

# Enunciados

## Capítulo 1: Física de partículas y relatividad especial

### 1. A quantum field

- (a) Is a field with quanta that are operators
- (b) Is a field parameterized by the position operator
- © Commutes with the Hamiltonian
- (d) Is an operator that can create or destroy particles

### 2. The particle generations

- (a) Are in some sense duplicates of each other, with each generation having increasing mass
- (b) Occur in pairs of three particles each
- © Have varying electrical charge but the same mass
- (d) Consists of three leptons and three quarks each

### 3. In relativistic situations

- (a) Particle number and type is not fixed
- (b) Particle number is fixed, but particle types are not
- © Particle number can vary, but new particle types cannot appear
- (d) Particle number and types are fixed

### 4. In quantum field theory

- (a) Time is promoted to an operator
- (b) Time and momentum satisfy a commutation relation
- © Position is demoted from being an operator
- (d) Position and momentum continue to satisfy the canonical commutation relation

### 5. Leptons experience

- (a) The strong force, but not the weak force
- (b) The weak force and electromagnetism
- © The weak force only

- (d) The weak force and the strong force

## 6. The number of force-carrying particles is

- (a) Equivalent to the number of generators for the fields gauge group
- (b) Random
- © Proportional to the number of fundamental matter particles involved in the interaction
- (d) Proportional to the number of generators minus one

## 7. The gauge group of the strong force is:

- (a) SU(2)
- (b) U(1)
- © SU(3)
- (d) SU(1)

## 8. Antineutrinos

- (a) Have charge -1 and lepton number 0
- (b) Have lepton number +1 and charge 0
- © Have lepton number -1 and charge 0
- (d) Are identical to neutrinos, since they carry no charge

# Capitulo 2: Teoría de campos lagrangiana

## 1. Find the equation of motion for a forced harmonic oscillator with Lagrangian

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\omega^2 x^2 + \alpha x$$

Here  $\alpha$  is a constant

## 2. Consider a Lagrangian given by

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 - V(\varphi)$$

- (a) Write down the field equations for this system.
- (b) Find the canonical momentum density  $\pi(x)$ .
- © Write down the Hamiltonian.

3. Consider a free scalar field with Lagrangian  $L = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi$  and suppose that the field varies according to  $\phi \rightarrow \phi + \alpha$ , where  $\alpha$  is a constant. Determine the conserved current.
4. Refer to the Lagrangian for a complex scalar field Eq. (2.37). Determine the equations of motion obeyed by the fields  $\phi$  and  $\phi^\dagger$ .
5. Refer to Eq. (2.37) and calculate the conserved charge.
6. Consider the action  $S = \int d^4x \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ . Vary the potential according to  $A_\mu \rightarrow A_\mu + \partial_\mu \phi$  where  $\phi$  is a scalar field. Determine the variation in the action.

### Capítulo 3: Una Introducción a la teoría de grupos

1. Consider an element of  $SU(2)$  given by  $U = e^{i\alpha \sigma_x/2}$ . By writing down the power series expansion, write  $U$  in terms of trigonometry functions.
2. Consider  $SU(3)$  and calculate  $\text{tr}(T_a T_b) = \frac{1}{2} \delta_{ab}$
3. How many casimir operators are there for  $SU(2)$ ?
4. Write down the casimir operators for  $SU(2)$ . A Lorentz transformation can be described by boost matrices with rapidity defined by  $\tanh \eta = v/c$ . A boost in the  $x$  direction is represented by the matrix
5. Find the generator  $K_x$

6. Knowing that  $[J_x, J_y] = i\hbar J_z$ , where  $J_z$  is the angular momentum operator written in four dimensions as find  $K_y$ .

7. Do pure Lorentz boosts constitute a group?

## Capitulo 4: Simetrías discretas y números cuánticos

1. Angular momentum states transform under the parity operator as

- (a)  $P|l, m\rangle = (-1)^m |l, m\rangle$
- (b)  $P|l, m\rangle = |l, m\rangle$
- (c)  $P|l, m\rangle = (-1)^l |l, m\rangle$
- (d)  $P|l, m\rangle = (-1)^m |l, -m\rangle$

2. The interaction Lagrangian of electromagnetism is invariant under charge conjugation if

- (a)  $\mathcal{L}_{int} = -e\bar{\psi}\gamma^\mu\psi A_\mu$
- (b) It is not invariant under charge conjugation
- (c)  $\mathcal{L}_{int} = e\bar{\psi}\gamma^\mu\psi A_\mu$
- (d)  $\mathcal{L}_{int} = -e\bar{\psi}\gamma^\mu\psi A_\mu$

3. Parity is

- (a) Conserved in weak and electromagnetic interactions, but is violated in the strong interaction
- (b) Conserved in strong interactions, but is violated in weak and electromagnetic interactions
- (c) Not conserved
- (d) Conserved in the strong and electromagnetic interactions, but is violated in the weak interaction

4. The eigenvalues of charge conjugation are

- (a)  $c = \pm 1$
- (b)  $c = \pm 0.1$
- (c)  $c = \pm i$
- (d)  $c = \pm 0$

5. An operator is antiunitary if

## Capitulo 5: La ecuación de Dirac

### Quiz

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$$L = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} m \omega^2 x^2 + \alpha x$$

Here  $\alpha$  is a constant.



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### Quantum Field Theory Demystified

2. Consider a Lagrangian given by

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - V(\phi)$$

- (a) Write down the field equations for this system.
  - (b) Find the canonical momentum density  $\pi(x)$ .
  - (c) Write down the Hamiltonian.
3. Consider a free scalar field with Lagrangian  $\mathcal{L} = \partial_\mu \phi \partial^\mu \phi$  and suppose that the field varies according to  $\phi \rightarrow \phi + \alpha$ , where  $\alpha$  is a constant. Determine the conserved current.
4. Refer to the Lagrangian for a complex scalar field Eq. (2.37). Determine the equations of motion obeyed by the fields  $\phi$  and  $\phi^\dagger$ .
5. Refer to Eq. (2.37) and calculate the conserved charge.
6. Consider the action  $S = \frac{1}{4} \int F_{\mu\nu} F^{\mu\nu} d^4x$ . Vary the potential according to  $A_\mu \rightarrow A_\mu + \partial_\mu \phi$  where  $\phi$  is a scalar field. Determine the variation in the action.

1. Given the Lagrangian  $\mathcal{L} = \bar{\psi} \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi$  vary  $\psi$  to find the equation of motion obeyed by  $\psi$ .

2. Calculate  $\{\gamma^\mu, \gamma^\nu\}$ .

3. Consider the solution of the Dirac equation with  $E_k \Rightarrow \omega_0$ . Find a relationship between the  $u$  and  $v$  components of the Dirac field.
4. Find the normalization of the free space solutions of the Dirac equation using the density  $\bar{\psi}\gamma^0\psi$ .
5. Find  $S_{01}$ , the generator of a boost in the  $x$  direction.
6. We can introduce an electromagnetic field with a vector potential  $A_\mu$ . Let the source charge be  $q$ . Using the substitution  $\partial_\mu \rightarrow \partial_\mu + iqA_\mu$ , determine the form of the Dirac equation in the presence of an electromagnetic field.

## Capitulo 6: Campos escalares

1. Compute  $\langle 0 | \hat{\phi}(x) \hat{\phi}^\dagger(x') | 0 \rangle$  for the real scalar field.
2. Find  $\langle 0 | \hat{\phi}(x) \hat{\phi}^\dagger(x') | 0 \rangle$
3. Find  $\langle 0 | \hat{N} | 0 \rangle$
4. Consider the complex scalar field. Determine if charge is conserved by examining the Heisenberg equation of motion for the charge operator  $\hat{Q}$ .  $[Q, H] = 0$ , Do this computation by writing out the operators using Eqs. (6.68) and (6.70) and using the commutation relations Eq. (6.64)

## Quiz

1. Given the Lagrangian

$$\mathcal{L} = \bar{\psi}(x)[i\gamma^\mu \partial_\mu - m]\psi$$

vary  $\psi(x)$  to find the equation of motion obeyed by  $\bar{\psi}(x)$ .

2. Calculate  $\{\gamma_5, \gamma^\mu\}$ .
3. Consider the solution of the Dirac equation with  $E = \omega_k > 0$ . Find a relationship between the  $u$  and  $v$  components of the Dirac field.
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## Capitulo 7: Las reglas de Feynman

1. What is the amplitude for the process shown in Fig. 7.16?
2. Find the lifetime for the decay as shown in Fig. 7.17.
3. An internal line corresponds to a spin-0 boson of mass  $m$ . The propagator is
  - (a)  $i/(q^2 - m^2)$
  - (b)  $i/(q^2 + m^2)$
  - (c)  $i/(q^2 - m^2)$
  - (d)  $\delta(q^2 - m^2)$
4. In the interaction picture,
  - (a) The time evolution of states is governed by the free Hamiltonian
  - (b) States are stationary, operators evolve according to the interaction part of the Lagrangian
  - (c) States evolve according to the interaction part of the Hamiltonian, fields evolve according to the free part of the Hamiltonian
  - (d) States obey the Heisenberg equation of motion

5. Each vertex in a Feynman diagram requires the addition of

- (a) One factor of the coupling constant  $-ig$
- (b) One factor of the coupling constant  $-g$
- © One factor of the coupling constant  $-ig^2$
- (d) One factor of the coupling constant  $-ig$

6. What number is the coupling constant for quantum electrodynamics related to?

## Capítulo 8: Electrodinámica cuántica

1. Compute  $\int d^4x D_{\mu\nu}$ .

2. The Lagrangian of quantum electrodynamics can be best described as (a) Admitting a local  $U(1)$  symmetry (b) Admitting a global  $U(1)$  symmetry © Admitting a local  $SU(2)$  symmetry (d) Admitting a local  $SU(1)$  symmetry

3. Write down the amplitude for electron-positron scattering as shown in Fig. 8.11.

4. The minimal coupling prescription for the QED Lagrangian is (a)  $\mathcal{L} = \bar{\psi} \gamma^\mu \partial_\mu \psi - \frac{1}{2} F_{\mu\nu} F^{\mu\nu}$  (b)  $\mathcal{L} = \bar{\psi} \gamma^\mu \partial_\mu \psi - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - ig \bar{\psi} \gamma^\mu \psi A_\mu$  ©  $\mathcal{L} = \bar{\psi} \gamma^\mu \partial_\mu \psi - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + ig \bar{\psi} \gamma^\mu \psi A_\mu$  (d)  $\mathcal{L} = \bar{\psi} \gamma^\mu \partial_\mu \psi - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + ig \bar{\psi} \gamma^\mu \psi A_\mu$

5. In a QED process an incoming antiparticle state is written as (a)  $v(p, s)$  (b)  $u(p, s)$  ©  $\bar{u}(p, s)$  (d)  $\bar{v}(p, s)$



# Soluciones

## Capitulo 1: Física de partículas y relatividad especial

1. d

- 6. a
- 2. a
- 7. c
- 3. a
- 8. c
- 4. c
- 9. c
- 5. b
- 10. d

## Capitulo 2: Teoría de campos lagrangiana

## Capitulo 3: Una Introducción a la teoría de grupos

## Capitulo 4: Simetrías discretas y números cuánticos

- 1. c
- 4. a
- 2. a
- 5. c
- 3. d

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