A Predictor-Controller Approach for Q-Law 6th Element Targeting in Low-Thrust Trajectory Design

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Take home message: Q-Law 6th element targeting capabilities can be obtained through the addition of a simple predictor-controller stage evaluated throughout the transfer propagation. This permits to generate a suitable initial guess for further optimisation of rendezvous trajectories or to perform fast preliminary rendezvous mission design.

The Q-Law guidance algorithm can generate near-optimal low-thrust trajectories with minimal computational effort, which can be used as an initial guess for further optimisation or for early mission design. However, the classical formulation cannot target a specific moving-position in the target orbit. Such constraint could be a geographical longitude in GEO, a position in a LEO or MEO constellation, or a rendezvous mission. Here, we present a novel algorithm extending the capability of the Q-Law using a predictor-controller stage evaluated throughout the transfer. This method opens the door to fast rendezvous mission design and significantly improves the initial guess quality for further trajectory optimisation.

Keywords: Q-law, low-thrust transfer, rendezvous mission design

Case Studies

- **Case A:** SSO circular raise transfer increasing the semi-major axis by 200 km from 7028.0 km and an eccentricity change from 0.0075 to 0.001.
- **Case C:** High eccentricity transfer increasing the semi-major axis from 9222.7 km to 30000.0 km and an eccentricity change from 0.2 to 0.7.

Approach

Based on the predictor-controller approach introduced by Locoche *et al.* (2021):

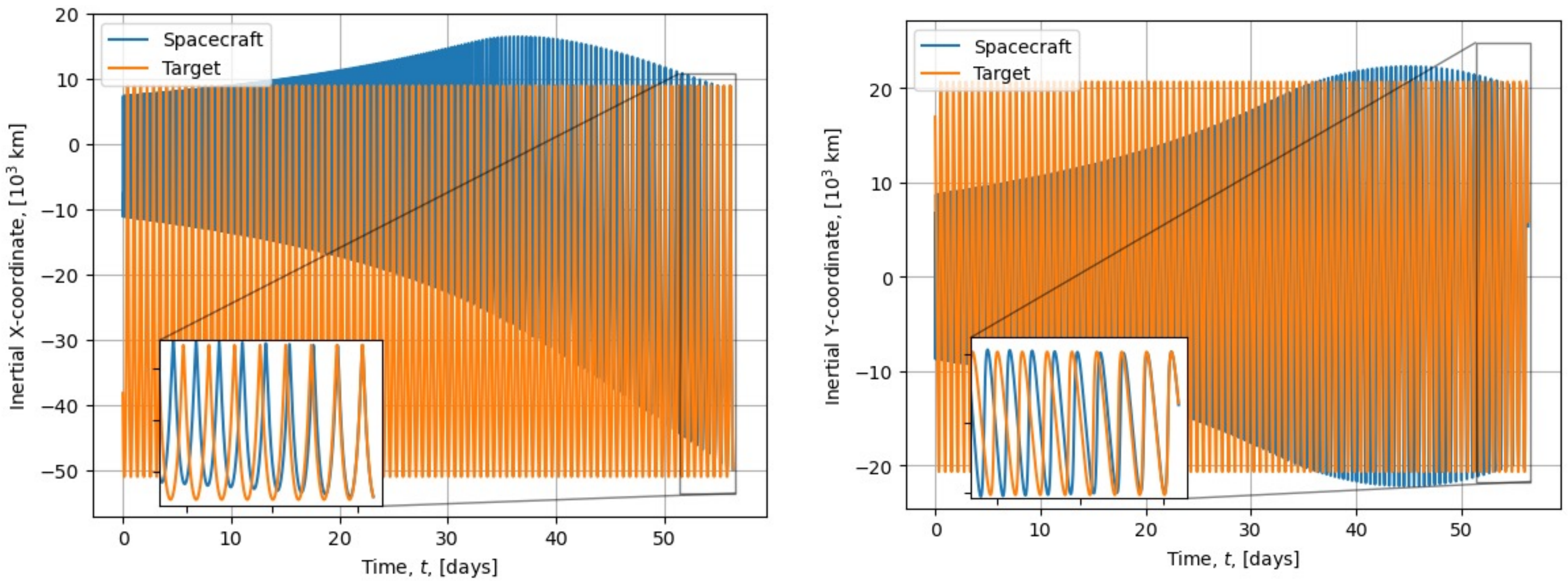
1. Propagate Q-Law until $t = t_{start}$
2. Evaluate predictor stage: $t_{eval} = t$
   a) Propagate from state by targeting a_{ref} until convergence
   b) Evaluate phase error $\Delta\Phi$
3. Update target semi-major axis according to,

$$a_t = \left(a_{ref}^{-3/2} - \frac{\alpha\,\Delta\Phi}{T_r\sqrt{\mu}}\right)^{-3/2}$$

4. Propagate until $t = t_{eval} + t_{int}$

Generalising, the phase angle error $\Delta\Phi$ must be constant throughout the unperturbed orbit.

- **Circular equatorial**: inertial longitude, $\Delta\Phi = \lambda_{I_t} - \lambda_I$.
- **Circular inclined**: argument of latitude $\Delta\Phi = u_t - u$.
- **Eccentric**: mean anomaly. $\Delta\Phi = M_t - M$.



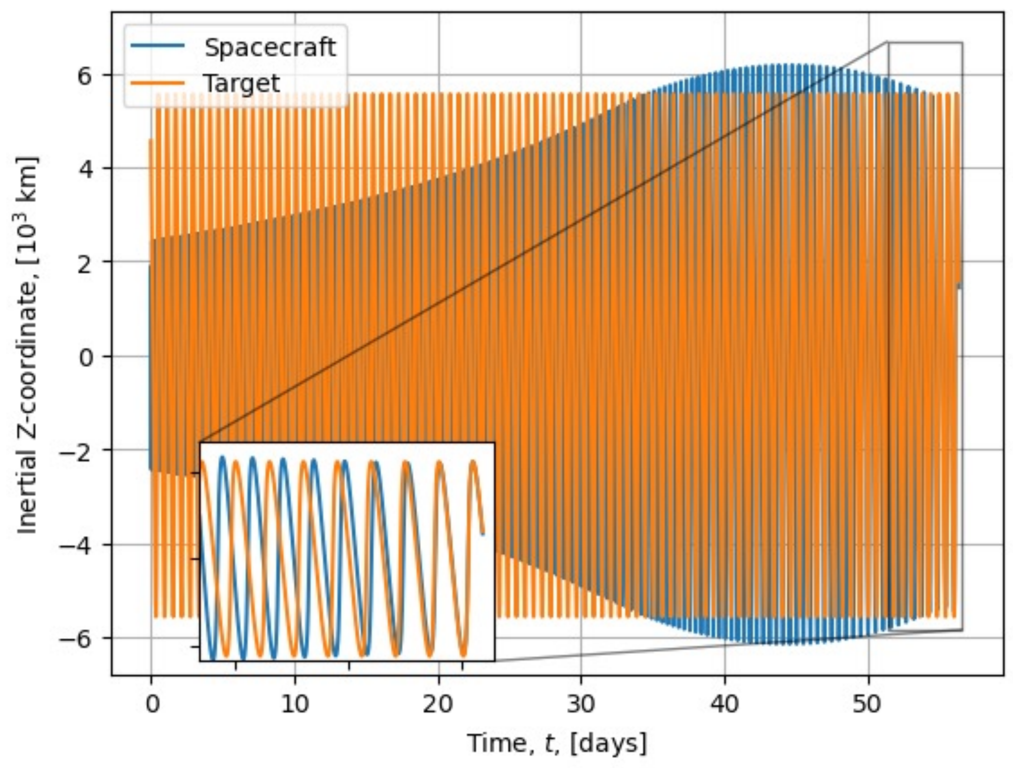


Fig. 1: X-Y-Z Cartesian coordinates of the spacecraft and target position as a function of time during the case C transfer.

Results

- Converging combinations match the inertial position and velocity of the target at the end of the transfer, giving near-optimal feasible solutions.
- The targeting capabilities only result in an increase of 1.03% and 0.25% in propellant mass consumption for cases A and C respectively.
- Case A best combination: $\alpha = 9$, $t_{start} = 0$, and $t_{int} = 0.4$ days.
- Case C best combination: $\alpha = 6$, $t_{start} = 0$, and $t_{int} = 3$ days.
- Thrust chattering inhibits convergence in 40% of the combinations for both circular and eccentric cases.

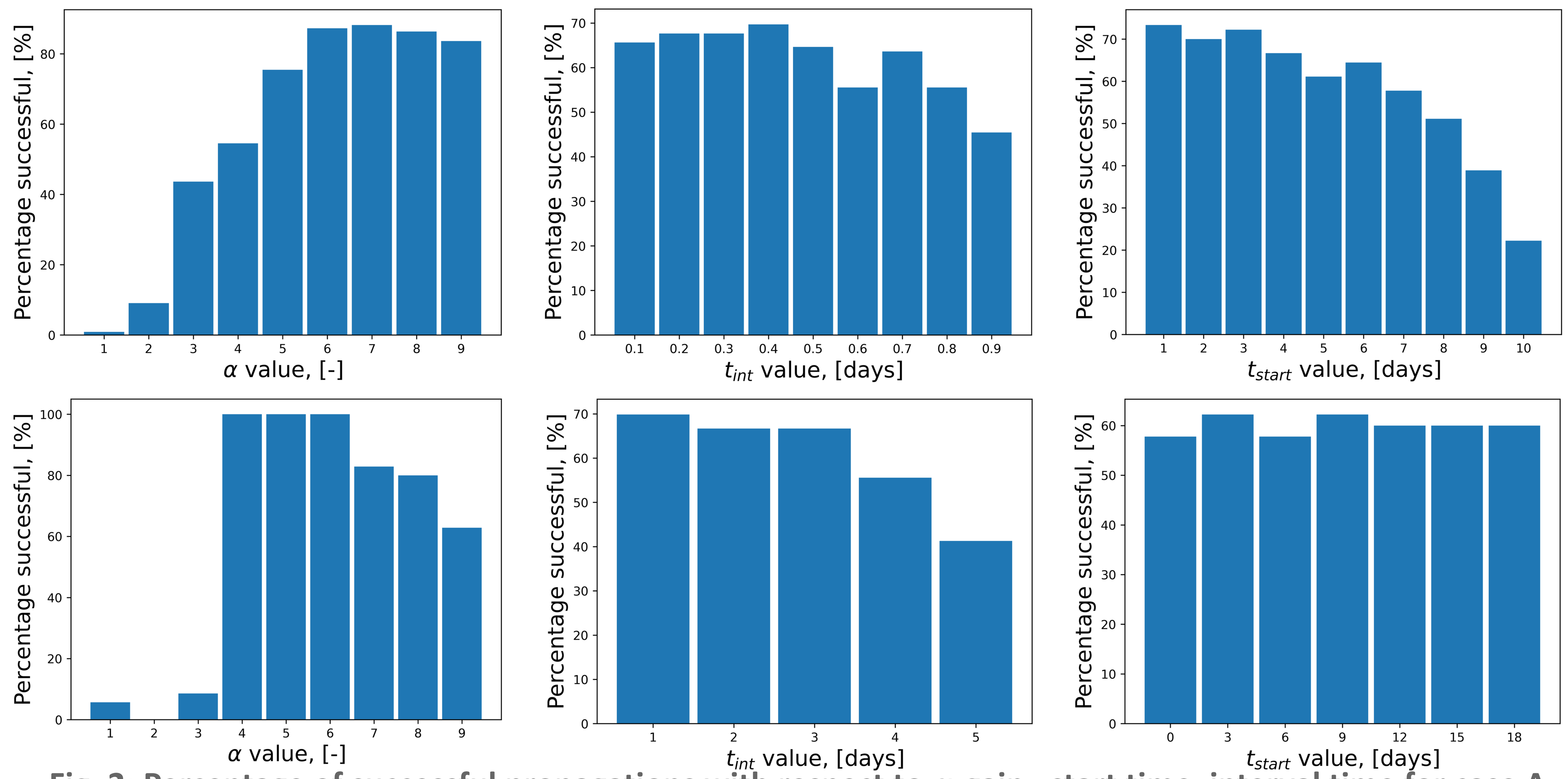


Fig. 2: Percentage of successful propagations with respect to α-gain, start time, interval time for case A (top row) and C (bottom row).

Discussion

- The optimal value of the α-gain is heavily case dependent.
- Lower t_{start} and t_{int} yield better convergence at the cost of an increased computational time, without guaranty of a more optimal trajectory.
- The algorithm shows a drop in performance for the circular case compared to the eccentric case.

Outlook

- Testing of the approach in a perturbed environment on more trajectories.
- Investigating the performance drop for the circular case.
- Selecting the method inputs through a global optimisation method, such as an evolutionary algorithm.

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