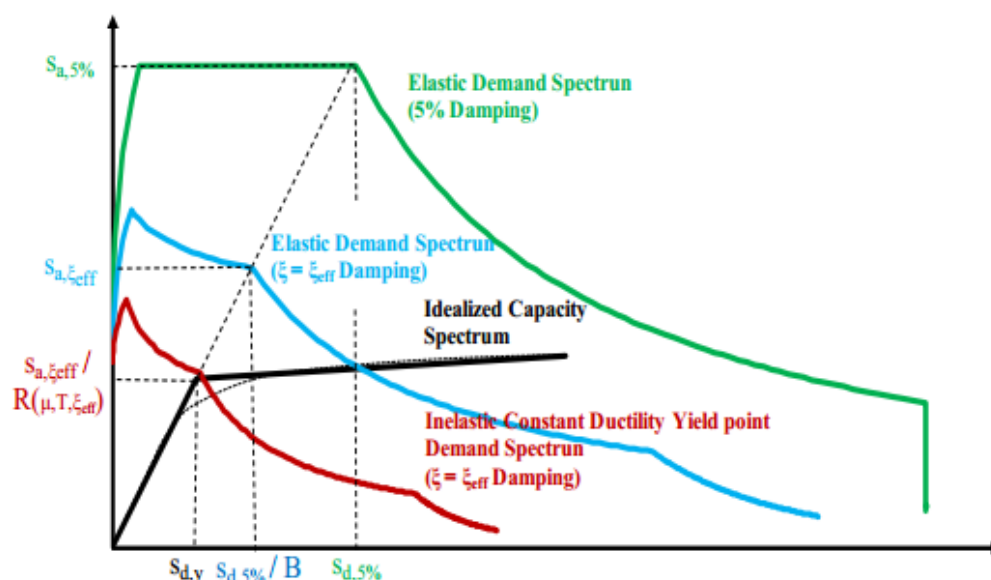


Figure 7: Shifts of the demand curve with different damping ratios on Capacity Spectrum

#### 4. Formation of the seismic response analysis system

The traditional non-linear structural analysis uses Pushover analysis as a significant indicator to predict the seismic response during an earthquake. Nonetheless, applying the Pushover analysis independently is not sufficient to deal with real-life complexities. The paper introduces a complementary approach which is CSM to improve the reliabilities of the non-linear structural analysis further. The unification of two methods pushes the non-linear structural analysis into a broader scope. Extensions on monitoring hinge states on each joint of every member allow a more accurate evaluation of structural behavior. Visual observation on vulnerable points helps structural engineers quickly locate where the retrofiting is required as well as how much it needs. However, in reality, the examined building may contain tons of members and joints, and manually calculating individual member is time-consuming and even hard to accomplish. Also, the inflexibility of manual calculation restricts the possibility of adjusting variables. Thus, SAP2000 is introduced as a useful tool to merge two methods with flexibilities for adjusting variables and explores a more effective way to analyze the seismic response of a structure.

#### 5. Case Study ---- Application of Pushover analysis and Capacity Spectrum Method

This section contains a Pushover analysis with complementary Capacity Spectrum done on a 3-story reinforced-concrete building located in Richmond, BC, CA by using the structural application SAP2000. The underlying design code is followed by NBCC2015 and Design textbook, which is provided from the CIVE 318 (McGill). As seismic forces are unstable and sometimes fluctuate dramatically, the analysis used the displacement control with a monitored magnitude of 0.5m. All plastic hinges are formulated as per ATC 40 guideline

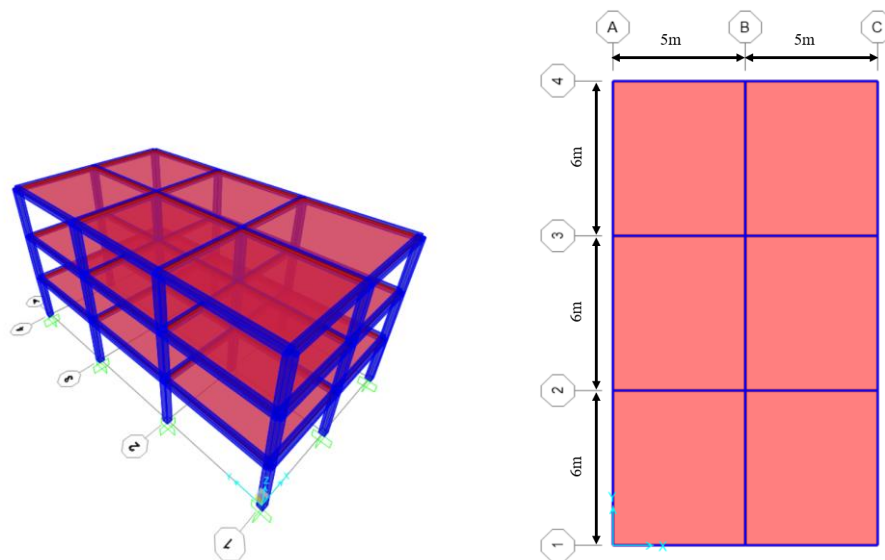


Figure 8: Case Study building overview & Dimensions on grids

### 5.1 Modeling of the building

The model was bare frame having columns, beams, and slabs. All the slabs were designed as a shell element of 125mm thickness. Diaphragm action was applied to all floor slabs to maintain integral lateral action of beams in each floor. The geometrical properties of the structure are listed in Table 1. The frame was designed according to the current NBCC2015 with a moderate seismicity region with peak ground acceleration 0.15g. For simplicity, the evaluation assumed that all concretes and rebars were uniform types, which were  $f_c 4000$  psi and ASTM615 Grade 60, respectively. The reinforced concrete itself was assumed to have a strength of 4000psi with yield strength 56000psi where Modulus of Elasticity is 3600 psi.

The residential building was 18m by 10m in plan dimension, and the typical floor height is 3m. Additionally, the building is symmetrical in x and y directions (Fig 7), and the end support conditions at the base are assumed to be fixed.

Table 1: Column Size, confinement bar size, beam size and live loads

Floor	Column Size (mm×mm)	Confinement Bar Size	Clear Cover for Confinement Bars (mm)	Beam Size (mm×mm)	Live load (KN/m <sup>2</sup> )
Ground Floor	350×350	8d	30	230×350	0
1st Floor	350×350	8d	30	230×350	2
2nd Floor	350×350	8d	30	230×350	2
3rd Floor	350×350	8d	30	230×350	1.5

The point load was applied in a pattern of that 1<sup>st</sup> DOF shape in the transverse direction of the building at joint 4 (see the Fig), with an intensity for Design Base Earthquake as per ATC-40, the soil type follows the default setting (medium soil) of SAP2000. Moment hinges (M3) were applied to both ends of the selected beams, and axial hinges (PMM) were assigned to the column ends (excepting the ends at base) as per FEMA 356. Geometric non-linearity (P-delta) and large displacement are considered with the full dead load. The gravity loads(self-weight) of the members, loads of floor finish and live loads were combined by the formula referred from NBCC2015. The programming assumed that all partition walls were loaded directly on the beams. The ultimate displacement

capacity of the structure was calculated at each step of the analyzing process followed by guidelines of ATC-40 and FEMA 356

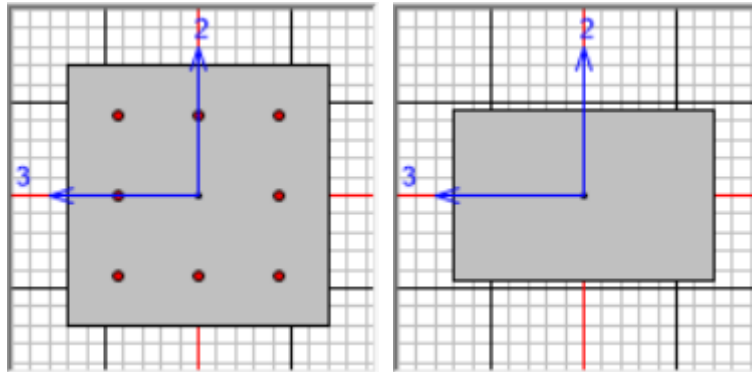


Figure 9: Dimensions for beams (left) and columns (right) for the structure

## 6. Result & Discussion

From the CSM plot (Fig 9), the performance point of the frame can be located from the intersection of the capacity curve and the demand curve, as shown in Figure 11 and  $S_a$  and  $S_d$  are 0.221g and 0.044m, respectively. The corresponding frame base shear ( $V_b$ ) and top roof displacement ( $\Delta_{\text{rooftop}}$ ) are 1139.042 kN and 5.9 cm. The value of effective  $T$  is 0.894s. The effective damping ratio ( $\beta$ ) which met the performance point is 5.2% The plastic hinges obtained at different performance levels are shown in Fig 12

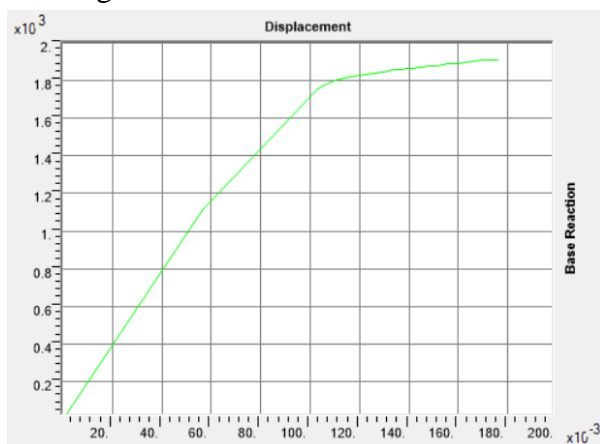


Figure 10: Pushover curve(green)

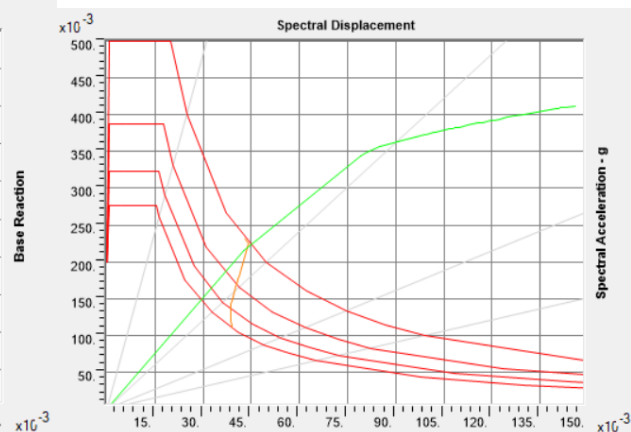


Figure 11: Capacity Spectrum

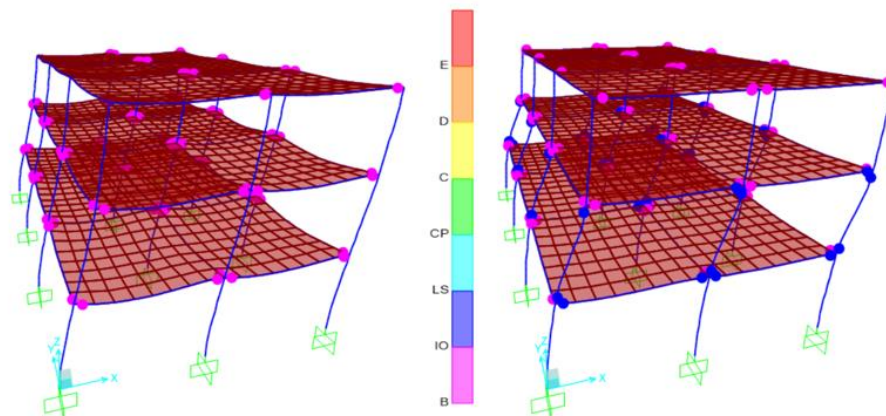


Figure 12: Hinge states in the structure model before & after the performance point during the pushover analysis, with color codes of hinge states



Table 2: Hinge states of the pushover analysis

TABLE: Pushover Capacity Curve												
LoadCase	Step	Displacement	BaseForce	AtoB	BtoIO	IOtoLS	LStoCP	CPtoC	CtoD	DtoE	BeyondE	Total
Text	Unitless	m	KN	Unitless	Unitless	Unitless	Unitless	Unitless	Unitless	Unitless	Unitless	Unitless
Pushover x	0	0.000009435	0	72	78	0	0	0	0	0	0	150
Pushover x	1	0.000011	0.027	72	78	0	0	0	0	0	0	150
Pushover x	2	0.056609	1110.126	64	86	0	0	0	0	0	0	150
Pushover x	3	0.102707	1752.566	38	92	20	0	0	0	0	0	150
Pushover x	4	0.10524	1775.506	36	92	22	0	0	0	0	0	150
Pushover x	5	0.109919	1801.901	34	88	28	0	0	0	0	0	150
Pushover x	6	0.111417	1806.331	34	88	28	0	0	0	0	0	150
Pushover x	7	0.111464	1805.52	34	88	28	0	0	0	0	0	150
Pushover x	8	0.111471	1805.719	34	88	28	0	0	0	0	0	150
Pushover x	9	0.111474	1805.752	34	88	28	0	0	0	0	0	150
Pushover x	10	0.115913	1819.756	34	86	30	0	0	0	0	0	150

Table 1 shows how hinge states change in the process of the analysis. The performance point is the intersection of the capacity curve and the demand curve (Fig 3). This point indicates the maximizing performance of the structure under a specified load. According to Table 2, taken as step 3 (or between steps 2 and 3 by linear extrapolation) 100% of the hinges are falling into the range of LS and 86.7% within the IO performance level. Figure 11 (left) shows the hinge states of the building before passing the performance point as all hinges within the Immediate Occupancy (IO) level (all pink), and it means the residential building is quickly able to provide shelter and continue functioning post-event. Figure 11 (right) shows the hinge states after passing the performance point. Obviously, some of the hinges turn to blue as they undergo a transformation from the IO level to Life Safety (LS) level, it means that the building is under severe damages, but its margin remains against collapse. Moreover, it can be observed that hinges started to behavior plastically (Blue dots occurs) with beams and columns of lower stories, then propagated to upper stories and continue with yielding of interior columns in the upper stories.

A roof displacement of 5.9 cm, with the height of the building up to rooftop  $h$  being 9 m, gives a roof displacement to height ratio being 0.00655 ( $=0.655\%$ ). The ratio of less than 1% at the performance point indicates the building is capable of resisting the designed earthquake ground motions. Thus, the output result is reliable as the structure behaves an expected safety at the performance point. For the further verification of results from the composed seismic analyzing method, the drift ratio for each floor is calculated (Fig 13), the drift ratio increases within the increasing floor level and the maximum shear force occurs at the 1st-floor column. Both confirm that the evaluation result is reasonable.

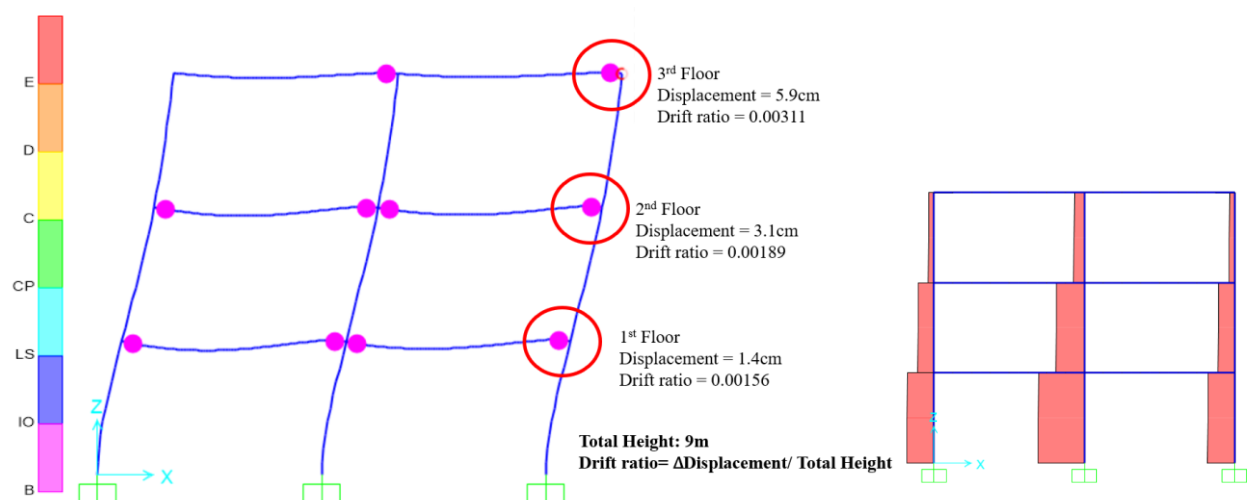


Figure 13: Roof Drift Ratio for each story for the verification of analyzing results (left) & Shear loads on the column by the pushover load (right)

## 7. Conclusion

The main objective of this study is to develop a logical approach to merge the Pushover analysis and Capacity Spectrum method. The paper explains the mechanism of the Pushover analysis and Capacity Spectrum method and confirms the interdependency of two methods by referring relevant formulas that convert the Pushover curve to the Capacity Spectrum that enables a graphic presentation of the structural response. The importance of hinge states is clearly stated as it plays a significant role in identifying vulnerable members as well as indicates the state of the building (IO, LS, and CP). Additionally, the paper describes the reason for the combination of two methods by investigating potential shortcomings if utilizing two methods independently. Finally, a case study of a 2-story building following the ATC 40 guideline validates the accuracy of the seismic analyzing modal by passing the drift ratio test and shear force test. Thus, it can be concluded that the combination of two methods complements the accuracy of analyzing results and eliminates the potential drawbacks of using each method independently.

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