Problem description

Given input

$L_{\max}$	Maximum length of concatenated rolls
$r_{\min}$	Factor that determines minimum length of concatenated rolls i.e. $L_{\min} = r_{\min} L_{\max}$
$a_1, \dots, a_n$	Lengths of rolls of material A
$b_1, \dots, b_m$	Lengths of rolls of material B
$d_1, \dots, d_k$	Lengths of agglutinated target rolls
$c^{\mathrm{a}} = 5$	Time in minutes to agglutinate two rolls of the same material
$c^{\mathrm{n}} = 8$	Time in minutes to put on a new target roll
$c^{\mathrm{s}} = 7$	Time in minutes paid for rolls less than $L_{\min}$
$c^{\mathrm{r}} = 7$	Time in minutes paid for unused rolls

Optimization problem

$$\min x^{\mathrm{a}}c^{\mathrm{a}} + x^{\mathrm{n}}c^{\mathrm{n}} + x^{\mathrm{s}}c^{\mathrm{s}} + x^{\mathrm{r}}c_{\mathrm{r}} \tag{1}$$

s.t.

Restriction on the length of the resulting rolls (2)

$$\forall i = 1, \ldots, k: \; c_i \leq L_{\max} \tag{3}$$

$$\textstyle\sum_{i=1}^{k} d_i \leq \sum_{i=1}^{n} a_i \tag{4}$$

$$\textstyle\sum_{i=1}^{k} d_i \leq \sum_{i=1}^{m} b_i \tag{5}$$

Each resulting roll contains the same length of material A and B

$$\forall i = 1, \ldots, k, \; j = 1, \ldots, n: \; x_{ij} \in [0,1] \tag{6}$$

$$\forall i = 1, \ldots, k, \; j = 1, \ldots, m: \; y_{ij} \in [0,1] \tag{7}$$

$$\forall i = 1, \ldots, k: \; d_i = \textstyle\sum_{j=1}^{n} x_{ij}^{a} a_j \tag{8}$$

$$\forall i = 1, \ldots, k: \; d_i = \textstyle\sum_{j=1}^{m} x_{ij}^{b} b_j \tag{9}$$

Each roll can only be used to 100%

$$\forall i = 1, \ldots, n: \; \textstyle\sum_{j=1}^{k} x_{ij} \leq 1 \tag{10}$$

$$\forall i = 1, \ldots, m: \; \textstyle\sum_{j=1}^{k} y_{ij} \leq 1 \tag{11}$$

Binary variables from continuous variables

$$\forall i = 1, \ldots, k, \; j = 1, \ldots, n: \; y_{ij}^{a} \geq x_{ij}^{a}, \; y_{ij}^{a} \in \{0,1\} \tag{12}$$

$$\forall i = 1, \ldots, k, \; j = 1, \ldots, m: \; y_{ij}^{b} \geq x_{ij}^{b}, \; y_{ij}^{b} \in \{0,1\} \tag{13}$$

A roll is only used for subsequent rolls

$$\forall i = 1, \ldots, k, \; j = 1, \ldots, n: \; s_{ij}^{a}, t_{ij}^{a} \in \{0,1\} \tag{14}$$

$$s_{ij}^{a} \geq \tfrac{1}{2k} \textstyle\sum_{j'=1}^{j} y_{ij'}^{a} \tag{15}$$

$$t_{ij}^{a} \geq \tfrac{1}{2k} \textstyle\sum_{j'=j}^{n} y_{ij'}^{a} \tag{16}$$

$$y_{ij}^{a} \geq s_{ij}^{a} + t_{ij}^{a} - 1 \tag{17}$$

$$\forall i = 1, \ldots, k, \; j = 1, \ldots, m: \; s_{ij}^{b}, t_{ij}^{b} \in \{0,1\} \tag{18}$$

$$s_{ij}^{b} \geq \tfrac{1}{2k} \textstyle\sum_{j'=1}^{j} y_{ij'}^{b} \tag{19}$$

$$t_{ij}^{b} \geq \tfrac{1}{2k} \textstyle\sum_{j'=j}^{m} y_{ij'}^{b} \tag{20}$$

$$y_{ij}^{b} \geq s_{ij}^{b} + t_{ij}^{b} - 1 \tag{21}$$

Number of resulting rolls

$$\forall i = 1, \ldots, k: \; z_i^{n} \geq \tfrac{1}{n} \textstyle\sum_{j=1}^{n} y_{ij}^{a}, \; z_i \in \{0,1\} \tag{22}$$

$$x^{n} = \textstyle\sum_{i=1}^{k} z_i^{n} \tag{23}$$

Number of agglutinations

$$\forall i = 1, \ldots, k: \; z_i^{a,a} = \textstyle\sum_{j=1}^{n} y_{ij}^{a} - 1, \; z_i^{a,a} \in \mathbb{Z} \tag{24}$$

$$\forall i = 1, \ldots, k: \; z_i^{a,b} = \textstyle\sum_{j=1}^{n} y_{ij}^{b} - 1, \; z_i^{a,b} \in \mathbb{Z} \tag{25}$$

$$x^{a} = \textstyle\sum_{i=1}^{k} z_i^{a,a} + z_i^{a,b} \tag{26}$$

Number of rolls with lengths lass than $L_{\min}$

$$\forall i = 1, \ldots, k: \; z_i^{s} \geq \tfrac{1}{L_{\max}} (L_{\min} - c_i), \; z_i^{s} \in \{0,1\} \tag{27}$$

$$x^{s} = \textstyle\sum_{i=1}^{k} z_i^{s} - (k - x^{n}) \tag{28}$$

Number of not completely used rolls

$$\forall j = 1, \ldots, n z_j^{r,a} \geq 1 - \textstyle\sum_{i=1}^{k} x_{ij}^{a}, \; z_j^{r,a} \in \{0,1\} \tag{29}$$

$$\forall j = 1, \ldots, m z_j^{r,b} \geq 1 - \textstyle\sum_{i=1}^{k} x_{ij}^{b}, \; z_j^{r,b} \in \{0,1\} \tag{30}$$

$$x^{r} = \textstyle\sum_{j=1}^{n} z_j^{r,a} + \sum_{j=1}^{m} z_j^{r,b} \tag{31}$$