

Problem description

Given input

L_{\max}	Maximum length of concatenated rolls
r_{\min}	Factor that determines minimum length of concatenated rolls i.e. $L_{\min} = r_{\min} L_{\max}$
a_1, \dots, a_n	Lengths of rolls of material A
b_1, \dots, b_m	Lengths of rolls of material B
d_1, \dots, d_k	Lengths of agglutinated target rolls
$c^a = 5$	Time in minutes to agglutinate two rolls of the same material
$c^n = 8$	Time in minutes to put on a new target roll
$c^s = 7$	Time in minutes paid for rolls less than L_{\min}
$c^r = 7$	Time in minutes paid for unused rolls

Optimization problem

$$\min x^a c^a + x^n c^n + x^s c^s + x^r c^r \quad (1)$$

s.t.

$$\text{Restriction on the length of the resulting rolls} \quad (2)$$

$$\forall i = 1, \dots, k : c_i \leq L_{\max} \quad (3)$$

$$\sum_{i=1}^k d_i \leq \sum_{i=1}^n a_i \quad (4)$$

$$\sum_{i=1}^k d_i \leq \sum_{i=1}^m b_i \quad (5)$$

Each resulting roll contains the same length of material A and B

$$\forall i = 1, \dots, k, j = 1, \dots, n : x_{ij} \in [0, 1] \quad (6)$$

$$\forall i = 1, \dots, k, j = 1, \dots, m : y_{ij} \in [0, 1] \quad (7)$$

$$\forall i = 1, \dots, k : d_i = \sum_{j=1}^n x_{ij} a_j \quad (8)$$

$$\forall i = 1, \dots, k : d_i = \sum_{j=1}^m x_{ij} b_j \quad (9)$$

Each roll can only be used to 100%

$$\forall i = 1, \dots, n : \sum_{j=1}^k x_{ij} \leq 1 \quad (10)$$

$$\forall i = 1, \dots, m : \sum_{j=1}^k y_{ij} \leq 1 \quad (11)$$

Binary variables from continuous variables

$$\forall i = 1, \dots, k, j = 1, \dots, n : y_{ij}^a \geq x_{ij}^a, y_{ij}^a \in \{0, 1\} \quad (12)$$

$$\forall i = 1, \dots, k, j = 1, \dots, m : y_{ij}^b \geq x_{ij}^b, y_{ij}^b \in \{0, 1\} \quad (13)$$

A roll is only used for subsequent rolls

$$\forall i = 1, \dots, k, j = 1, \dots, n : s_{ij}^a, t_{ij}^a \in \{0, 1\} \quad (14)$$

$$s_{ij}^a \geq \frac{1}{2k} \sum_{j'=1}^j y_{ij'}^a \quad (15)$$

$$t_{ij}^a \geq \frac{1}{2k} \sum_{j'=j}^n y_{ij'}^a \quad (16)$$

$$y_{ij}^a \geq s_{ij}^a + t_{ij}^a - 1 \quad (17)$$

$$\forall i = 1, \dots, k, j = 1, \dots, m : s_{ij}^b, t_{ij}^b \in \{0, 1\} \quad (18)$$

$$s_{ij}^b \geq \frac{1}{2k} \sum_{j'=1}^j y_{ij'}^b \quad (19)$$

$$t_{ij}^b \geq \frac{1}{2k} \sum_{j'=j}^m y_{ij'}^b \quad (20)$$

$$y_{ij}^b \geq s_{ij}^b + t_{ij}^b - 1 \quad (21)$$

Number of resulting rolls

$$\forall i = 1, \dots, k : z_i^n \geq \frac{1}{n} \sum_{j=1}^n y_{ij}^a, z_i^n \in \{0, 1\} \quad (22)$$

$$x^n = \sum_{i=1}^k z_i^n \quad (23)$$

Number of agglutinations

$$\forall i = 1, \dots, k : z_i^{a,a} = \sum_{j=1}^n y_{ij}^a - 1, z_i^{a,a} \in \mathbb{Z} \quad (24)$$

$$\forall i = 1, \dots, k : z_i^{a,b} = \sum_{j=1}^m y_{ij}^b - 1, z_i^{a,b} \in \mathbb{Z} \quad (25)$$

$$x^a = \sum_{i=1}^k z_i^{a,a} + z_i^{a,b} \quad (26)$$

Number of rolls with lengths less than L_{\min}

$$\forall i = 1, \dots, k : z_i^s \geq \frac{1}{L_{\max}} (L_{\min} - c_i), z_i^s \in \{0, 1\} \quad (27)$$

$$x^s = \sum_{i=1}^k z_i^s - (k - x^n) \quad (28)$$

Number of not completely used rolls

$$\forall j = 1, \dots, n : z_j^{r,a} \geq 1 - \sum_{i=1}^k x_{ij}^a, z_j^{r,a} \in \{0, 1\} \quad (29)$$

$$\forall j = 1, \dots, m : z_j^{r,b} \geq 1 - \sum_{i=1}^k x_{ij}^b, z_j^{r,b} \in \{0, 1\} \quad (30)$$

$$x^r = \sum_{j=1}^n z_j^{r,a} + \sum_{j=1}^m z_j^{r,b} \quad (31)$$