Ocean Engineering 222 (2021) 108439

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Observer-based adaptive robust stabilization of dynamic positioning ship with delay via Hamiltonian method

Jiankuo Cui, Renming Yang *, Chengcheng Pang, Qiang Zhang

Shandong Jiaotong University, Jinan, 250357, PR China

ARTICLE INFO

Keywords:
Time-delay dynamic positioning ship
Hamiltonian model
Observer design
Adaptive robust control

ABSTRACT

In this paper, by applying the Hamiltonian function method, we study the observer-based adaptive robust stabilization problem for dynamic positioning (DP) ship with time delay, and present several new results on the issue. Firstly, the three degree of freedoms DP ship model with time delay is transformed into a Port-Controlled Hamiltonian (PCH) one, based on which we design its observer system. Then, by using the augmented technology and the Lyapunov stability theory, several observer-based robust stabilization controllers and observer-based adaptive robust stabilization controllers are designed for the DP ship with time delay, and some delay-independent and delay-dependent robust stabilization results are obtained. Finally, the simulation results show the effectiveness of the observer-based robust stabilization controller proposed in this paper.

1. Introduction

With the scarcity of terrestrial resources, people are gradually turning more attentions to the ocean, which is a source of abundant. As a shipping tool, the ship is getting more and more important, its control problem has received extensive attention from research and engineering staff (Du et al., 2015; Li et al., 2020; Ding et al., 2015; Zwierzewicz, 2015; Jiang and Bian, 2016; Jin et al., 2017; Fossen, 2002; Zhang et al., 2020a, 2020b). Due to different work tasks, some specific requirements are imposed on ships such as exploration, tracking and replenishment operations, etc. Among them, the mining, salvaging and collaborative work of multiple ships are some main tasks for the marine vessels, which implies that the DP problem is a key issue. The so-called dynamic positioning means that the ship uses its own propulsion device to keep a specific location. In the past decades, the DP problem has attracted the attention of many scholars and some significant results have been obtained in (Du et al., 2015; Li et al., 2020; Donaire and Perez, 2012; Skulstad et al., 2018; Ding et al., 2015; Xia et al., 2013; Zhang et al., 2020c). However, due to the complexity and variability of the marine environment, the danger for marine ship may happen at any time. To reduce this, some robust stabilization control techniques are proposed (Du et al., 2015; Li et al., 2020; Ding et al., 2015). The DP ship under the robust stabilization controller can effectively resist the external interference and make it stable at a target position (Ferial, 2000).

In addition to robustness, another key issue is the observer design problem for the ship. In fact, the position and heading information of the marine ship under study can be directly measured by the Beidou navigation system respectively. While, compared with the position and heading, the measurement of speed is difficult. Although one can use corresponding sensors, more sensors can lead to the system's redundancy and increase the risk of operational errors. Therefore, it is a meaningful work to design an observer for the marine ship under study. In recent years, many scholars have applied modern control theory to study the observer-based control problem for the DP ship, and have made some achievements (Du et al., 2015; Zwierzewicz, 2015; Jiang and Bian, 2016; Jin et al., 2017). In (Du et al., 2015), the authors designed a high-gain observer for the DP ship. In (Zwierzewicz, 2015), some control synthesis results was obtained by means of an output feedback linearization method combined a nonlinear observer. Ref (Jiang and Bian, 2016). designed a controller by constructing a linear expansion state observer and applying the proportional and differential control technology for the DP ship. In Ref. (Jin et al., 2017), considering the existence of unknown external disturbances and the uncertainty of ship model parameters in the system, the authors designed an inner loop observer and an outer loop controller for the system under study, respectively. However, the paper (Jiang and Bian, 2016) used a linear observer so that the design scheme has some limitations. Moreover, the above-mentioned papers (Du et al., 2015; Zwierzewicz, 2015; Jiang and Bian, 2016; Jin et al., 2017) are all based on linear form or the approximate linearization technique. In view of these, some scholars

* Corresponding author.
E-mail address: renmingyang0222@163.com (R. Yang).

https://doi.org/10.1016/j.oceaneng.2020.108439
Received 16 July 2020; Received in revised form 12 October 2020; Accepted 29 November 2020
Available online 29 January 2021
0029-8018/© 2020 Elsevier Ltd. All rights reserved.

have devoted to using sliding mode technology and backstepping method in the ship control system. Ref (Du et al., 2012). constructed a nonlinear state observer for the DP ship system. In Ref. (Xia and Shao, 2014), the authors presented a passive nonlinear robust observer design method for a surface ship with a special structure by using acceleration feedback and sliding mode technique. Based on the structural principle of Luenberger observer and the Lyapunov stability theory, in (Kim et al., 2012), (Ihle et al., 2006) and (Lin et al., 2018), the nonlinear output feedback controllers were derived on the basis of the developed observer by using the backstepping technique. Under unknown constant environmental disturbances and uncertain ship dynamics, some global robust adaptive output feedback control scheme were developed for DP ships based on Lyapunov's direct method and backstepping technique (Du et al., 2015; Francesco et al., 2003; Deng et al., 2017; Tong et al., 2011).

Even so, there are few results on the DP ship system with time delay. As well known, the time delay is inevitable because there exist measurement, transmission and transport lags in the system. Its existence may lead to instability and poor performance for the systems under study (Gu et al., 2003), such as longer response time, and bigger overshoot, and so on. Therefore, it is necessary to study the ship control system with time delay. To solve this problem, the authors in (Zhao et al., 2015) considered a input delay system. By designing a state-derivative control law, the system under study was turned to a neutral time-delay system, and then by applying the delay-decomposition approach and linear matrix inequality (LMI), some less conservative results were derived for the neutral system with state-derivative feedback. In (Xia et al., 2017), by using a fuzzy approximation method, an inversion controller was proposed to solve the problem of input time-delay and disturbance in the DP ship. Based on the upper bound of time delay, Ref (Xia et al., 2013). designed a robust sliding mode controller to reduce the time delay influence for the DP ship. However, the results obtained in (Zhao et al., 2015; Xia et al., 2013, 2017) are not observer-based ones, mainly because it is a very difficult task to develop some observer-based results for a given nonlinear delay system. In fact, one cannot apply the traditional error method like studying linear systems or linear approximation systems. Therefore, it is an urgent need to develop a new method to study the observer-based control problem for nonlinear DP ship system with time delay.

Recently, the Hamiltonian function method has received a lot of attention from scholars since a Hamiltonian function in the system can be chosen as Lyapunov candidate, and some mature design methods have been given for the system in (Wang, 2007; Sun and Fu, 2016; Ortega and Borja, 2014). Particularly, in Refs. (Wang, 2007; Wang et al., 2003; Yang and Wang, 2012), the authors proposed some realization methods on general nonlinear (time-delay) systems. Using the realization method, one can easily transform some systems under study into their Hamiltonian forms by applying the realization methods. In view of these, Refs (Donaire and Perez, 2012; Muhammad and DoRia-Cerezo, 2012; Zhou et al., 2019). extended the Hamiltonian function method to investigate control problems of ships. In (Donaire and Perez, 2012), the authors designed several controllers on the DP ship system with actuator input saturation. In Ref. (Muhammad and DoRia-Cerezo, 2012), a family of passivity-based controllers for DP ship were presented. Ref (Zhou et al., 2019). studied simultaneously stability problem on two DP ships based on the Hamiltonian method, and presented some corresponding results. It should be pointed out that the aforementioned papers are not based on the observer results. More recently, by promoting the Hamiltonian function method, Ref (Yang et al., 2020). investigated the observer-based finite-time robust control problem for a general class of nonlinear time-delay systems.

In short, although the authors in Refs (Du et al., 2015; Francesco et al., 2003; Deng et al., 2017; Tong et al., 2011). have considered the observer-based adaptive control problem for the DP ship system, the systems under study in the papers do not contain time-delay term, and the methods used are backstepping and the error ones. As well known, the error method is suitable for the linear or approximate linear model, while for a DP ship system with nonlinear term, it is very difficult to obtain its error model. Moreover, as shown in Subsection 4.2 below of the paper, the error method will lead to conservative results. To this end, by applying Hamiltonian method and augmented technique, Ref (Yang et al., 2020). studied the observer-based robust control problem for a class of nonlinear time-delay systems. However, the system under study in Ref (Yang et al., 2020). do not contain uncertain term and is a purely theoretical model, that is to say, it is an ideal one. Therefore, it is still an open issue to investigate the observer-based adaptive control problem and present corresponding less conservative results for the real DP ship system, which motivated the present paper.

In this paper, we consider the observer-based adaptive robust stabilization problem for a class of DP ship with time delay, propose several adaptive robust control results, give delay-independent and delay-dependent stabilization results. The main contributions of this paper are as follows: (1) Unlike the existing papers like (Zhao et al., 2015; Zheng et al., 2017), the present article studies the observer design problem and develops several robust control results based on the observer. While Refs (Zhao et al., 2015; Zheng et al., 2017). only proposed some robust stabilization results. (2) Although the authors in Refs (Du et al., 2015; Francesco et al., 2003; Deng et al., 2017; Tong et al., 2011). have considered the adaptive observer design problem for the ship system, the systems under study in the mentioned papers do not contain time-delay term. Obviously, the studied system in the present paper is more general. (3) Unlike the traditional observer design method in (Du et al., 2015; Zwierzewicz, 2015; Jiang and Bian, 2016; Jin et al., 2017), the present paper applies expansion technology rather than the error method. That is to say, the method of the present paper can be applied to study a general nonlinear systems not only linear models and/or approximate linear models, and the results obtained have less conservatism. (4) Different from recent work on observer-based control result for time-delay system (Yang et al., 2020) (**Note**: In (Yang et al., 2020), the authors studied a class of nonlinear time-delay system without uncertainties and applied the orthogonal decomposition method to develop the equivalent Hamiltonian model of the original system), by applying the coordinate transformation method, the present paper develops an equivalent ship Hamiltonian model and investigates a general class of DP ship systems with time-delay, external disturbance and uncertainties simultaneously.

The remainder of the paper is organized as follows. Section 2 is the problem formulation and preliminaries. In Section 3, the observer is designed and several main results are presented. Section 4 gives an illustrative example to support our new result, which is followed by the conclusion in Section 5.

2. Problem formulations

This section gives preliminaries and some lemmas.

In the paper, we first consider the following model of DP ship (Fossen, 2002):

$$\begin{cases} \dot{\eta} = R(\phi)\upsilon, \\ M\dot{\upsilon} = u - D\upsilon + w, \end{cases} \tag{1}$$

where, $\eta = [e, n, \phi]^T$ denotes the positions (e, n) and the yaw angle ϕ in the earth-fixed frame, $\upsilon = [\mu, \nu, r]^T$ expresses the surge, sway velocity and yaw angular velocity in the body-fixed frame respectively. $\eta \in \Omega$ and $\upsilon \in \Omega$ denote the state vector with Ω being some bounded convex neighborhood of the space $\mathbb{R}^3$. $u \in \mathbb{R}^{3\times 1}$ is a control quantity, $M \in \mathbb{R}^{3\times 3}$ is a positive definite inertia matrix, $D \in \mathbb{R}^{3\times 3}$ is a positive definite linear damp matrix. The external interference is $w \in \mathbb{R}^{3\times 1}$ and $R(\phi) \in \mathbb{R}^{3\times 3}$ is the rotation matrix given by

$$R(\phi)=\begin{bmatrix} cos(\phi) & -sin(\phi) & 0 \\ sin(\phi) & cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{2}$$

Next, to develop several observer-based results via Hamiltonian method, we transform system (1) into its Hamilton form. To do this, let x_1 and x_2 as

$$x_1 = v - \alpha_1, \tag{3}$$

$$x_2 = \eta - \eta_d, \tag{4}$$

where η_d denotes the constant expected position and yaw angle, so that $\dot{x}_2 = \dot{\eta}$, and $\alpha_1 \in \mathbb{R}^3$ satisfies

$$\alpha_1 = -k_1 x_2, \tag{5}$$

with $k_1 \in \mathbb{R}^{3\times 3}$ being a constant positive definite matrix.

Substituting (3),(4) and (5) into (1), we have

$$\begin{cases} M\dot{x}_1 = u - Dx_1 + Dk_1x_2 - M\dot{\alpha}_1 + w, \\ \dot{x}_2 = R(\phi)x_1 - R(\phi)k_1x_2. \end{cases} \tag{6}$$

Computing the derivative of α_1 and using (6), we have

$$\dot{\alpha}_1 = -k_1\dot{x}_2 = -k_1R(\phi)x_1 + k_1R(\phi)k_1x_2. \tag{7}$$

Substitute (7) into (6), it is easy to obtain

$$\begin{cases} M\dot{x}_1(t) = u + ax_1(t) + bx_2(t) + w, \\ \dot{x}_2(t) = R(\phi)x_1(t) - R(\phi)k_1x_2(t), \end{cases} \tag{8}$$

where. $a = -D + Mk_1R(\phi), b = Dk_1 - Mk_1R(\phi)k_1$.

Obviously, the response time of the ship speed is affected by the delay response, namely, the state x_1 contains time delay. Therefore, similar to (Fosseen, 2002), in the paper, we consider a more general ship model with time delay as follows:

$$\begin{cases} M\dot{x}_1(t) = u + ax_1(t) - D_1x_1(t-h) + bx_2(t) + w, \\ \dot{x}_2(t) = R(\phi)x_1(t) - R(\phi)k_1x_2(t), \\ x(s) = \varphi(s),\ \forall s \in [-h,\ 0] \end{cases} \tag{9}$$

where, $\varphi(\tau)$ is a vector-valued initial-condition function and $D_1 \in \mathbb{R}^{3\times 3}$ is a weighted matrix.

Remark 2.1. From (1), it is easy to know the equation $\dot{\eta}$does not contain the control quantity u, which implies that one cannot design its damping-injection controller in applying Hamiltonian function method. This is a difficulty in studying the ship system (1) via the Hamiltonian method. Therefore, to develop a standard Hamiltonian form of system (1), motivated by Refs. (Du et al., 2015) and (Zhou et al., 2019), we have introduced the coordinate transforming relations (3) and (4), and obtain an equivalent form (9) of system (1). Thanks for the equivalent form, so that we can get some asymptotic stability results without the damping injection in applying Hamiltonian function method, which is an advantage of the paper. Further details, please see Refs. (Du et al., 2015; Zhou et al., 2019) and Section 3 below.

The main goal of the paper is to apply Hamiltonian method to study the observer-based adaptive robust stabilization problem for DP ship with time delay. Next, we present the Definition and methods on Hamiltonian realization.

Consider the following general nonlinear system:

$$\dot{x}(t) = f(x) + g(x)u, \tag{10}$$

where $x(t) \in \mathbb{R}^n$ is the state, $f(x)$ is a vector field satisfying $f(0) = 0$, u is the control input.

Definition 1((Wang et al., 2003; Yang and Wang, 2012)): If there exists a continuous differentiable function $H(x)$ and a class-K function β such that $H(x) \geq \beta(\|x\|)$, $H(0) = 0$, $\frac{\partial H}{\partial x}\Big|_{x=0} = 0$ and $\frac{\partial H}{\partial x}\Big|_{x\neq 0} \neq 0$, then $H(x)$ is called a regular positive definite one on x. For example, $H(x) = x_1^2 + x_2^2$ is a regular positive definite function with respect to x, where ∂ represents the partial derivative.

Definition 2((Sun and Peng, 2014)): system (10) is said to have a Port-Controlled Hamiltonian realization form if there exist a suitable coordinate chart and a regular positive definite function $H(x)$ such that system (10) can be expressed as the following Hamiltonian form:

$$\dot{x}(t) = (J(x) + R(x))\nabla H(x(t)) + g(x)u, \tag{11}$$

where.

$\nabla_x H(x) := \frac{\partial H(x)}{\partial x}$, $J(x) = -J^T(x) \in \mathbb{R}^{n\times n}$, $R(x) \in \mathbb{R}^{n\times n}$ is a symmetric matrix, $H(x)$ is the Hamiltonian function with $x = 0$ as its minimum point and $H(0) = 0$, $\nabla H(x)$ is the gradient vector of $H(x)$ at x.

Noting that, in applying Hamiltonian method, the key problem is that how to express the system under study into its Hamiltonian form. In the following, we present two Generalized Hamilton Realization(GHR) methods under f(x) being smooth and non-smooth cases respectively (Further details, please see (Wang et al., 2003). In addition, for other realization methods, also see (Wang et al., 2003)).

Lemma 2.2. (Wang et al., 2003) For a given regular positive definite function $H(x)$, the system has the following orthogonal decomposition Hamiltonian realization:

$$\dot{x} = [J(x) + R(x)]\nabla_x H(x) + g(x)u, \tag{12}$$

where,

$$J(x) := \frac{1}{||\nabla_x H||^2}\left[f_{td}\nabla_x H^T - \nabla_x H f_{td}^T\right],$$

$$R(x) := \frac{< f, \nabla_x H >}{\left\|\nabla_x \overline{H}\right\|^2} I_n,$$

$$f_{gd}(x) := \frac{< f, \nabla_x H >}{||\nabla_x H||^2}\nabla H,\ f_{td}(x) := f(x) - f_{gd}(x).$$

When $f(x)$ in the system (10) is smooth, we have the following GHR result.

Lemma 2.3. ((Wang et al., 2003)) *Consider the system (10). If $f(x)(= (f_1(x), \cdots, f_n(x))^T)$is a smooth vector field and its Jacobi matrix $J_f$is non-singular, then the system (10) has the following Hamiltonian realization:*

$$\dot{x} = [J(x) + R(x)]\nabla_x H(x) + g(x)u,$$

where $H(x) = \frac{1}{2}\sum_{i=1}^n f_i^2(x)$, $\nabla_x H(x) = \left[\sum_{i=1}^n f_i(x)\frac{\partial f_i(x)}{\partial x_1}, \cdots, \sum_{i=1}^n f_i(x)\frac{\partial f_i(x)}{\partial x_n}\right]^T$ *and* $J(x) + R(x) := J_f^{-T}(x)$. *In addition, if $f(x)$ is a smooth vector field and its Jacobi matrix J_f is singular with a fixed reversible sub-block, then there exist a matrix $N(x)$ and a vector field $g(x)(= (g_1(x), \cdots, g_n(x))^T)$ with non-singular Jacobi matrix J_g such that the system (10) has following Hamiltonian realization:*

$$\dot{x} = [J(x) + R(x)]\nabla_x H(x) + g(x)u$$

with $H(x) = \frac{1}{2}\sum_{i=1}^n g_i^2(x)$, $\nabla_x H(x) = \left[\sum_{i=1}^n g_i(x)\frac{\partial g_i(x)}{\partial x_1}, \cdots, \sum_{i=1}^n g_i(x)\frac{\partial g_i(x)}{\partial x_n}\right]^T$ *and* $J(x) + R(x) := N(x)J_{\overline{g}}^{-T}(x)$.

Remark 2.4. *Different from Lemma 2.3, Lemma 2.2 presents a general realization method for a given system under study. That is to say, by applying Lemma 2.2, whether $f(x)$is smooth or non-smooth, one can always choose a suitable Hamiltonian function to transform the system under study into its Hamiltonian form. In the paper, we only present two Hamiltonian realization* *methods. Note that the GHR method is not unique for a given system. For specific details and other realization methods, please see* (Wang, 2007; Wang et al., 2003; Yang and Wang, 2012).

Now, we transform the system (9) *into its Hamilton form. To do this, based on the structure of the system* (9), *we choose a special Hamiltonian function as*:

$$H(x)=\frac{1}{2}x_1^Tx_1+\frac{1}{2}x_2^Tx_2, \tag{13}$$

then the system (9) *can be expressed as*:

$$\dot{x}(t)=A_1(x)\nabla H(x(t))+A_2(x)\nabla H(x(t-h))+g(x)u+g(x)w, \tag{14}$$

where $x(t)=[x_1^T(t),\ x_2^T(t)]^T$, $A_1(x)=\begin{bmatrix} M^{-1}a & M^{-1}b \\ R(\phi) & -R(\phi)k_1 \end{bmatrix}$, $A_2(x)=\begin{bmatrix} -M^{-1}D_1 & 0 \\ 0 & 0 \end{bmatrix}$, $g(x)=\begin{bmatrix} M^{-1} \\ 0_{3\times3} \end{bmatrix}$, *and it is obvious that* $g(x)$ *has the full column rank. In addition, in the paper, we let the output* $y(t)=g^T(x)\nabla H(x(t))$.

Remark 2.5. *It should be pointed out that the system* (14) *is not a linear one due to containing* $R(\phi)$, *a and b in* $A_1(x)$, *which implies that one cannot use the traditional error method to develop its observer-based results.*

Lemma 2.6. (Liao et al., 2003) *For real matrices* X, Y *and a scalar* $\iota>0$, *the following inequality is true*:

$$X^TY+Y^TX\le \iota X^TX+\iota^{-1}Y^TY. \tag{15}$$

Lemma 2.7. (Gu et al., 2003) For constant matrix $Z>0$, constant number $h>0$and vector function $x:[0,h]\to\mathbb{R}^n$, the following inequality is true:

$$h\int_0^h x^T(s)Zx(s)\mathrm{d}s\geqslant\left(\int_0^h x(s)\mathrm{d}s\right)^T Z\left(\int_0^h x(s)\mathrm{d}s\right). \tag{16}$$

From the above, we know that the system (14) *is equivalent to the system* (9). *In* Section 3, *we will investigate the observer-based robust stabilization of the system* (14) *and the observer-based adaptive robust stabilization of the system* (14) *with uncertainty*.

3. Main results

In the section, we first present several observer-based robust stabilization results for the equivalent system (14) in Subsection 3.1 below, and then further study the observer-based adaptive robust stabilization problem of system (14) with uncertainty and disturbance in Subsection 3.2.

3.1. Robust stabilization based on observer

In this subsection, we will design an observer of the PCH model (14) and study its observer-based robust stabilization problem. In the subsection, unlike the traditional error system method, we adopt a new approach to develop observer-based robust stabilization result for system (14).

Based on (14), we design an observer system of system (14) as follows:

$$\begin{cases} \dot{\widehat{x}}(t)=A_1(\widehat{x})\nabla H(\widehat{x}(t))+A_2(\widehat{x})\nabla H(\widehat{x}(t-h))+g(\widehat{x})u+k_2^TM^T[y(t)-\widehat{y}(t)] \\ +g(\widehat{x})w, \\ \widehat{y}(t)=g^T(\widehat{x})\nabla H(\widehat{x}(t)), \end{cases} \tag{17}$$

from which and system (14), by applying the augmented technique (Wang, 2007), one can obtain

$$\dot{X}(t)=\overline{B}(X)\nabla\overline{H}(X(t))+F(X)\nabla\overline{H}(X(t-h))+\overline{g}(X)u+\overline{g}(X)w, \tag{18}$$

where $k_2\in\mathbb{R}^{3\times6}$ is a weighted matrix, $X(t):=[x^T,\ \widehat{x}^T]^T$, $\overline{H}(X(t))=H(x(t))+H(\widehat{x}(t)),\overline{H}(X(t-h))=H(x(t-h))+H(\widehat{x}(t-h))$, $\overline{g}(X)=[g^T(x),\ g^T(\widehat{x})]^T$,

$$\overline{B}(X)=\begin{bmatrix} A_1(x) & 0 \\ k_2^TM^Tg^T(x) & A_1(\widehat{x})-k_2^TM^Tg^T(\widehat{x}) \end{bmatrix}, F(X)=\begin{bmatrix} A_2(x) & 0 \\ 0 & A_2(\widehat{x}) \end{bmatrix}.$$

Remark 3.1. It should be pointed out that, for studying the observer-based control problem, one usually constructs an error system by using the original and observer systems. However, the method is suitable for the linear model or approximate linear model (Du et al., 2012; Zwierzewicz, 2015), while for a general system, it is very difficult to obtain its error one(**Note**:system (14) of the paper is not linear model or approximate linear model since there exists $R(\phi)$, a and b in $A_1(x)$). Therefore, motivated by (Wang, 2007), we adopt an augmented technique to develop a dimension expansion system(18), which is different from the ones in (Du et al., 2012; Zwierzewicz, 2015). Therefore, the method presented in this paper is more general.

To study the robust control problem of system (18) based on observer method, we give the Definition and two assumptions.

The observer-based robust control problem of system (18): Design an observer-based output feedback control law as: $u=a(\widehat{x},y)$ with the observer $\dot{\widehat{x}}=f(\widehat{x},y,u)$ such that all the states of system (18) under the control law are asymptotic stabilization when $w=0$. Moreover, for any non-zero $w\in\Xi$(Ξ is a bounded region on the disturbance w, which will be defined below) and under the zero state response condition

$$\varphi(s)=0, w(s)=0, s\in[-h,0], \tag{19}$$

the closed-loop system satisfies

$$\int_0^t \parallel z\left(s\right)\parallel^2 ds\le\gamma^2\int_0^t \parallel w\left(s\right)\parallel^2 ds, \infty<t<0, \tag{20}$$

where $\gamma>0$ is the disturbance attenuation level, and z is the penalty signal, which is designed as

$$z(t)=\rho g_1^T(x)\nabla H(x(t)), \tag{21}$$

with $\rho\in\mathbb{R}^{3\times3}$ being a weighted matrix, $g_1(x)=g(x)M=\begin{bmatrix} I_{3\times3} \\ 0_{3\times3} \end{bmatrix}$, and $g_1(x)$ has the full column rank.

To clearly show the process, we give the control principle structure diagram in Fig. 1 below:

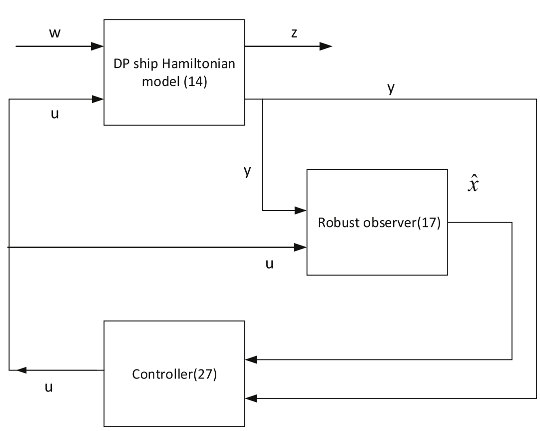

Fig. 1. The control principle structure diagram.

Assumption S1. Hamiltonian function $H(x)$ and its gradient $\nabla H(x)$ satisfy:

$$(1)\quad \varepsilon_1(||x||) \le H(\|x\|) \le \varepsilon_2(x), \tag{22}$$

$$(2)\quad l_1(\ \| x \|\ ) \le \nabla^T H(x))\ \nabla H(x) \le l_2(\|x\|) \tag{23}$$

where $\varepsilon_1, \varepsilon_2, l_1, l_2$ are some functions of K-type.

Remark 3.2. The Assumption is true for the Hamiltonian function(13).

Assumption S2. (Coutinho and Souza, 2008): Assume that the disturbance w belongs to the following set

$$\Xi = \left\{ w \in \mathbb{R}^q : c^2 \int_0^{+\infty} w^T w dt \le 1 \right\}, \tag{24}$$

where c is a positive real number.

For the observer-based robust stabilization problem of system (18), we have the following main results.

Theorem 3.3. *Under* Assumptions S1 *and S2, consider the system (18). For the given $\gamma > 0$, assume that there exist a positive definite symmetric matrice P with appropriate dimension, a positive number ι, and a constant matrix k_2 such that*

$$\gamma^2 - \iota^{-1} \ge 0, \tag{25}$$

$$\Theta_1 : = \begin{bmatrix} E + P + \iota \overline{g}(X)\overline{g}(X)^T & F(X) \\ F^T(X) & -P \end{bmatrix} \le 0, \tag{26}$$

where, $E = Diag\{E_{11}, E_{22}\}$,$E_{11} = A_1(x) + A_1^T(x) - \frac{1}{\gamma^2}g_1(x)g_1^T(x)$, $E_{22} = A_1(\widehat{x}) + A_1^T(\widehat{x}) + g_1(\widehat{x})\rho^T\rho g_1^T(\widehat{x}) + \frac{1}{\gamma^2}g_1(\widehat{x})g_1^T(\widehat{x}) - 2k_2^T g_1^T(\widehat{x}) - 2g_1(\widehat{x})k_2$, *then an H_∞stabilization controller of the system (18)* can be designed as

$$u = -Mk_2\nabla H(\widehat{x}(t)) + M\beta(t), \tag{27}$$

$$\beta(t) = -\Lambda\left[M^T y(t) - M^T \widehat{y}(t)\right], \tag{28}$$

where $\Lambda = \frac{\rho^T\rho}{2} + \frac{1}{2\gamma^2}I_m(m = 3)$.

Remark 3.4. *According to the Definition of the robust control problem, the Proof of Theorem* 3.3 *should be divided into two steps*: 1) *Under the controller* (27), *we prove that, for any non-zero $w \in \Xi$and under the zero state response condition* (19), *the closed-loop system* (18) *satisfies γ-dissipative inequality* (20). 2) *When the disturbance w is equal to zero, the system is asymptotically stable.*

Proof: Substituting (27) and (28) into (18), one can get:

$$\dot{X}(t) = B(X)\nabla\overline{H}(X(t)) + F(X)\nabla\overline{H}(X(t-h)) + \overline{g}(X)w, \tag{29}$$

where

$$B(X) = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix},$$

$$F(X) = \begin{bmatrix} A_2(x) & 0 \\ 0 & A_2(\widehat{x}) \end{bmatrix},$$

$$B_{11} = A_1(x) - \frac{1}{2}g_1(x)\rho^T\rho g_1^T(x) - \frac{1}{2\gamma^2}g_1(x)g_1^T(x),$$

$$B_{12} = \frac{1}{2}g_1(x)\rho^T\rho g_1^T(\widehat{x}) + \frac{1}{2\gamma^2}g_1(x)g_1^T(\widehat{x}) - g_1(x)k_2,$$

$$B_{21} = -\frac{1}{2}g_1(\widehat{x})\rho^T\rho g_1^T(x) - \frac{1}{2\gamma^2}g_1(\widehat{x})g_1^T(x) + k_2^T g_1^T(x),$$

$$B_{22} = A_1(\widehat{x}) + \frac{1}{2}g_1(\widehat{x})\rho^T\rho g_1^T(\widehat{x}) + \frac{1}{2\gamma^2}g_1(\widehat{x})g_1^T(\widehat{x}) - k_2^T g_1^T(\widehat{x}) - g_1(\widehat{x})k_2.$$

Construct the following Lyapunov function as:

$$V(X(t)) = V_1 + V_2, \tag{30}$$

where $V_1 = 2\overline{H}(X(t))$, $V_2 = \int_{t-h}^t \nabla^T\overline{H}(X(\tau))P\nabla\overline{H}(X(\tau))d\tau$, and let

$$Q\left(X\left(t\right)\right) = V\left(X\left(t\right)\right) + \int_0^t \| z\left(s\right)\|^2 - \gamma^2 \| w\left(s\right)\|^2 ds \tag{31}$$

Obviously, $V(X(t))$ satisfies the inequality

$$2\varepsilon_1(||X||) \le V(X(t)) \le 2\varepsilon_2(||X||) + h\pi l_2(||X||), \tag{32}$$

where $\pi = \lambda_{max}(P) > 0$, and let $\varepsilon = 2\varepsilon_1(||X||), l = 2\varepsilon_2(||X||) + h\pi l_2(||X||)$, from which we obtain that there exist K-type functions ε and l such that

$$\varepsilon \parallel X \parallel) \le V(X(t)) \le l(\|X\|. \tag{33}$$

First, we prove $Q(X(t)) \le 0$, namely,

$$\int_0^t \|z(s)\|^2 ds \le \gamma^2 \int_0^t \|w(s)\|^2 ds.$$

Computing the derivative of V_1 along the trajectory of the closed-loop system (29), it is easy to obtain that

$$\begin{aligned}\dot{V}_1 &= 2\nabla^T\overline{H}(X(t))\dot{X}(t) \\ &= 2\nabla^T\overline{H}(X(t))\left(B(X)\nabla\overline{H}(X(t)) + F(X)\nabla\overline{H}(X(t-h)) + \overline{g}(X)w\right) \\ &= 2\nabla^T\overline{H}(X(t))B(X)\nabla\overline{H}(X(t)) + 2\nabla^T\overline{H}(X(t))F(X)\nabla\overline{H}(X(t-h)) \\ &\quad + 2\nabla^T\overline{H}(X(t))\overline{g}(X)w \quad = \nabla^T\overline{H}(X(t))\left[B(X) + B^T(X)\right]\nabla\overline{H}(X(t)) \\ &\quad + 2\nabla^T\overline{H}(X(t))\overline{g}(X)w + 2\nabla^T\overline{H}(X(t))F(X)\nabla\overline{H}(X(t-h)).\end{aligned} \tag{34}$$

In addition, noting that $B(X) = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$ and $B_{12} = -B_{21}^T$, one can obtain $B(X) + B^T(X) = Diag\{B_{11} + B_{11}^T, B_{22} + B_{22}^T\}$ and we have $B_{11} + B_{11}^T = A_1(x) + A_1^T(x) - g_1(x)\rho^T\rho g_1^T(x) - \frac{1}{\gamma^2}g_1(x)g_1^T(x)$, $B_{22} + B_{22}^T = A_1(\widehat{x}) + A_1^T(\widehat{x}) + g_1(\widehat{x})\rho^T\rho g_1^T(\widehat{x}) + \frac{1}{\gamma^2}g_1(\widehat{x})g_1^T(\widehat{x}) - 2k_2^T g_1^T(\widehat{x}) - 2g_1(\widehat{x})k_2$, from which, $z = \rho g_1^T(x)\nabla H(x(t))$ and (34), one can obtain

$$\begin{aligned}\dot{V}_1 &= \nabla^T H(x(t))\left[A_1(x) + A_1^T(x) - g_1(x)\rho^T\rho g_1^T(x) - \frac{1}{\gamma^2}g_1(x)g_1^T(x)\right]\nabla H(x(t)) \\ &\quad + \nabla^T H(\widehat{x}(t))\left[A_1(\widehat{x}) + A_1^T(\widehat{x}) + g_1(\widehat{x})\rho^T\rho g_1^T(\widehat{x}) + \frac{1}{\gamma^2}g_1(\widehat{x})g_1^T(\widehat{x})\right. \\ &\quad \left. - 2k_2^T g_1^T(\widehat{x}) - 2g_1(\widehat{x})k_2\right]\nabla H(\widehat{x}(t)) + 2\nabla^T\overline{H}(X(t))F(X)\nabla\overline{H}(X(t-h)) \\ &\quad + 2\nabla^T\overline{H}(X(t))\overline{g}(X)w \\ &= \nabla^T H(x(t))\left[A_1(x) + A_1^T(x) - \frac{1}{\gamma^2}g_1(x)g_1^T(x)\right]\nabla H(x(t)) \\ &\quad + \nabla^T H(\widehat{x}(t))\left[A_1(\widehat{x}) + A_1^T(\widehat{x}) + g_1(\widehat{x})\rho^T\rho g_1^T(\widehat{x}) + \frac{1}{\gamma^2}g_1(\widehat{x})g_1^T(\widehat{x})\right. \\ &\quad \left. - 2k_2^T g_1^T(\widehat{x}) - 2g_1(\widehat{x})k_2\right]\nabla H(\widehat{x}(t)) + 2\nabla^T\overline{H}(X(t))F(X)\nabla\overline{H}(X(t-h)) \\ &\quad + 2\nabla^T\overline{H}(X(t))\overline{g}(X)w - \|z(t)\|^2.\end{aligned} \tag{35}$$

According to Lemma 2.6, we obtain

$$2\nabla^T\overline{H}(X(t))\overline{g}w \le \iota\nabla^T\overline{H}(X(t))\overline{g}\overline{g}^T\nabla\overline{H}(X(t)) + \iota^{-1}w^T w, \tag{36}$$

where ι is a positive number, such that

$$\dot{V}_1 \le \nabla^T H\Big(x\Big(t\Big)\Big)\Big[A_1\Big(x\Big) + A_1^T\Big(x\Big) - \frac{1}{\gamma^2} g_1\Big(x\Big) g_1^T\Big(x\Big)\Big]\nabla H\Big(x\Big(t\Big)\Big) + \nabla^T H\Big(\widehat{x}\Big(t\Big)\Big)\Big[A_1\Big(\widehat{x}\Big) + A_1^T\Big(\widehat{x}\Big) + g_1\Big(\widehat{x}\Big)\rho^T\rho g_1^T\Big(\widehat{x}\Big) + \frac{1}{\gamma^2} g_1\Big(x\Big) g_1^T\Big(x\Big) - 2K_2^T g_1^T\Big(\widehat{x}\Big) - 2g_1\Big(\widehat{x}\Big)k_2\Big]\nabla H\Big(\widehat{x}\Big(t\Big)\Big) + 2\nabla^T\overline{H}\Big(X\Big(t\Big)\Big)F\Big(X\Big)\nabla\overline{H}\Big(X\Big(t-h\Big)\Big) + \iota\nabla^T\overline{H}\Big(X\Big(t\Big)\Big)\overline{g}\Big(X\Big)\overline{g}^T\Big(X\Big)\nabla\overline{H}\Big(X\Big(t\Big)\Big) + \iota^{-1}w^T w - \|z\Big(s\Big)\|^2 \tag{37}$$

Computing the derivative of V_2, we have

$$\dot{V}_2 = \nabla^T\overline{H}(X(t))P\nabla\overline{H}(X(t)) - \nabla^T\overline{H}(X(t-h))P\nabla\overline{H}(X(t-h)), \tag{38}$$

with which, and substituting $\dot{V}_1, \dot{V}_2$ into the derivative of $Q(X(t))$, one can obtain $\dot{Q}(X(t))$

$$\begin{aligned}
&= \dot{V}(X(t)) + ||z(t)||^2 - \gamma^2||w(t)||^2 = \dot{V}_1 + \dot{V}_2 + ||z(t)||^2 - \gamma^2||w(t)||^2 \\
&\le \nabla^T H\Big(x\Big(t\Big)\Big)\Big[A_1\Big(x\Big) + A^T{}_1\Big(x\Big) - \frac{1}{\gamma^2} g_1\Big(x\Big) g_1^T\Big(x\Big)\Big]\nabla H\Big(x\Big(t\Big)\Big) \\
&+ \nabla^T H\Big(\widehat{x}\Big(t\Big)\Big)\Big[A_1\Big(\widehat{x}\Big) + A_1^T\Big(\widehat{x}\Big) + g_1\Big(\widehat{x}\Big)\rho^T\rho g_1^T\Big(\widehat{x}\Big) \\
&+ \frac{1}{\gamma^2} g_1\Big(x\Big) g_1^T\Big(x\Big) - 2K_2^T g_1^T\Big(\widehat{x}\Big) - 2g_1\Big(\widehat{x}\Big)k_2\Big]\nabla H\Big(\widehat{x}\Big(t\Big)\Big) \\
&+ 2\nabla^T\overline{H}\Big(X\Big(t\Big)\Big)F\Big(X\Big)\nabla\overline{H}\Big(X\Big(t-h\Big)\Big) \\
&+ \iota\nabla^T\overline{H}\Big(X\Big(t\Big)\Big)\overline{g}\Big(X\Big)\overline{g}^T\Big(X\Big)\nabla\overline{H}\Big(X\Big(t\Big)\Big) + \iota^{-1}w^T w - \gamma^2 w^T w \\
&+ \nabla^T\overline{H}\Big(X\Big(t\Big)\Big)P\nabla\overline{H}\Big(X\Big(t\Big)\Big) - \nabla^T\overline{H}\Big(X\Big(t-h\Big)\Big)P\nabla\overline{H}\Big(X\Big(t-h\Big)\Big) \\
&= \nabla^T\overline{H}\Big(X\Big(t\Big)\Big)E\nabla\overline{H}\Big(X\Big(t\Big)\Big) + 2\nabla^T\overline{H}\Big(X\Big(t\Big)\Big)F\Big(X\Big)\nabla\overline{H}\Big(X\Big(t-h\Big)\Big) \\
&+ \nabla^T\overline{H}\Big(X\Big(t\Big)\Big)P\nabla\overline{H}\Big(X\Big(t\Big)\Big) - \nabla^T\overline{H}\Big(X\Big(t-h\Big)\Big)P\nabla\overline{H}\Big(X\Big(t-h\Big)\Big) \\
&+ \iota\nabla^T\overline{H}\Big(X\Big(t\Big)\Big)\overline{g}\Big(X\Big)\overline{g}^T\Big(X\Big)\nabla\overline{H}\Big(X\Big(t\Big)\Big) + \iota^{-1}w^T w - \gamma^2 w^T w \\
&= -\Big(\gamma^2 - \iota^{-1}\Big)w^T w + \xi^T\Big(t\Big)\Theta_1\xi\Big(t\Big)
\end{aligned} \tag{39}$$

where. $\xi(t) = \left[\nabla^T\overline{H}(X(t)) \quad \nabla^T\overline{H}(X(t-h)) \quad\right]^T$.

Using the conditions of the theorem, we have $\dot{Q}(X(t)) \le 0$.

Integrating (39) from 0 to t and using the zero state response condition (19), we obtain

$$V(X(t)) + \int_0^t (\|z(s)\|^2 - \gamma^2\|w(s)\|^2)\mathrm{d}s \le 0, \tag{40}$$

from which and the fact that $V(X(t)) \ge 0$, (20) holds.

Second, we prove that the closed-loop system (29) converges to 0 when $w = 0$.

If $w = 0$, we represent the ship system as follows:

$$\dot{X}(t) = B(X)\nabla\overline{H}(X(t)) + F(X)\nabla\overline{H}(X(t-h)). \tag{41}$$

Computing the derivative of $V(X(t))$ along the trajectory of the closed-loop system (41), similar to (34), we obtain $\dot{V}(X(t)) = \dot{V}_1 + \dot{V}_2$

$$\begin{aligned}
&= \nabla^T H(x(t))\left[A_1(x) + A_1^T(x) - \tfrac{1}{\gamma^2} g_1(x) g_1^T(x)\right]\nabla H(x(t)) \\
&+\nabla^T H(\widehat{x}(t))\Big[A_1(\widehat{x} + A_1^T(\widehat{x} + g_1(\widehat{x}\rho^T\rho g_1^T(\widehat{x} + \frac{1}{\gamma^2} g_1(\widehat{x} g_1^T(\widehat{x} - 2k_2^T g_1^T(\widehat{x} \\
&-2g_1(\widehat{x}k_2\Big]\nabla H(\widehat{x}(t)) + 2\nabla^T\overline{H}(X(t))F(X)\nabla\overline{H}(X(t-h)) - ||z(t)||^2 \\
&+\nabla^T\overline{H}(X(t))P\nabla\overline{H}(X(t)) - \nabla^T\overline{H}(X(t-h))P\nabla\overline{H}(X(t-h)) \\
&= \nabla^T\overline{H}(X(t))E\nabla\overline{H}(X(t)) + 2\nabla^T\overline{H}(X(t))F(X)\nabla\overline{H}(X(t-h)) - ||z(t)||^2 \\
&+\nabla^T\overline{H}(X(t))P\nabla\overline{H}(X(t)) - \nabla^T\overline{H}(X(t-h))P\nabla\overline{H}(X(t-h)) \\
&= \xi^T(t)\Theta_1\xi(t) - \iota\nabla^T\overline{H}(X(t))\overline{g}(X)\overline{g}^T(X)\nabla\overline{H}(X(t)) - ||z(t)||^2,
\end{aligned}$$

from Theorem 3.3 and Assumption S1, one can obtain

$$\begin{aligned}
\dot{V}(X(t)) &\le \xi^T(t)\Theta_1\xi(t) - \|z(t)\|^2 \le -\|z(t)\|^2 \quad \le -\left\|\rho g_1^T(x)\nabla H(x(t))\right\|^2 \\
&\le -\lambda_{min}\left\{g_1(x)\rho^T\rho g_1^T(x)\right\} \| l_2(\ \| x \|\ ) \le -\varpi l_2(\|x\|),
\end{aligned} \tag{42}$$

where $\varpi = \lambda_{min}(\{g_1(x)\rho^T\rho g_1^T(x)\}) > 0$, thus we obtain

$$\dot{V}(X(t)) \le -\varpi l_2(\|x\|). \tag{43}$$

Base on Lyapunov-Krasovskii stability Theory, the closed-loop system (29) is asymptotically stable when $w = 0$. Therefore, the Proof is completed.

In Theorem 3.3, we have given a delay-independent result on system (29), which implies that the result has conservatism for small delay system (Gu et al., 2003). In the following, we present a delay-dependent result.

Theorem 3.5. *Under Assumptions S1 and S2, consider the system* (18). *For the given $\gamma > 0$, if there exist positive definite symmetric matrices P, Z with appropriate dimensions, a positive number ι, and a constant matrix k_2 such that*

$$\gamma^2 - \iota^{-1} \ge 0, \tag{44}$$

$$\Theta_2 := \begin{bmatrix} E + P + h^2 Z + \iota\overline{g}(X)\overline{g}^T(X) & F(X) & 0 \\ F^T(X) & -P & 0 \\ 0 & 0 & -Z \end{bmatrix} \le 0, \tag{45}$$

then an H_∞stabilization controller of the system (18) *can be designed as*

$$u = -Mk_2\nabla H(\widehat{x}(t)) + M\beta(t), \tag{46}$$

$$\beta(t) = -\Lambda\left[M^T y(t) - M^T\widehat{y}(t)\right], \tag{47}$$

where E, P, k_2, and $\Lambda$are the same as Theorem 3.3.

Proof: *Construct the following Lyapunov function as*

$$V(X(t)) = V_1 + V_2 + V_3, \tag{48}$$

where

$$V_1 = 2\overline{H}(X(t)), V_2 = \int_{t-h}^t \nabla^T\overline{H}(X(\tau))P\nabla\overline{H}(X(\tau))d\tau,$$

$$V_3 = h\int_{-h}^0\int_{t+\beta}^t \nabla^T\overline{H}(X(\alpha))Z\nabla\overline{H}(X(\alpha))d\alpha d\beta,$$

and let

$$Q(X(t)) = V(X(t)) + \int_0^t \|z(s)\|^2 - \gamma^2\|w(s)\|^2\mathrm{d}s. \tag{49}$$

Obviously, $V(X(t))$ satisfies the inequality

$$2\varepsilon_1(||X||) \le V(X(t)) \le 2\varepsilon_2(||X||) + h(\pi + \kappa)l_2(||X||), \tag{50}$$

where $\pi = \lambda_{max}(P) > 0, \kappa = \lambda_{max}(Z) > 0$ and $\varepsilon = 2\varepsilon_1(||X||), l = 2\varepsilon_2(||X||) + h(\pi + \kappa)l_2(||X||)$, from which it is easy to know that ε and l belong to K-type functions satisfying

$$\varepsilon(\|X\|) \le V(X(t)) \le l(||X||). \tag{51}$$

First, *we prove $Q(X(t)) \le 0$, namely,*

$$\int_0^t ||z(s)||^2 ds \le \gamma^2\int_0^t ||w(s)||^2\mathrm{d}s.$$

Similar to Theorem 3.3, one can obtain that the system (29). *Computing the derivative of $V(X(t))$ along the trajectory of the closed-loop system*(29), *we have*

$$
\begin{aligned}
\dot{V}_1 \leq & \nabla^T H(x(t))\left[A_1(x)+A_1^T(x)-\tfrac{1}{\gamma^2}g_1(x)g_1^T(x)\right]\nabla H(x(t)) \\
& +\nabla^T H(\widehat{x}(t))\left[A_1(\widehat{x}+A_1^T(\widehat{x}+g_1(\widehat{x}\rho^T\rho g_1^T(\widehat{x}+\frac{1}{\gamma^2}g_1(\widehat{x}g_1^T(\widehat{x}\right. \\
& \left.-2k_2^Tg_1^T(\widehat{x}-2g_1(\widehat{x}k_2\right]\nabla H(\widehat{x}(t))+2\nabla^T\overline{H}(X(t))F(X)\nabla\overline{H}(X(t-h)) \\
& +\iota^{-1}w^Tw+\iota\nabla^T\overline{H}(X(t))\overline{g}(X)\overline{g}^T(X)\nabla\overline{H}(X(t))-\|z(t)\|^2,
\end{aligned} \tag{52}
$$

and,

$$
\dot{V}_2=\nabla^T\overline{H}(X(t))P\nabla\overline{H}(X(t))-\nabla^T\overline{H}(X(t-h))P\nabla\overline{H}(X(t-h)), \tag{53}
$$

Using Lemma 2.7, we obtain that the derivative of V_3:

$$
\begin{aligned}
\dot{V}_3 &= h^2\nabla^T\overline{H}(X(t))Z\nabla\overline{H}(X(t))-h\int_{t-h}^{t}\nabla^T\overline{H}(X(\alpha))Z\nabla\overline{H}(X(\alpha))d\alpha \\
&\leq h^2\nabla^T\overline{H}(X(t))Z\nabla\overline{H}(X(t))-\int_{t-h}^{t}\nabla^T\overline{H}(X(\alpha))d\alpha Z\int_{t-h}^{t}\nabla\overline{H}(X(\alpha))d\alpha.
\end{aligned} \tag{54}
$$

Substituting $\dot{V}_1$, $\dot{V}_2$ and $\dot{V}_3$ into $\dot{V}$, it is easy to obtain $\dot{V}=\dot{V}_1+\dot{V}_2+\dot{V}_3$

$$
\begin{aligned}
\leq & \nabla^T H(x(t))\left[A_1(x)+A_1^T(x)-\tfrac{1}{\gamma^2}g_1(x)g_1^T(x)\right]\nabla H(x(t)) \\
& +\nabla^T H(\widehat{x}(t))\left[A_1(\widehat{x}+A_1^T(\widehat{x}+g_1(\widehat{x}\rho^T\rho g_1^T(\widehat{x}+\frac{1}{\gamma^2}g_1(\widehat{x}g_1^T(\widehat{x}\right. \\
& \left.-2k_2^Tg_1^T(\widehat{x}-2g_1(\widehat{x}k_2\right]\nabla H(\widehat{x}(t))+2\nabla^T\overline{H}(X(t))F(X)\nabla\overline{H}(X(t-h)) \\
& +\iota\nabla^T\overline{H}(X(t))\overline{g}(X)\overline{g}^T(X)\nabla\overline{H}(X(t))+\iota^{-1}w^Tw \\
& +\nabla^T\overline{H}(X(t))P\nabla\overline{H}(X(t))-\nabla^T\overline{H}(X(t-h))P\nabla\overline{H}(X(t-h))-\|z(t)\|^2 \\
& +h^2\nabla^T\overline{H}(X(t))Z\nabla\overline{H}(X(t))-\int_{t-h}^{t}\nabla^T\overline{H}(X(\alpha))d\alpha\int_{t-h}^{t}\nabla\overline{H}(X(\alpha))d\alpha,
\end{aligned} \tag{55}
$$

with which, and computing the derivative of $Q(X(t))$, we obtain $\dot{Q}(X(t))$

$$
\begin{aligned}
&= \dot{V}(t,X(t))+\|z(t)\|^2-\gamma^2\|w(t)\|^2 \\
&\leq \nabla^T H(x(t))\left[A_1(x)+A_1^T(x)-\frac{1}{\gamma^2}g_1(x)g_1^T(x)\right]\nabla H(x(t)) \\
&+\nabla^T H(\widehat{x}(t))\left[A_1(\widehat{x}+A_1^T(\widehat{x}+g_1(\widehat{x}\rho^T\rho g_1^T(\widehat{x}+\frac{1}{\gamma^2}g_1(\widehat{x}g_1^T(\widehat{x}-2k_2^Tg_1^T(\widehat{x}\right. \\
&\left.-2g_1(\widehat{x}k_2\right]\nabla H(\widehat{x}(t))+2\nabla^T\overline{H}(X(t))F(X)\nabla\overline{H}(X(t-h)) \\
&+\iota\nabla^T\overline{H}(X(t))\overline{g}(X)\overline{g}^T(X)\nabla\overline{H}(X(t))+\iota^{-1}w^Tw-\gamma^2w^Tw \\
&+\nabla^T\overline{H}(X(t))P\nabla\overline{H}(X(t))-\nabla^T\overline{H}(X(t-h))P\nabla\overline{H}(X(t-h)) \\
&+h^2\nabla^T\overline{H}(X(t))Z\nabla\overline{H}(X(t))-\int_{t-h}^{t}\nabla^T\overline{H}(X(\alpha))d\alpha Z\int_{t-h}^{t}\nabla\overline{H}(X(\alpha))d\alpha,
\end{aligned} \tag{56}
$$

$$
\begin{aligned}
&= \nabla^T\overline{H}(X(t))E\nabla\overline{H}(X(t))+2\nabla^T\overline{H}(X(t))F(X)\nabla\overline{H}(X(t-h)) \\
&+\nabla^T\overline{H}(X(t))P\nabla\overline{H}(X(t))-\nabla^T\overline{H}(X(t-h))P\nabla\overline{H}(X(t-h)) \\
&+\iota\nabla^T\overline{H}(X(t))\overline{g}\overline{g}^T\nabla\overline{H}(X(t))+\iota^{-1}w^Tw-\gamma^2w^Tw \\
&+h^2\nabla^T\overline{H}(X(t))Z\nabla\overline{H}(X(t))-\int_{t-h}^{t}\nabla^T\overline{H}(X(\alpha))d\alpha Z\int_{t-h}^{t}\nabla\overline{H}(X(\alpha))d\alpha, \\
&= -(\gamma^2-\iota^{-1})w^Tw+\zeta^T(t)\Theta_2\zeta(t),
\end{aligned}
$$

where $\zeta(t)=\left[\nabla^T\overline{H}(X(t)) \quad \nabla^T\overline{H}(X(t-h)) \quad \int_{t-h}^{t}\nabla^T\overline{H}(X(\alpha))d\alpha\right]^T$.

Using the conditions of Theorem 3.5, one can obtain $\dot{Q}(X(t))\leq 0$. Integrating (56) from 0 to t and using the zero state response condition (19), we obtain

$$
V(X(t))+\int_0^t\|z(s)\|^2-\gamma^2\|w(s)\|^2\mathrm{d}s\leq 0, \tag{57}
$$

from which and $V(X(t))\geq 0$, we have

$$
\int_0^t\|z(s)\|^2\mathrm{d}s\leq\gamma^2\int_0^t\|w(s)\|^2\mathrm{d}s,
$$

namely, (52)holds.

***Second**, we prove that the closed-loop system (29) converges to 0 when $w=0$. To do this, computing the derivative of $V(X(t))$ along the trajectory of the closed-loop system (41), and similar to the Proof of Theorem 3.3, one can obtain the result.*

Corollary 3.6. *Under Assumption S1, consider the system (18) with $w=0$. If there exist positive definite symmetric matrices P, Z with appropriate dimensions such that*

$$
\Theta_3:=\begin{bmatrix}\overline{E}+P+h^2Z & F(X) & 0 \\ F^T(X) & -P & 0 \\ 0 & 0 & -Z\end{bmatrix}\leq 0,
$$

then a stabilization controller of system (18) with $w=0$can be designed as

$$
u=-Mk_2\nabla H(\widehat{x}(t)), \tag{58}
$$

where $\overline{E}=Diag\{\overline{E}_{11},\overline{E}_{22}\}$, $\overline{E}_{11}=A_1(x)+A_1^T(x)$, $\overline{E}_{22}=A_1(\widehat{x})+A_1^T(\widehat{x})-2k_2^Tg_1^T(\widehat{x})-2g_1(\widehat{x})k_2$, and $k_2$is the same as Theorem 3.5.

Remark 3.7. *In Corollary 3.6, we give a delay-dependent stabilization result based on observer for the system (18) without the disturbance.*

Remark 3.8. *Different from existing results on the DP ship (Du et al., 2012; Jin et al., 2017), we consider a more general form and present some delay-independent and delay-dependent results. In addition, it should be pointed out that the matrices D and D_1in the ship system (9) may contain the state variable not just constant one, which implies that the results obtained of the paper have broader application than existing results on the DP ship system.*

3.2. Adaptive robust stabilization based on observer

In this subsection, we will study observer-based adaptive robust stabilization problem of the DP ship system (14) with parametric uncertainties in their structure matrix and Hamiltonian function. Under the case, the system can be expressed as

$$
\begin{cases}\dot{x}(t)=A_1(x,q)\nabla H(x(t),q)+A_2(x)\nabla H(x(t-h))+g(x)u+g(x)w, \\ y(t)=g(x)^T\nabla H(x(t)),\end{cases} \tag{59}
$$

Moreover, similar to (Yang and Guo, 2018; Sun and Fu, 2016), we assume the q is a bounded uncertainty of systems (14), and $A_1(x,0)=A_1(x)$, $\nabla H(x,q)=\nabla H(x)+\triangle_H(x,q)$.

Assumption S3. There exists a constant matrix Φ such that

$$
A_1(x,q)\triangle_H(x,q)=g(x)\Phi\theta \tag{60}
$$

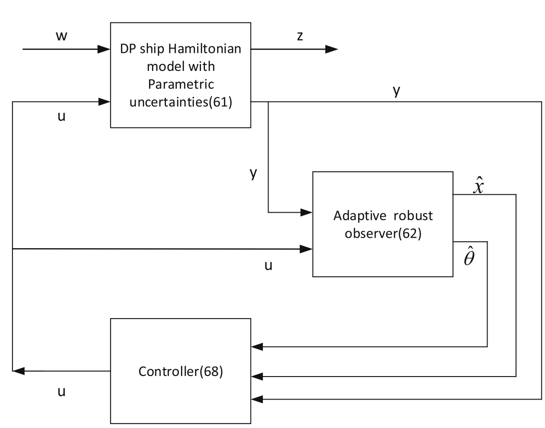

Fig. 2. The control principle structure diagram.

holds for all $x \in \Omega$, where θ represents a uncertain vector on q.

Remark 3.9. Noting that (60) is a matching condition in studying adaptive control problem, similar conditions can be seen in (Yang and Guo, 2018; Sun and Fu, 2016). Moreover, many real systems satisfy the Assumption in existing literature (Wang, 2007; Zhou et al., 2019; Yang and Guo, 2018; Sun and Fu, 2016). Particularly, for the ship system, Assumption S3 is also true in studying adaptive control problem. Further details, also see (Zhou et al., 2019) and the simulation example in Section 4 below.

Under Assumption S3, system (59) can be transformed into the following form:

$$\begin{cases} \dot{x}(t) = A_1(x,q)\nabla H(x(t)) + A_2(x)\nabla H(x(t-h)) + g(x)u + g(x)w + g(x)\Phi\theta, \\ y(t) = g^T(x)\nabla H(x(t)). \end{cases} \tag{61}$$

Based on (61), we design its observer system as follows:

$$\begin{cases} \dot{\widehat{x}}(t) = A_1(\widehat{x},0)\nabla H(\widehat{x}(t)) + A_2(\widehat{x})\nabla H(\widehat{x}(t-h)) + k_2^T M^T[y(t)-\widehat{y}(t)] + g(\widehat{x})u + g(\widehat{x})w + g(\widehat{x})\Phi\widehat{\theta}, \\ \widehat{y}(t) = g^T(\widehat{x})\nabla H(\widehat{x}(t)), \end{cases} \tag{62}$$

from which and system (61), one can obtain

$$\dot{X}(t) = \overline{B}(X,q)\nabla\overline{H}(X(t)) + F(X)\nabla\overline{H}(X(t-h)) + \begin{bmatrix} g(x)\Phi\theta \\ g(\widehat{x})\Phi\widehat{\theta} \end{bmatrix} + \overline{g}(X)u + \overline{g}(X)w, \tag{63}$$

where

$$\overline{B}(X,q) = \begin{bmatrix} A_1(x,q) & 0 \\ k_2^T M^T g^T(x) & A_1(\widehat{x}) - k_2^T M^T g^T(\widehat{x}) \end{bmatrix}, F(X) = \begin{bmatrix} A_2(x) & 0 \\ 0 & A_2(\widehat{x}) \end{bmatrix}.$$

The observer-based adaptive robust control problem of system (63): Design an observer-based output feedback control law as: $u = a(\widehat{x},y,\widehat{\theta})$ with the observer $\dot{\widehat{x}} = f(\widehat{x},y,u)$ such that all the states of system (63) under the control law are adaptive asymptotic stabilization when $w = 0$. Moreover, for any non-zero $w \in \Xi$ and under the zero state response condition

$$\varphi(s) = 0, w(s) = 0, \widehat{\theta}(s) = 0, \theta(s) = 0 \; s \in [-h, 0], \tag{64}$$

the closed-loop system satisfies

$$\int_0^t \|z(s)\|^2 ds \le \gamma^2 \int_0^t \|w(s)\|^2 ds, \infty > t > 0. \tag{65}$$

The observer-based adaptive robust control principle structure diagram (Fig. 2) is shown as follows:

For the adaptive robust stabilization problem based on observer of systems (63), we have the following some results.

Theorem 3.10. *Under Assumptions S1, S2, and S3, consider system (63). For the given $\gamma > 0$, assume that there exist positive definite symmetric matrices P, Z with appropriate dimensions, a positive number ι, and a constant matrix $k_2$such that*

$$\gamma^2 - \iota^{-1} \ge 0, \tag{66}$$

$$\Theta_4 := \begin{bmatrix} N + P + h^2 Z + \iota\overline{g}(X)\overline{g}^T(X) & F(X) & 0 \\ F^T(X) & -P & 0 \\ 0 & 0 & -Z \end{bmatrix} \le 0, \tag{67}$$

where $N = Diag\{N_{11}, N_{22}\}$,

$$N_{11} = A_1(x,q) + A_1^T(x,q) - \frac{1}{\gamma^2} g_1(x) g_1^T(x),$$

$$N_{22} = A_1(\widehat{x}) + A_1^T(\widehat{x}) + g_1(\widehat{x})\rho^T\rho g_1^T(\widehat{x}) + \frac{1}{\gamma^2} g_1(\widehat{x}) g_1^T(\widehat{x}) - 2k_2^T g_1^T(\widehat{x}) - 2g_1(\widehat{x})k_2,$$

then an H_∞adaptive stabilization controller of the system (63) can be designed as

$$u = -Mk_2\nabla H(\widehat{x}(t)) + M\beta(t) - \Phi\widehat{\theta}, \tag{68}$$

$$\beta(t) = -\Lambda\left[M^T y(t) - M^T\widehat{y}(t)\right], \tag{69}$$

$$\dot{\widehat{\theta}} = \Gamma\Phi^T G^T(X)\nabla\overline{H}(X(t)), \tag{70}$$

where $\Lambda = \frac{\rho^T\rho}{2} + \frac{1}{2\gamma^2} I_m (m = 3)$, $\Gamma \in \mathbb{R}^{6\times 6}$ is a weighted matrix, and $G(X) = [g^T(x), \; 0_{3\times 6}]^T$.

Remark 3.11. According to the Definition of the adaptive robust control problem, the Proof of Theorem 3.10 *is divided into two steps:* 1) we prove, for any non-zero $w \in \Xi$and under the zero state response condition *(64),* the closed-loop system *(63)* satisfies γ-dissipative inequality *(65).* 2) When the disturbance is zero, the system is asymptotically stable.

proof Substituting (68), (69) *and* (70) *into* (63), *one can get*:

$$\dot{X}(t) = B(X,q)\nabla\overline{H}(X(t)) + F(X)\nabla\overline{H}(X(t-h)) + \overline{g}(X)w + G(X)\Phi\widetilde{\theta}, \tag{71}$$

where $\widetilde{\theta} = \theta - \widehat{\theta}$,

$$B(X,q) = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}, F(X) = \begin{bmatrix} A_2(x) & 0 \\ 0 & A_2(\widehat{x}) \end{bmatrix}, \text{and}$$

$$B_{11} = A_1(x,q) - \frac{1}{2} g_1(x)\rho^T\rho g_1^T(x) - \frac{1}{2\gamma^2} g_1(x) g_1^T(x),$$

$$B_{12} = \frac{1}{2} g_1(x)\rho^T\rho g_1^T(\widehat{x}) + \frac{1}{2\gamma^2} g_1(x) g_1^T(\widehat{x}) - g_1(x)k_2,$$

$$B_{21} = -\frac{1}{2} g_1(\widehat{x})\rho^T\rho g_1^T(x) - \frac{1}{2\gamma^2} g_1(\widehat{x}) g_1^T(x) + k_2^T g_1^T(x),$$

$$B_{22} = A_1(\widehat{x}) + \frac{1}{2} g_1(\widehat{x})\rho^T\rho g_1^T(\widehat{x}) + \frac{1}{2\gamma^2} g_1(\widehat{x}) g_1^T(\widehat{x}) - k_2^T g_1^T(\widehat{x}) - g_1(\widehat{x})k_2.$$

Construct the following Lyapunov function as:

$$V(X(t)) = V_1 + V_2 + V_3, \tag{72}$$

where, $V_1 = 2\overline{H}(X(t))$, $V_2 = \int_{t-h}^t \nabla^T\overline{H}(X(\tau))P\nabla\overline{H}(X(\tau))d\tau$,

$$V_3 = h\int_{-h}^0 \int_{t+\beta}^t \nabla^T\overline{H}(X(\alpha))Z\nabla\overline{H}(X(\alpha))d\alpha d\beta,$$

and let

$$Q(X(t)) = V(X(t)) + (\theta - \hat{\theta})^T \Gamma^{-1} (\theta - \hat{\theta}) + \int_0^t \left(\|z(s)\|^2 - \gamma^2 \|w(s)\|^2 \right) ds. \tag{73}$$

Obviously, $V(X(t))$ satisfies the inequality

$$2\varepsilon_1(\|X\|) \le V(X(t)) \le 2\varepsilon_2(\|X\|) + h(\pi + \kappa) l_2(\|X\|), \tag{74}$$

where $\pi = \lambda_{max}(P) > 0$, $\kappa = \lambda_{max}(Z) > 0$ and $\varepsilon = 2\varepsilon_1(\|X\|)$, $l = 2\varepsilon_2(\|X\|) + h(\pi + \kappa) l_2(\|X\|)$, from which it is easy to know that ε and l belong to K-type functions satisfying

$$\varepsilon(\|X\|) \le V(X(t)) \le l(\|X\|). \tag{75}$$

Now, we show $Q(X(t)) \le 0$, which implies that

$$\int_0^t \|z(s)\|^2 ds \le \gamma^2 \int_0^t \|w(s)\|^2 ds.$$

Computing the derivative of V_1 along the trajectory of the closed-loop system(71), it is easy to obtain that $\dot{V}_1$

$$\begin{aligned}
&= 2\nabla^T \overline{H}(X(t)) \dot{X}(t) \\
&= 2\nabla^T \overline{H}(X(t)) B(X,q) \nabla \overline{H}(X(t)) + 2\nabla^T \overline{H}(X(t)) F(X) \nabla \overline{H}(X(t-h)) \\
&\quad + 2\nabla^T \overline{H}(X(t)) \overline{g}(X) w + 2\nabla^T \overline{H}(X(t)) G(X) \Phi \widetilde{\theta} = \nabla^T \overline{H}(X(t)) \big[B(X,q) \\
&\quad + B^T(X,q) \big] \nabla \overline{H}(X(t)) + 2\nabla^T \overline{H}(X(t)) G(X) \Phi \widetilde{\theta} \\
&\quad + 2\nabla^T \overline{H}(X(t)) F(X) \nabla \overline{H}(X(t-h)) + 2\nabla^T \overline{H}(X(t)) \overline{g}(X) w.
\end{aligned} \tag{76}$$

In addition, noting that $B(X,q) = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$ and $B_{12} = -B_{21}^T$, one can get $B(X,q) + B^T(X,q) = Diag\{B_{11} + B_{11}^T, B_{22} + B_{22}^T\}$, and we have $B_{11} + B_{11}^T = A_1(x,q) + A_1^T(x,q) - g_1(x)\rho^T \rho g_1^T(x) - \frac{1}{\gamma^2} g_1(x) g_1^T(x)$,

$B_{22} + B_{22}^T = A_1(\hat{x}) + A_1^T(\hat{x}) + g_1(\hat{x})\rho^T \rho g_1^T(\hat{x}) + \frac{1}{\gamma^2} g_1(\hat{x}) g_1^T(\hat{x}) - 2k_2^T g_1^T(\hat{x}) - 2g_1(\hat{x}) k_2$, from which, $z = \rho g_1^T(x) \nabla H(x(t))$ and (76), one can obtain $\dot{V}_1$

$$\begin{aligned}
&= \nabla^T H(x(t)) \left[A_1(x,q) + A_1^T(x,q) - g_1(x)\rho^T \rho g_1^T(x) - \frac{1}{\gamma^2} g_1(x) g_1^T(x) \right] \nabla H(x(t)) \\
&\quad + \nabla^T H(\hat{x}(t)) \Big[A_1(\hat{x}) + A_1^T(\hat{x}) + g_1(\hat{x})\rho^T \rho g_1^T(\hat{x}) \\
&\quad + \frac{1}{\gamma^2} g_1(\hat{x}) g_1^T(\hat{x}) - 2k_2^T g_1^T(\hat{x}) - 2g_1(\hat{x}) k_2 \Big] \nabla H(\hat{x}(t)) \\
&\quad + 2\nabla^T \overline{H}(X(t)) F(X) \nabla \overline{H}(X(t-h)) + 2\nabla^T \overline{H}(X(t)) \overline{g}(X) w \\
&\quad + 2\nabla^T \overline{H}(X(t)) G(x) \Phi \widetilde{\theta} \\
&= \nabla^T H(x(t)) \left[A_1(x,q) + A_1^T(x,q) - \frac{1}{\gamma^2} g_1(x) g_1^T(x) \right] \nabla H(x(t)) \\
&\quad + \nabla^T H(\hat{x}(t)) \left[A_1(\hat{x}) + A_1^T(\hat{x}) + g_1(\hat{x})\rho^T \rho g_1^T(\hat{x}) + \frac{1}{\gamma^2} g_1(\hat{x}) g_1^T(\hat{x}) \right. \\
&\quad \left. - 2k_2^T g_1^T(\hat{x}) - 2g_1(\hat{x}) k_2 \right] \nabla H(\hat{x}(t)) + 2\nabla^T \overline{H}(X(t)) F(X) \nabla \overline{H}(X(t-h)) \\
&\quad + 2\nabla^T \overline{H}(X(t)) \overline{g}(X) w + 2\nabla^T \overline{H}(X(t)) G(X) \Phi \widetilde{\theta} - \|z(t)\|^2.
\end{aligned}$$

Using Lemma 2.6 and (77), we obtain $\dot{V}_1$

$$\begin{aligned}
&\le \nabla^T H(x(t)) \left[A_1(x,q) + A_1^T(x,q) - \frac{1}{\gamma^2} g_1(x) g_1^T(x) \right] \nabla H(x(t)) - \|z(t)\|^2 \\
&+ \nabla^T H(\hat{x}(t)) \left[A_1(\hat{x} + A_1^T(\hat{x} + g_1(\hat{x}\rho^T \rho g_1^T(\hat{x} + \frac{1}{\gamma^2} g_1(\hat{x} g_1^T(\hat{x} \right. \\
&\left. - 2k_2^T g_1^T(\hat{x} - 2g_1(\hat{x} k_2 \right] \nabla H(\hat{x}(t)) + 2\nabla^T \overline{H}(X(t)) F(X) \nabla \overline{H}(X(t-h)) \\
&+ \iota \nabla^T \overline{H}(X(t)) \overline{g}(X) \overline{g}^T(X) \nabla \overline{H}(X(t)) + \iota^{-1} w^T w + 2\nabla^T \overline{H}(X(t)) G(X) \Phi \widetilde{\theta}.
\end{aligned} \tag{77}$$

Computing the derivative of V_2, we have

$$\dot{V}_2 = \nabla^T \overline{H}(X(t)) P \nabla \overline{H}(X(t)) - \nabla^T \overline{H}(X(t-h)) P \nabla \overline{H}(X(t-h)). \tag{78}$$

Using Lemma 2.7, we obtain that the derivative of V_3:

$$\begin{aligned}
\dot{V}_3 &= h^2 \nabla^T \overline{H}(X(t)) Z \nabla \overline{H}(X(t)) - h \int_{t-h}^t \nabla^T \overline{H}(X(\alpha)) Z \nabla \overline{H}(X(\alpha)) d\alpha \\
&\le h^2 \nabla^T \overline{H}(X(t)) Z \nabla \overline{H}(X(t)) - \int_{t-h}^t \nabla^T \overline{H}(X(\alpha)) d\alpha Z \int_{t-h}^t \nabla \overline{H}(X(\alpha)) d\alpha.
\end{aligned} \tag{79}$$

Computing the derivative of $Q(X(t))$, we obtain

$$\dot{Q}(X(t)) = \dot{V}(t, X(t)) + \|z(t)\|^2 - \gamma^2 \|w(t)\|^2 - 2(\theta - \hat{\theta})^T \Gamma^{-1} \dot{\hat{\theta}}. \tag{80}$$

Nothing that $\dot{\hat{\theta}} = \Gamma \Phi^T G^T(X) \nabla \overline{H}(X(t))$, (77), (78), (79), (80) and $\theta - \hat{\theta} = \widetilde{\theta}$, it is easy to obtain that $\dot{Q}(X(t))$

$$\begin{aligned}
&= \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \|z(t)\|^2 - \gamma^2 \|w(t)\|^2 - 2(\theta - \hat{\theta})^T \Gamma^{-1} \dot{\hat{\theta}} \qquad (81) \\
&\le \nabla^T H(x(t)) \left[A_1(x,q) + A_1^T(x,q) - \frac{1}{\gamma^2} g_1(x) g_1^T(x) \right] \nabla H(x(t)) \\
&\quad + \nabla^T H(\hat{x}(t)) \left[A_1(\hat{x}) + A_1^T(\hat{x}) + g_1(\hat{x})\rho^T \rho g_1^T(\hat{x}) + \frac{1}{\gamma^2} g_1(\hat{x}) g_1^T(\hat{x}) \right. \\
&\quad \left. - 2k_2^T g_1^T(\hat{x}) - 2g_1(\hat{x}) k_2 \right] \nabla H(\hat{x}(t)) + 2\nabla^T \overline{H}(X(t)) F(X) \nabla \overline{H}(X(t-h)) \\
&\quad + \iota \nabla^T \overline{H}(X(t)) \overline{g}(X) \overline{g}^T(X) \nabla \overline{H}(X(t)) + \iota^{-1} w^T w - \gamma^2 w^T w \\
&\quad + \nabla^T \overline{H}(X(t)) P \nabla \overline{H}(X(t)) - \nabla^T \overline{H}(X(t-h)) P \nabla \overline{H}(X(t-h)) \\
&\quad + h^2 \nabla^T \overline{H}(X(t)) Z \nabla \overline{H}(X(t)) - \int_{t-h}^t \nabla^T \overline{H}(X(\alpha)) d\alpha Z \int_{t-h}^t \nabla \overline{H}(X(\alpha)) d\alpha \\
&\quad + 2\nabla^T \overline{H}(X(t)) G(X) \Phi \widetilde{\theta} - 2(\theta - \hat{\theta})^T \Gamma^{-1} \dot{\hat{\theta}} \\
&= \nabla^T H(x(t)) \left[A_1(x,q) + A_1^T(x,q) - \frac{1}{\gamma^2} g_1(x) g_1^T(x) \right] \nabla H(x(t)) \\
&\quad + \nabla^T H(\hat{x}(t)) \left[A_1(\hat{x}) + A_1^T(\hat{x}) + g_1(\hat{x})\rho^T \rho g_1^T(\hat{x}) + \frac{1}{\gamma^2} g_1(\hat{x}) g_1^T(\hat{x}) \right. \\
&\quad \left. - 2k_2^T g_1^T(\hat{x}) - 2g_1(\hat{x}) k_2 \right] \nabla H(\hat{x}(t)) + 2\nabla^T \overline{H}(X(t)) F(X) \nabla \overline{H}(X(t-h)) \\
&\quad + \iota \nabla^T \overline{H}(X(t)) \overline{g}(X) \overline{g}^T(X) \nabla \overline{H}(X(t)) + \iota^{-1} w^T w - \gamma^2 w^T w \\
&\quad + \nabla^T \overline{H}(X(t)) P \nabla \overline{H}(X(t)) - \nabla^T \overline{H}(X(t-h)) P \nabla \overline{H}(X(t-h)) \\
&\quad + h^2 \nabla^T \overline{H}(X(t)) Z \nabla \overline{H}(X(t)) - \int_{t-h}^t \nabla^T \overline{H}(X(\alpha)) d\alpha Z \int_{t-h}^t \nabla \overline{H}(X(\alpha)) d\alpha \\
&= \nabla^T \overline{H}(X(t)) N \nabla \overline{H}(X(t)) + 2\nabla^T \overline{H}(X(t)) F(X) \nabla \overline{H}(X(t-h)) \\
&\quad + \nabla^T \overline{H}(X(t)) P \nabla \overline{H}(X(t)) - \nabla^T \overline{H}(X(t-h)) P \nabla \overline{H}(X(t-h)) \\
&\quad + \iota \nabla^T \overline{H}(X(t)) \overline{g} \overline{g}^T \nabla \overline{H}(X(t)) + \iota^{-1} w^T w - \gamma^2 w^T w \\
&\quad + h^2 \nabla^T \overline{H}(X(t)) Z \nabla \overline{H}(X(t)) - \int_{t-h}^t \nabla^T \overline{H}(X(\alpha)) d\alpha Z \int_{t-h}^t \nabla \overline{H}(X(\alpha)) d\alpha \\
&= -\left(\gamma^2 - \iota^{-1} \right) w^T w + \zeta^T(t) \Theta_4 \zeta(t),
\end{aligned}$$

where $\zeta(t) = \left[\nabla^T \overline{H}(X(t)) \quad \nabla^T \overline{H}(X(t-h)) \quad \int_{t-h}^t \nabla^T \overline{H}(X(\alpha)) d\alpha \right]^T$.

Using the conditions of Theorem 3.10, one can obtain $\dot{Q}(X(t)) \le 0$. Integrating (81) from 0 to t and using the zero state response condition (64), we obtain

$$Q(X(t)) = V(X(t)) + (\theta-\widehat{\theta})^T\Gamma^{-1}(\theta-\widehat{\theta}) + \int_0^t\left(||z(s)||^2-\gamma^2\|w(s)\|^2\right)ds \le 0,$$

from which and $V(X(t)) \ge 0$, *we have*

$$\int_0^t ||Z(s)||^2 \mathrm{d}s \le \gamma^2 \int_0^t \|w(s)\|^2 \mathrm{d}s,$$

namely, (65) *holds.*

Second*, we prove that the closed-loop system* (71) *converges to* 0 *when* $w = 0$.

If $w = 0$, *we represent the ship system as follows*:

$$\dot{X}(t) = B(X,q)\nabla\overline{H}(X(t)) + F(X)\nabla\overline{H}(X(t-h)) + G(X)\Phi\widetilde{\theta}, \tag{82}$$

and construct the following Lyapunov function as

$$V(X(t)) = V_1 + V_2 + V_3 + (\theta-\widehat{\theta})^T\Gamma^{-1}(\theta-\widehat{\theta}). \tag{83}$$

Computing the derivative of $V(X(t))$ *along the trajectory of the closed-loop system* (82), *we obtain*

$$\dot{V}(X(t)) = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 - 2(\theta-\widehat{\theta})^T\Gamma^{-1}\dot{\widehat{\theta}} \tag{84}$$

$$\le \nabla^T H(x(t))\left[A_1(x,q) + A_1^T(x,q) - \frac{1}{\gamma^2}g_1(x)g_1^T(x)\right]\nabla H(x(t))$$

$$+\nabla^T H(\widehat{x}(t))\left[A_1(\widehat{x}) + A_1^T(\widehat{x}) + g_1(\widehat{x})\rho^T\rho g_1^T(\widehat{x}) + \frac{1}{\gamma^2}g_1(\widehat{x})g_1^T(\widehat{x})\right.$$

$$\left. -2k_2^T g_1^T(\widehat{x}) - 2g_1(\widehat{x})k_2\right]\nabla H(\widehat{x}(t)) + 2\nabla^T\overline{H}(X(t))F(X)\nabla\overline{H}(X(t-h))$$

$$+\nabla^T\overline{H}(X(t))P\nabla\overline{H}(X(t)) - \nabla^T\overline{H}(X(t-h))P\nabla\overline{H}(X(t-h))$$

$$+h^2\nabla^T\overline{H}(X(t))Z\nabla\overline{H}(X(t)) - \int_{t-h}^t \nabla^T\overline{H}(X(\alpha))d\alpha Z\int_{t-h}^t \nabla\overline{H}(X(\alpha))d\alpha$$

$$+2\nabla^T\overline{H}(X(t))G(X)\Phi\widetilde{\theta} - 2(\theta-\widehat{\theta})^T\Gamma^{-1}\dot{\widehat{\theta}} - ||z(t)||^2$$

$$= \nabla^T\overline{H}(X(t))N\nabla\overline{H}(X(t)) + 2\nabla^T\overline{H}(X(t))F(X)\nabla\overline{H}(X(t-h))$$

$$R(\eta_2) = \begin{bmatrix} \cos\vartheta\cos\psi & -\sin\vartheta\cos\psi+\cos\vartheta\sin\psi\sin\phi & \sin\vartheta\sin\psi+\cos\vartheta\sin\psi\cos\phi \\ \sin\vartheta\cos\psi & \cos\vartheta\cos\psi+\sin\vartheta\sin\psi\sin\phi & -\cos\vartheta\sin\psi+\cos\vartheta\sin\psi\sin\phi \\ -\sin\psi & \cos\psi\sin\phi & \cos\psi\cos\phi \end{bmatrix} \text{ and}$$

$$T(\eta_2) = \begin{bmatrix} 1 & \sin\phi\tan\psi & \cos\phi\tan\psi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\psi & \cos\phi/\cos\psi \end{bmatrix}$$

$$+\nabla^T\overline{H}(X(t))P\nabla\overline{H}(X(t)) - \nabla^T\overline{H}(X(t-h))P\nabla\overline{H}(X(t-h)) - ||z(t)||^2$$

$$+h^2\nabla^T\overline{H}(X(t))Z\nabla\overline{H}(X(t)) - \int_{t-h}^t \nabla^T\overline{H}(X(\alpha))d\alpha Z\int_{t-h}^t \nabla\overline{H}(X(\alpha))d\alpha$$

$$= \xi^T(t)\Theta_4\xi(t) - \iota\nabla^T\overline{H}(X(t))\overline{g}(X)\overline{g}^T(X)\nabla\overline{H}(X(t)) - \|z(t)\|^2.$$

From Theorem 3.10 *and* Assumption S1, *one can obtain*

$$\begin{aligned}\dot{V}(X(t)) &\le \xi^T(t)\Theta_4\xi(t) - z(t)^2 \le -||z(t)||^2 \\ &\le -\left\|\rho g_1^T(x)\nabla H(x(t))\right\|^2 \\ &\le -\rho\left\|g_1^T(x)\right\|^2 l_2(\|x\|) \\ &\le -\varpi l_2(\|x\|).\end{aligned} \tag{85}$$

The remainder of the Proof is similar to Theorem 3.3, *and thus is omitted.*

In the following, we give an adaptive stabilization results under $w = 0$.

Corollary 3.12. *Under* Assumptions S1 *and S*3, *consider the system* (63) *with* $w = 0$. *If there exist positive definite symmetric matrices P, Z with appropriate dimensions such that*

$$\Theta_5 := \begin{bmatrix} \overline{N}+P & F(X) & 0 \\ F^T(X) & -P & 0 \\ 0 & 0 & -Z \end{bmatrix} \le 0,$$

then an adaptive stabilization controller of the system *(63) with* $w =$ 0*can be designed as*

$$u = -Mk_2\nabla H(\widehat{x}(t)) - \Phi\widehat{\theta}, \tag{86}$$

where $\overline{N} = Diag\{\overline{N}_{11}, \overline{N}_{22}\}$, $\overline{N}_{11} = A_1(x,q) + A_1^T(x,q)$, $\overline{N}_{22} = A_1(\widehat{x}) + A_1^T(\widehat{x}) - 2k_2^T g_1^T(\widehat{x}) - 2g_1(\widehat{x})k_2$, *and* k_2, $\widehat{\theta}$*are the same as Theorem* 3.10.

Remark 3.13. *In Corollary* 3.12*, we have given a delay-dependence result for system (63) without the disturbance.*

According to the previous discussion, we obtain the adaptive robust stabilization results for the 3-DOF ship's model based on the observer. Now, we generalize the above results to the 6-DOF ship model, give an adaptive robust stabilization result based on the observer.

The DP ship model with 6-DOF can be described as (Fossen, 2002):

$$\begin{cases} \dot{\eta} = R(\eta)\upsilon, \\ M\dot{\upsilon} = u - D\upsilon + w, \end{cases} \tag{87}$$

where, $\eta = [\eta_1^T, \eta_2^T]^T (\eta_1 = [e,n,d]^T, \eta_2 = [\vartheta,\psi,\phi]^T)$ *denotes the positions and orientation vector with coordinates in the earth-fixed frame,* $\upsilon = [\mu,\nu,j,p,i,r]^T$ *expresses the surge, sway, heave, roll, pitch, yew velocity in the body-fixed frame respectively.* $u \in \mathbb{R}^{6\times1}$ *is a control quantity,* $M \in \mathbb{R}^{6\times6}$ *is a positive definite inertia matrix,* $D \in \mathbb{R}^{6\times6}$ *is a positive definite linear damp matrix. The external interference is* $w \in \mathbb{R}^{6\times1}$ *and* $R(\eta) \in \mathbb{R}^{6\times6}$ *is the rotation matrix given by*

$$R(\eta) = \begin{bmatrix} R(\eta_2) & 0_{3\times3} \\ 0_{3\times3} & T(\eta_2) \end{bmatrix},$$

where,

Next, to develop the observer-based result via Hamiltonian function method, we transform system (87) *into its Hamilton form. To do this, similar to* Section 2*, let* x_1, x_2 *and* α_1 *as*

$$x_1 = \nu - \alpha_1, \tag{88}$$

$$x_2 = \eta - \eta_d, \tag{89}$$

$$\alpha_1 = -k_3x_2, \tag{90}$$

with $k_3 \in \mathbb{R}^{6\times6}$ *being a constant positive definite matrix.*

Substituting (88), (89) *and* (90) *into* (87), *we have*

$$\begin{cases} M\dot{x}_1(t) = u + ax_1(t) + bx_2(t) + w, \\ \dot{x}_2(t) = R(\phi)x_1(t) - R(\phi)k_1x_2(t), \end{cases} \tag{91}$$

where $a = -D + Mk_3R(\eta), b = Dk_3 - Mk_3R(\eta)k_3$.

Similar to (9), *we consider a more general ship model with time delay as follows*:

$$\begin{cases} M\dot{x}_1(t) = u + ax_1(t) - D_2x_1(t-h) + bx_2(t) + w, \\ \dot{x}_2(t) = R(\eta)x_1(t) - R(\eta)k_1x_2(t), \\ x(s) = \varphi(s),\ \forall s \in [-h,\ 0] \end{cases} \tag{92}$$

where, $D_2 \in \mathbb{R}^{6\times 6}$ *is a weighted matrix.*

Similar to Section 2, *choose the Hamiltonian function as*:

$$H(x) = \frac{1}{2}x_1^Tx_1 + \frac{1}{2}x_2^Tx_2, \tag{93}$$

then the system (92) *can be expressed as*:

$$\dot{x}(t) = A_3(x)\nabla H(x(t)) + A_4(x)\nabla H(x(t-h)) + g(x)u + g(x)w, \tag{94}$$

where $A_3(x) = \begin{bmatrix} M^{-1}a & M^{-1}b \\ R(\phi) & -R(\phi)k_3 \end{bmatrix}$, $A_4(x) = \begin{bmatrix} -M^{-1}D_2 & 0 \\ 0 & 0 \end{bmatrix}$, $g(x) = \begin{bmatrix} M^{-1} \\ 0_{6\times 6} \end{bmatrix}$,.

System (92) *with uncertainty can be expressed as*

$$\begin{cases} \dot{x}(t) = A_3(x,q)\nabla H(x(t),q) + A_4(x)\nabla H(x(t-h)) + g(x)u + g(x)w, \\ y(t) = g(x)^T\nabla H(x(t)), \end{cases} \tag{95}$$

Assumption S4. *There exists a constant matrix* Φ *such that*

$$A_3(x,q)\triangle_H(x,q) = g(x)\Phi\theta, \tag{96}$$

holds for all $x \in \Omega$, *where* θ *represents a uncertain vector on* q.

Under Assumption S4, *the system* (95) *can be transformed into the following form*:

$$\begin{cases} \dot{x}(t) = A_3(x,q)\nabla H(x(t)) + A_4(x)\nabla H(x(t-h)) + g(x)u + g(x)w + g(x)\Phi\theta, \\ y(t) = g^T(x)\nabla H(x(t)). \end{cases} \tag{97}$$

From (97), *we design its observer system as follows*:

$$\begin{cases} \dot{\widehat{x}}(t) = A_3(\widehat{x},0)\nabla H(\widehat{x}(t)) + A_4(\widehat{x})\nabla H(\widehat{x}(t-h)) + k_4^TM^T[y(t) - \widehat{y}(t)] + g(\widehat{x})u + g(\widehat{x})w + g(\widehat{x})\Phi\widehat{\theta}, \\ \widehat{y}(t) = g^T(\widehat{x})\nabla H(\widehat{x}(t)), \end{cases} \tag{98}$$

where $k_4 \in \mathbb{R}^{6\times 12}$ *is a weighted matrix, from which and the system* (97), *one can obtain*

$$\dot{X}(t) = \overline{B}(X,q)\nabla\overline{H}(X(t)) + F(X)\nabla\overline{H}(X(t-h)) + \begin{bmatrix} g(x)\Phi\theta \\ g(\widehat{x})\Phi\widehat{\theta} \end{bmatrix} + \overline{g}(X)u + \overline{g}(X)w, \tag{99}$$

where

$$\overline{B}(X,q) = \begin{bmatrix} A_3(x,q) & 0 \\ k_4^TM^Tg^T(x) & A_3(\widehat{x}) - k_4^TM^Tg^T(\widehat{x}) \end{bmatrix}, F(X) = \begin{bmatrix} A_4(x) & 0 \\ 0 & A_4(\widehat{x}) \end{bmatrix}.$$

For the adaptive robust stabilization problem based on observer of the systems (99), *we have the following result.*

Theorem 3.14. Under *Assumptions S1, S2, and S4, consider system (99). For the given* $\gamma > 0$, *assume that there exist positive definite symmetric matrices P, Z with appropriate dimensions, a positive number* ι, *and a constant matrix* k_4 *such that*

$$\gamma^2 - \iota^{-1} \geq 0, \tag{100}$$

$$\Theta_5 := \begin{bmatrix} N + P + h^2Z + \iota\overline{g}(X)\overline{g}^T(X) & F(X) & 0 \\ F^T(X) & -P & 0 \\ 0 & 0 & -Z \end{bmatrix} \leq 0, \tag{101}$$

where $N = Diag\{N_{11}, N_{22}\}$,

$$N_{11} = A_3(x,q) + A_3^T(x,q) - \frac{1}{\gamma^2}g_1(x)g_1^T(x),$$

$$N_{22} = A_3(\widehat{x}) + A_3^T(\widehat{x}) + g_1(\widehat{x})\rho^T\rho g_1^T(\widehat{x}) + \frac{1}{\gamma^2}g_1(\widehat{x})g_1^T(\widehat{x}) - 2k_4^Tg_1^T(\widehat{x}) - 2g_1(\widehat{x})k_4,$$

then an H_∞ adaptive stabilization controller of the system *(99)* can be designed as

$$u = -Mk_4\nabla H(\widehat{x}(t)) + M\beta(t) - \Phi\widehat{\theta}, \tag{102}$$

$$\beta(t) = -\Lambda\left[M^Ty(t) - M^T\widehat{y}(t)\right], \tag{103}$$

$$\dot{\widehat{\theta}} = \Gamma\Phi^TG^T(X)\nabla\overline{H}(X(t)), \tag{104}$$

where $\Lambda = \frac{\rho^T\rho}{2} + \frac{1}{2\gamma^2}I_m(m=6)$, $\Gamma \in \mathbb{R}^{12\times 12}$ *is a weighted matrix, and* $G(X) = [g^T(x),\ 0_{6\times 12}]^T$.

Proof The proof is similar to Theorem 3.10, *and thus is omitted.*

4. Simulation examples

In this section, an illustrative example is given to verify the effectiveness of the proposed control scheme.

4.1. Performance analysis of proposed DP control law

Consider the following DP ship model (Fossen, 2002):

$$\begin{cases} \dot{\eta} = R(\phi)\upsilon, \\ M\dot{\upsilon} = u - D\upsilon + w, \end{cases} \tag{105}$$

By applying the method of the paper, the ship model (105) is transformed into

$$\begin{cases} M\dot{x}_1(t) = u - (D - Mk_1R(\phi))x_1(t) + (Dk_1 - Mk_1R(\phi)k_1)x_2(t) + w, \\ \dot{x}_2(t) = R(\phi)x_1(t) - R(\phi)k_1x_2(t), \\ x(s) = \varphi(s),\ \forall s \in [-h,\ 0] \end{cases} \tag{106}$$

Since the complexity of DP problem, one cannot develop its exact model, which implies that D should be divided into two part, namely, $D = D_0 + D_\delta$, where D_0 is the normal part and D_δ is the unknown part. In addition, it is obvious that the velocity state exists time delay. Therefore, the general form of the model (106) can be expressed as:

$$\begin{cases} M\dot{x}_1(t) = u - (D_0 - Mk_1R(\phi))x_1(t) + D_\delta x_1 - D_1x_1(t-h) \\ +(D_0k_1 - Mk_1R(\phi)k_1)x_2(t) + D_\delta k_1x_2(t) + w, \\ \dot{x}_2(t) = R(\phi)x_1(t) - R(\phi)k_1x_2(t), \\ x(s) = \varphi(s),\ \forall s \in [-h,\ 0] \end{cases} \tag{107}$$

Now, by using Theorem 3.10, we study the adaptive robust control problem based on observer for the model (107).

Obviously, Assumptions S1 and S2 are satisfied easily. Now, we show Assumption S3 also holds for the ship system. Let $A_1(x,q)\triangle_H(x,q) = \begin{bmatrix} -M^{-1}D_\delta x_1 + M^{-1}D_\delta k_1 x_2 \\ 0_{3\times3} \end{bmatrix}$, then

$$\begin{aligned} A_1(x,q)\triangle_H(x,q) &= \begin{bmatrix} -M^{-1}D_\delta x_1 + M^{-1}D_\delta k_1 x_2 \\ 0_{3\times3} \end{bmatrix} \\ &= \begin{bmatrix} M^{-1} \\ 0_{3\times3} \end{bmatrix} \begin{bmatrix} -I & I \end{bmatrix} \begin{bmatrix} D_\delta x_1 \\ D_\delta k_1 x_2 \end{bmatrix} = g(x)\Phi\theta, \end{aligned} \tag{108}$$

thus we obtain $\Phi = \begin{bmatrix} -I & I \end{bmatrix}$ and $\theta = \begin{bmatrix} D_\delta x_1 \\ D_\delta k_1 x_2 \end{bmatrix}$, which implies that Assumption S3 holds.

The model data in the simulations are mainly based on the offshore oil 299 dynamic positioning ship and the matrices M, D_0 and D_1 are given as follows (Fossen, 2002):

$M = 10^6 \times \begin{bmatrix} 6.2632 & 0 & 0 \\ 0 & 8.0901 & 15.721 \\ 0 & 15.721 & 2512.6 \end{bmatrix}, D_1 = 10^4 \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, D_0 = 10^4 \times \begin{bmatrix} 33.306 & 0 & 0 \\ 0 & 4.9457 & -47.649 \\ 0 & -47.649 & 4709.6 \end{bmatrix}$. In the example, to facilitate the simulation, similar to (Zhou et al., 2019; Xia et al., 2019), let $D_\delta = 0.2D$.

Applying Theorem 3.10 and solving linear matrix inequalities, the parameters P, Z are in appendix, and the others parameters are obtained as follows:

$\imath = 10,\ k_1 = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix},$

$k_2 = \begin{bmatrix} 0.001 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.001 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.001 & 0 & 0 & 0 \end{bmatrix}$, and the parameters of the adaptive controller are chosen as $\Gamma = I_{6\times6}$.

To show the control effectiveness, we give several simulation results with the following choices. The initial velocity of the ship: $v(0) = [1.6m/s\ 0.8m/s\ 1(\circ)/s]^T$, the initial velocity of the estimator law: $\hat{v}(0) = [0m/s\ 0m/s\ 0(\circ)/s]^T$, the initial position of the ship: $\eta(0) = [10m\ 10m\ 45(\circ)]^T$ and the time delay: $h = 3$. To test the robustness of the controller with respect to external disturbances, a disturbance of amplitude $[1\times10^6 m/s,\ 1\times10^6 m/s,\ 1\times10^6(\circ)/s]^T$ is added to the system in the time duration $[4s \sim 6s]$, and assume the disturbance attenuation level $\gamma = \sqrt{0.2}$ and $\rho = 0.0001$. In addition, we set up two different tasks to test whether the ship can reach the desired position at the same initial speed. The desired positions are: $\eta_{d1} = [0m\ 0m\ 0(\circ)]^T$, $\eta_{d2} = [20m\ 20m\ 0(\circ)]^T$.

Under the observer-based adaptive robust stabilization controller (68) of the paper, the simulation results of the ship speed on the actual value and estimated value are shown in Fig. 3a)–3c). Simulation results of two speed differences are shown in Fig. 4a)–4c). In Fig. 5a)–5b), we

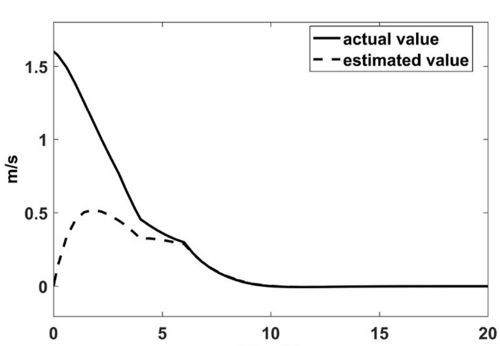

Fig. 3a. velocity in surge motion.

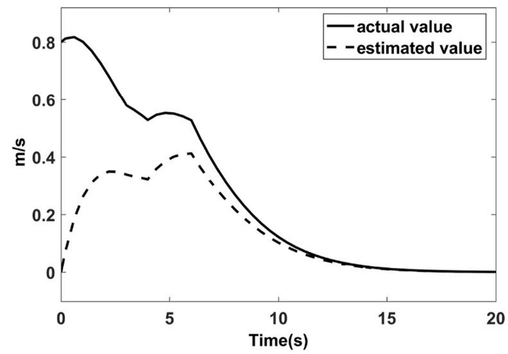

Fig. 3b. velocity in sway motion.

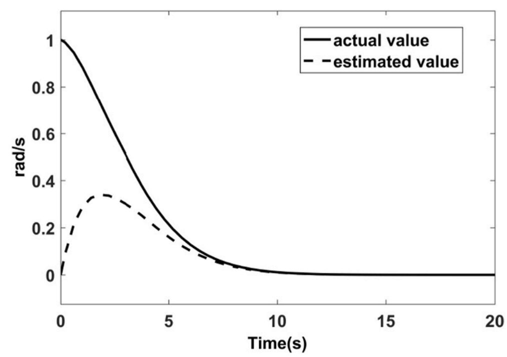

Fig. 3c. velocity in yaw motion.

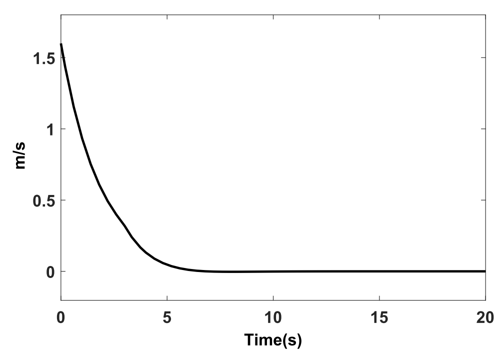

Fig. 4a. speed difference in surge.

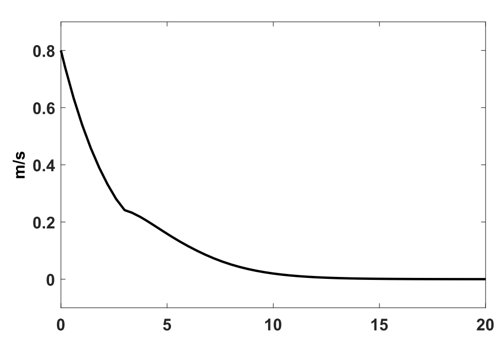

Fig. 4b. speed difference in sway.

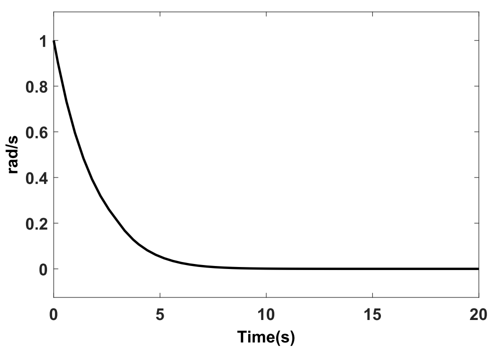

Fig. 4c. speed difference in yaw.

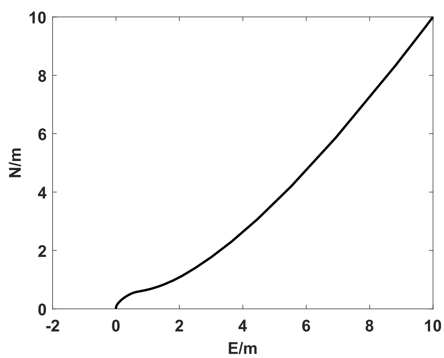

Fig. 5a. the trajectory of the ship motion ($\eta(0)\rightarrow\eta_{d1}$).

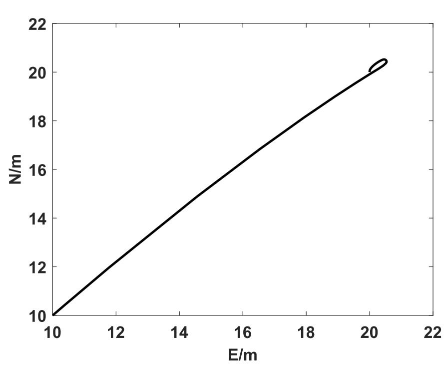

Fig. 5b. the trajectory of the ship motion ($\eta(0)\rightarrow\eta_{d2}$).

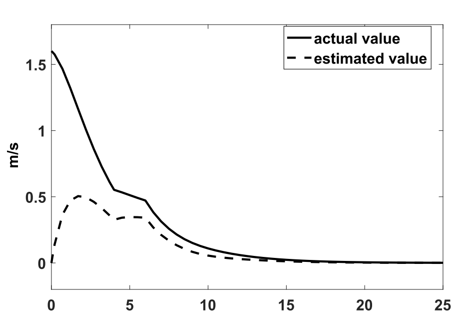

Fig. 6a. velocity in surge motion.

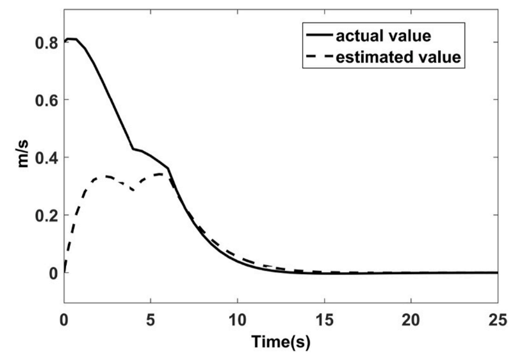

Fig. 6b. velocity in sway motion.

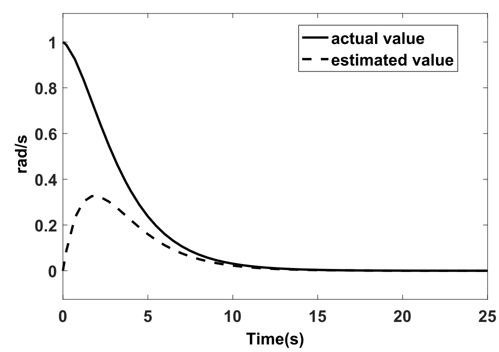

Fig. 6c. velocity in yaw motion.

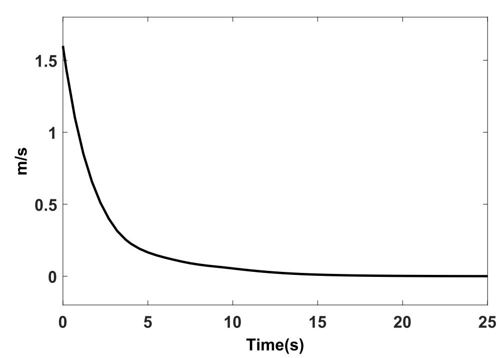

Fig. 7a. speed difference in surge.

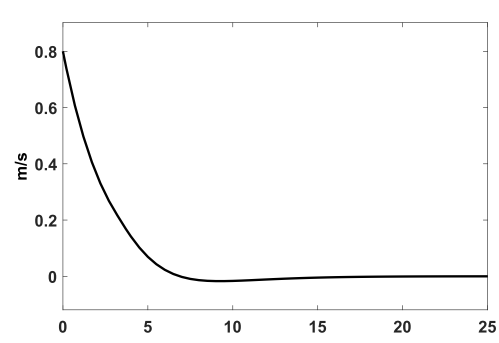

Fig. 7b. speed difference in sway.

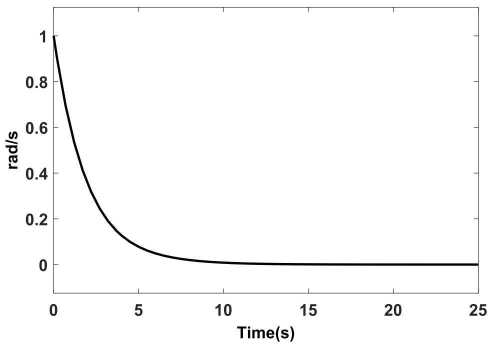

Fig. 7c. speed difference in yaw.

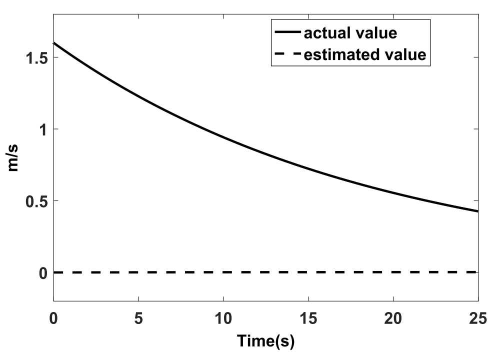

Fig. 8a. velocity in surge motion.

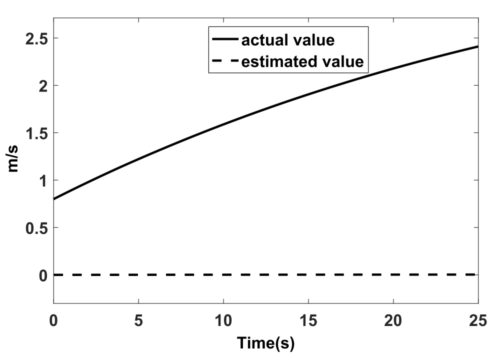

Fig. 8b. velocity in sway motion.

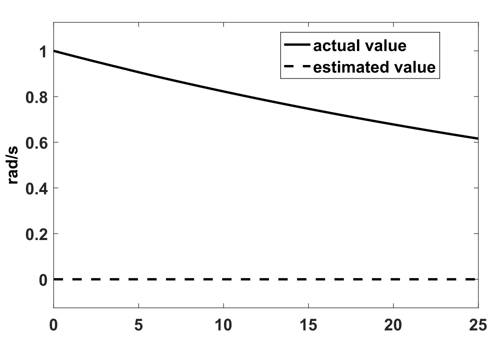

Fig. 8c. velocity in yaw motion.

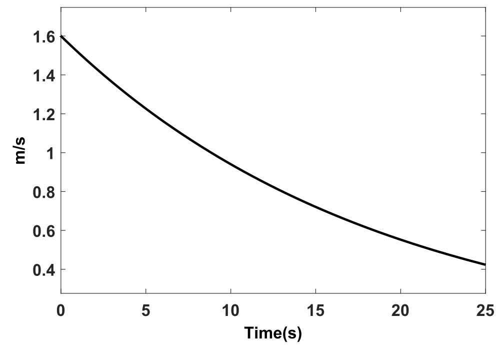

Fig. 9a. speed difference in surge.

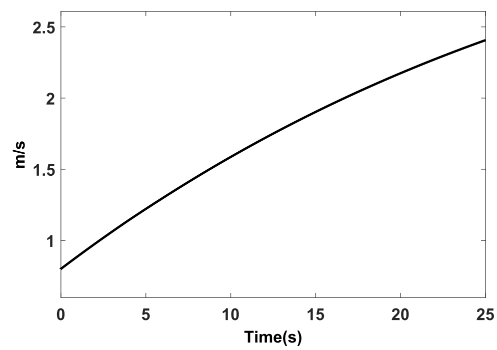

Fig. 9b. speed difference in sway.

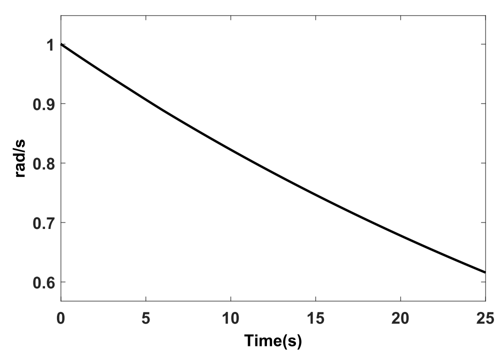

Fig. 9c. speed difference in yaw.

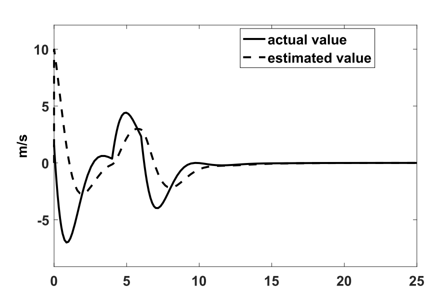

Fig. 10a. velocity in surge motion.

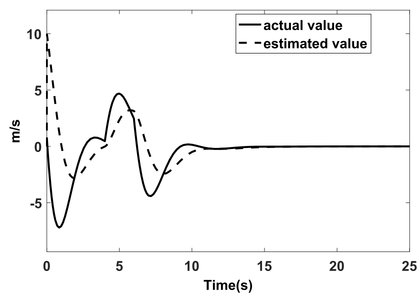

Fig. 10b. velocity in sway motion.

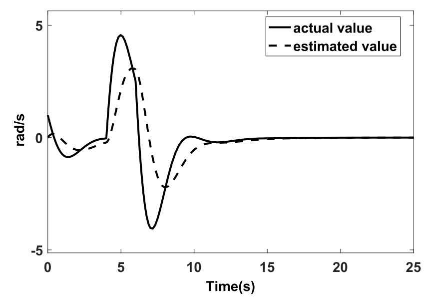

Fig. 10c. velocity in yaw motion.

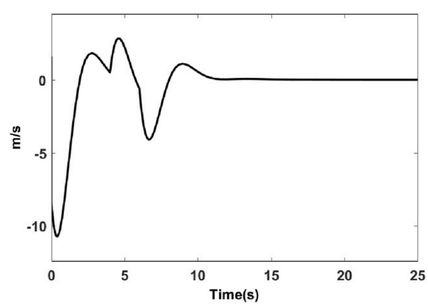

Fig. 11a. speed difference in surge.

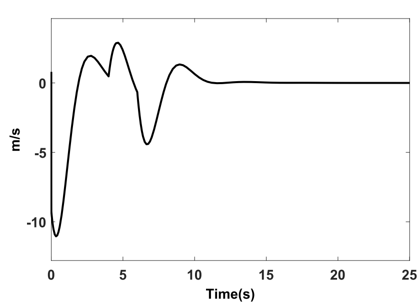

Fig. 11b. speed difference in sway.

give the simulation results on the trajectories of ship motion with two different tasks respectively.

From Fig. 3a)–3c), it can be seen that, under the observer-based adaptive robust stabilization controller (68) of the paper, when the ship system is disturbed by the environment in 4–6 s, the speed estimate values increase slightly, and the states of the original system and observer system tend to be consistent within 15 s when the disturbance w is removed, and all converge to equilibrium. From Fig. 4a)–4c), we can see that the error curves are in a downward trend and tend to zero within 15 s. These show that the observer values in this paper can quickly approximate the actual values. From Fig. 5a)–5b), it is obvious that, under the disturbance input, the DP ship system with two different tasks can reach the desired positions and the curves are smooth respectively. It

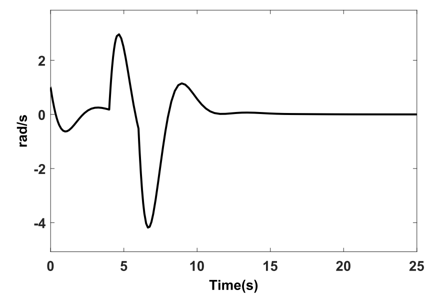

Fig. 11c. speed difference in yaw.

is well worth pointing out that, since our initial angle is 45(∘), while the expected angle is 0(∘) in two tasks, the ship will make a circle at (20, 20) and stabilize at (20, 20) in Fig. 5b. Therefore, the above simulation results show that the observer-based controllers designed of the paper are very effective in the adaptive robust stabilization the ship system.

4.2. Comparisons with existing DP control law

In the following, to show the superiority of the method presented in this paper, we give several comparison results with (Du et al., 2012) and (Ngongi and Du, 2014) respectively under the same initial conditions (the initial condition settings are the same as in Subsection 4.1 of the paper) and same DP model without time-delay(**Note**: In Ref. (Du et al., 2012), the authors applied the error method, and in Ref (Ngongi and Du, 2014)., the authors proposed a high-gain observer based PD controller method). The simulation results are shown in Figs.4.4a) ˜4.5c) (using our method in the paper), and Figs.4.6a) ˜4.7c) (using the method in (Du et al., 2012)) and Figs.4.8a) ˜4.9c) (using the method in (Ngongi and Du, 2014)).

From Fig. 6a)–7c), it is easy to see that, by using our method in the paper, the errors between the estimated value of the observer and the actual value are gradually reduced, and the actual values can be accurately estimated within 20 s. While the stabilization time by using (Du et al., 2012) (Fig. 8a)–9c)) is longer and the observer can not accurately estimate the actual value in a short time. In addition, we know, from Fig. 10a)–11c), by using the method in Ref. (Ngongi and Du, 2014), although the errors converge to zero in a short time, when the disturbance is added in 4–6 s, the motion speed in (Ngongi and Du, 2014) increases instantaneously. Compared with the method in our paper (Fig. 6a)–7c), the PD controller designed in (Ngongi and Du, 2014) has poor robustness. Moreover, the error curves in our paper (Fig. 7a–7c) is uniformly decreasing rather than the repeated fluctuations like (Ngongi and Du, 2014) (Fig. 11a)–11c). Therefore, under the observer system and controller designed in this paper, the DP ship system has faster estimation speed, smaller overshoot and better robustness than the ones in (Du et al., 2012; Ngongi and Du, 2014) respectively.

5. Conclusion

This paper has investigated the observer-based adaptive robust stabilization problem for the DP ship system with time-delay, and presented several delay-independent and delay-dependent results. Unlike traditional methods, we have adopted a novel idea to design robust stabilization controller. Namely, by transforming the DP ship system into its Hamiltonian model, and applying Hamiltonian method and augmented technology, we have developed an dimension expansion system. Based on which, the paper has presented several robust stabilization and adaptive robust stabilization results based on the observer method. The simulation results have shown the effectiveness of the designed controller in the paper. It is well worth pointing out that the ship system under study of the paper is more general and the method used of the paper can be applied to study other nonlinear system not just linear and/or approximate linear system. In the future, we will consider some more complex ship models, such as dynamic positioning ships with actuator faults Refs (Zhang et al., 2020c). and under actuated cable-laying ship Refs. (Zhang et al., 2020b), and so on.

Funding

This work was partially supported by the National Natural Science Foundation of China (G61773015, G61877028, G61803230), The key research and development plan of Shandong province (2019JZZY020712), Natural Science Foundation of Shandong Province (ZR2019MF032), A Project of Shandong Province Higher Educational Science and Technology Program(J18KA330, J18KA348, J18KB144), Jinan Engineering Laboratory-Intelligent Control and Cloud Platform and Implementation Plan of “1251” Talent Training Project of Shandong Jiaotong University.

7. Appendix

$$P=\begin{bmatrix}
0.3101 & 0 & 0 & -0.0153 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0.3094 & -0.0049 & 0 & -0.0154 & 0.0040 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -0.0049 & 0.3109 & 0 & -0.0012 & -0.0154 & 0 & 0 & 0 & 0 & 0 & 0\\
-0.0153 & 0 & 0 & 0.2486 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -0.0154 & -0.0012 & 0 & 0.2486 & 0.0007 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0.0040 & -0.0154 & 0 & 0.0007 & 0.2482 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0.2763 & 0 & 0 & -0.0203 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2759 & -0.0112 & 0 & -0.0205 & 0.0056\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0112 & 0.2793 & 0 & -0.0014 & -0.0206\\
0 & 0 & 0 & 0 & 0 & 0 & -0.0203 & 0 & 0 & 0.2485 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0205 & -0.0014 & 0 & 0.2485 & 0.0007\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0056 & -0.0206 & 0 & 0.0007 & 0.2480
\end{bmatrix},$$

$$Z=\begin{bmatrix}
0.3766 & 0 & 0 & -0.0791 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0.3821 & -0.0574 & 0 & -0.0803 & 0.0282 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -0.0574 & 0.3991 & 0 & 0.0004 & -0.0847 & 0 & 0 & 0 & 0 & 0 & 0\\
-0.0791 & 0 & 0 & 0.0606 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -0.0803 & 0.0004 & 0 & 0.0606 & -0.0002 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0.0282 & -0.0847 & 0 & -0.0002 & 0.0607 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0.1653 & 0 & 0 & -0.0776 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1706 & -0.0555 & 0 & -0.0789 & 0.0277\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0555 & 0.1870 & 0 & 0.0005 & -0.0832\\
0 & 0 & 0 & 0 & 0 & 0 & -0.0776 & 0 & 0 & 0.0606 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0789 & 0.0005 & 0 & 0.0606 & -0.0002\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0277 & -0.0832 & 0 & -0.0002 & 0.0607
\end{bmatrix}.$$

CRediT authorship contribution statement

Jiankuo Cui: Conceptualization, Methodology, Software, Writing - original draft. **Renming Yang:** Writing - review & editing, Data curation, Supervision, Project administration, Funding acquisition. **Chengcheng Pang:** Investigation, Writing - original draft, Visualization, Software. **Qiang Zhang:** Formal analysis, Resources, Data curation, Funding acquisition.

Declaration of competing interest

No conflict of interest exits in the submission of this manuscript, and manuscript is approved by all authors for publication. The work described is original research that has not been published previously.

References

Coutinho, D.F., Souza, C.E.D., 2008. Delay-dependent robust stability and gain analysis of a class of nonlinear time-delay systems. Automatica 44, 2006–2018, 08.

Deng, F., Wang, L., Jiao, D., 2017. Adaptive observer based backstepping controller design for dynamic ship positioning. China Ocean Eng. 31, 639–645, 05.

Ding, F., Ma, Y., Wang, Y., Li, J., 2015. Robust synchronization control of multiple vessels with state observer. J. Harbin Eng. Univ. 34, 789–794, 03.

Donaire, A., Perez, T., 2012. Dynamic positioning of marine craft using a port-Hamiltonianian framework. Automatica 48, 851–856, 05.

Du, J., Wang, S., Zhang, X., 2012. Nonlinear observer design for ship dynamic positioning system. Ship engineering 34, 58–61, 03.

Du, J., Hu, X., Liu, H., Chen, C., 2015. Adaptive robust output feedback control for a marine dynamic positioning system based on a high-gain observer. IEEE Transactions on Neural Networks and Learning Systems 26 (11), 2775–2786.

Ferial, E.H., 2000. The Ocean Engineering Handbook. CRC Press, pp. 12–28, 2000.

Fosseen, T.I., 2002. Marine Control Systems. TaPier Trtkkeri, Trondheim.

Fossen, T.I., 2002. Marine Control Systems: Guidance, Navigation and Control of Ships. *Rigs and Underwater Vehicles*.

Francesco, C., Anders, R., Rolf, J., 2003. An adaptive observer for dynamical ship position control using vectorial observer backstepping. 42th IEEE Conference on Decision and Control (CDC 03) 27 (9), 771–780.

Gu, K., Vladimir, L. Kharitonov, Chen, J., 2003. Stability of Time-Delay Systems. Springer, Berlin.

Ihle, I.A.F., Skjetne, R., Fossen, T.I., 2006. Output feedback control for maneuvering systems using observer backstepping. In: 20th IEEE International Symposium on Intelligent Control (ISIC'05), and the 13th Mediterranean Conference on Control and Automation (MED'05), pp. 149–154.

Jiang, F., Bian, H., 2016. Research on ship formation control algorithm based on linear active disturbance rejection technology. Digital technology and applications 6, 148–149.

Jin, Y., Yv, M., Yuan, W., 2017. Back-stepping sliding mode control of ship dynamic positioning system based on extended state observer. Ship science and technology 39, 103–107, 03.

Kim, M.H., Wang, S., Zhang, X., 2012. Nonlinear observer design for ship dynamic positioning system. Ship engineering 34, 58–61, 03.

Li, J., Du, J., Xin, H., 2020. Robust adaptive prescribed performance control for dynamic positioning of ships under unknown disturbances and input constraints. Ocean. Eng. 206, 107254.

Liao, X., Chen, G., Sanchez, E.N., 2003. Delay-dependent exponential stability analysis of delayed neural networks:an *LMI approach*. Neural Network. 16 (10), 1401–1402.

Lin, X., Nie, J., Jiao, Y., Liang, K., Li, H., 2018. Nonlinear adaptive fuzzy output-feedback controller design for dynamic positioning system of ships. Ocean. Eng. 158, 186–195.

Muhammad, S., DoRia-Cerezo, A., 2012. Passivity-based control applied to the dynamic positioning of ships. IET Control Theory & Appl. 6 (5), 680–688.

Ngongi, W.E., Du, J., 2014. A high-gain observer-based PD controller design for dynamic positioning of ships. Appl. Mech. Mater. 490–491, 803–808.

Ortega, R., Borja, L.P., 2014. New results on control by interconnection and energy-balancing passivity-based control of port-Hamiltonian systems. Decis. Contr. 2015, 2346–2351.

Skulstad, R., Li, G., Zhang, H., 2018. A neural network approach to control allocation of ships for dynamic positioning. IFAC PapersOnLine 36, 128–133, 06.

Sun, W., Fu, B., 2016. Adaptive control of time-varying uncertain nonlinear systems with input delay: a Hamiltonian approach. IET Control Theory & Appl. 10 (15), 1844–1858.

Sun, W., Peng, L., 2014. Observer-based robust adaptive control for uncertain stochastic Hamiltonian systems with state and input delays. Nonlinear Anal. Model Contr. 19, 626–645, 04.

Tong, S., Li, Y., Feng, G., Li, T., 2011. Observer-based adaptive fuzzy backstepping dynamic surface control for a class of MIMO nonlinear systems. IEEE Trans. Syst. Man Cybern. B Cybern. *: a publication of the IEEE Systems, Man, and Cybernetics Society* 41, 1121–1135, 04.

Wang, Y., 2007. Generalized Hamiltonian Control System Theory . *Implementation, Control and Application*, vol. 247. Science Press, Beijing, CN.

Wang, Y., Li, C., Cheng, D., 2003. Generalized Hamiltonian realization of time-invariant nonlinear systems. Automatica 39 (8), 1437–1443.

Xia, G., Shao, X., 2014. Design of robust passive nonlinear observer based on acceleration for surface ships. China shipbuilding 55, 12–24, 03.

Xia, G., Fu, M., Ding, F., zhang, J., 2013. Robust sliding mode control for dynamic positioning ship with input delay. J. Harbin Eng. Univ. 48, 851–856, 05.

Xia, G., Xue, J., Jiao, J., 2017. Dynamic positioning control system with input time-delay using fuzzy approximation approach. Int. J. Fuzzy Syst. 20, 630–639, 02.

Xia, G., Liu, C., Zhao, B., Chen, X., Shao, X., 2019. Finite time output feedback control for ship dynamic positioning assisted mooring positioning system with disturbances. Int. J. Contr. Autom. Syst. 17 (X), 1–13, 2019.

Yang, R., Guo, R., 2018. Adaptive finite-time robust control of nonlinear delay Hamiltonian systems via Lyapunmov-Krasovskii method. Asian J. Contr. 20 (2), 1–11.

Yang, R., Wang, Y., 2012. Stability for a class of nonlinear time-delay systems via Hamiltonian functional method. Sci. China Inf. Sci. 55, 1218–1228, 05.

Yang, R., Zhang, G., Sun, L., 2020. Observer-based finite-time robust control of nonlinear time-delay systems via Hamiltonian function method. Int. J. Contr. 2020 (4), 1–32.

Zhang, G., Chu, S., Jin, X., Zhang, W., 2020a. Composite neural learning fault-tolerant control for underactuated vehicles with event-triggered input. IEEE Transactions on Cybernetics (99), 1–12.

Zhang, G., Zhang, C., Li, J., Zhang, X., 2020b. Improved composite learning path-following control for the underactuated cable-laying ship via the double layers logical guidance. Ocean. Eng. 207, 107342.

Zhang, G., Yao, M., Xu, J., Zhang, w., 2020c. Robust neural event-triggered control for dynamic positioning ships with actuator faults. Ocean. Eng. 207, 107292.

Zhao, D., Ding, F., Zhou, L., Zhang, W., Xu, H., 2015. Robust H_∞ control of neutral system with time-delay for dynamic positioning ships. M*ath. Probl Eng.* 2015 (PT.5), 1–11, 1667-1694.

Zheng, M., Zhou, Y., Yang, S., Suo, Y., Li, L., 2017. Robust H_∞ control for sampled-data dynamic positioning ships. C*ontrol Engineering and Applied Informatics* 4, 84–92.

Zhou, P., Yang, R., Zhang, G., Han, Y., 2019. Adaptive robust simultaneous stabilization of two dynamic positioning vessels based on a port-controlled Hamiltonian (PCH) model. Energies 12 (20), 1–14, 3936.

Zwierzewicz, Z., 2015. The design of ship autopilot via observer based adaptive feedback linearization. In: 20th International Conference on Methods and Models in Automation and Robotics (MMAR).