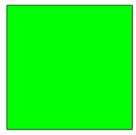
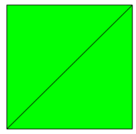
DEE-34306 Numerical Modeling of Electrical Machines December 9 2020
Exam

1. You are modelling a radial-flux electrical machine with 2-D finite element analysis using different types of meshes. Symmetry sectors are not used, but the whole cross section is modelled. The Dirichlet boundary corresponds to the outer boundary of the stator. The following arrays are used for storing the mesh data:
   - Matrix ***C*** containing the nodal point coordinates
   - Matrix ***E*** containing the elements
   - Vector ***d*** containing the indices of the nodes which lie on the Dirichlet boundary

   What are the sizes of ***C***, ***E*** and ***d*** (number of rows/columns) in the following cases:





   a) The mesh consists of 12128 linear quadrilateral elements (top figure) with 3140 nodes and 6168 edges, of which 97 edges lie on Dirichlet boundary.
   b) The quadrilateral elements are split in half to obtain a mesh with linear triangular elements (bottom figure).
   c) The interpolation order is increased so that the linear triangular elements are changed to quadratic ones.

2. You are modelling the eddy-current distribution in a conducting sheet with a 1-D finite element model using quadratic elements and a magnetic vector potential formulation given by

$$\frac{\partial}{\partial z}\left(\nu \frac{\partial A(z,t)}{\partial z}\right) - \sigma \frac{\partial A(z,t)}{\partial t} = 0$$

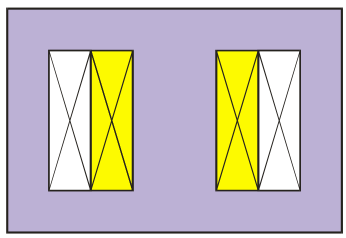
   How many Gauss integration points per element are needed (at least) to calculate the stiffness matrix? How about for calculating the damping matrix related to the eddy currents? Justify your answer. The reluctivity ν and conductivity σ of the material are constant.

3. A solution for the system of equations

$$\begin{cases} x = \cos y \\ e^x = y \end{cases}$$

   is sought iteratively using the Newton-Raphson method. At iteration step i, an approximate solution (x_i, y_i) has been obtained. Derive the matrix equation for the next solution (x_{i+1}, y_{i+1}).

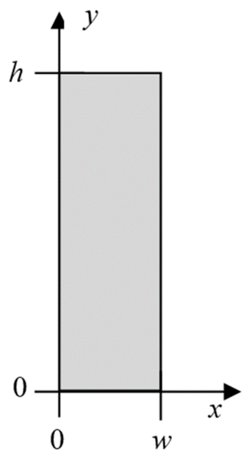
4. Your task is to model a single-phase transformer using a magnetic vector potential formulation. The magnetic field is assumed to be two-dimensional. The basic geometry is shown in the figure.



   a) How can you apply symmetry when modelling this transformer?
   b) How would you define the boundary conditions?
   c) How would you formulate the problem when the coils are supplied by sinusoidal voltage and the core material is assumed to be linear?
   d) How would you formulate the problem when the coils are supplied by sinusoidal current and the core material is assumed to be nonlinear?

   The eddy currents in the windings are assumed to be small.

5. You have modeled the magnetic field in a steel sheet shown on the right using a magnetic vector potential formulation. The dimensions of the sheet are w = 30 mm, h = 300 mm, and the thickness (in z-direction) is d = 0.5 mm. The solution of the vector potential is $A(x, y, t) = ax/w \cdot \sin(\omega t)$, where a = 0.0450 Wb/m and $\omega = 2\pi \cdot 100$ Hz.



You are now interested in calculating the power losses which occur in the sheet. The local power-loss density (in W/kg) can be divided into hysteresis (hy) and eddy-current (ed) losses as $p_{\mathrm{Fe}} = c_{\mathrm{hy}} f B^2 + c_{\mathrm{ed}} f^2 B^2$, where f is the frequency and B is the amplitude of a sinusoidally varying flux density. From earlier measurements for the same material, you know the following:

at B = 1 T and f = 10 Hz, the loss density is p_{Fe} = 0.6 W/kg

at B = 1 T and f = 50 Hz, the loss density is p_{Fe} = 5 W/kg

The mass density of the material is 7800 kg/m^3. What are the hysteresis, eddy-current and total power losses (in watts or milliwatts) in the sheet in the case of your field solution?