

归结算法实验报告

Honor Code

- 同王浩算法的课题，参考了互联网上对表达式进行解析的思路和数据结构。
- 获得了程设助教在一个关于深拷贝与浅拷贝的bug上的帮助。

1.表达式解析

与王浩算法的实现中的实现一模一样，麻烦老师批改的时候翻一下王浩算法的实验报告。

2.表达式化简

分三步将表达式化为一个合取范式。

第一步：消除蕴含连接词，按照蕴含等值式对表达式二叉树进行处理

如果是蕴含连接词，那么将节点换为 \neg ，然后拼上否定的左子树，右子树不变

```
if(treenode->val=='>') {
    treenode->val='v';
    tree* temp=new tree;
    temp->val='!';
    temp->right=treenode->left;
    treenode->left=temp;
}
```

如果是双蕴含，将节点换为 \wedge ，左侧是左子树推出右子树，右侧是右子树推出左子树。这里要注意要进行深拷贝，否则在后续处理会出现改左子树却把右子树也修改了的问题。

```
else if(treenode->val=='<') {
    treenode->val='^';
    tree* temp1=new tree;
    tree* temp2=new tree;
    tree* temp3=deepCopy(treenode);
    tree* temp4=deepCopy(treenode);
    temp1->val='>';
    temp1->left=temp3->left;
    temp1->right=temp3->right;
    temp2->val='>';
    temp2->left=temp4->right;
    temp2->right=temp4->left;
    treenode->left=temp1;
    treenode->right=temp2;
}
```

深拷贝函数如下

```

tree* deepCopy(tree* root) {
    if(root==nullptr) return nullptr;
    tree* newRoot =new tree;
    newRoot->val=root->val;
    newRoot->left=deepCopy(root->left);
    newRoot->right=deepCopy(root->right);
    return newRoot;
}

```

如果这个节点的左右子树非空，递归处理，结束之后返回根节点

```

if(treenode->left!=nullptr) {
    treenode->left=elimImp(treenode->left);
}
if(treenode->right!=nullptr) {
    treenode->right=elimImp(treenode->right);
}
return treenode;

```

第二步：否定词内移

根据DeMorgan律，将否定词内移，其中要注意双重否定的消去

```

if(treenode->val=='!') {
    if(treenode->right->val=='!') {
        treenode->val=treenode->right->right->val;
        treenode->left=treenode->right->right->left;
        treenode->right=treenode->right->right->right;
    } else if(treenode->right->val=='v') {
        treenode->val='^';
        treenode->right->val='!';
        tree* temp=new tree;
        temp->val='!';
        temp->right=treenode->right->left;
        treenode->left=temp;
        treenode->right->left=nullptr;
    } else if(treenode->right->val=='^') {
        treenode->val='v';
        treenode->right->val='!';
        tree* temp=new tree;
        temp->val='!';
        temp->left=nullptr;
        temp->right=treenode->right->left;
        treenode->left=temp;
        treenode->right->left=nullptr;
    }
}
}

```

递归处理左右子树

```
if(treenode->left!=nullptr) {
    treenode->left=pushNeg(treenode->left);
}
if(treenode->right!=nullptr) {
    treenode->right=pushNeg(treenode->right);
}
return treenode;
```

第三步：利用分配律将表达式简化为范式。寻找所有或，考虑左右，使用分配律，然后递归处理。

```
tree* distriLaw(tree* treenode) {
    if(treenode->val=='v') {
        if(treenode->left!=nullptr) {
            if(treenode->left->val=='^') {
                treenode->val='^';
                tree* temp1=new tree;
                tree* temp2=new tree;
                temp1->val='v';
                temp1->left=treenode->left->left;
                temp1->right=treenode->right;
                temp2->val='v';
                temp2->left=treenode->left->right;
                temp2->right=treenode->right;
                treenode->left=temp1;
                treenode->right=temp2;
            }
        }
        if(treenode->right!=nullptr) {
            if(treenode->right->val=='^') {
                treenode->val='^';
                tree* temp1=new tree;
                tree* temp2=new tree;
                temp1->val='v';
                temp1->left=treenode->left;
                temp1->right=treenode->right->left;
                temp2->val='v';
                temp2->left=treenode->left;
                temp2->right=treenode->right->right;
                treenode->left=temp1;
                treenode->right=temp2;
            }
        }
    }
    if(treenode->left!=nullptr) {
        treenode->left=distriLaw(treenode->left);
    }
    if(treenode->right!=nullptr) {
        treenode->right=distriLaw(treenode->right);
    }
}
```

```

    return treenode;
}

```

3.提取子句集

引入 `vector<set<atom>> clauses` 表示子句集合

其中 `atom` 定义为，表示一个原子命题

```

struct atom {
    char val;
    bool neg=false;

    //重载set使用到的的比较运算符，根据val的字典序排序
    bool operator<(const atom& other) const {
        if (val != other.val) {
            return val<other.val;
        }
        return neg>other.neg;
    }
};

```

对根节点建立字句集，函数原型为 `void constructClauses(tree* root,vector<set<atom>> clauses)`

如果是 `^`，则对两个子树分别建立子句集，放入vector中

```

if(root->val=='^') {
    constructClauses(root->left,clauses);
    constructClauses(root->right,clauses);
}

```

如果是 `v`，对两个子树分别找子句集，然后放入同一个set中，再放入vector

```

else if(root->val=='v') {
    set<atom> clause;
    vector<set<atom>> leftclauses;
    vector<set<atom>> rightclauses;
    constructClauses(root->left,leftclauses);
    constructClauses(root->right,rightclauses);
    if(!leftclauses.empty()) clause.insert(leftclauses[0].begin(), leftclauses[0].end());
    if(!rightclauses.empty()) clause.insert(rightclauses[0].begin(),
rightclauses[0].end());
    clauses.push_back(clause);
}

```

如果是根节点的 `val` 是原子命题，新建一个子句，然后放入子句集

```

else {
    set<atom> clause;
    atom tempatom;
    if(root->val=='!') {
        tempatom.neg=true;
        tempatom.val=root->right->val;
    } else {
        tempatom.val=root->val;
    }
    clause.insert(tempatom);
    clauses.push_back(clause);
}

```

到达 `nullptr` 的递归边界后返回

```

if(root==nullptr) {
    return;
}

```

4.归结推理

遍历子句中的所有原子命题，判断子句是否可以归结，记录可以归结的原子命题 `find`。

```

bool ifCanResolve(set<atom>& clause1,set<atom>& clause2,atom& find) {
    for(auto it=clause1.begin();it!=clause1.end();++it) {
        atom negIt;
        negIt.neg=(find.neg ? false : true);
        negIt.val=it->val;
        if(clause2.find(negIt)!=clause2.end()) {
            find=*it;
            return true;
        }
    }
    return false;
}

```

根据找到的可以归结的原子命题，归结出新子句。首先对子句1擦除可以归结的原子命题，然后插入子句2中的原子命题

```

set<atom> resolve(set<atom>& clause1,set<atom>& clause2,atom& find) {
    set<atom> result=clause1;
    result.erase(find);
    for(auto it=clause2.begin();it!=clause2.end();++it) {
        if(find.neg) {
            if(!(it->neg&&it->val==find.val)) {
                result.insert(*it);
            }
        } else {
            if(!(it->neg&&it->val==find.val)) {

```

```

        result.insert(*it);
    }
}
return result;
}

```

引入 `set<set<atom>> beenInVector`，表示已经放在子句集中的子句

通过两重循环遍历所有子句，进行归结推理

如果能够找到，进行一次归结，判断归结出的子句是否为空，如果为空，`return true`，找到空子句，命题正确。

反之如果不为空，判断是否已经出现过，如果没有出现过，就加入子句集，同时把 `newClauseFound` 改为真

```

for(int i=0;i<clauses.size();++i) {
    for(int j=0;j<clauses.size();++j) {
        if(j==i) continue;
        atom find;
        if(ifCanResolve(clauses[i],clauses[j],find)) {
            set<atom> newClause=resolve(clauses[i],clauses[j],find);
            if (newClause.empty()) {
                return true;
            }
            if (beenInVector.find(newClause)==beenInVector.end()) {
                clauses.push_back(newClause);
                beenInVector.insert(newClause);
                newClauseFound=true;
            }
        }
    }
}
}

```

如果最后不能找到新子句，归结无法进行，`return false`，命题无法证明

```

if (clauses.size()==iniSize) {
    return false;
}

```

5.输入处理逻辑

与王浩算法不同，归结推理是直接处理推理的，我们不能要求用户输入一个完整的命题，因此我们采用如下输入方式。

我们首先让用户输入条件，在真实情景中，条件可以有很多个。同时用户可能想判断一个命题是不是重言式，因此可以不输入条件，直接在之后输入结论。用户输入结束之后程序 `cout<<"Continue or END:"`；用户可以继续输入，或者输入 `END` 来结束输入。

我们引入变量 `emptyReturn`，对每一个条件输入，如果确实是一个空的输入，那我们不认为这是一个错误输入，直接放过。如果不是一个空的条件输入，但是却返回了一个空子句，那么表明这是一个错误输入，程序终止。

我们引入变量 `notEmptyReturn`，只要有不是一个空的条件输入，那么我们就把这变量的值改为真。

第一次输入时，也就是说 `preconditionInput` 为空，那么直接把输入赋给 `preconditionInput`。反之，如果不为空，那么我们需要把已有的条件和新输入的条件拼起来。

然后让用户输入结论，与条件不同，结论有且只有一个。这里我们采取与王浩算法完全相同的错误判定方法，直接根据返回的树是否是空指针来判断输入的正确性，如果输入错误，程序直接 `cout<<"Empty Input or Not Well-Formed Formula";`，然后返回。

接下来程序需要将条件和结论拼起来，即 `条件 ^ !结论`，考虑到用户可能想直接证明是不是重言式，如果 `notEmptyReturn` 为假，表示没有输入任何一个条件，归结中要用到的命题就直接是 `! 结论`。

最后程序就调用 `simplify` 函数进行化简，其中依次调用 `elimImp`，`pushNeg`，`distriLaw` 进行化简。并用 `constructClauses` 从化简的归结使用的命题中提取子句集。

最后的最后，进行归结推理。

6.实验结果

- 输入测试

- 错误的命题输入，与王浩算法一样，我们就不再这里具体测试了，麻烦老师批改的时候翻一下我的王浩算法的实验报告
- 空条件输入的测试

```
Please enter precondition:END
Please enter conclusion:(a->B)->((c->D)->(a^c->B^D))
Conclusion Constructed:
(a->B)->((c->D)->((a^c)->(B^D)))
```

- 重言式（正确的命题）

这里采用与王浩算法相同的测例，由于不用输出化简过程，所以显得非常简练

```
Please enter precondition:END
Please enter conclusion:(R^(! (P->Q) -> ! (RvS)) ^ ((Q->P)v!R)) -> (P<->Q)
Conclusion Constructed:
((R^(! (P->Q) -> ! (RvS))) ^ ((Q->P)v(!R))) -> (P<->Q)
Proposition Used in Revolution:
((R^(((P)vQ)v(!R)) ^ (((P)vQ)v(!S)))) ^ (((!Q)vP)v(!R)) ^ (((PvQ) ^ (Pv(!P))) ^ (((!Q)vQ) ^ (!Q)v(!P))))
Simplified Version:
((R^(((P)vQ)v(!R)) ^ (((P)vQ)v(!S)))) ^ (((!Q)vP)v(!R)) ^ (((PvQ) ^ (Pv(!P))) ^ (((!Q)vQ) ^ (!Q)v(!P))))
Clauses Constructed:
{R}
{!P,Q,!R}
{!P,Q,!S}
{P,!Q,!R}
{P,Q}
{!P,P}
{!Q,Q}
```

```
{!P,!Q}
Empty clause found. It's true!
```

```
Please enter precondition:END
Please enter conclusion:(P->(Q->R))<->((P^Q)->R)
Conclusion Constructed:
(P->(Q->R))<->((P^Q)->R)
Proposition Used in Revolution:
((((!P)v(!Q)vR))v(((!P)v(!Q))vR))^((((!P)v(!Q)vR))vP)^((((!P)v(!Q)vR))vQ)^((((!P)v(!Q)vR))v(!R))))^(((Pv(((!P)v(!Q))vR))^((PvP)^((PvQ)^((Pv(!R))))))^(Qv(((!P)v(!Q))vR))^(QvP)^((QvQ)^((Qv(!R))))))^(((!R)v(((!P)v(!Q))vR))^(((!R)vP)^(((!R)vQ)^(((!R)v(!R))))))
Simplified Version:
((((!P)v(!Q)vR))v(((!P)v(!Q))vR))^((((!P)v(!Q)vR))vP)^((((!P)v(!Q)vR))vQ)^((((!P)v(!Q)vR))v(!R))))^(((Pv(((!P)v(!Q))vR))^((PvP)^((PvQ)^((Pv(!R))))))^(Qv(((!P)v(!Q))vR))^(QvP)^((QvQ)^((Qv(!R))))))^(((!R)v(((!P)v(!Q))vR))^(((!R)vP)^(((!R)vQ)^(((!R)v(!R))))))
Clauses Constructed:
{!P,!Q,R}
{!P,P,!Q,R}
{!P,!Q,Q,R}
{!P,!Q,!R,R}
{!P,P,!Q,R}
{P}
{P,Q}
{P,!R}
{!P,!Q,Q,R}
{P,Q}
{Q}
{Q,!R}
{!P,!Q,!R,R}
{P,!R}
{Q,!R}
{!R}
Empty clause found. It's true!
```

```
Please enter precondition:END
Please enter conclusion:((P->(Q->R))^(Q->(R->A)))->(P->(Q->A))
Conclusion Constructed:
((P->(Q->R))^(Q->(R->A)))->(P->(Q->A))
Proposition Used in Revolution:
((((!P)v(!Q)vR))^((!Q)v(!R)vA))^((P^(Q^(!A))))
Simplified Version:
((((!P)v(!Q)vR))^((!Q)v(!R)vA))^((P^(Q^(!A))))
Clauses Constructed:
{!P,!Q,R}
{A,!Q,!R}
{P}
{Q}
{!A}
```


Empty clause found. It's true!

Please enter precondition:END

Please enter conclusion: $((S \rightarrow !Q) \wedge (P \rightarrow Q) \wedge (R \vee S) \wedge (R \rightarrow !Q)) \rightarrow !P$

Conclusion Constructed:

$((((S \rightarrow !Q) \wedge (P \rightarrow Q) \wedge (R \vee S)) \wedge (R \rightarrow !Q)) \rightarrow !P)$

Proposition Used in Revolution:

$(((((!S) \vee (!Q)) \wedge ((!P) \vee Q)) \wedge (R \vee S)) \wedge ((!R) \vee (!Q))) \wedge P$

Simplified Version:

$(((((!S) \vee (!Q)) \wedge ((!P) \vee Q)) \wedge (R \vee S)) \wedge ((!R) \vee (!Q))) \wedge P$

Clauses Constructed:

$\{!Q, !S\}$

$\{!P, Q\}$

$\{R, S\}$

$\{!Q, !R\}$

$\{P\}$

Empty clause found. It's true!

Please enter precondition:END

Please enter conclusion: $(a \rightarrow B) \rightarrow ((c \rightarrow D) \rightarrow (a \wedge c \rightarrow B \wedge D))$

Conclusion Constructed:

$(a \rightarrow B) \rightarrow ((c \rightarrow D) \rightarrow ((a \wedge c) \rightarrow (B \wedge D)))$

Proposition Used in Revolution:

$((!a) \vee B) \wedge (((!c) \vee D) \wedge ((a \wedge c) \wedge ((!B) \vee (!D))))$

Simplified Version:

$((!a) \vee B) \wedge (((!c) \vee D) \wedge ((a \wedge c) \wedge ((!B) \vee (!D))))$

Clauses Constructed:

$\{B, !a\}$

$\{D, !c\}$

$\{a\}$

$\{c\}$

$\{!B, !D\}$

Empty clause found. It's true!

Please enter precondition:END

Please enter conclusion: $((!Q \rightarrow R) \wedge (R \rightarrow P) \wedge (P \wedge p) \wedge (!P \wedge !p) \wedge (!P \rightarrow Q)) \rightarrow Q$

Conclusion Constructed:

$(((((!Q) \rightarrow R) \wedge (R \rightarrow P)) \wedge (P \wedge p)) \wedge ((!P) \wedge (!p))) \wedge ((!P) \rightarrow Q)) \rightarrow Q$

Proposition Used in Revolution:

$(((((Q \vee R) \wedge ((!R) \vee P)) \wedge (P \wedge p)) \wedge ((!P) \wedge (!p))) \wedge (P \vee Q)) \wedge (!Q)$

Simplified Version:

$(((((Q \vee R) \wedge ((!R) \vee P)) \wedge (P \wedge p)) \wedge ((!P) \wedge (!p))) \wedge (P \vee Q)) \wedge (!Q)$

Clauses Constructed:

$\{Q, R\}$

$\{P, !R\}$

$\{P\}$

$\{p\}$

$\{!P\}$

$\{!p\}$

$\{P, Q\}$

Empty clause found. It's true!

Please enter precondition:END

Conclusion Constructed:

$$(((((((!Q) \rightarrow R) \wedge (R \rightarrow P)) \wedge (P \wedge p)) \wedge ((!P) \wedge (!p))) \wedge ((!P) \rightarrow Q)) \leftrightarrow Q$$

Proposition Used in Revolution:

[illegible]

Simplified Version:

[illegible]

Clauses Constructed:

 $\{Q, R\}$
$$\{\textcolor{red}{!}P, P, \textcolor{red}{!}Q, Q, R, \textcolor{red}{!}p, p\}$$
 $\{P, Q, \textcolor{red}{!}R\}$
$$\{\neg P, P, \neg Q, \neg R, R, \neg p, p\}$$
$$\{\neg P, P, \neg Q, \neg R, R, \neg p, p\}$$
 $\{P, Q\}$
$$\{\neg P, P, \neg Q, R, \neg p, p\}$$
$$\{\neg P, P, \neg Q, R, \neg p, p\}$$
$$\{\neg P, P, \neg Q, \neg p, p\}$$
$$\{\neg P, P, \neg Q, \neg R, R, \neg p, p\}$$
$$\{\neg P, P, \neg Q, \neg R, \neg p, p\}$$
 $\{Q, p\}$
$$\{\neg P, P, \neg Q, R, \neg p, p\}$$
$$\{\neg P, P, \neg Q, \neg p, p\}$$
$$\{\textcolor{red}{!}P, P, \textcolor{red}{!}R, R, \textcolor{red}{!}p, p\}$$

```

{!P,P,!R,!p,p}
{!P,P,!Q,R,!p,p}
{!P,P,!Q,!p,p}
{!P,P,!Q,!R,R,!p,p}
{!P,P,!Q,!R,!p,p}
{!P,Q}
{!P,P,!Q,R,!p,p}
{!P,P,!Q,!p,p}
{!P,P,!R,R,!p,p}
{!P,P,!R,!p,p}
{!P,P,!Q,R,!p,p}
{!P,P,!Q,!p,p}
{!P,P,!Q,!R,R,!p,p}
{!P,P,!Q,!R,!p,p}
{Q,!p}
{!P,P,!Q,R,!p,p}
{!P,P,!Q,!p,p}
{!P,P,!R,R,!p,p}
{!P,P,!R,!p,p}
{!P,P,!Q,R,!p,p}
{!P,P,!Q,!p,p}
{!P,P,!Q,!R,R,!p,p}
{!P,P,!Q,!R,!p,p}
{P,Q}
{!P,P,!Q,Q,R,!p,p}
{!P,P,!Q,Q,!p,p}
{!P,P,Q,!R,R,!p,p}
{!P,P,Q,!R,!p,p}
{!P,P,!Q,Q,R,!p,p}
{!P,P,!Q,Q,!p,p}
{!P,P,!Q,Q,!R,R,!p,p}
{!P,P,!Q,Q,!R,!p,p}
{!Q,Q}
{!P,P,!Q,R,!p,p}
{!P,P,!Q,!p,p}
{!P,P,!Q,!R,R,!p,p}
{!P,P,!Q,!R,!p,p}
{!P,P,!Q,R,!p,p}
{!P,P,!Q,!p,p}
{!P,P,!Q,!R,R,!p,p}
{!P,P,!Q,!R,!p,p}
Resolution failed. Can't be proved.

```

Please enter precondition:END

Please enter conclusion: $((a \rightarrow b) \wedge (c \rightarrow d) \wedge (!b \vee !d) \leftrightarrow (!a \vee !c)) \vee ((A \leftrightarrow B) \leftrightarrow ((C \leftrightarrow D) \rightarrow ((A \leftrightarrow C) \rightarrow (B \leftrightarrow D))))$

Conclusion Constructed:

$((((a \rightarrow b) \wedge (c \rightarrow d)) \wedge (!b \vee !d)) \leftrightarrow (!a \vee !c)) \vee ((A \leftrightarrow B) \leftrightarrow ((C \leftrightarrow D) \rightarrow ((A \leftrightarrow C) \rightarrow (B \leftrightarrow D))))$

Proposition Used in Revolution:

$(((((!a \vee b) \vee (!a \vee !c))) \wedge (((!a \vee b) \vee (((avc) \vee b) \wedge (av(!d) \vee b)) \wedge (((!b \vee c) \vee b) \wedge (((!b \vee$

[illegible]

[illegible]

[illegible]

$$\begin{aligned}
& (!B \vee D)) \wedge (((!D \vee D) \vee ((!D \vee D) \vee ((!C \vee !A) \vee (!B \vee D)))) \wedge ((((!D \vee D) \vee ((!D \vee D) \vee (AvC \vee \\
& \vee (!D \vee B)))) \wedge (((!D \vee D) \vee ((!D \vee D) \vee (Av(!A) \vee (!D \vee B)))) \wedge ((((!D \vee D) \vee ((!D \vee D) \vee ((!C \vee C \\
&) \vee (!D \vee B)))) \wedge (((!D \vee D) \vee ((!D \vee D) \vee ((!C \vee !A) \vee (!D \vee B)))) \wedge ((((!D \vee D) \vee ((!D \vee D) \vee (!C \\
&)) \vee (AvC \vee (!B \vee D)))) \wedge (((!D \vee D) \vee ((!D \vee D) \vee (!C) \vee (Av(!A) \vee (!B \vee D)))) \wedge ((((!D \vee D) \vee ((!D \\
& \vee !C)) \vee ((!C \vee C) \vee (!B \vee D)))) \wedge (((!D \vee D) \vee ((!D \vee D) \vee (!C) \vee ((!C \vee !A) \vee (!B \vee D)))) \wedge ((((!D \\
& \vee D) \vee ((!D \vee D) \vee (!C) \vee (AvC \vee (!D \vee B)))) \wedge (((!D \vee D) \vee ((!D \vee D) \vee (!C) \vee (Av(!A) \vee (!D \vee B)))) \wedge (\\
& (((!D \vee D) \vee ((!D \vee D) \vee (!C) \vee ((!C \vee C) \vee (!D \vee B)))) \wedge (((!D \vee D) \vee ((!D \vee D) \vee (!C) \vee ((!C \vee !A) \vee (!D \\
& \vee B)))))) \wedge (((((((!D \vee !B) \vee ((!D \vee D) \vee (AvC \vee (!B \vee D))) \wedge (((!D \vee !B) \vee ((!D \vee D) \vee (Av \\
& (!A) \vee (!B \vee D)))) \wedge ((((!D \vee !B) \vee ((!D \vee D) \vee ((!C \vee C) \vee (!B \vee D))) \wedge (((!D \vee !B) \vee ((!D \vee D) \vee \\
& D) \vee ((!C \vee !A) \vee (!B \vee D)))) \wedge (((((((!D \vee !B) \vee ((!D \vee D) \vee (AvC \vee (!D \vee B))) \wedge (((!D \vee !B) \\
&) \vee ((!D \vee D) \vee (Av(!A) \vee (!D \vee B)))) \wedge ((((!D \vee !B) \vee ((!D \vee D) \vee ((!C \vee C) \vee (!D \vee B))) \wedge ((((!D \\
& \vee !B) \vee ((!D \vee D) \vee ((!C \vee !A) \vee (!D \vee B)))) \wedge (((((((!D \vee !B) \vee ((!D \vee D) \vee (!C) \vee (AvC \vee (\\
& !B \vee D)))) \wedge ((((!D \vee !B) \vee ((!D \vee D) \vee (!C) \vee (Av(!A) \vee (!B \vee D)))) \wedge (((((((!D \vee !B) \vee ((!D \vee D) \vee (!C) \\
&) \vee ((!C \vee C) \vee (!B \vee D))) \wedge ((((!D \vee !B) \vee ((!D \vee D) \vee (!C) \vee ((!C \vee !A) \vee (!B \vee D)))) \wedge (((((((!D \vee \\
& !B) \vee ((!D \vee D) \vee (!C) \vee (AvC \vee (!D \vee B))) \wedge ((((!D \vee !B) \vee ((!D \vee D) \vee (!C) \vee (Av(!A) \vee (!D \vee B)))) \\
&) \wedge (((((((!D \vee !B) \vee ((!D \vee D) \vee (!C) \vee ((!C \vee C) \vee (!D \vee B))) \wedge ((((!D \vee !B) \vee ((!D \vee D) \vee (!C) \vee (\\
& !A) \vee (!D \vee B)))))) \wedge (((((((BvD) \vee (AvB)) \wedge (BvD) \vee (Av(!A))) \wedge ((BvD) \vee (!B \vee B)) \wedge (BvD) \vee \\
& (!B \vee !A)))) \wedge (((Bv(!B) \vee (AvB)) \wedge (Bv(!B) \vee (Av(!A))) \wedge ((Bv(!B) \vee (!B \vee B)) \wedge (Bv(!B) \vee \\
& (!B \vee !A)))) \wedge (((((((!D \vee D) \vee (AvB)) \wedge ((!D \vee D) \vee (Av(!A))) \wedge ((((!D \vee D) \vee (!B \vee B)) \wedge ((!D \\
& \vee D) \vee (!B \vee !A)))) \wedge (((((((!D \vee !B) \vee (AvB)) \wedge ((!D \vee !B) \vee (Av(!A))) \wedge (((((((!D \vee !B) \vee (!B \vee \\
& B)) \wedge ((((!D \vee !B) \vee (!B \vee !A)))))))))
\end{aligned}$$

Simplified Version:

$$\begin{aligned}
& (((((((!a) \vee b) \vee (!a \vee !c)) \wedge (((!a) \vee b) \vee (((avc) \vee b) \wedge (av(!d) \vee b)) \wedge (((!b) \vee c) \vee b) \wedge (((!b) \vee \\
& (!d) \vee b)) \wedge (((avc) \vee d) \wedge (av(!d) \vee d)) \wedge ((((!b) \vee c) \vee d) \wedge (((!b) \vee (!d) \vee d)))) \wedge ((((!c) \vee d) \vee (\\
& !a \vee !c)) \wedge ((((!c) \vee d) \vee (((avc) \vee b) \wedge (av(!d) \vee b)) \wedge ((((!b) \vee c) \vee b) \wedge (((!b) \vee (!d) \vee b)))) \wedge (((\\
& (!c) \vee d) \vee (avc \vee d)) \wedge ((((!c) \vee d) \vee (av(!d) \vee d)) \wedge ((((!c) \vee d) \vee ((!b) \vee c) \vee d)) \wedge ((((!c) \vee d) \vee ((!b) \\
& \vee (!d) \vee d)))) \wedge ((((!b) \vee (!d) \vee d) \vee (!a \vee !c)) \wedge (((((((!b) \vee (!d) \vee (avc) \vee b) \wedge ((!b) \vee (!d) \vee (\\
& av(!d) \vee b)) \wedge ((((!b) \vee (!d) \vee ((!b) \vee c) \vee b) \wedge (((!b) \vee (!d) \vee ((!b) \vee (!d) \vee b)) \wedge (((((((!b) \vee (!d \\
&)) \vee (avc \vee d)) \wedge (((!b) \vee (!d) \vee (av(!d) \vee d)) \wedge ((((!b) \vee (!d) \vee ((!b) \vee c) \vee d)) \wedge (((!b) \vee (!d) \vee (\\
& (!b) \vee (!d) \vee d)))) \wedge (((av((!a) \vee !c)) \wedge (((av((avc) \vee b) \wedge (av((av(!d) \vee b)) \wedge (av((!b) \vee c \\
&) \vee b) \wedge (av((!b) \vee (!d) \vee b)) \wedge (((av((avc) \vee d) \wedge (av((av(!d) \vee d)) \wedge (av((!b) \vee c) \vee d)) \wedge (av((\\
& (!b) \vee (!d) \vee d)))) \wedge ((cv((!a) \vee !c)) \wedge (((cv((avc) \vee b) \wedge (cv((av(!d) \vee b)) \wedge ((cv((!b) \vee c) \vee \\
& b) \wedge (cv((!b) \vee (!d) \vee b)) \wedge (((cv((avc) \vee d) \wedge (cv((av(!d) \vee d)) \wedge ((cv((!b) \vee c) \vee d)) \wedge (cv((!b) \\
& \vee (!d) \vee d)))) \wedge (((((((!A) \vee B) \vee (((((CvD) \vee (AvC) \vee (!B \vee D)) \wedge (CvD) \vee (Av(!A) \vee (!B \vee \\
& D))) \wedge ((CvD) \vee ((!C) \vee C) \vee (!B \vee D)) \wedge ((CvD) \vee ((!C) \vee !A) \vee (!B \vee D))) \wedge (((CvD) \vee (AvC) \vee (\\
& (!D \vee B)) \wedge ((CvD) \vee (Av(!A) \vee (!D \vee B)) \wedge ((CvD) \vee ((!C) \vee C) \vee (!D \vee B)) \wedge ((CvD) \vee ((!C) \vee !A \\
&)) \vee (!D \vee B)))) \wedge (((((Cv(!C) \vee (AvC) \vee (!B \vee D)) \wedge (Cv(!C) \vee (Av(!A) \vee (!B \vee D))) \wedge ((Cv \\
& (!C) \vee ((!C) \vee C) \vee (!B \vee D)) \wedge ((Cv(!C) \vee ((!C) \vee !A) \vee (!B \vee D)))) \wedge (((Cv(!C) \vee (AvC) \vee (! \\
& D \vee B)) \wedge ((Cv(!C) \vee (Av(!A) \vee (!D \vee B)) \wedge ((Cv(!C) \vee ((!C) \vee C) \vee (!D \vee B)) \wedge ((Cv(!C) \vee ((\\
& !C) \vee !A) \vee (!D \vee B)))) \wedge (((((((!D \vee D) \vee (AvC) \vee (!B \vee D)) \wedge ((!D \vee D) \vee (Av(!A) \vee (!B \vee D) \\
&)) \wedge ((((!D \vee D) \vee ((!C) \vee C) \vee (!B \vee D)) \wedge ((!D \vee D) \vee ((!C) \vee !A) \vee (!B \vee D)))) \wedge (((((((!D \vee D) \vee (\\
& AvC) \vee (!D \vee B)) \wedge ((!D \vee D) \vee (Av(!A) \vee (!D \vee B)) \wedge ((((!D \vee D) \vee ((!C) \vee C) \vee (!D \vee B)) \wedge ((((!D \\
& \vee D) \vee ((!C) \vee !A) \vee (!D \vee B)))) \wedge (((((((!D \vee D) \vee (!C) \vee (AvC) \vee (!B \vee D)) \wedge ((!D \vee D) \vee (!C) \vee (Av(\\
& !A) \vee (!B \vee D)) \wedge ((((!D \vee D) \vee (!C) \vee ((!C) \vee C) \vee (!B \vee D)) \wedge ((((!D \vee D) \vee (!C) \vee ((!C) \vee !A) \vee (!B \vee D \\
&)))) \wedge (((((((!D \vee D) \vee (!C) \vee (AvC) \vee (!D \vee B)) \wedge ((!D \vee D) \vee (!C) \vee (Av(!A) \vee (!D \vee B)) \wedge ((((!D \vee D) \vee (!C \\
&)) \vee ((!C) \vee C) \vee (!D \vee B)) \wedge ((((!D \vee D) \vee (!C) \vee ((!C) \vee !A) \vee (!D \vee B)))))) \wedge ((((!A) \vee B) \vee ((AvB) \wedge \\
& (Av(!A)) \wedge ((!B) \vee B) \wedge ((!B) \vee !A)))) \wedge ((((!B) \vee A) \vee (((((((CvD) \vee (AvC) \vee (!B \vee D)) \wedge (CvD) \vee (\\
& Av(!A) \vee (!B \vee D)) \wedge ((CvD) \vee ((!C) \vee C) \vee (!B \vee D)) \wedge ((CvD) \vee ((!C) \vee !A) \vee (!B \vee D)))) \wedge ((\\
& (CvD) \vee (AvC) \vee (!D \vee B)) \wedge ((CvD) \vee (Av(!A) \vee (!D \vee B)) \wedge ((CvD) \vee ((!C) \vee C) \vee (!D \vee B)) \wedge ((\\
& CvD) \vee ((!C) \vee !A) \vee (!D \vee B)))) \wedge (((((Cv(!C) \vee (AvC) \vee (!B \vee D)) \wedge (Cv(!C) \vee (Av(!A) \vee (!B \vee D) \\
&)) \wedge ((Cv(!C) \vee ((!C) \vee C) \vee (!B \vee D)) \wedge ((Cv(!C) \vee ((!C) \vee !A) \vee (!B \vee D)))) \wedge (((Cv(\\
& !C) \vee (AvC) \vee (!D \vee B)) \wedge ((Cv(!C) \vee (Av(!A) \vee (!D \vee B)) \wedge ((Cv(!C) \vee ((!C) \vee C) \vee (!D \vee B))
\end{aligned}$$

$$\begin{aligned}
&)^{\wedge}((Cv(!C))v(((!C)v(!A))v((!D)vB))))^{\wedge}(((((!D)vD)v((AvC)v((!B)vD)))^{\wedge}((!D)vD)v((A \\
& v(!A))v((!B)vD))))^{\wedge}(((!D)vD)v((!C)vC)v((!B)vD)))^{\wedge}((!D)vD)v(((!C)v(!A))v((!B)vD))) \\
&)^{\wedge}(((((!D)vD)v((AvC)v((!D)vB)))^{\wedge}((!D)vD)v((Av(!A))v((!D)vB)))^{\wedge}(((!D)vD)v(((!C)vC)v \\
& ((!D)vB))))^{\wedge}(((!D)vD)v(((!C)v(!A))v((!D)vB))))^{\wedge}(((((!D)v(!C))v((AvC)v((!B)vD)))^{\wedge}((\\
& !D)v(!C))v((Av(!A))v((!B)vD)))^{\wedge}(((!D)v(!C))v(((!C)vC)v((!B)vD)))^{\wedge}((!D)v(!C))v(((!C \\
&)v(!A))v((!B)vD))))^{\wedge}(((((!D)v(!C))v((AvC)v((!D)vB)))^{\wedge}((!D)v(!C))v((Av(!A))v((!D)vB) \\
&)))^{\wedge}(((!D)v(!C))v(((!C)vC)v((!D)vB)))^{\wedge}((!D)v(!C))v(((!C)v(!A))v((!D)vB))))^{\wedge}(((\\
& (!B)vA)v(AvB))^{\wedge}((!B)vA)v(Av(!A)))^{\wedge}(((!B)vA)v((!B)vB))^{\wedge}((!B)vA)v((!B)v(!A))))^{\wedge} \\
& ((((((((!C)vD)v(((CvD)v((AvC)v((!B)vD)))^{\wedge}((CvD)v((Av(!A))v((!B)vD)))^{\wedge}((CvD)v((!C \\
&)vC)v((!B)vD)))^{\wedge}((CvD)v((!C)v(!A))v((!B)vD))))^{\wedge}(((((!C)vD)v((CvD)v((AvC)v((!D)vB) \\
&)))^{\wedge}((!C)vD)v((CvD)v((Av(!A))v((!D)vB))))^{\wedge}(((((!C)vD)v((CvD)v(((!C)vC)v((!D)vB) \\
&)))^{\wedge}((!C)vD)v((CvD)v((!C)v(!A))v((!D)vB))))^{\wedge}(((((!C)vD)v((Cv(!C))v((AvC)v((!B)vD)))^{\wedge} \\
& ((!C)vD)v((Cv(!C))v((Av(!A))v((!B)vD))))^{\wedge}(((((!C)vD)v((Cv(!C))v(((!C)vC)v((!B)vD))) \\
&)^{\wedge}((!C)vD)v((Cv(!C))v((!C)v(!A))v((!B)vD))))^{\wedge}(((((!C)vD)v((Cv(!C))v((AvC)v((!D)vB) \\
&)))^{\wedge}((!C)vD)v((Cv(!C))v((!C)vC)v((!D)vB))))^{\wedge}((\\
& !C)vD)v((CvD)v((!C)v(!A))v((!D)vB))))^{\wedge}(((((!C)vD)v((Cv(!C))v((AvC)v((!B)vD)))^{\wedge} \\
& ((!C)vD)v((Cv(!C))v((Av(!A))v((!B)vD))))^{\wedge}(((((!C)vD)v((Cv(!C))v((!C)vC)v((!D)vB \\
&)))^{\wedge}((!C)vD)v((Cv(!C))v((!C)v(!A))v((!D)vB))))^{\wedge}(((((!C)vD)v((!D)vD)v((AvC)v \\
& ((!B)vD)))^{\wedge}((!C)vD)v((!D)vD)v((Av(!A))v((!B)vD))))^{\wedge}(((((!C)vD)v((!D)vD)v((!C)vC) \\
& v((!B)vD)))^{\wedge}((!C)vD)v((!D)vD)v((!C)v(!A))v((!B)vD))))^{\wedge}(((((!C)vD)v((!D)vD)v((A \\
& vC)v((!D)vB)))^{\wedge}((!C)vD)v((!D)vD)v((Av(!A))v((!D)vB))))^{\wedge}(((((!C)vD)v((!D)vD)v((!C \\
&)vC)v((!D)vB)))^{\wedge}((!C)vD)v((!D)vD)v((!C)v(!A))v((!D)vB))))^{\wedge}(((((!C)vD)v((!D)v \\
& (!C))v((AvC)v((!B)vD)))^{\wedge}((!C)vD)v((!D)v(!C))v((Av(!A))v((!B)vD))))^{\wedge}(((((!C)vD)v((\\
& !D)v(!C))v((!C)vC)v((!B)vD)))^{\wedge}((!C)vD)v((!D)v(!C))v((!C)v(!A))v((!B)vD))))^{\wedge}(((\\
& (!C)vD)v((!D)v(!C))v((AvC)v((!D)vB)))^{\wedge}((!C)vD)v((!D)v(!C))v((Av(!A))v((!D)vB))) \\
&)^{\wedge}(((((!C)vD)v((!D)v(!C))v((!C)vC)v((!D)vB)))^{\wedge}((!C)vD)v((!D)v(!C))v((!C)v(!A))v(\\
& (!D)vB))))^{\wedge}(((((!C)vD)v(AvB))^{\wedge}((!C)vD)v(Av(!A)))^{\wedge}(((((!C)vD)v((!B)vB))^{\wedge}((!C)vD \\
&)v((!B)v(!A))))^{\wedge}(((((((((!D)vC)v((CvD)v((AvC)v((!B)vD)))^{\wedge}((!D)vC)v((CvD)v((Av(!A) \\
&)v((!B)vD)))^{\wedge}(((((!D)vC)v((CvD)v((!C)vC)v((!B)vD)))^{\wedge}((!D)vC)v((CvD)v((!C)v(!A))v \\
& ((!B)vD))))^{\wedge}(((((!D)vC)v((CvD)v((AvC)v((!D)vB)))^{\wedge}((!D)vC)v((CvD)v((Av(!A))v((!D)v \\
& B))))^{\wedge}(((((!D)vC)v((CvD)v((!C)vC)v((!D)vB)))^{\wedge}((!D)vC)v((CvD)v((!C)v(!A))v((!D)vB) \\
&))))^{\wedge}(((((!D)vC)v((Cv(!C))v((AvC)v((!B)vD)))^{\wedge}((!D)vC)v((Cv(!C))v((Av(!A))v((!B)v \\
& D))))^{\wedge}(((((!D)vC)v((Cv(!C))v((!C)vC)v((!B)vD)))^{\wedge}((!D)vC)v((Cv(!C))v((!C)v(!A))v((\\
& !B)vD))))^{\wedge}(((((!D)vC)v((Cv(!C))v((AvC)v((!D)vB)))^{\wedge}((!D)vC)v((Cv(!C))v((Av(!A))v((\\
& !D)vB))))^{\wedge}(((((!D)vC)v((Cv(!C))v((!C)vC)v((!D)vB)))^{\wedge}((!D)vC)v((Cv(!C))v((!C)v(!A) \\
&)v((!D)vB))))^{\wedge}(((((((((!D)vC)v((!D)vD)v((AvC)v((!B)vD)))^{\wedge}((!D)vC)v((!D)vD)v((Av \\
& (!A))v((!B)vD))))^{\wedge}(((((!D)vC)v((!D)vD)v((!C)vC)v((!B)vD)))^{\wedge}((!D)vC)v((!D)vD)v((\\
& !C)v(!A))v((!B)vD))))^{\wedge}(((((!D)vC)v((!D)vD)v((AvC)v((!D)vB)))^{\wedge}((!D)vC)v((!D)vD)v \\
& ((Av(!A))v((!D)vB))))^{\wedge}(((((!D)vC)v((!D)vD)v((!C)vC)v((!D)vB)))^{\wedge}((!D)vC)v((!D)vD) \\
& v((!C)v(!A))v((!D)vB))))^{\wedge}(((((((((!D)vC)v((!D)v(!C))v((AvC)v((!B)vD)))^{\wedge}((!D)vC)v(\\
& ((!D)v(!C))v((Av(!A))v((!B)vD))))^{\wedge}(((((!D)vC)v((!D)v(!C))v((!C)vC)v((!B)vD)))^{\wedge}(((\\
& !D)vC)v((!D)v(!C))v((!C)v(!A))v((!B)vD))))^{\wedge}(((((!D)vC)v((!D)v(!C))v((AvC)v((!D)vB) \\
&)))^{\wedge}((!D)vC)v((!D)v(!C))v((Av(!A))v((!D)vB))))^{\wedge}(((((!D)vC)v((!D)v(!C))v((!C)vC)v \\
& ((!D)vB)))^{\wedge}(((((!D)vC)v((!D)v(!C))v((!C)v(!A))v((!D)vB))))^{\wedge}(((((((((!D)vC)v(AvB))^{\wedge} \\
& ((!D)vC)v(Av(!A)))^{\wedge}(((((!D)vC)v((!B)vB))^{\wedge}((!D)vC)v((!B)v(!A))))^{\wedge}(((((((((!A)vC) \\
& v((CvD)v((AvC)v((!B)vD)))^{\wedge}((!A)vC)v((CvD)v((Av(!A))v((!B)vD))))^{\wedge}(((((!A)vC)v((CvD)v \\
& ((!C)vC)v((!B)vD)))^{\wedge}((!A)vC)v((CvD)v((!C)v(!A))v((!B)vD))))^{\wedge}(((((!A)vC)v((CvD)v \\
& ((AvC)v((!D)vB)))^{\wedge}((!A)vC)v((CvD)v((Av(!A))v((!D)vB))))^{\wedge}(((((!A)vC)v((CvD)v((!C)vC \\
&)v((!D)vB)))^{\wedge}((!A)vC)v((CvD)v((!C)v(!A))v((!D)vB))))^{\wedge}(((((((((!A)vC)v((Cv(!C))v((A \\
& vC)v((!B)vD)))^{\wedge}((!A)vC)v((Cv(!C))v((Av(!A))v((!B)vD))))^{\wedge}(((((!A)vC)v((Cv(!C))v((!C \\
&)vC)v((!B)vD)))^{\wedge}((!A)vC)v((Cv(!C))v((!C)v(!A))v((!B)vD))))^{\wedge}(((((((((!A)vC)v((Cv(!C)) \\
& v((AvC)v((!D)vB)))^{\wedge}((!A)vC)v((Cv(!C))v((Av(!A))v((!D)vB))))^{\wedge}(((((!A)vC)v((Cv(!C))v(\\
& ((!C)vC)v((!D)vB))))^{\wedge}((!A)vC)v((Cv(!C))v((!C)v(!A))v((!D)vB))))^{\wedge}(((((((((!A)vC)v(\\
& ((!D)vD)v((AvC)v((!B)vD)))^{\wedge}((!A)vC)v((!D)vD)v((Av(!A))v((!B)vD))))^{\wedge}(((((!A)vC)v((
\end{aligned}$$

$$\begin{aligned}
& (!D)vD)v(((!C)vC)v(!B)vD)))^(((!A)vC)v(!D)vD)v(!C)v(!A))v(!B)vD)))^((((!A)v \\
& C)v(!D)vD)v(AvC)v(!D)vB)))^(((!A)vC)v(!D)vD)v(Av(!A))v(!D)vB)))^((((!A)vC) \\
& v(!D)vD)v(!C)vC)v(!D)vB)))^(((!A)vC)v(!D)vD)v(!C)v(!A))v(!D)vB)))^((((\\
& (!A)vC)v(!D)v(!C))v(AvC)v(!B)vD)))^(((!A)vC)v(!D)v(!C))v(Av(!A))v(!B)vD))) \\
&)^((((!A)vC)v(!D)v(!C))v(!C)vC)v(!B)vD)))^(((!A)vC)v(!D)v(!C))v(!C)v(!A))v(\\
& (!B)vD)))^((((!A)vC)v(!D)v(!C))v(AvC)v(!D)vB)))^(((!A)vC)v(!D)v(!C))v(Av(\\
& !A))v(!D)vB)))^((((!A)vC)v(!D)v(!C))v(!C)vC)v(!D)vB)))^(((!A)vC)v(!D)v(!C) \\
&)v(!C)v(!A))v(!D)vB)))^((((!A)vC)v(AvB))^(((!A)vC)v(Av(!A)))^((((!A)vC)v(! \\
& !B)vB))^(((!A)vC)v(!B)v(!A))))^(((((!C)vA)v(CvD)v(AvC)v(!B)vD)))^(((!C)vA) \\
& v(CvD)v(Av(!A))v(!B)vD)))^((((!C)vA)v(CvD)v(!C)vC)v(!B)vD)))^(((!C)vA)v(Cv \\
& D)v(!C)v(!A))v(!B)vD)))^((((!C)vA)v(CvD)v(AvC)v(!D)vB)))^(((!C)vA)v(CvD)v \\
& (Av(!A))v(!D)vB)))^((((!C)vA)v(CvD)v(!C)vC)v(!D)vB)))^(((!C)vA)v(CvD)v(!C \\
&)v(!A))v(!D)vB)))^(((((!C)vA)v(Cv(!C))v(AvC)v(!B)vD)))^(((!C)vA)v(Cv(!C))v \\
& (Av(!A))v(!B)vD)))^((((!C)vA)v(Cv(!C))v(!C)vC)v(!B)vD)))^(((!C)vA)v(Cv(!C)) \\
& v(!C)v(!A))v(!B)vD)))^((((!C)vA)v(Cv(!C))v(AvC)v(!D)vB)))^(((!C)vA)v(Cv(! \\
& C))v(Av(!A))v(!D)vB)))^((((!C)vA)v(Cv(!C))v(!C)vC)v(!D)vB)))^(((!C)vA)v(Cv(\\
& !C))v(!C)v(!A))v(!D)vB)))^(((((!C)vA)v(!D)vD)v(AvC)v(!B)vD)))^(((!C)vA \\
&)v(!D)vD)v(Av(!A))v(!B)vD)))^((((!C)vA)v(!D)vD)v(!C)vC)v(!B)vD)))^(((!C)v \\
& A)v(!D)vD)v(!C)v(!A))v(!B)vD)))^((((!C)vA)v(!D)vD)v(AvC)v(!D)vB)))^(((! \\
& C)vA)v(!D)vD)v(Av(!A))v(!D)vB)))^((((!C)vA)v(!D)vD)v(!C)vC)v(!D)vB)))^(((\\
& !C)vA)v(!D)vD)v(!C)v(!A))v(!D)vB)))^(((((!C)vA)v(!D)v(!C))v(AvC)v(!B)vD \\
&))^(((!C)vA)v(!D)v(!C))v(Av(!A))v(!B)vD)))^((((!C)vA)v(!D)v(!C))v(!C)vC)v \\
& (!B)vD)))^(((!C)vA)v(!D)v(!C))v(!C)v(!A))v(!B)vD)))^(((((!C)vA)v(!D)v(!C) \\
&)v(AvC)v(!D)vB)))^(((!C)vA)v(!D)v(!C))v(Av(!A))v(!D)vB)))^((((!C)vA)v(!D)v \\
& (!C))v(!C)vC)v(!D)vB)))^(((!C)vA)v(!D)v(!C))v(!C)v(!A))v(!D)vB))))^((((\\
& (!C)vA)v(AvB))^(((!C)vA)v(Av(!A)))^((((!C)vA)v(!B)vB))^(((!C)vA)v(!B)v(!A))))^((\\
& (((((((BvD)^(Bv(!B)))^(((!D)vD)^(!D)v(!B)))v(CvD)v(AvC)v(!B)vD)))^(((BvD)v(\\
& CvD)v(Av(!A))v(!B)vD)))^((Bv(!B))v(CvD)v(Av(!A))v(!B)vD)))^((((!D)vD)v(CvD)v \\
& (Av(!A))v(!B)vD)))^(((!D)v(!B))v(CvD)v(Av(!A))v(!B)vD)))^((((BvD)v(CvD)v(\\
& (!C)vC)v(!B)vD)))^((BvD)v(CvD)v(!C)v(!A))v(!B)vD)))^((Bv(!B))v(CvD)v(!C) \\
& vC)v(!B)vD)))^((Bv(!B))v(CvD)v(!C)v(!A))v(!B)vD)))^(((((!D)vD)v(CvD)v(!C) \\
& vC)v(!B)vD)))^(((!D)vD)v(CvD)v(!C)v(!A))v(!B)vD)))^((((!D)v(!B))v(CvD)v(!C \\
&)vC)v(!B)vD)))^(((!D)v(!B))v(CvD)v(!C)v(!A))v(!B)vD))))^((((BvD)v(CvD)v(\\
& AvC)v(!D)vB)))^((BvD)v(CvD)v(Av(!A))v(!D)vB)))^((BvD)v(CvD)v(!C)vC)v(!D) \\
& vB)))^((BvD)v(CvD)v(!C)v(!A))v(!D)vB)))^((((Bv(!B))v(CvD)v(AvC)v(!D)vB))) \\
& ^((Bv(!B))v(CvD)v(Av(!A))v(!D)vB)))^((Bv(!B))v(CvD)v(!C)vC)v(!D)vB)))^((Bv \\
& (!B))v(CvD)v(!C)v(!A))v(!D)vB)))^(((((!D)vD)v(CvD)v(AvC)v(!D)vB)))^(((!D \\
&)vD)v(CvD)v(Av(!A))v(!D)vB)))^((((!D)vD)v(CvD)v(!C)vC)v(!D)vB)))^(((!D)vD)v \\
& (CvD)v(!C)v(!A))v(!D)vB)))^(((((!D)v(!B))v(CvD)v(AvC)v(!D)vB)))^(((!D)v(!B \\
&))v(CvD)v(Av(!A))v(!D)vB)))^(((((!D)v(!B))v(CvD)v(!C)vC)v(!D)vB)))^(((!D)v(! \\
& B))v(CvD)v(!C)v(!A))v(!D)vB))))^((((BvD)v(Cv(!C))v(AvC)v(!B)vD)))^((B \\
& vD)v(Cv(!C))v(Av(!A))v(!B)vD)))^((BvD)v(Cv(!C))v(!C)vC)v(!B)vD)))^((BvD)v(\\
& Cv(!C))v(!C)v(!A))v(!B)vD)))^((BvD)v(Cv(!C))v(AvC)v(!D)vB)))^((BvD)v(Cv \\
& (!C))v(Av(!A))v(!D)vB)))^((BvD)v(Cv(!C))v(!C)vC)v(!D)vB)))^((BvD)v(Cv(!C)) \\
& v(!C)v(!A))v(!D)vB))))^((((Bv(!B))v(Cv(!C))v(AvC)v(!B)vD)))^((Bv(!B))v(Cv \\
& (!C))v(Av(!A))v(!B)vD)))^((Bv(!B))v(Cv(!C))v(!C)vC)v(!B)vD)))^((Bv(!B))v(C \\
& v(!C))v(!C)v(!A))v(!B)vD)))^((((Bv(!B))v(Cv(!C))v(AvC)v(!D)vB)))^((Bv(!B))v \\
& (Cv(!C))v(Av(!A))v(!D)vB)))^((Bv(!B))v(Cv(!C))v(!C)vC)v(!D)vB)))^((Bv(!B)) \\
& v(Cv(!C))v(!C)v(!A))v(!D)vB))))^(((((!D)vD)v(Cv(!C))v(AvC)v(!B)vD)))^((\\
& !D)vD)v(Cv(!C))v(Av(!A))v(!B)vD)))^((((!D)vD)v(Cv(!C))v(!C)vC)v(!B)vD)))^((\\
& !D)vD)v(Cv(!C))v(!C)v(!A))v(!B)vD))))^(((((!D)vD)v(Cv(!C))v(AvC)v(!D)vB))) \\
&)^(((!D)vD)v(Cv(!C))v(Av(!A))v(!D)vB)))^((((!D)vD)v(Cv(!C))v(!C)vC)v(!D)vB))
\end{aligned}$$

{!b,!c,!d,d}
{!a,!b,!c,!d}
{a,!b,b,c,!d}
{a,!b,b,!d}
{!b,b,c,!d}
{!b,b,!d}
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{a,!b,!d,d}
{!b,c,!d,d}
{!b,!d,d}
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{a,b,!d}
{a,!b,b,c}
{a,!b,b,!d}
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{a,!b,!d,d}
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 { !B, B, D}
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 {A, !B, B}
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 { !B, B}
 { !A, !B, B}
 {A, B, !D, D}
 { !A, A, !D, D}
 { !B, B, !D, D}

$\{\neg A, \neg B, \neg D, D\}$

$\{A, \neg B, B, \neg D\}$

$\{\neg A, A, \neg B, \neg D\}$

$\{\neg B, B, \neg D\}$

$\{\neg A, \neg B, \neg D\}$

Resolution failed. Can't be proved.