Delft University of Technology

Faculty of Aerospace Engineering

AE4870B

Re-Entry Systems

Assignment II: Aerodynamics

Lecturer: Dr. E. Mooij

Hours spent: 2.5h

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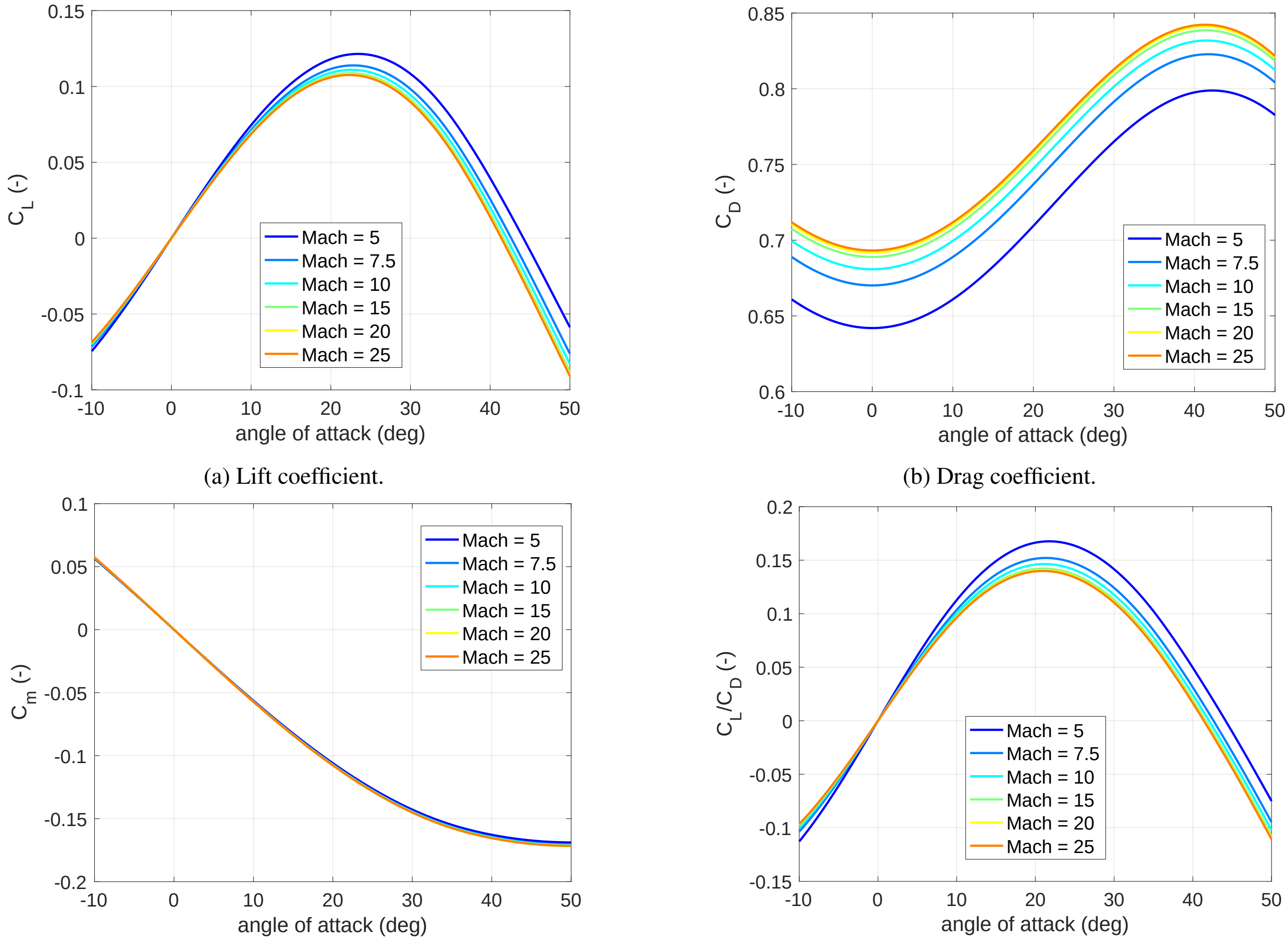


(a) Lift coefficient.

(b) Drag coefficient.

(c) Moment coefficient.

(d) Lift-to-drag ratio.

Figure 1: Aerodynamic coefficients of the capsule, as functions of the angle of attack and Mach number.

1 Vehicle Geometry and Maximum Lift-to-Drag Ratio

The geometry chosen was obtained from my student number: 5075211: the nose radius $R_N = 0.7$ m, the semi-cone angle $\theta_c = 1.7 \cdot 21 = 35.7°$, and the base radius $r_{c,2} = 1.5$ m (note that the digits were combined in different manners here, such that some factors are formed; eg: digits 1 and 7 to form 1.7). For the geometry to be geometrically feasible, under the assumption that all parameters are positive ($\theta_c \geq 0$), the base radius needs to be larger than the nose radius. Furthermore, it is desired that the shape resembles as much as possible an actual entry vehicle, it is clear from Figure 3 that the capsule is a blunt body with a significant drag coefficient (which is desirable for re-entry applications). The volume of the body is approximately 3.5240 m^3 (approximated by neglecting the smooth transition from nose to body), which permits to carry a payload to the surface of the planet (such as a rover for a Martian entry, even if the useful volume only a portion of that). At last, the largest diameter of the body is 3 m, which is similar to Mars entry vehicles. Having defined the geometry of the capsule, the code provided is used to estimate its aerodynamic coefficients as a function of the Mach number and angle of attack, based on the Modified Newtonian Flow method. The basic aerodynamic coefficients are given as a function of the Mach number and angle of attack in Figure 1.

First considering the 'standard' aerodynamic coefficients, it can be seen that both the drag, C_D (Figure 1b), and lift, C_L (Figure 1a), coefficients have a significant dependency on the March number. On the one hand, the C_D increases for larger Mach numbers at the same Angle of Attack (AoA), over the entire range of considered. While, on the other hand, the lift coefficient only shows a dependence on the Mach number for AoAs above $\approx 7°$: a larger Mach number results in a smaller $C_{L_{max}}$[1]. Additionally, the change of those coefficients for a certain ΔM decreases with increasing Mach numbers. This is a direct effect from the Mach independence of aerodynamic coefficients in hypersonic flows: as the Mach number increases, the aerodynamic coefficients convergence towards a finite value. The moment coefficient, C_m (Figure 1c, computed with the center of mass as reference point), however, does not experience major changes from one Mach number to another. Even at an AoA of 50°, the change in C_m is of 1.62% only between $M = 5$ and $M = 25$. It can also be noted that the slope of the C_m curve is negative, indicating that the vehicle is statically stable.

[1] Note that the same behaviour would be seen from the standpoint of the negative AoA's, but opposite, meaning that the most negative C_L is reached at the lowest Mach number in the hypersonic regime.

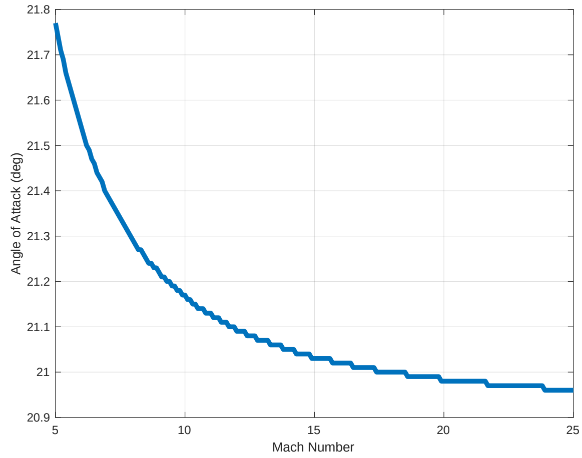


Figure 2: Maximum angle of attack.

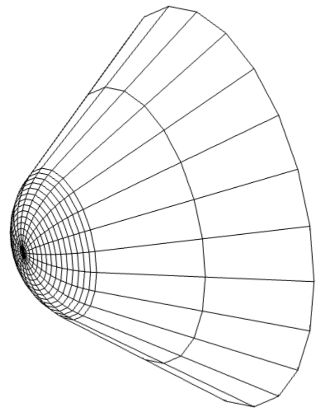


Figure 3: Entry Vehicle used in this report.

From a design standpoint, it is important to note that a lager AoA results in a significant increase in drag coefficient, which will be linked to a large increase in aeroheating. However, such an increase in drag will result in a higher deceleration of the vehicle, which might be desirable depending on the application. Additionally, an atmospheric entry with a certain (trim) angle of attack results in a non-zero lift coefficient (which is zero at a 0 AoA due to the symmetry of the vehicle), which would change the trajectory of the capsule. This means that there is a need for a trade-off between the overall trajectory, flight time, and flight loads/aeroheating; when a trajectory is designed with this entry vehicle.

The lift-to-drag ratio can be considered through Figure 1d, clearly, the maximum C_L/C_D ratio is reached at lower AoAs at larger Mach numbers in hypersonic flows. This can be explained from the points raised above: the drag coefficient is increased at larger Mach numbers, while the lift coefficient is reduced. This results in an overall decrease of the maximum lift-to-drag ratio of the vehicle. Furthermore, similarly as for the drag and lift coefficients independently, the $\left(\frac{C_L}{C_D}\right)_{max}$ shows a Mach independence, with converging values as the Mach number increases (the lines get closer together).

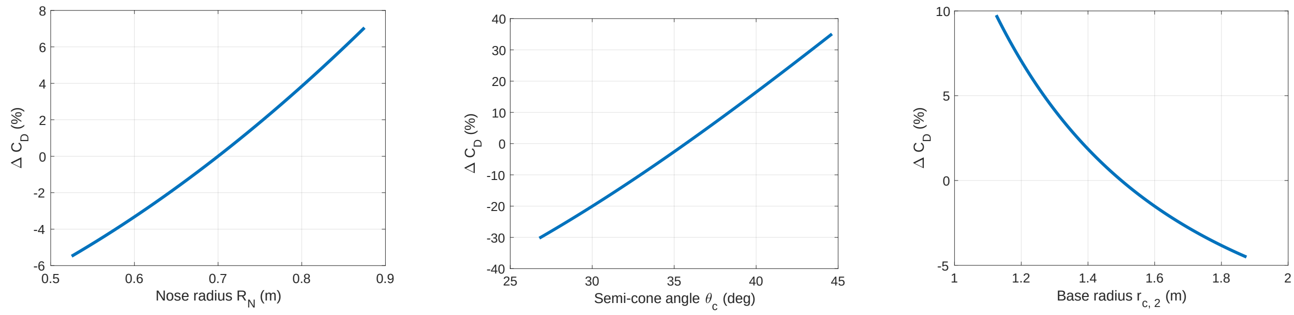
The angle of attack, α_{max}, for $\left(\frac{C_L}{C_D}\right)_{max}$ is given in Figure 2. Note that the step decrease is only the result of the Mach number resolution set to 0.1 to limit the computational time required. α_{max} decreases with increasing Mach number, but the change is only of the order of 0.9°. The Mach number being a direct indicator for the compressibility of the flow, this shift is only the result of the compressible effects at play. Indeed, the lower the compressibility (lower Mach) of the flow, the larger α_{max}, and inversely. This can also be seen from [2]. For design purposes, α_{max} provides the angle of attack that is the most suited for longer trajectories, which travel more in the air

2 Impact of Vehicle Shape on Drag

The effect of the geometry parameters on the drag coefficient of the vehicle can be investigated by considering a certain variation around the values presented earlier. The analysis is performed using an AoA of $\alpha = 0°$ and a Mach number of 12.5, using a range of 25% to vary around the nominal value (while keeping the other parameters at their value specified above). This permits to assess the (non-)linearity of the drag coefficient with respect to the parameters clearly, while being able to apply the same percentage to each (the limiting parameter is $r_{c,2}$ which needs to stay above R_N for all considered cases). Figure 4 gives the change in drag coefficient in percentage from the 'nominal' value. Each parameter is discussed below, note that the slenderness of the vehicle mentioned a few times in the discussion is defined as the vehicle length divided by the base diameter (largest 'width' dimension).

- It can be seen from Figure 4a that increasing the nose radius increases the drag in a non-linear fashion. An increase of 25% of the nose radius results in an increase of about 7% in C_D. This is expected as a larger nose radius reduces the slenderness of the vehicle and hence increases the drag), for the same base radius and semi-cone angle. On the other side of the range, reducing the nose radius by 25% yields a reduction of about 6% in the drag coefficient.
- It can be seen from Figure 4b that the relation of C_D on θ_c is also non-linear (although very slight here), and that increasing the parameter results in an increase in the drag coefficient. This makes sense as a larger semi-cone angle (while keeping the rest constant) approaches a flat plate (if $\theta_c \to 90°$), and reduces the slenderness of the vehicle. An increase of 25% of θ_c results in an increase of $\approx 35\%$ in C_D, the opposite (decrease) results in a decrease of $\approx 30\%$ in C_D.
- It can be seen from Figure 4c that an increase in the base radius results in a decrease in coefficient drag. This can seem counter intuitive as the area exposed to the airflow is larger, however, the drag coefficient is considered here:

[2]Gudmundsson, Snorri (2014). "Chapter 8 - The Anatomy of the Airfoil". In: General Aviation Aircraft Design. Ed. by Snorri Gudmundsson. Boston: Butterworth-Heinemann, pp. 235–297. ISBN: 978-0-12-397308-5. DOI: https : / / doi . org / 10 . 1016 / B978 - 0 - 12 - 397308 - 5 . 00008 - 8. URL: https://www.sciencedirect.com/science/article/pii/B9780123973085000088.



(a) Nose radius variation. (b) Semi-cone angle variation. (c) Base radius variation.

Figure 4: Effect of shape on the drag coefficient considered by varying each parameter by $\pm 25\%$ at maximum.

it is likely that the drag would increase, but the C_D decreases by rendering the vehicle more slender overall. As the semi-cone angle, and the nose radius stay constant, resulting in a larger increase of the vehicle's length than the increase in the base diameter (and hence a more slender vehicle). The dependence of the drag coefficient on the base radius is highly non-linear in comparison to the other parameters. Furthermore, a decrease of 25% in the base radius results in an increase of about 10% in C_D, an increase of 25% results in a decrease of $\approx 5\%$ in C_D.

Considering the design of such an entry vehicle, this parameter variations gives insights into the aerodynamics of blunt vehicles with shapes close to the one considered in this report. First, combining a smaller nose radius, with a smaller semi-cone angle and a larger base radius permits to reduce the drag significantly (yielding an overall longer decelerate but smaller aerodynamic loading). That is because the shape of a rocket cone is being approached essentially. A similar but opposite combination results in an increased drag, resulting in a more effective deceleration but a larger aerodynamic loading. Furthermore, for a given desired drag coefficient, this analysis permits to vary the different parameters in an educated manner to reduce the aeodynamic heating and/or loading on the vehicle, by keeping the same vehicle drag.

3 Center of Mass Location Effect on Moment Coefficient

The moment coefficient is computed around the center of mass, chosen as reference point. The effect of varying the position of the center of mass can be investigated by considering the following equation, where L_{ref} is the reference length of the vehicle.

$$\Delta C_m = \frac{\Delta \vec{r}}{L_{ref}} \times \begin{bmatrix} C_D \cos(\alpha) - C_L \sin(\alpha) \\ 0 \\ C_D \sin(\alpha) + C_L \cos(\alpha) \end{bmatrix} \tag{1}$$

The moment coefficient at any point on the body can then be simply computed as $C_m = \Delta C_m + C_{m_0}$, where C_{m_0} is the moment coefficient from the reference point used in the code (the 'current' center of mass, at half of the length of the body). Note that the AoA considered is zero, and the Mach number is 10 in this assignment. The effect of the position of the center of mass is considered in both the x and y coordinates independently:

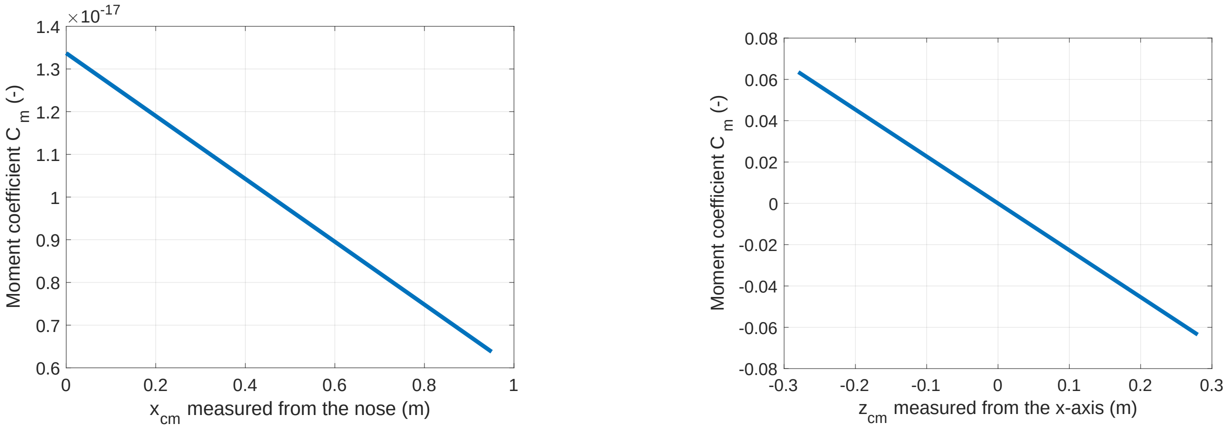
1. In the x-direction, along the velocity vector of the capsule in this case, the range of 'non-unreasonable'[3]values for the position of the center of mass is between the nose and 60% length. The center of mass of most re-entry vehicles is close to the heatshield for stability reasons.
2. In the z-direction, perpendicular to the velocity vector for the case at hand, the range of 'non-unreasonable'[3] shifts from the original position (0 m in z), is about 40% of the nose radius (although that is already quite high, a too large offset might render the vehicle inherently unstable). Note that such shift of the center of mass would permit to obtain lift from a symmetrical vehicle.

Note that when changing the center of mass of the body in one direction, the other components are kept to the reference position $[0.5L_{vehicule}, 0, 0]$ (in (x, y, z) coords, negative sign from the implementation omitted here), although this does not matter in this case as the lift is zero for angles of attack of zero. With $\alpha = 0°$ and M=12.5, this yields Figure 5, which presents the moment coefficient at different positions of the center of mass.

First, for the conditions described above, a shift in the x position of the center of mass does not change the moment coefficient of the original reference point, as the lift is zero and the drag is aligned with the x-axis. The moment coefficient is then zero everywhere on that axis, and the x-position of the center does mass does not contribute to the moment coefficient of the vehicle (again, in this situation of $\alpha = 0°$).

Following, it can be seen from Figure 5b that the z position of the center of mass has a linear effect on the moment coefficient of the vehicle. This plot indicates that for an angle of attack of zero (at Mach 10), the vehicle will be in moment equilibrium around its center of mass if the latter is on the x-axis. In case the center of mass is offset from the x-axis, the vehicle will rotate until it finds the trim angle of attack α_{trim}, at which the moment coefficient is zero (supposedly, as

[3]non-unreasonable: most re-entry vehicles of this type will not go as far as the boundaries of the range but it is worth investigating the effect of having such as shift.



(a) X position.

(b) Z positions.

Figure 5: Effect of changing the position of the center of mass on the moment coefficient.

dynamical effects could be unstable). It can be concluded that shifting the center of mass from the main body axis permits to ensure a natural response of the vehicle to rotate towards a certain angle of attack, and an associated lift coefficient. This can be used for trajectory design, such that a different type of trajectory than a ballistic entry is used.

4 Trim Angle

The trim angle was discussed above to be a 'result' of the shift in center of mass of the vehicle from the x-axis of the body. It is possible to determine the trim angle as a function of the shift in the center of mass Z-location (while keeping the X and Y locations the same). For computational efficiency, however, the inverse problem is solved. The moment coefficient at an arbitrary angle of attack, at any point with an offset from the body main axis, Δz, is given by,

$$C_m = C_{m_0} + \frac{\Delta z}{L_{ref}} \left(C_D \cos(\alpha) - C_L \sin(\alpha) \right) \tag{2}$$

Where C_{m_0} is the moment coefficient at a reference point, as computed by the program. By setting $C_m = 0$, and for a given α, one can solve for Δz. This relation can then simply be inversed, such that for a given Δz, the angle of attack associated is the trim AoA, α_{trim}. This results in the relation shown by Figure 6.

It can be seen from Figure 6 that the relation between the trim angle and the off-set from the x-axis is non-linear. This is due to the non-linear behaviour of the aerodynamic coefficients at high angles of attack (see the lift and moment coefficients curves in Figure 1, eg.). Following this assumption, this means that angles of attack larger than $\approx 13°$ cannot be obtained as trim AoA's by design. Furthermore, it was seen earlier that the α_{max}, for maximum lift-to-drag ratio, is around $21°$, which relates to an offset of 2.4 m. This would lie outside the considered range however.

Now considering a case where the center of mass can be put freely within the entry vehicle (outside of the range of 'non-unreasonable' values defined), this graph can be used to place some ballasts in the entry vehicle to shift the position of the center of gravity along the z-direction. Such a shift then results in a natural response of the vehicle to take the trim angle of attack during entry, and then produce some lift despite being a symmetrical shape. This, then, enhances the possibilities for trajectory design with the given entry vehicle. The overall conclusion is then that a symmetrical vehicle can produce lift, from an offset of the center of mass with respect to the main axis of symmetry (x-axis), during its atmospheric entry.

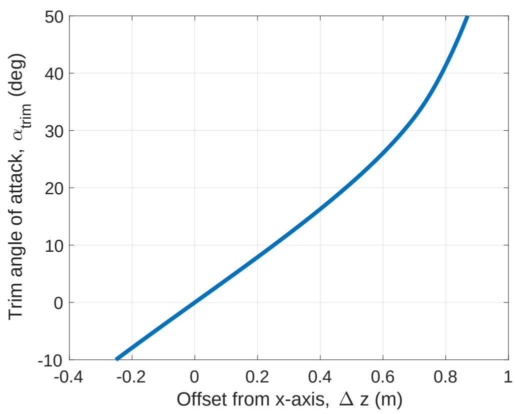


Figure 6: Trim angle of attack as a function of the center of mass offset with respect to the z-axis.