

Enunciados

Capítulo 1: Física de partículas y relatividad especial

1. A quantum field

- (a) Is a field with quanta that are operators
- (b) Is a field parameterized by the position operator
- © Commutes with the Hamiltonian
- (d) Is an operator that can create or destroy particles

2. The particle generations

- (a) Are in some sense duplicates of each other, with each generation having increasing mass
- (b) Occur in pairs of three particles each
- © Have varying electrical charge but the same mass
- (d) Consists of three leptons and three quarks each

3. In relativistic situations

- (a) Particle number and type is not fixed
- (b) Particle number is fixed, but particle types are not
- © Particle number can vary, but new particle types cannot appear
- (d) Particle number and types are fixed

4. In quantum field theory

- (a) Time is promoted to an operator
- (b) Time and momentum satisfy a commutation relation
- © Position is demoted from being an operator
- (d) Position and momentum continue to satisfy the canonical commutation relation

5. Leptons experience

- (a) The strong force, but not the weak force
- (b) The weak force and electromagnetism
- © The weak force only
- (d) The weak force and the strong force

6. The number of force-carrying particles is

- (a) Equivalent to the number of generators for the field's gauge group

- (b) Random
- © Proportional to the number of fundamental matter particles involved in the interaction
- (d) Proportional to the number of generators minus one

7. The gauge group of the strong force is:

- (a) $SU(2)$
- (b) $U(1)$
- © $SU(3)$
- (d) $SU(1)$

8. Antineutrinos

- (a) Have charge -1 and lepton number 0
- (b) Have lepton number +1 and charge 0
- © Have lepton number -1 and charge 0
- (d) Are identical to neutrinos, since they carry no charge

Capítulo 2: Teoría de campos lagrangiana

Quiz

1. Find the equation of motion for a forced harmonic oscillator with Lagrangian

$$L = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} m \omega^2 x^2 + \alpha x$$

Here α is a constant.



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Quantum Field Theory Demystified

2. Consider a Lagrangian given by

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - V(\phi)$$

- Write down the field equations for this system.
 - Find the canonical momentum density $\pi(x)$.
 - Write down the Hamiltonian.
3. Consider a free scalar field with Lagrangian $\mathcal{L} = \partial_\mu \phi \partial^\mu \phi$ and suppose that the field varies according to $\phi \rightarrow \phi + \alpha$, where α is a constant. Determine the conserved current.
4. Refer to the Lagrangian for a complex scalar field Eq. (2.37). Determine the equations of motion obeyed by the fields ϕ and ϕ^\dagger .
5. Refer to Eq. (2.37) and calculate the conserved charge.
6. Consider the action $S = \frac{1}{4} \int F_{\mu\nu} F^{\mu\nu} d^4x$. Vary the potential according to $A_\mu \rightarrow A_\mu + \partial_\mu \phi$ where ϕ is a scalar field. Determine the variation in the action.

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5. Refer to Eq. (2.37) and calculate the conserved charge.
6. Consider the action $S = \int d^4x \mathcal{L}$. Vary the potential according to $V(\phi) \rightarrow V(\phi) + \delta V$ where ϕ is a scalar field. Determine the variation in the action.

Chapter 2

1. $\frac{d^2 x}{dt^2} + \omega^2 x = -\frac{\alpha}{m}$
2. (a) $\partial_\mu \partial^\mu \phi - m^2 \phi = \frac{\partial V}{\partial \phi}$
 (b) $\pi = \dot{\phi}$
 (c) $H = \int d^3x \left(\frac{1}{2} \pi^2 + \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} m^2 \phi^2 + V(\phi) \right)$
3. $J^\mu = \partial^\mu \phi$
4. Each field separately satisfies a Klein-Gordon equation, that is, $\partial_\mu \partial^\mu \phi + m^2 \phi = 0$, $\partial_\mu \partial^\mu \phi^\dagger + m^2 \phi^\dagger = 0$. To get this result apply Eq. (2.14) to the Lagrangian Eq. (2.37).
5. $Q = i \int d^3x \left(\phi^\dagger \frac{\partial \phi}{\partial t} - \phi \frac{\partial \phi^\dagger}{\partial t} \right)$
6. The action is invariant under that transformation.

Capítulo 3: Una Introducción a la teoría de grupos

Quiz

1. Consider an element of $SU(2)$ given by $U = e^{i\sigma_x \alpha/2}$. By writing down the power series expansion, write U in terms of trigonometry functions.
2. Consider $SU(3)$ and calculate $\text{tr}(\lambda_i \lambda_j)$.
3. How many casimir operators are there for $SU(2)$?
4. Write down the casimir operators for $SU(2)$.

A Lorentz transformation can be described by boost matrices with rapidity defined by $\tanh \phi = v/c$. A boost in the x direction is represented by the matrix

$$\begin{pmatrix} \cosh \phi & \sinh \phi & 0 & 0 \\ \sinh \phi & \cosh \phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

5. Find the generator K_x .

CHAPTER 3 An Introduction to Group Theory

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6. Knowing that $[K_x, K_y] = -iJ_z$, where J_z is the angular momentum operator written in four dimensions as

$$J_z = -i \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

find K_y .

7. Do pure Lorentz boosts constitute a group?

1. Consider an element of $SU(2)$ given by $U = e^{i\sigma_x \alpha/2}$. By writing down the power series expansion, write U in terms of trigonometry functions.

2. Consider $SU(3)$ and calculate $\text{tr}(\lambda_i \lambda_j)$.

3. How many casimir operators are there for $SU(2)$?

4. Write down the casimir operators for $SU(2)$. A Lorentz transformation can be described by boost matrices with rapidity defined by $\tanh \phi = v/c$. A boost in the x direction is represented by the matrix

5. Find the generator K_x

6. Knowing that $[J_i, K_j] = i\epsilon_{ijk}J_k$, where J_i is the angular momentum operator written in four dimensions as find K_y .

7. Do pure Lorentz boosts constitute a group?

Chapter 3

1. $U = \cos \alpha + i\sigma_x \sin \alpha$

2. $2\delta_{ij}$

3. 1

4. Try $\vec{\sigma}^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2$

5. $K_x = -i \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

6. $K_y = -i \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

7. No, because the algebra among the generators requires the introduction of the angular momentum operators. Therefore, Lorentz transformations together with rotations form a group.

Quiz

1. Angular momentum states transform under the parity operator as
 - (a) $P|L, m_z\rangle = -|L, m_z\rangle$
 - (b) $P|L, m_z\rangle = L|L, m_z\rangle$
 - (c) $P|L, m_z\rangle = (-1)^L|L, m_z\rangle$
 - (d) $P|L, m_z\rangle = |L, m_z\rangle$
2. The interaction Lagrangian of electromagnetism is invariant under charge conjugation if
 - (a) $CA^\mu C^{-1} = -A^\mu$
 - (b) It is not invariant under charge conjugation
 - (c) $CJ^\mu C^{-1} = J^\mu$
 - (d) $CA^\mu C^{-1} = A^\mu$
3. Parity is
 - (a) Conserved in weak and electromagnetic interactions, but is violated in the strong interaction
 - (b) Conserved in strong interactions, but is violated in weak and electromagnetic interactions
 - (c) Not conserved
 - (d) Conserved in the strong and electromagnetic interactions, but is violated in the weak interaction
4. The eigenvalues of charge conjugation are
 - (a) $c = \pm 1$
 - (b) $c = 0, \pm 1$
 - (c) $c = \pm q$
 - (d) $c = 0, \pm q$
5. An operator is antiunitary if
 - (a) $\langle A\phi | A\psi \rangle = -\langle \phi | \psi \rangle$
 - (b) $\langle A\phi | A\psi \rangle = \langle \phi | \psi \rangle$
 - (c) $\langle A\phi | A\psi \rangle = \langle \phi | \psi \rangle^*$
 - (d) $\langle A\phi | A\psi \rangle = -\langle \phi | \psi \rangle^*$

1. Angular momentum states transform under the parity operator as

- (a) $PLmLm_{zz} = -$
- (b) $PLmLLm_{zz} =$
- (c) $PLmLm_zL_z = -()$
- (d) $PLmLm_{zz} =$

2. The interaction Lagrangian of electromagnetism is invariant under charge conjugation if

- (a) $CA_{\mu\mu} = -1$
- (b) It is not invariant under charge conjugation
- © $CJ_{\mu\mu} = 1$
- (d) $CA_{\mu\mu} = 1$

3. Parity is

- (a) Conserved in weak and electromagnetic interactions, but is violated in the strong interaction
- (b) Conserved in strong interactions, but is violated in weak and electromagnetic interactions
- © Not conserved
- (d) Conserved in the strong and electromagnetic interactions, but is violated in the weak interaction

4. The eigenvalues of charge conjugation are

- (a) $c = \pm 1$
- (b) $c = \pm 01$,
- © $cq = \pm$
- (d) $cq = \pm 0$,

5. An operator is antiunitary if

Sols T4

- 1. c
 - 4. a
 - 2. a
 - 5. c
 - 3. d
-

Capitulo 5: La ecuación de Dirac

Quiz

1. Given the Lagrangian

$$\mathcal{L} = \bar{\psi}(x)[i\gamma^\mu \partial_\mu - m]\psi$$

vary $\psi(x)$ to find the equation of motion obeyed by $\bar{\psi}(x)$.

2. Calculate $\{\gamma_5, \gamma^\mu\}$.
3. Consider the solution of the Dirac equation with $E = \omega_k > 0$. Find a relationship between the u and v components of the Dirac field.
4. Find the normalization of the free space solutions of the Dirac equation using the density $\bar{\psi}\gamma^0\psi$.
5. Find S^{01} , the generator of a boost in the x direction.
6. We can introduce an electromagnetic field with a vector potential A_μ . Let the source charge be q . Using the substitution $p_\mu \rightarrow p_\mu - qA_\mu$, determine the form of the Dirac equation in the presence of an electromagnetic field.

1. Given the Lagrangian $\mathcal{L} = \bar{\psi} \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi$ vary $\psi(x)$ to find the equation of motion obeyed by $\bar{\psi}(x)$.

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Chapter 5

1. $i \frac{\partial \bar{\psi}}{\partial x^\mu} \gamma^\mu + m \bar{\psi}$
2. 0
3. $v = \frac{\vec{k} \cdot \vec{\sigma}}{\omega_k + m} u$
4. $\psi(0) = \sqrt{2m} \begin{pmatrix} u \\ v \end{pmatrix}$
5. $-\frac{i}{2} \begin{pmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix}$
6. $\gamma^\mu (i\partial_\mu - qA_\mu)\psi - m\psi = 0$

Capitulo 6: Campos escalares

Quiz

1. Given the Lagrangian

$$\mathcal{L} = \bar{\psi}(x)[i\gamma^\mu \partial_\mu - m]\psi$$

vary $\psi(x)$ to find the equation of motion obeyed by $\bar{\psi}(x)$.

2. Calculate $\{\gamma_5, \gamma^\mu\}$.
3. Consider the solution of the Dirac equation with $E = \omega_k > 0$. Find a relationship between the u and v components of the Dirac field.
4. Find the normalization of the free space solutions of the Dirac equation using the density $\bar{\psi}\gamma^0\psi$.
5. Find S^{01} , the generator of a boost in the x direction.
6. We can introduce an electromagnetic field with a vector potential A_μ . Let the source charge be q . Using the substitution $p_\mu \rightarrow p_\mu - qA_\mu$, determine the form of the Dirac equation in the presence of an electromagnetic field.

1. Compute $\langle 0 | \hat{\psi}^\dagger(\mathbf{k}) \hat{\psi}(\mathbf{k}) | 0 \rangle$ for the real scalar field.

2. Find $\langle 0 | \hat{\psi}^\dagger(\mathbf{k}) \hat{\psi}(\mathbf{k}) | 0 \rangle$

3. Find $\langle 0 | \hat{N} | 0 \rangle$

4. Consider the complex scalar field. Determine if charge is conserved by examining the Heisenberg equation of motion for the charge operator \hat{Q} . $[\hat{Q}, H] = 0$, Do this computation by writing out the operators using Eqs. (6.68) and (6.70) and using the commutation relations Eq. (6.64)

Chapter 6

1. 0
2. $[n(\vec{k}) + 1] \hat{a}^\dagger(\vec{k}) |n(\vec{k})\rangle$
3. 0
4. $[H, Q] = 0$

Capitulo 7: Las reglas de Feynman

Quiz

1. What is the amplitude for the process shown in Fig. 7.16?

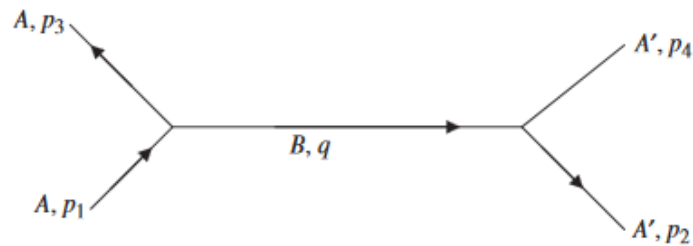


Figure 7.16 Feynman diagram for Question 1.

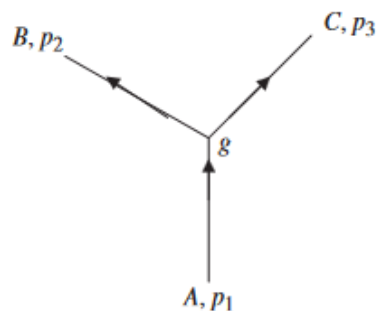


Figure 7.17 Feynman diagram for Question 1.

Figure 7.17 Feynman diagram for Question 1.

2. Find the lifetime for the decay as shown in Fig. 7.17.
3. An internal line corresponds to a spin-0 boson of mass m . The propagator is
 - (a) $i \frac{q + m}{q^2 - m^2}$
 - (b) $\frac{i}{q^2 - m^2}$
 - (c) $\frac{i}{q - m}$
 - (d) $\delta(q^2 - m^2)$
4. In the interaction picture,
 - (a) The time evolution of states is governed by the free Hamiltonian
 - (b) States are stationary, operators evolve according to the interaction part of the Lagrangian

- (c) States evolve according to the interaction part of the Hamiltonian, fields evolve according to the free part of the Hamiltonian
 - (d) States obey the Heisenberg equation of motion
5. Each vertex in a Feynman diagram requires the addition of
- (a) One factor of the coupling constant $-ig$
 - (b) One factor of the coupling constant $-g$
 - (c) One factor of the coupling constant $-ig^2$
 - (d) One factor of the coupling constant $-i\sqrt{g}$
6. What number is the coupling constant for quantum electrodynamics related to?

Quiz

1. Compute $[D_\mu, D_\nu]$.
2. The Lagrangian of quantum electrodynamics can be best described as
 - (a) Admitting a local $U(1)$ symmetry
 - (b) Admitting a global $U(1)$ symmetry
 - (c) Admitting a local $SU(2)$ symmetry
 - (d) Admitting a local $SU(1)$ symmetry
3. Write down the amplitude for electron-positron scattering as shown in Fig. 8.11.



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Quantum Field Theory Demystified

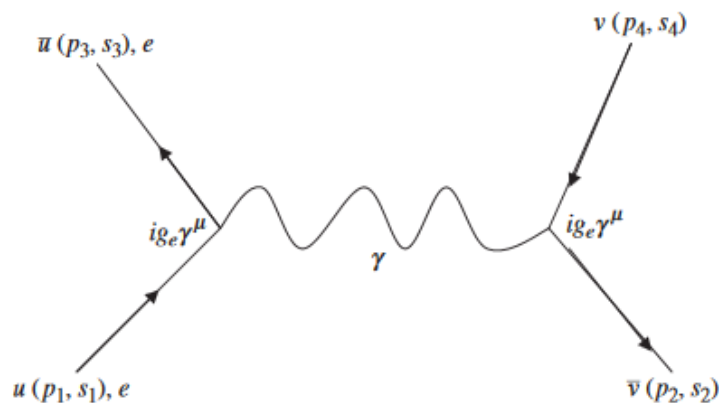


Figure 8.11 Electron-muon scattering to lowest order.

4. The minimal coupling prescription for the QED Lagrangian is
 - (a) $D_\mu = \partial_\mu + ig_e A_\mu$

4. The minimal coupling prescription for the QED Lagrangian is
 - (a) $D_\mu = \partial_\mu + ig_e A_\mu$
 - (b) $D_\mu = \partial_\mu - ig_e A_\mu$
 - (c) $D_\mu = \partial_\mu + iq A_\mu$
 - (d) $D_\mu = \partial_\mu + iq \gamma^\mu A_\mu$
5. In a QED process an incoming antiparticle state is written as
 - (a) $\bar{v}(p, s)$
 - (b) $\bar{u}(p, s)$
 - (c) $u(p, s)$
 - (d) $v(p, s)$

1. What is the amplitude for the process shown in Fig. 7.16?

2. Find the lifetime for the decay as shown in Fig. 7.17.

3. An internal line corresponds to a spin-0 boson of mass m . The propagator is

- (a) $i q m q m / + - 2 2$
- (b) $i q m 2 2 -$
- © $i q m / -$
- (d) $\delta () q m 2 2 -$

4. In the interaction picture,

- (a) The time evolution of states is governed by the free Hamiltonian
- (b) States are stationary, operators evolve according to the interaction part of the Lagrangian
- © States evolve according to the interaction part of the Hamiltonian, fields evolve according to the free part of the Hamiltonian
- (d) States obey the Heisenberg equation of motion

5. Each vertex in a Feynman diagram requires the addition of

- (a) One factor of the coupling constant $-ig$
- (b) One factor of the coupling constant $-g$
- © One factor of the coupling constant $-ig^2$
- (d) One factor of the coupling constant $-ig$

6. What number is the coupling constant for quantum electrodynamics related to?

Chapter 7

1. $-i \frac{g^2}{(p_2 - p_4)^2 - m_B^2}$
2. $\frac{1}{g^2}$
3. b
4. c
5. a
6. $\alpha = 1/137$

Chapter 8

1. $iqF_{\mu\nu}$
2. a
3. $-g_e^2 [\bar{v}(k') \gamma^\mu \bar{v}(k)] \frac{g_{\mu\nu}}{(p - p')^2} \bar{u}(p') \gamma^\mu u(p)$
4. c
5. a

Capitulo 8: Electrodinámica cuántica

1. Compute $\langle 0 | T \{ \bar{\psi}(x) \psi(y) \} | 0 \rangle$.
2. The Lagrangian of quantum electrodynamics can be best described as (a) Admitting a local $U(1)$ symmetry (b) Admitting a global $U(1)$ symmetry (c) Admitting a local $SU(2)$ symmetry (d) Admitting a local $SU(3)$ symmetry
3. Write down the amplitude for electron-positron scattering as shown in Fig. 8.11.
4. The minimal coupling prescription for the QED Lagrangian is (a) $\mathcal{L} = \bar{\psi} \gamma^\mu \psi A_\mu$ (b) $\mathcal{L} = \bar{\psi} \gamma^\mu \psi \partial_\mu A$ (c) $\mathcal{L} = \bar{\psi} \gamma^\mu \psi \partial_\mu A + \bar{\psi} \gamma^\mu \psi \partial_\mu A$ (d) $\mathcal{L} = \bar{\psi} \gamma^\mu \psi \partial_\mu A + \bar{\psi} \gamma^\mu \psi \partial_\mu A$
5. In a QED process an incoming antiparticle state is written as (a) $\bar{v}(p)$ (b) $\bar{u}(p)$ (c) $\bar{u}(p)$ (d) $\bar{v}(p)$

Soluciones

Capitulo 1: Física de partículas y relatividad especial

1. d

- 6. a
 - 2. a
 - 7. c
 - 3. a
 - 8. c
 - 4. c
 - 9. c
 - 5. b
 - 10. d
-

Capitulo 2: Teoría de campos lagrangiana

Capitulo 3: Una Introducción a la teoría de grupos

Capitulo 4: Simetrías discretas y números cuánticos

- 1. c
- 4. a
- 2. a
- 5. c
- 3. d

Capitulo 5: La ecuación de Dirac

Capitulo 6: Campos escalares

Chapter 6

1. 0
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3. 0
4. $[H,Q]=0$

Capitulo 7: Las reglas de Feynman

Capitulo 8: Electrodinámica cuántica