Multimedia

§5 Data reduction

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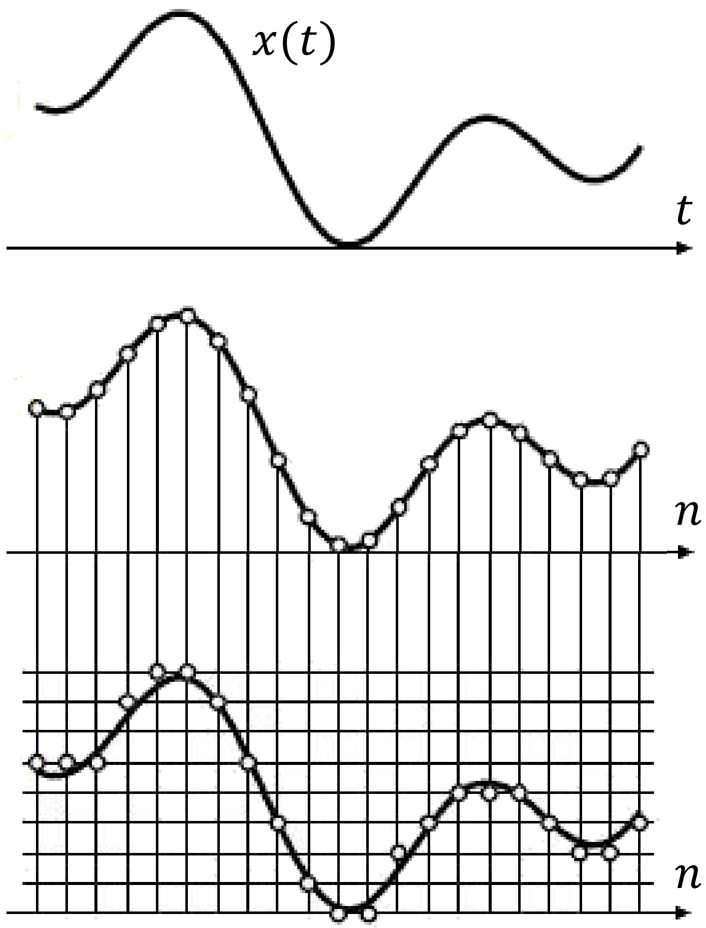
Content

§5.1 Basics

From analog to digital signals

- Analog signal $x(t)$



- **Sampling**
  - Time (or space) is discretized
- **Quantization**
  - Values are discretized

Confusion of terms in different applications

	Audio	Graphics/Video/Image
Sampling	Is called sampling	Is called resolution
Quantization	Is called resolution	Is called color depth

Example: Typical setting for audio data

Quantization	Application
8-12 bit	PC
16 bit	CD/PC
24 bit	Sound studio

Sampling rate	Application
8 kHz	Video conference, ISDN
11 kHz	PC (Voice & Games)
22 kHz	PC (Voice & Music)
32 kHz	Broadcasting
44,1 kHz	CD, digital video
48 kHz	DAT, DVD
96-192 kHz	Sound studio, DVD, dolby-surround

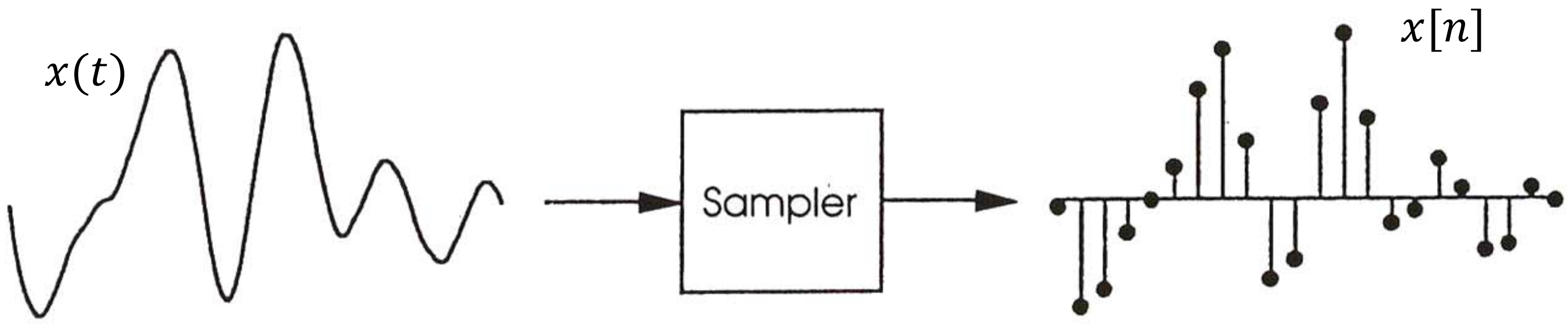
- Data size for a 60 minute stereo recording:

$$\underbrace{44{,}1\text{ kHz}}_{\text{Sampling rate}} \cdot \underbrace{2}_{\text{Stereo}} \cdot \underbrace{16\text{ bit}}_{\text{Quantization}} \cdot \underbrace{60 \cdot 60\text{ sec}}_{\text{Duration}} = 5{,}1\text{ Gbit} = 635\text{ MB}$$

Content

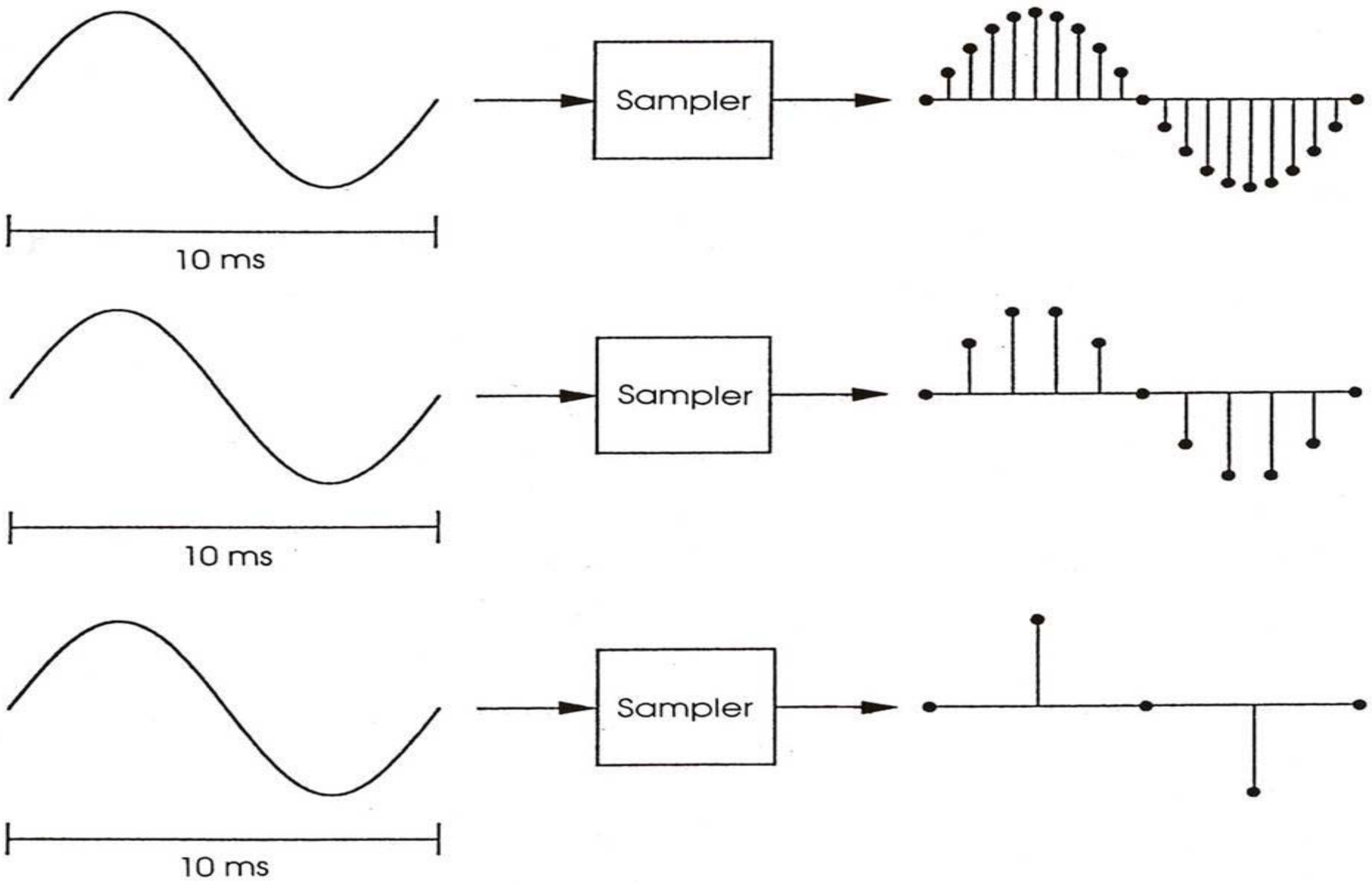
Sampling

- **Input:** Time- and value-continuous (band-limited) signal $x(t)$, that contains only frequencies up to a maximal frequency $f_{\max}$.
- **Output:** Time discrete and value continuous signal $x[n]$.
- **Sampling rate, sampling frequency** = number of samples per second



§5.2 Sampling and sampling theorem

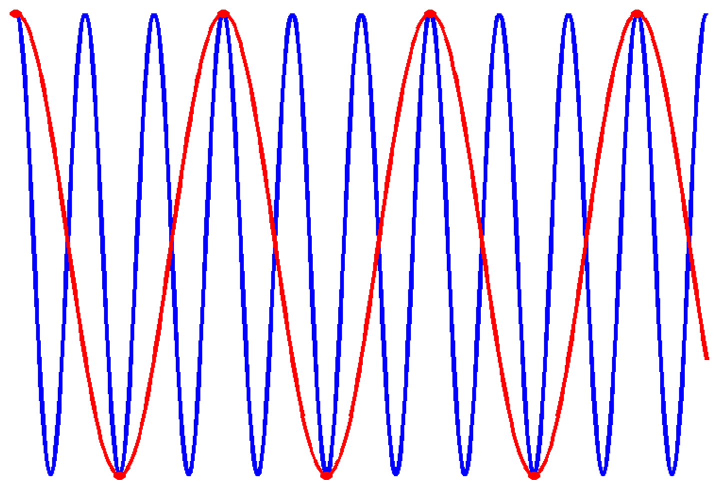
Sampling frequency



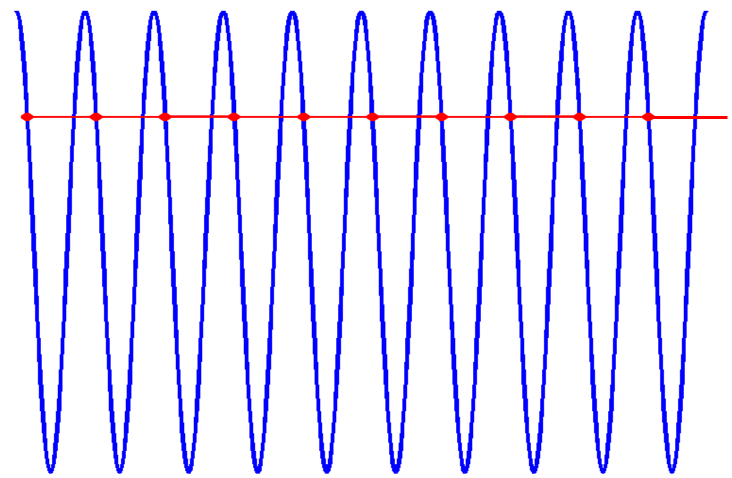
§5.2 Sampling and sampling theorem

Reconstruction from samples (1)

- $f_{\text{sample}} < f_{\max}$
  - Example: $f_{\text{sample}} = \frac{1}{3} f_{\max}$

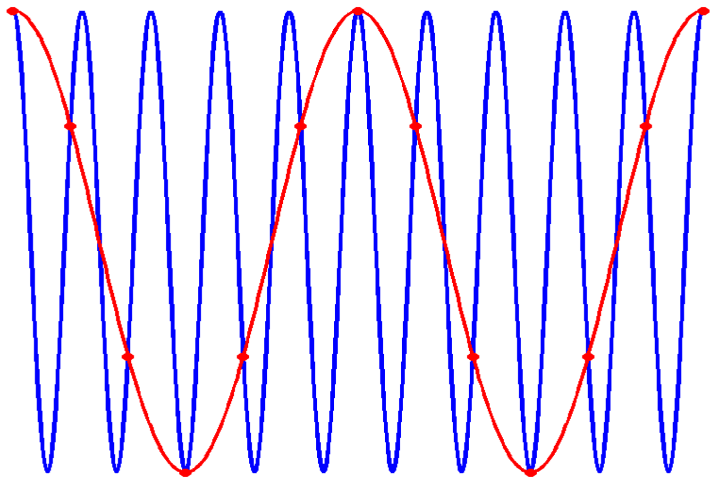


- $f_{\text{sample}} = f_{\max}$

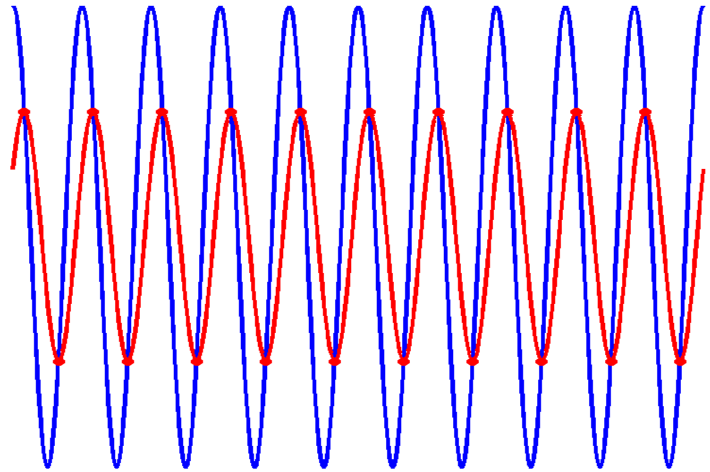


Reconstruction from samples (2)

- $f_{\max} < f_{\text{sample}} < 2 f_{\max}$
  - Example: $f_{\text{sample}} = \frac{6}{5} f_{\max}$

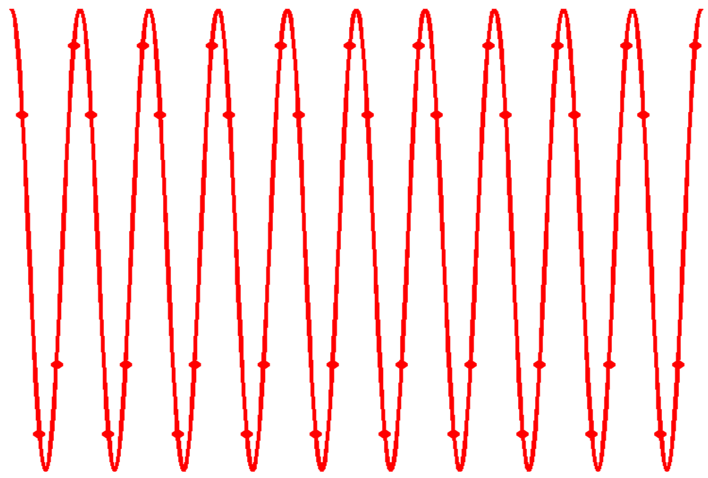


- $f_{\text{sample}} = 2 f_{\max}$



Reconstruction from samples (3)

- $f_{\text{sample}} > 2f_{\max}$
  - Example: $f_{\text{sample}} = 4f_{\max}$



Sampling theorem (Nyquist/Shannon) (1)

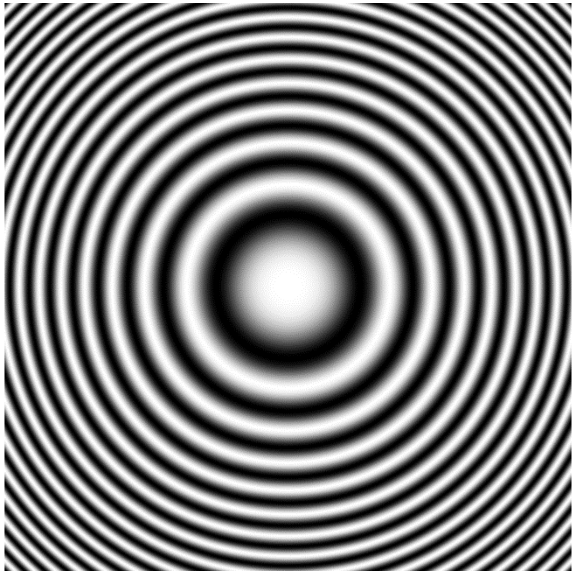
- An analog (band-limited) signal $x(t)$ can be perfectly reconstructed from the samples, if the sampling frequency f_{sample} is larger than the double maximal signal frequency $f_{\max}$

$$f_{\text{sample}} > 2 \cdot f_{\max}\,.$$

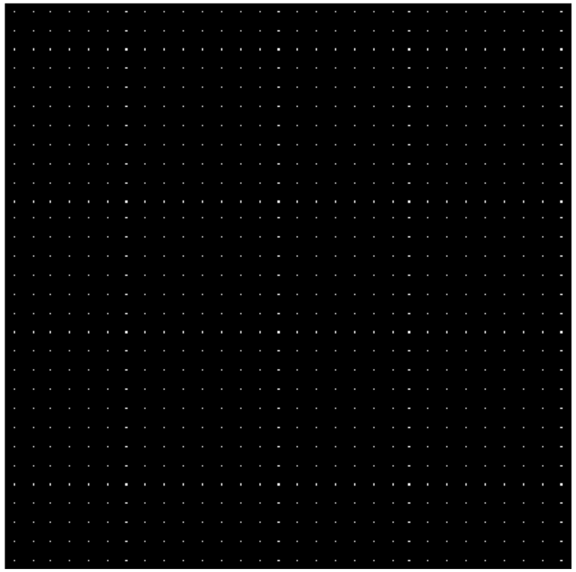
  - ➡ Otherwise aliasing will occur, e.g. high frequencies will be interpreted as low frequencies.
    - ➡ Signal needs to be filtered with an **ideal low-pass filter** before the sampling.
  - ➡ Samples have to be exact.
    - ➡ **Ideal sampling** is necessary.

Sampling theorem (Nyquist/Shannon) (2)

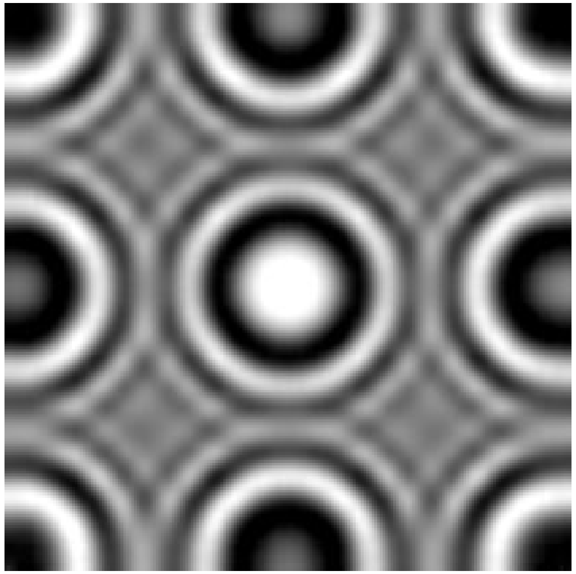
- **Example 1:** For speech recognition the sampling frequency is usually 16 kHz, because speech signals contain only frequencies up to 7 kHz.
- **Example 2:**



Cosine-circle pattern



30x30 samples



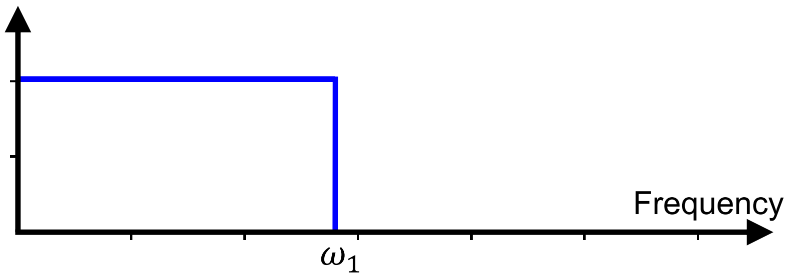
Reconstruction

Source: Wikipedia

§5.2 Sampling and sampling theorem

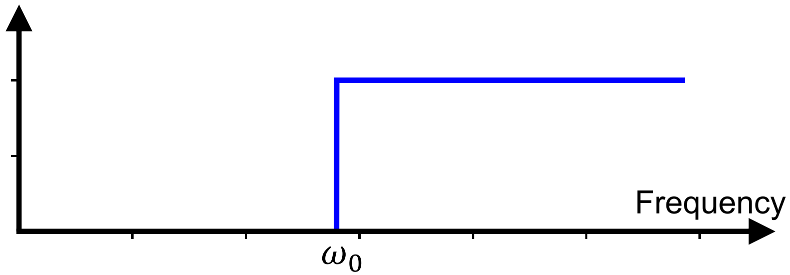
Ideal low-/high-pass filter (1)

- The **transfer function** characterizes the shape of the filter in the frequency domain.
- **Low-pass filter:** Frequencies **larger** than a limit frequency ω_1 are removed, lower frequencies can pass the filter.
  - ➡ The transfer function has the form: $H(\omega) = \begin{cases} 1, \text{for } |\omega| \leq \omega_1 \\ 0, \text{for } |\omega| > \omega_1 \end{cases}$.



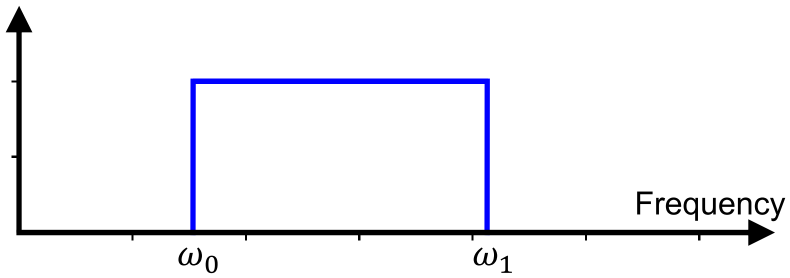
Ideal low-/high-pass filter (2)

- **High-pass filter:** Frequencies **smaller** than a limit frequency ω_0 are removed, higher frequencies can pass the filter.
  - ➡ The transfer function has the form: $H(\omega) = \begin{cases} 1, \text{for } |\omega| \geq \omega_0 \\ 0, \text{for } |\omega| < \omega_0 \end{cases}$.



- **Band-pass filter:** Frequencies **outside** a frequency band $[\omega_0, \omega_1]$ are removed, frequencies inside the band can pass the filter.
  - ➡ The transfer function has the form:

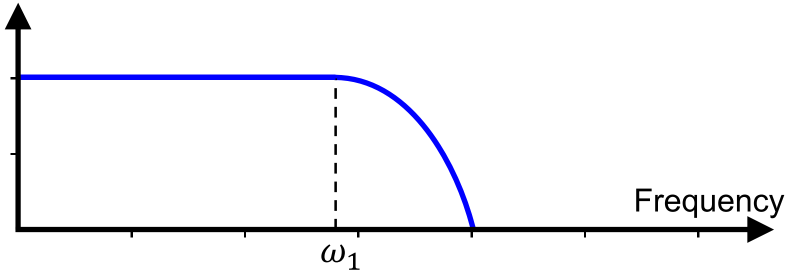
$$H(\omega) = \begin{cases} 1, \text{for } |\omega| \in [\omega_0, \omega_1] \\ 0, \text{for } |\omega| \notin [\omega_0, \omega_1] \end{cases}.$$



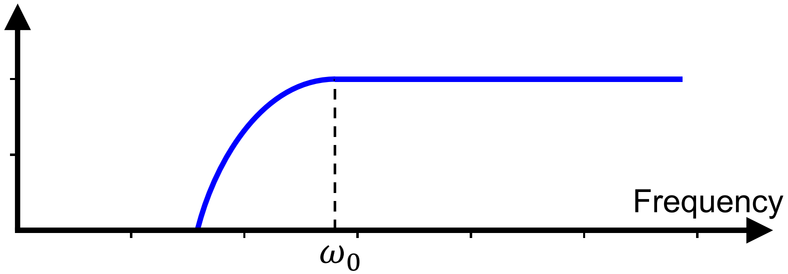
Real low-/high-pass filter (1)

➡ Real **transfer functions** do not have vertical flanks.

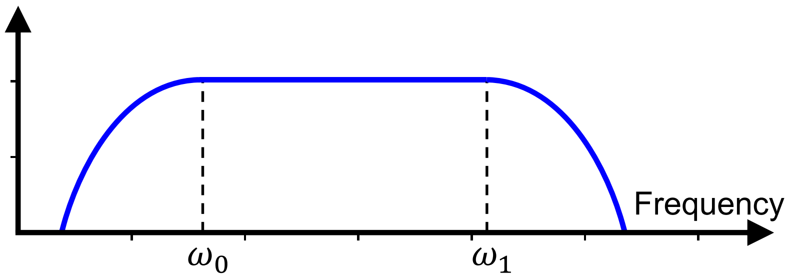
- **Low-pass filter:** Frequencies **above** a limit frequency ω_1 are **damped**, lower frequencies can pass the filter.



- **High-pass filter:** Frequencies **below** a limit frequency ω_0 are **damped**, higher frequencies can pass the filter.



- **Band-pass filter:** Frequencies **outside** a frequency band $[\omega_0, \omega_1]$ are **damped**, frequencies inside the band can pass the filter.



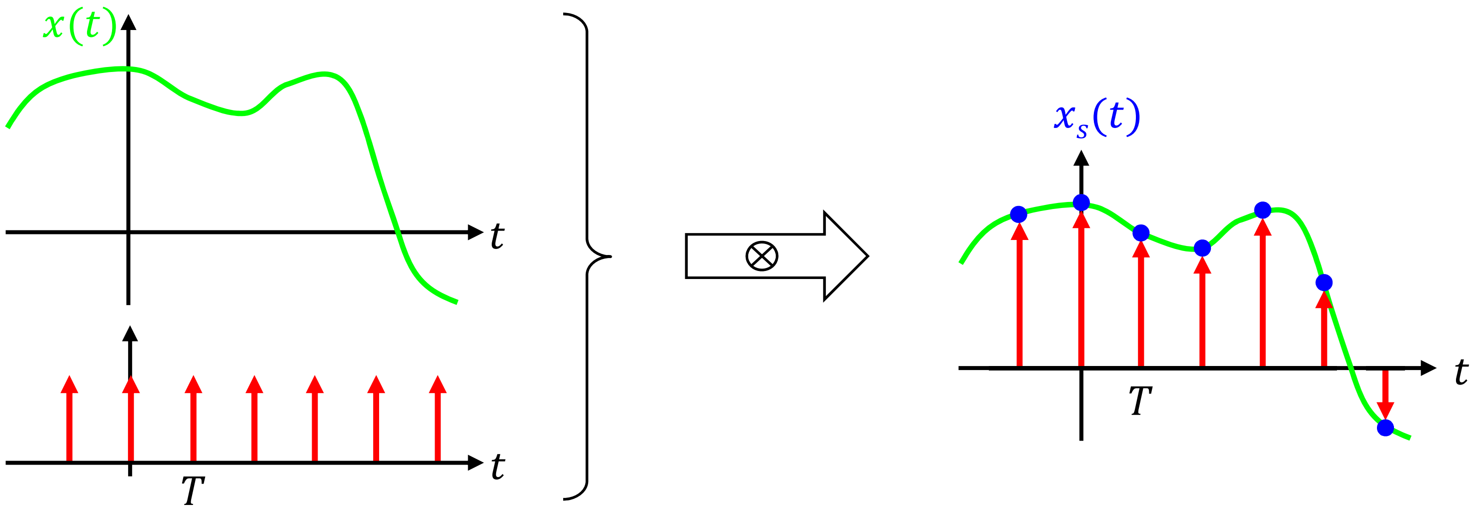
Real low-/high-pass filter (2)

- Real **transfer functions** do not have vertical flanks.
- Perfect reconstruction from the frequency space representation is not possible, because ideal low-pass filters do not exist
  - The slopes of the flanks are bounded.
  - Over-sampling.

Ideal sampling

- The signal is evaluated exactly at sampling time T.
  - Mathematically this corresponds to a multiplication of the signal $x(t)$ with a function δ, that attains its only non-zero value at time T:

$$x_s(t) = x(t) \cdot \sum_n \delta(t - nT) \quad \text{with} \quad \delta(t) = \begin{cases} 1, & \text{for } t = 0 \\ 0, & \text{otherwise} \end{cases}.$$

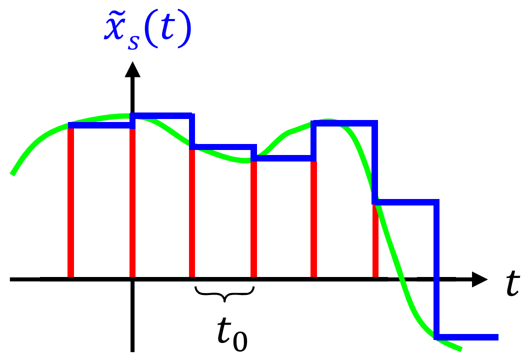


Real sampling

- Signal is accumulated over a short period of time around the ideal sampling time (sample-and-hold).
  - The Dirac-delta δ is replaced with a rectangular function ρ of width t_0

$$\tilde{x}_s(t) = \sum_n x(nT) \cdot \rho((t - nT)/t_0)$$

with $\rho(t) = \begin{cases} 1, & \text{for } t \in [0, t_0) \\ 0, & \text{otherwise} \end{cases}$.



Over-sampling

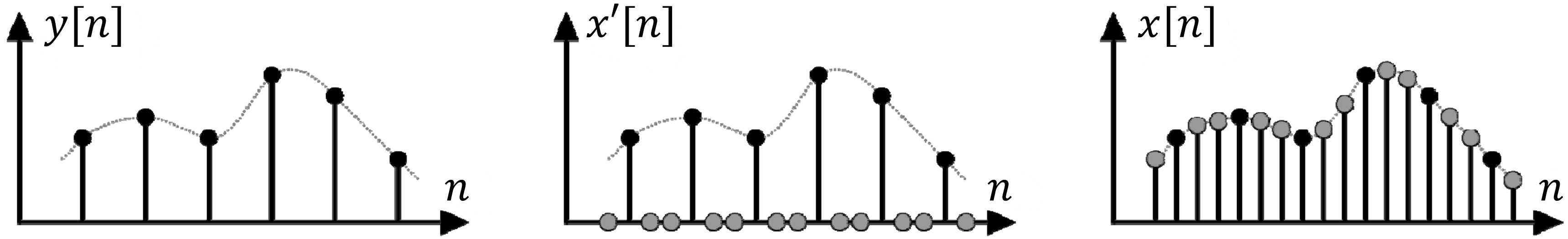
- Sampling of a signal with **more** than the double signal frequency.

Up-sampling

- Conversion of a digital signal with low sampling frequency to a digital signal with higher sampling frequency

$$x'[n] = \begin{cases} y[n/L], & \text{for } n = m \cdot L, m \in \mathbb{N} \\ 0, & \text{otherwise} \end{cases}$$

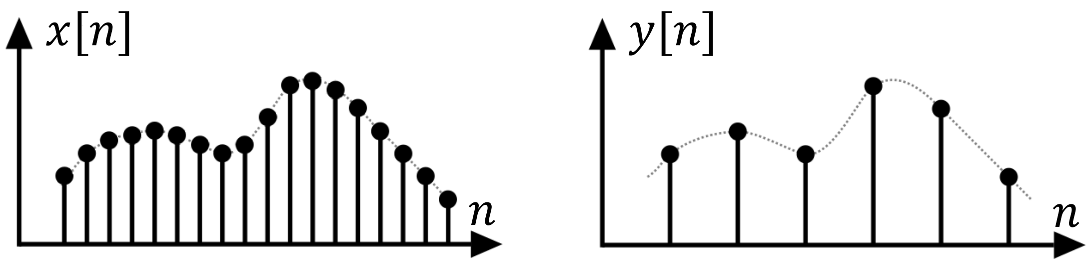
and subsequent interpolation.



Quelle: wikipedia

Sub-sampling

- Sampling of a signal with **less** than the double signal frequency.
  - Copy from signal $x[n]$ only every M-th value: $y[n] = x[n \cdot M]$.
  - ➡ The sampling frequency is decreased to $f_y = f_x / M$.
  - ➡ Might yield aliasing.



Down-sampling

- Sampling of a signal with **less** than the double signal frequency.
  - Copy from signal $x[n]$ only every M-th value: $y[n] = x[n \cdot M]$.
  - ➡ The sampling frequency is decreased to $f_y = f_x/M$.
  - ➡ Use band-pass filter to limit frequency $f_g = (f/2)/M$ to avoid aliasing.
  - ➡ The new values $y[n]$ are averages of the old values.
  - ➡ This approach is called **decimation**.

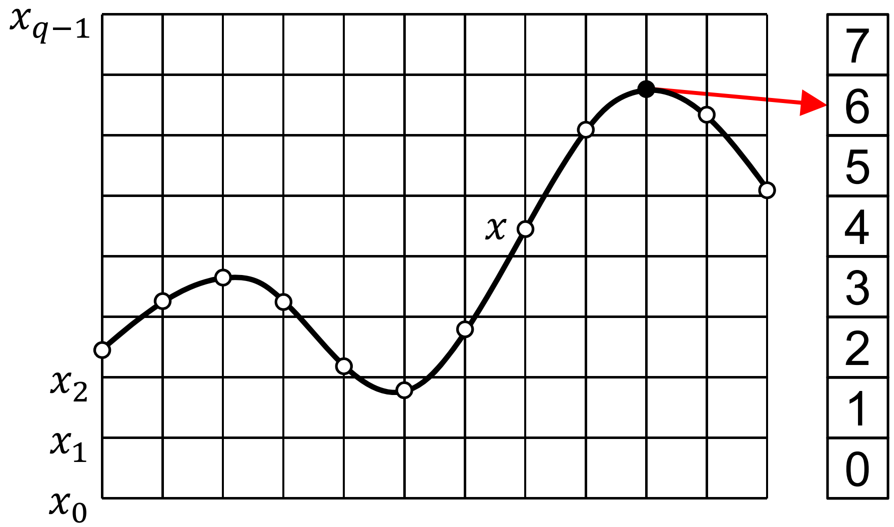
Content

§5.3 Quantization

- **Goal:**
  - Representation of continuous values on the computer
  - Reduction of irrelevant information.
- **Assumption:** Amplitude of the signals is bounded.
- **Quantization** is a two-stage mapping Q of a value-continuous signal $x \in \mathbb{R}$ to a value-discrete signal:

  1. The range of the values is subdivided into sub-intervals $[x_i, x_{i+1})$.
  2. Each signal value is mapped to its interval index:

$$Q: \mathbb{R} \to \underbrace{\{x_0, x_1, \dots, x_{q-1}\}}_{\subset \mathbb{R}} \to \mathbb{Z}.$$



§5.3 Quantization

- The values x_i are called **quantization stages**.
- They define the **quantization intervals** $[x_i, x_{i+1}), i = 0, \dots, q-2$.
- The interval index i of the interval, that contains x, is the so-called **quantization symbol**.
- The width of the quantization intervals is given by $\Delta_i = x_{i+1} - x_i$.
- The accuracy of the quantization is given by the number of quantization stages q and the width of the quantization intervals Δ_i.
  - Audio: q is called **resolution** [in bits].
  - Images: q is called **color depth** [in bits].

§5.3 Quantization

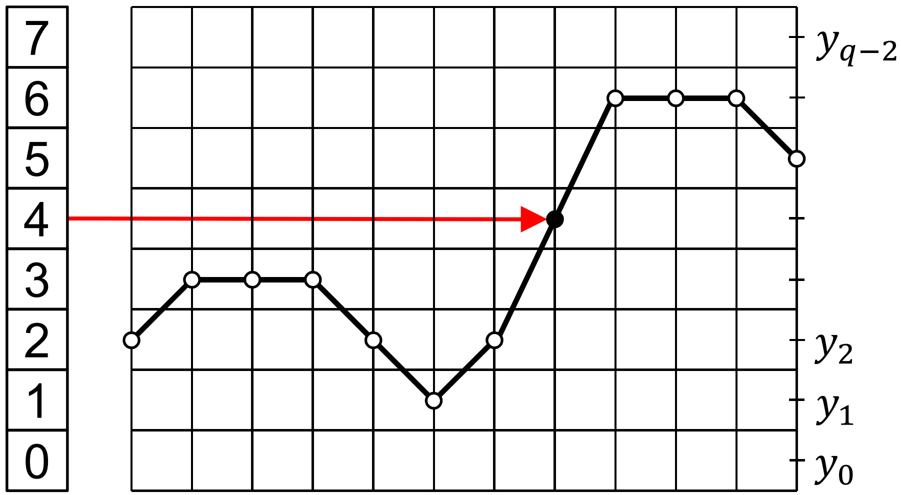
- **De-quantization** is a mapping Q^{-1} of a value-discrete signal to a value-continuous:
  - The interval index i is mapped to a representative **reconstruction value** y_i from the corresponding quantization interval

$$Q^{-1}: \mathbb{Z} \rightarrow \mathbb{R}, i \mapsto y_i \in [x_i, x_{i+1}).$$

- The **quantization error** of a signal x is given by

$$e(x) = |x - Q^{-1}(Q(x))|$$

with $Q^{-1}\big(Q(x)\big) = y_i$.



§5.3 Quantization

Uniform quantization

- **Quantization:** The signal values x are mapped to q intervals with equal widths $\Delta = x_{i+1} - x_i$, for $i = 0, \dots, q-1$, i.e.

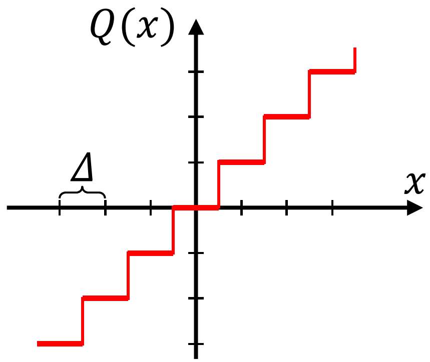
$$Q:\ x \ \mapsto\ Q(x) = \left\lfloor \frac{|x|}{\Delta} + \frac{1}{2} \right\rfloor \cdot \operatorname{sign}(x).$$

➡ All quantization intervals have the same width Δ.

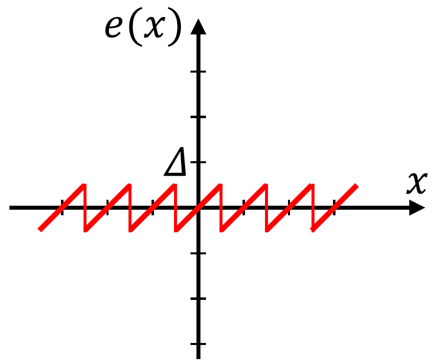
- **De-quantization:**

$$Q^{-1}(y) = y \cdot \Delta.$$

➡ The reconstruction value is the midpoint of the quantization intervals.



Characteristic curve



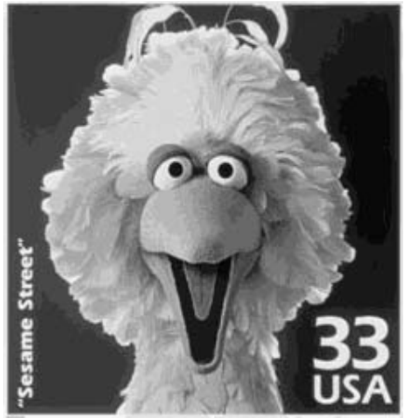
Quantization error

§5.3 Quantization

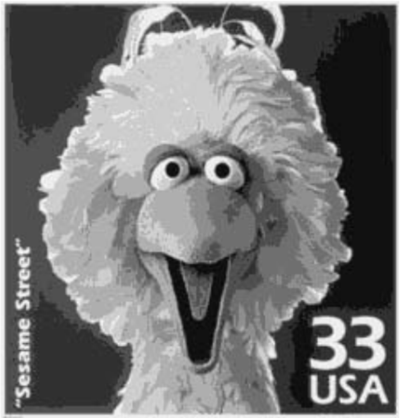
Uniform quantization (example)

256 levels

32 levels



16 levels



8 levels

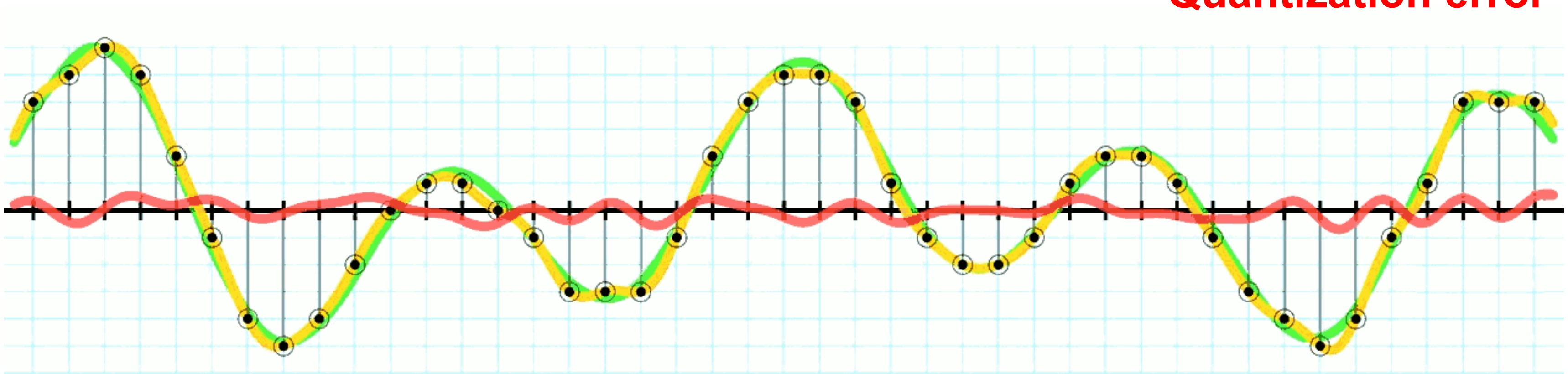
4 levels

2 levels

Source: Paul W. Cuff

§5.3 Quantization

- Because quantization is lossy, the exact de-quantization is **impossible**.
  - ➡ There is no analog to the sampling theorem for quantization .
- Because in practice every signal is noisy, exact de-quantization is **not necessary**.



Source: wikipedia

§5.3 Quantization

- **But:** If the signal x is noisy, does the information about the noise in the signal gets lost in the quantization error?
  - How do you chose the quantization, such that noise information does not get lost?
- What is the optimal ratio $\gamma = \sigma / \Delta$ of
  - signal noise (with standard deviation σ) and
  - quantization accuracy Δ?

§5.3 Quantization

- What is the optimal ratio $\gamma = \sigma / \Delta$ of signal noise (with standard deviation σ) and quantization accuracy Δ?
  - For $\gamma \ll 1$ the noise is much smaller than the quantization intervals.
    - ➡ The noise information is lost.
    - ➡ The average of the values has an error of up to $\pm\Delta/2$.
  - For $\gamma \gg 1$ the noise is much larger than the quantization intervals.
    - ➡ The noise information is preserved.
    - ➡ How fine do we have to sample the noise to capture all noise information?

§5.3 Quantization

The quantization theorem

- The distribution of a **band-limited** signal with limit frequency f can be completely reconstructed from the quantized signal, if $\frac{2\pi}{\Delta} \geq 2f$.

➡ **Results:**

- After quantization the signal contains signal noise and pseudo-noise from the quantization.
  - ➡ The pseudo-noise has variance of approximately $0.3 \cdot \Delta$.
- The optimal uniform quantization is at $\Delta \approx 2 \cdot \sigma$, i.e. $\gamma = 0.5$.
- Uniform Quantization with n bits yields a SNR of ca. $6n$ dB.
  - **Example:** CD-player quantization: 16 bit, i.e. SNR $6 \cdot 16 = 96$dB.
    - Perception up to 130 dB, depending on the spectrum and amplitude.

§5.3 Quantization

Non-uniform quantization

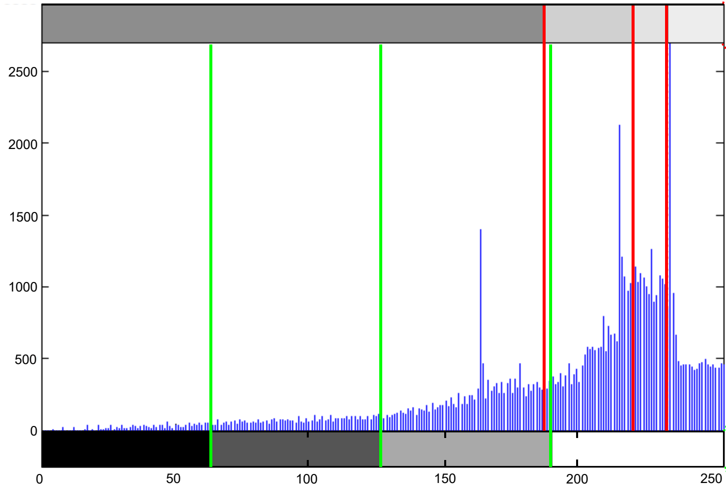
- **Perception based:** Quantization error does not correspond necessarily to perceived signal distortion.
  - Perception of distortion depends on amplitude.
  - Determine quantization intervals by experiment to minimize perceived signal distortion.

- **pdf-optimized:** Adapt interval widths to probability distribution function (pdf) to minimize quantization error energy.
  - Frequent signal values are quantized more densely.

§5.3 Quantization

Non-uniform quantization (example)

original



Histogram of original image and
non-uniform/uniform breakpoints

4 non-uniform levels

4 uniform levels

Source: Paul W. Cuff

§5.3 Quantization

Vector quantization

- Combine n values to a **feature vector** and quantize these vectors.
  1. Assign to each feature vector one vector from a table (**code book**), that is most similar to the feature vector.
  2. Store/send the index of the most similar vector.
- **Approach:**
  - **Training:** Generate code book from most frequent feature vectors.
  - **Quantization:** Determine for each vector the closest code book vector.
- **De-coder:** Requires the same code book.

Goals

- What is sampling?
- What is the difference between sampling and quantization?
- What is the essence of the sampling theorem?
- What happens if a signal is sampled using a too small/large sampling frequency?
- What is a low pass filter and what is the shape of its transfer function?
- What is quantization ?
- What is uniform quantization ?
- What is the essence of the quantization theorem?
- What is perception based quantization ?