9th IFAC Conference on Control Applications in Marine Systems
The International Federation of Automatic Control
September 17-20, 2013. Osaka, Japan

Dynamic Positioning Based on Nonlinear MPC

Margarita V. Sotnikova, Evgeny I. Veremey

Faculty of Applied Mathematics and Control Processes,
Saint-Petersburg State University, Saint-Petersburg, Russia.
(e-mail: s_margosha@mail.ru)

Abstract: This paper is devoted to nonlinear model predictive control (MPC) application for dynamic positioning (DP) problem. The MPC approach has prominent features among other nonlinear control schemes. They include the explicit way for using nonlinear models and taking into account constraints imposed on controlled and manipulated variables while provide optimal performance with respect to a given cost functional. Dynamic positioning is a nonlinear control process with three-dimensional input and output, constraints and external disturbances that allows to state that MPC strategy is a quite suitable approach to be used here. In this paper MPC based control algorithm for DP problem is proposed. This algorithm includes two stages: the nonlinear stage, where nonlinear predictive model is used, and the linear stage, which is switch on in the vicinity of desired position. The algorithm provides offset-free performance. The effectiveness of the MPC approach is demonstrated by the simulation study examples.

Keywords: dynamic positioning, model predictive control, constraints, external disturbances.

1. INTRODUCTION

There are a lot of control processes with essentially nonlinear dynamics, constraints and uncertainties, where traditional linear systems control synthesis can't be used. In such cases it is necessary to use more sophisticated control design approaches in order to provide high-quality performance subject to all constraints and disturbances, which influence system dynamic. Nowadays available computational resources allow us to implement quite complicated control algorithms in real-time.

This paper is focused on the Model Predictive Control (MPC) scheme application for the ship dynamic positioning (DP) problem. The DP systems design is one of the most important problems nowadays from practical point of view, because such systems are widely used for different marine vehicles in a lot of applications. The detailed survey of modern DP control systems is done in Sørensen, 2011, the theoretical and practical background for DP control can be found in Fossen and Strand, 1999, Fossen, 2011, Loria et al., 2000.

DP problem is a nonlinear control process with three-dimensional input and output. Mathematical model of the process incorporate external slowly-varying (winds and currents) and wave disturbances, and constraints on input and output variables. So, control design problem is quite complicated due to nonlinear model, uncertainties, disturbances and constraints. Now, there are a number of papers addresses this problem, where different nonlinear control design approaches are suggested. But, the question we will pay a special attention in this paper is how to design optimal feedback control with respect to some given cost functional. The most suitable approach in this way is the MPC control design scheme.

MPC provides high-performance control in the case when an accurate mathematical model of the plant to be controlled is not fully known. In addition, these systems allow to take into account constraints imposed both on input and output variables and to predict disturbances influence on future process evolution. Furthermore, MPC algorithms can be based on both linear and nonlinear mathematical models of the plant (Camacho and Bordons, 2004, Maciejowski, 2002).

It is well-known that the MPC algorithms are time-consuming since they require the repeated on-line solution of the optimization problems at each sampling instant. This drawback prevents the wide expansion of MPC algorithms in practical applications, especially for systems with fast dynamic. But in most cases sufficient computational capabilities are available nowadays and MPC algorithms can be successfully implemented in real-time.

Existing MPC implementations for the DP problem are mostly based on the simplified linear models, representing ship dynamic in the vicinity of desired position. Another more sophisticated approach is based on feedback linearization in order to eliminate mathematical model nonlinearity. Both these approaches do not allow us fully utilize nonlinear model of the process and actuator resources. Thus, in this paper we propose two stages MPC based control algorithm. First stage is nonlinear, where nonlinear predictive model is used. The second stage with linear predictive model is started when the ship is entered given vicinity of the desired position. The proposed algorithm provides offset-free performance with respect to slowly-varying disturbances.

The paper is organized in the following way. Firstly, brief description of the MPC control scheme is given and its main features are discussed. Secondly, the DP nonlinear optimal control problem is formulated taking into account constraints and slowly-varying disturbances. Thirdly, using MPC control strategy for DP problem is considered. Here two stages MPC control algorithm is described. In the last section the results obtained in the simulation study are presented.

2. PREDICTIVE CONTROL SCHEME

Let we have a mathematical model of the control plant represented by the system of nonlinear difference equations of the form:

$$\begin{aligned} &\bar{\mathbf{x}}[k+1] = \bar{\mathbf{f}}(\bar{\mathbf{x}}[k], \bar{\mathbf{u}}[k], \mathbf{w}[k]), \\ &\bar{\mathbf{y}}[k] = \mathbf{C}\bar{\mathbf{x}}[k] + \mathbf{v}[k]. \end{aligned} \tag{1}$$

Here $\bar{\mathbf{x}}[k] \in \mathbf{E}^n$, $\bar{\mathbf{u}}[k] \in \mathbf{E}^m$, $\bar{\mathbf{y}}[k] \in \mathbf{E}^l$, $\mathbf{w}[k] \in \mathbf{E}^{n_e}$ are the vectors of states, control inputs, output variables and external disturbances correspondently, $\mathbf{v}[k]$ is a measurement noise. On the base of the equations (1) the following predictive model could be constructed:

$$\begin{aligned} &\mathbf{x}[i+1] = \mathbf{f}(\mathbf{x}[i], \mathbf{u}[i]), \quad i = k, k+1, \ldots., \quad \mathbf{x}[k] = \tilde{\mathbf{x}}[k], \\ &\mathbf{y}[i] = \mathbf{C}\mathbf{x}[i], \end{aligned} \tag{2}$$

where $\mathbf{x}[k] \in \mathbf{E}^n$, $\mathbf{u}[k] \in \mathbf{E}^m$, $\mathbf{y}[k] \in \mathbf{E}^l$ are the state, input and output vectors as previously. In contrast to the equations (1), predictive model can be used to predict future outputs given the programmed input control sequence over some discrete time interval named a prediction horizon. As it follows from (2), the initial condition for the predictive model is defined by the actual state $\tilde{\mathbf{x}}[k]$ of the plant or by its estimation $\hat{\mathbf{x}}[k]$, obtained with the help of nonlinear observer.

As usually, a prediction is performed in the following way. Let assume that the programmed control over a prediction horizon is represented by the given sequence $\{\mathbf{u}[k], \mathbf{u}[k+1], \ldots, \mathbf{u}[k+P-1]\}$, where P is a prediction horizon. Then a correspondent sequence of the vectors $\{\mathbf{y}[k+1], \mathbf{y}[k+2], \ldots, \mathbf{y}[k+P]\}$ is obtained by integrating of the equations (2). This sequence represents the prediction of future plant behaviour. The scheme of the prediction is illustrated in Fig. 1. Here C is a control horizon, that is a number of steps where control input can vary, and for the remaining steps it must be constant.

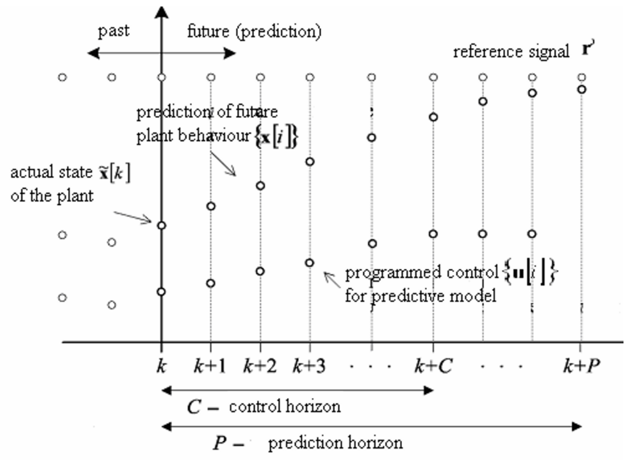

Fig. 1. Scheme of the prediction.

Observe that in a general case the function $\mathbf{f}$ for the equations (2) can be differed from the function $\bar{\mathbf{f}}$ in the system (1). This difference can be explained by the many reasons; in particular due to the fact that a predictive model must be integrated very fast to provide a possibility of real-time implementation, so the equations (2) should be essentially simplified in relation to the initial model (1). Besides that, a predictive model can include some additional components of the state vector which are used to model the external disturbances.

The idea of the MPC approach is to choose programmed control sequence $\{\mathbf{u}[k], \mathbf{u}[k+1], \ldots, \mathbf{u}[k+P-1]\}$ such that it minimizes a certain functional given on the motion of a predictive model. One of the most popular is the quadratic cost functional over the prediction horizon

$$\begin{aligned} J_k = J_k(\bar{\mathbf{y}}, \bar{\mathbf{u}}) = \sum_{j=1}^{P} \Big\{ &\left(\mathbf{y}[k+j] - \mathbf{r}^y_{k+j}\right)^T \mathbf{R}_{k+j} \left(\mathbf{y}[k+j] - \mathbf{r}^y_{k+j}\right) + \\ &+ \left(\mathbf{u}[k+j-1] - \mathbf{r}^u_{k+j-1}\right)^T \mathbf{Q}_{k+j} \left(\mathbf{u}[k+j-1] - \mathbf{r}^u_{k+j-1}\right) \Big\} \end{aligned} \tag{3}$$

Here $\mathbf{R}_{k+j}$ and $\mathbf{Q}_{k+j}$ are positive definite weight matrices, $\mathbf{r}^y_i$ and $\mathbf{r}^u_i$ are the output and input reference signals and

$$\begin{aligned} &\bar{\mathbf{u}} = (\mathbf{u}[k] \ \mathbf{u}[k+1] \ldots \mathbf{u}[k+P-1])^{\mathrm{T}} \in E^{mP}, \\ &\bar{\mathbf{y}} = (\mathbf{y}[k+1], \mathbf{y}[k+2], \ldots, \mathbf{y}[k+P])^{\mathrm{T}} \in E^{lP} \end{aligned}$$

are the auxiliary vectors. It is obvious that the programmed control sequence $\bar{\mathbf{u}}$ must satisfy to the complex of all constraints and conditions imposed on the state and control variables. Therefore, the programmed control $\bar{\mathbf{u}}$ over a prediction horizon can be chosen as a solution of the following optimization problem

$$J_k = J_k(\bar{\mathbf{x}}(\bar{\mathbf{u}}), \bar{\mathbf{u}}) = J_k(\bar{\mathbf{u}}) \to \min_{\bar{\mathbf{u}} \in \Omega \subseteq \mathbf{E}^{mP}}, \tag{4}$$

where $\Omega = \{\bar{\mathbf{u}} \in \mathbf{E}^{mP} : \mathbf{u}[k+j-1] \in \mathbf{U}, \mathbf{x}[k+j] \in \mathbf{X}, j = 1, 2, \ldots, P\}$ is an admissible set. Here $\mathbf{U} \subseteq \mathbf{E}^m$ is the set of feasible inputs and $\mathbf{X} \subseteq \mathbf{E}^n$ is the set of feasible states. The functional J_k (4) is a function of mP variables for the considered scheme of a prediction. Generally, this function is nonlinear and Ω is a non-convex set. Therefore, the optimization task (4) is a nonlinear programming problem.

Let the sequence $\bar{\mathbf{u}}^*$ be a solution of the optimization problem (4). According to the basic MPC idea, the obtained optimal programmed control $\bar{\mathbf{u}}^*$ is used as an input only for the current sample instant k, i.e. only the first component of $\bar{\mathbf{u}}^*$ is really implemented. At the next time instant the whole procedure of prediction and optimization is repeated again to find new optimal programmed control over time interval $[k+1, k+P]$. Summarizing, real-time MPC-algorithm works consists of the following actions:

- obtain the state estimation $\hat{\mathbf{x}}[k]$ based on measurements $\bar{\mathbf{y}}[k]$ using nonlinear observer;

- solve the nonlinear programming problem (4) subject to prediction model (2) with initial conditions $\mathbf{x}[k] = \hat{\mathbf{x}}[k]$ and cost functional (3) (it should be noted that the value of the J_k is obtained by numerically integrating the prediction model (2) and then substituting the prediction behaviour $\overline{\mathbf{y}}$ into the cost functional (3) given the programmed control $\overline{\mathbf{u}}$ over the prediction horizon and initial conditions $\hat{\mathbf{x}}[k]$);
- implement only the first component $\mathbf{u}^*[k]$ of the solution $\overline{\mathbf{u}}^* = \left(\mathbf{u}^*[k], \mathbf{u}^*[k+1], ..., \mathbf{u}^*[k+P-1]\right)^T$ of the problem (4) at the current step k;
- repeat the whole procedure 1–3 for the next time instant $k+1$.

It is obvious that the scheme of MPC approach, presented above, realizes a feedback control loop, which has both significant advantages and certain drawbacks.

One of the main positive features is that MPC is an adaptive control algorithm, because the control input is adjusted to the changing conditions at each sample of discrete time. On the other hand, one of the essential disadvantages consists of that there is no guarantee of the closed-loop motion stability in general case.

Nevertheless, in order to avoid drawbacks of this approach, usually the following practical techniques are used. The time consumptions are reduced by decreasing the optimization problem order, for example, by means of control horizon. The stability property is provided by choosing enough long prediction horizon P. It can be noted that a practical implementation of MPC approach in real-time can be done using parallel processing (Mizuno et al., 2012).

3. DYNAMIC POSITIONING MODEL

Let consider nonlinear model of a surface ship in the following form (Fossen, 1999, Sørensen, 2005),

$$\begin{aligned} \mathbf{M}\dot{\mathbf{v}} &= -\mathbf{D}\mathbf{v} + \boldsymbol{\tau} + \mathbf{R}^T(\boldsymbol{\eta})\mathbf{b}, \\ \dot{\boldsymbol{\eta}} &= \mathbf{R}(\boldsymbol{\eta})\mathbf{v}, \end{aligned} \tag{5}$$

where $\mathbf{v} = (u\ v\ p)^T$ represents velocities in a vessel-fixed frame, $\boldsymbol{\eta} = (x\ y\ \psi)^T$ is a position (x, y) and a heading angle ψ relative to an Earth-fixed frame, $\boldsymbol{\tau} = (\tau_u, \tau_v, \tau_p)^T \in E^3$ is a control action generated by the propulsion system. Vector $\mathbf{b} \in R^3$ is a bias, representing slowly-varying external disturbances, such as wind and currents, and additional unmodeled dynamics. Matrices $\mathbf{M}$ and $\mathbf{D}$ with the constant elements are positive definite and $\mathbf{M} = \mathbf{M}^T$.

The system (5) is nonlinear due to the rotation matrix in yaw

$$\mathbf{R}(\boldsymbol{\eta}) = \mathbf{R}(\psi) = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{6}$$

The slowly-varying external forces and moment are represented by the linear model

$$\dot{\mathbf{b}} = -\mathbf{T}^{-1}\mathbf{b} + \boldsymbol{\Psi}\mathbf{n}, \tag{7}$$

where $\mathbf{T}$ is a 3x3 diagonal matrix, whose positive elements are bias time constants, $\mathbf{n} \in E^3$ is the driven zero-mean Gaussian white noise, $\boldsymbol{\Psi}$ is a 3x3 diagonal matrix scaling the amplitude of noise $\mathbf{n}$.

Let us remark that the wave-induced ship dynamic is not included in the model (5) – (7). This is because we do not take it into account in the process of predictive model based controller design. But, the negative wave influence on ship dynamic and observer estimations is partially overcomed by means of a special wave filter, presented in general form in Veremey, 2012. The wave disturbances model is also used for a closed loop system simulation.

As usually for DP problem, the available measurements include position (x, y) and heading angle ψ of the ship. Hence, the measurement vector is equal to $\boldsymbol{\eta}$.

Let $\boldsymbol{\eta}_d \in E^3$ be a desired constant position vector. An aim of the MPC-control is to achieve position $\boldsymbol{\eta}_d$. Let us introduce following constraints, which should be satisfied during the mentioned manoeuvre:

$$|\tau_u(t)| \le c_u,\ |\tau_v(t)| \le c_v,\ |\tau_p(t)| \le c_p,\ \forall t \in [0, t^*], \tag{8}$$

where c_u, c_v, c_p are given real positive numbers, t^* is a transient process settling time.

Let accept a control processes performance index, which is represented by the quadratic cost functional of the form

$$J = J(\boldsymbol{\tau}) = \int_0^\infty \left((\boldsymbol{\eta} - \boldsymbol{\eta}_d)^T \mathbf{Q}_\eta (\boldsymbol{\eta} - \boldsymbol{\eta}_d) + \boldsymbol{\tau}^T \mathbf{Q}_\tau \boldsymbol{\tau}\right) dt, \tag{9}$$

where $\mathbf{Q}_\eta$, $\mathbf{Q}_\tau$ are positive definite weight matrices.

The goal is to design a nonlinear feedback controller on the base of MPC approach in order to provide the desirable position $\boldsymbol{\eta}_d$ of a ship while minimizing cost functional (9) subject to mathematical model (5) – (7) and constraints (8). The application of MPC control strategy is quite suitable here, because the mathematical model is nonlinear, the control vector $\boldsymbol{\tau}$ is three-dimensional and the input constraints are imposed. Moreover, in contrary to other nonlinear control synthesis approaches, MPC provides optimal solution for the DP problem in the sense of a given cost functional (9).

4. USING MPC FOR DYNAMIC POSITIONING

The DP problem is nonlinear, therefore the nonlinear predictive model must be used for MPC implementation. But it is also reasonable to use linear predictive model in the vicinity of the desired position $\boldsymbol{\eta}_d$. So, let consider two different predictive models.

The nonlinear predictive model can be formed on the base of nonlinear equations (5) – (6) using two step procedure. Firstly, the discrete-time model is obtained by means of Euler discretization with the given sample time T. This value is determined by the digital control system sample time period and depends on the particular ship. As a result of discretization, we obtain

$$\begin{aligned} \mathbf{v}[i+1] &= \mathbf{v}[i] + T \cdot \bar{\mathbf{f}}_1(\mathbf{v}[i], \boldsymbol{\eta}[i], \boldsymbol{\tau}[i], \mathbf{b}[i]), \\ \boldsymbol{\eta}[i+1] &= \boldsymbol{\eta}[i] + T \cdot \bar{\mathbf{f}}_2(\mathbf{v}[i], \boldsymbol{\eta}[i]), \end{aligned} \tag{10}$$

where $\bar{\mathbf{f}}_1$ and $\bar{\mathbf{f}}_2$ are determined by the system of nonlinear equations (5), (6). Secondly, the nonlinear predictive model is formed on the base of difference equations (10)

$$\begin{aligned} &\mathbf{v}[i+1] = \mathbf{f}_1(\mathbf{v}[i], \boldsymbol{\eta}[i], \boldsymbol{\tau}[i], \mathbf{b}[i]), \qquad i = k, k+1, ..., \\ &\boldsymbol{\eta}[i+1] = \mathbf{f}_2(\mathbf{v}[i], \boldsymbol{\eta}[i]), \\ &\mathbf{b}[i+1] = \mathbf{b}[i], \qquad \mathbf{v}[k] = \tilde{\mathbf{v}}[k], \ \boldsymbol{\eta}[k] = \tilde{\boldsymbol{\eta}}[k], \ \mathbf{b} = \tilde{\mathbf{b}}[k]. \end{aligned} \tag{11}$$

Here $\tilde{\boldsymbol{\eta}}[k]$ are measurements at time instant k, $\tilde{\mathbf{b}}[k]$ and $\tilde{\mathbf{v}}[k]$ are bias and velocity vectors at time k estimated using asymptotic observer. These vectors are used as initial conditions for predictive model (11). It should be noted that the bias vector $\mathbf{b}[i]$ hold constant and equal to the last estimation $\tilde{\mathbf{b}}[k]$ over prediction horizon P. It allows us to predict slowly-varying disturbances influence on ship dynamic and trajectory.

The sampling time T_s for predictive model (11) is determined as $T_s = s \cdot T$, where $s > 0$ is an integer number determining the time interval T_s over which control input τ remains constant. Actually, the vector functions $\mathbf{f}_1$ and $\mathbf{f}_2$ in (11) can not be obtained analytically and are described only algorithmically, so that the future behaviour of the plant is predicted by means of numerical solution of the difference equations (10). At the same time, the programmed control sequence over prediction horizon P is defined with respect to predictive model (11) and, in consequence, the computational procedures with the model (10) is performed subject the remaining control input τ constant over s steps.

Second predictive model is linear and is used in the vicinity of desired position $\boldsymbol{\eta}_d$. Let us define the vicinity as a set

$$\boldsymbol{\Omega}_v = \left\{ \boldsymbol{\eta} \in E^3 \middle| \, |\psi - \psi_d| < \alpha, \, (x - x_d)^2 + (y - y_d)^2 < r^2 \right\},$$

where $\alpha > 0$ and $r > 0$ are given real numbers. A corresponding linear mode, representing dynamics in the vicinity $\boldsymbol{\Omega}_v$ is derived from equations (5), (6) using discretization procedure with the sample time T_s. As a result we obtain:

$$\begin{aligned} \mathbf{v}[k+1] &= \mathbf{A}_v \mathbf{v}[k] + \mathbf{B}_\tau \boldsymbol{\tau}[k] + \mathbf{H}_v(\boldsymbol{\eta}_d)\mathbf{b}[k], \\ \boldsymbol{\eta}[k+1] &= \mathbf{A}_{v\eta} \mathbf{v}[k] + \mathbf{A}_\eta \boldsymbol{\eta}[k]. \end{aligned} \tag{12}$$

Here $\mathbf{A}_v, \mathbf{B}_\tau, \mathbf{A}_{v\eta}, \mathbf{A}_\eta, \mathbf{H}_v$ are constant matrices with correspondent dimensions. It is convenient for further discussion to represent system (12) in state-space form. To this end, let introduce state vector $\mathbf{x} = (\mathbf{v} \ \boldsymbol{\eta})^T$ and form equations

$$\begin{aligned} \mathbf{x}[k+1] &= \mathbf{A}\mathbf{x}[k] + \mathbf{B}\boldsymbol{\tau}[k] + \mathbf{H}\mathbf{b}[k], \\ \mathbf{y}[k] &= \mathbf{C}\mathbf{x}[k], \end{aligned} \tag{13}$$

where $\mathbf{y}[k] = \boldsymbol{\eta}[k]$ is an output vector at time instant k and matrices $\mathbf{A}, \mathbf{B}, \mathbf{H}, \mathbf{C}$ are determined as follows

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_v & \mathbf{0} \\ \mathbf{A}_{v\eta} & \mathbf{A}_\eta \end{pmatrix}, \ \mathbf{B} = \begin{pmatrix} \mathbf{B}_\tau \\ \mathbf{0} \end{pmatrix}, \ \mathbf{H} = \begin{pmatrix} \mathbf{H}_\eta \\ \mathbf{0} \end{pmatrix}, \ \mathbf{C} = \begin{pmatrix} \mathbf{0} & \mathbf{I}_{3x3} \end{pmatrix}.$$

Let notice that the bias $\mathbf{b}$ time constant is much greater than prediction horizon length. So, we can assume in both linear and nonlinear case that the bias $\mathbf{b}$ is constant over prediction horizon P. Thus, the external disturbance in linear model (13) can be considered as a constant force $\mathbf{f} = \mathbf{H}\mathbf{b}$ over prediction horizon.

Now let us derive linear predictive model on the base of equations (13) in the form of augmentations. To this end, let define predictive model expanded state vector $\mathbf{p}[i] = (\Delta\mathbf{x}[i] \ \ \mathbf{y}[i])^T$, where $\Delta\mathbf{x}[i] = \mathbf{x}[i] - \mathbf{x}[i-1]$ is a state augmentation at time instant i. In accordance with the model (13) subject to slowly-varying disturbances, we obtain following linear predictive model:

$$\begin{aligned} &\mathbf{p}[i+1] = \bar{\mathbf{A}}\mathbf{p}[i] + \bar{\mathbf{B}}\Delta\boldsymbol{\tau}[i], \quad i = k, k+1, ... \\ &\mathbf{z}[i] = \bar{\mathbf{C}}\mathbf{p}[i], \qquad \mathbf{p}[k] = (\Delta\tilde{\mathbf{x}}[k] \ \ \tilde{\mathbf{y}}[k])^T. \end{aligned} \tag{14}$$

Here matrices $\bar{\mathbf{A}}, \bar{\mathbf{B}}, \bar{\mathbf{C}}$ are defined as

$$\bar{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{0}_{6x3} \\ \mathbf{CA} & \mathbf{I}_{3x3} \end{bmatrix}, \ \bar{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ \mathbf{CB} \end{bmatrix}, \ \ \bar{\mathbf{C}} = \begin{pmatrix} \mathbf{0}_{3x6} & \mathbf{I}_{3x3} \end{pmatrix},$$

vector $\Delta\boldsymbol{\tau}[i] = \boldsymbol{\tau}[i] - \boldsymbol{\tau}[i-1]$ is an input augmentation at time instant i, $\mathbf{z}[i] = \mathbf{y}[i] = \boldsymbol{\eta}[i]$ is an output vector, $\mathbf{p}[k]$ represents initial conditions calculating using measurements and asymptotic observer estimates.

The linear predictive model (14) is convenient in the presence of constant or slowly-varying disturbances, because it provide zero steady state error while allows easy prediction dynamics and cost function computations in real-time.

So, proposed here dynamic positioning control law based on MPC strategy use two different predictive models: nonlinear model (11) and linear model (14) in the vicinity $\boldsymbol{\Omega}_v$ of the desired position $\boldsymbol{\eta}_d$.

The control input over prediction horizon is computed subject to imposed constraints (8). Let define the corresponding admissible set $\boldsymbol{\Omega}$ of the programmed control sequences. This set comprises next constraints:

$$|\boldsymbol{\tau}_j[i]| \le c_j \, , i = k, ..., k+P-1, \ j \in \{u, v, p\} \, , \tag{15}$$

where index j denotes components of the control vector $\boldsymbol{\tau}$, c_j are the given values. The admissible set $\boldsymbol{\Omega}$ is determined by linear inequalities (11) and therefore can be represented as

$$\mathbf{\Omega} = \left\{ \bar{\boldsymbol{\tau}} \in E^{3P} \mid \mathbf{A}_c \bar{\boldsymbol{\tau}} \le \mathbf{b}_c \right\}, \tag{16}$$

taking in use notations introduced above. Here $\mathbf{A}_c$ is a diagonal identity matrix and vector $\mathbf{b}_c$ correspond to inequalities (15). The admissible set representation (16) is convenient to use in nonlinear stage. But, the linear predictive model (14) is operated in terms of augmentations $\Delta\boldsymbol{\tau}[i]$. So, let consider analogous admissible set for $\Delta\boldsymbol{\tau}[i]$:

$$\mathbf{\Omega}' = \left\{ \Delta\bar{\boldsymbol{\tau}} \in E^{3P} \mid \mathbf{A}_c' \Delta\bar{\boldsymbol{\tau}} \le \mathbf{b}_c'\left(\boldsymbol{\tau}[k-1]\right) \right\}, \tag{17}$$

where vector $\Delta\bar{\boldsymbol{\tau}} = \left(\Delta\boldsymbol{\tau}[k]\ \Delta\boldsymbol{\tau}[k+1] \ldots \Delta\boldsymbol{\tau}[k+P-1]\right)^{\mathrm{T}} \in E^{3P}$ is a programmed control input over prediction horizon, matrix $\mathbf{A}_c'$ and vector $\mathbf{b}_c'$ are obtained by substituting the vector $\Delta\bar{\boldsymbol{\tau}}$ dependence upon $\bar{\boldsymbol{\tau}}$ to inequalities (16). Let notice that the right part in inequalities (17) is not stationary.

Now, let introduce the discrete analogue of the quadratic cost functional (9) in the following form

$$J_k = J_k\left(\bar{\boldsymbol{\tau}}\right) = \sum_{i=1}^{P} \left\{ \left(\boldsymbol{\eta}[k+i] - \boldsymbol{\eta}_d\right)^T \mathbf{Q}_\eta \left(\boldsymbol{\eta}[k+i] - \boldsymbol{\eta}_d\right) + \right. \\ \left. + \Delta\boldsymbol{\tau}[k+i-1]^T \mathbf{Q}_\tau \Delta\boldsymbol{\tau}[k+i-1] \right\}. \tag{18}$$

Here input augmentations vector $\Delta\bar{\boldsymbol{\tau}}$ is used in the functional instead of $\bar{\boldsymbol{\tau}}$. This allows providing offset free performance in the presence of slowly-varying disturbances. So, the functional (18) is used to optimize MPC control input over prediction horizon P for nonlinear (11) and linear (14) predictive models.

In accordance with MPC basic idea, the programmed control sequence $\bar{\boldsymbol{\tau}}$ over the prediction horizon is chosen as a solution of the optimization problem. This optimization is a nonlinear programming problem in the case of nonlinear predictive model (11) and has the following form

$$J_k = J_k\left(\bar{\boldsymbol{\tau}}\right) \to \min_{\bar{\boldsymbol{\tau}} \in \mathbf{\Omega} \subset E^{3P}}, \tag{19}$$

where $\mathbf{\Omega}$ is the admissible set (16). However, it is easy to show [x] that in the case of linear predictive model (14) optimization task reduces to quadratic programming problem of the form

$$J_k = J_k\left(\Delta\bar{\boldsymbol{\tau}}\right) = \Delta\bar{\boldsymbol{\tau}}^T \mathbf{H} \Delta\bar{\boldsymbol{\tau}} + 2\mathbf{f}^T \Delta\bar{\boldsymbol{\tau}} + g \to \min_{\Delta\bar{\boldsymbol{\tau}} \in \mathbf{\Omega}' \subset E^{3P}}, \tag{20}$$

where $\mathbf{\Omega}'$ is the admissible set (17), matrix $\mathbf{H}$ and vector $\mathbf{f}$ are determined by the matrices of linear predictive model (14) and cost functional (18).

Finally, let us form nonlinear MPC based control algorithm for ship dynamic positioning. The algorithm consists of two stages, which are implemented in real time as follows:

- if $\boldsymbol{\eta} \notin \mathbf{\Omega}_v$ then nonlinear predictive model (11) is used and for this case control input $\boldsymbol{\tau}^*[k]$ at each sample instant k is calculated as a solution of nonlinear programming problem (19);
- if $\boldsymbol{\eta} \in \mathbf{\Omega}_v$ then linear predictive model (14) is used for the linear stage where control input $\boldsymbol{\tau}^*[k]$ is computed as a solution of quadratic programming problem (20).

Proposed nonlinear MPC based scheme provides an optimal solution for ship dynamic positioning problem with respect to a given cost functional (18), taking into account nonlinear model of the process (5) – (7) and imposed constraints (8). Nevertheless, it suffers from inherent nonlinear MPC shortcomings such as local minima's and significant computational consumptions.

5. PRACTICAL EXAMPLE

Let consider a practical example, demonstrating the proposed control design approach for the ship "Northern Clipper" with the model (5), taken from Fossen and Strand, 1999. The length of the ship is $L = 76.2$ m and the mass is $m = 4.59 \cdot 10^6$ kg. The matrices in equations (5) are equal to

$$\mathbf{M} = \begin{pmatrix} 5.31 \cdot 10^6 & 0 & 0 \\ 0 & 8.28 \cdot 10^6 & 0 \\ 0 & 0 & 3.75 \cdot 10^9 \end{pmatrix},$$

$$\mathbf{D} = \begin{pmatrix} 5.02 \cdot 10^4 & 0 & 0 \\ 0 & 2.72 \cdot 10^5 & -4.39 \cdot 10^6 \\ 0 & -4.39 \cdot 10^6 & 4.19 \cdot 10^8 \end{pmatrix}.$$

The matrix $\mathbf{T}$ in bias equation (7) is defined as

$$\mathbf{T} = \begin{pmatrix} 1000 & 0 & 0 \\ 0 & 1000 & 0 \\ 0 & 0 & 1000 \end{pmatrix}.$$

The measurement variables are position (x, y) and heading angle ψ. The velocity vector $\mathbf{v}$ and slowly-varying disturbances vector $\mathbf{b}$ are estimated using asymptotic observer. The additional wave filter is also used to reduce negative influence from the wave disturbances.

Let assume that the digital control system sample time period T for a particular ship is equal to 0.1s. The MPC design parameters are chosen as follows: prediction horizon $P = 20$, sampling time for predictive model $Ts = 5$ s. Hence, the prediction is computed over the time interval with the duration of 100 s in continues time. Let accept that the vicinity $\mathbf{\Omega}_v$ of the desired position $\boldsymbol{\eta}_d$ is determined by the parameters value $\alpha = 10^\circ$ and $r = 2$ m. The cost functional has a form (18) with the following weight matrices:

$$\mathbf{Q}_\eta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 100 \end{pmatrix}, \quad \mathbf{Q}_\tau = 10^{-12} \mathbf{I}_{3x3}.$$

Now let introduce input constraints (8), where $c_u = 200$ kN, $c_v = 200$ kN, $c_p = 5000$ kN·m. Firstly, let examine MPC controller performance in the simplest situation without external disturbances. Assume that the desired position is given by $\boldsymbol{\eta}_d = (x_d \; y_d \; \psi_d)^T$, $x_d = 30$ m, $y_d = 30$ m, $\psi_d = 45^{\circ}$. The corresponding transient processes are represented in Fig. 2. The settling time is 100 s. It is easy to see that all of the constraints are satisfied.

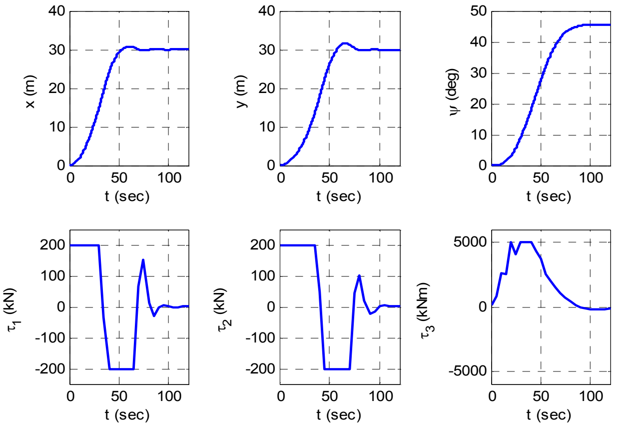

Fig.2. Transient processes for MPC without disturbances.

In order to reduce the computational consumptions, the experiments with control horizon C were carried out. The obtained results show that the control horizon with the value $C \geq 5$ allows us preserving the acceptable quality of the processes while simplifying the MPC controller real-time implementation.

Let consider MPC controller performance in the general case with the external disturbances and constraints imposed. Assume that a bias disturbance is represented by constant vector $\mathbf{b} = (-10kN \quad 30kN \quad 100kN \cdot m)$. Simulation results are shown on Fig. 3.

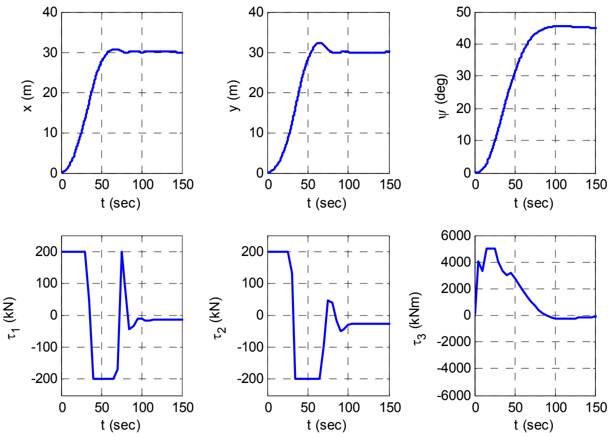

Fig.3. Transient processes for MPC with disturbances.

It is easy to see from figure 3 that the MPC algorithm provide offset-free performance. The settling time here is approximately the same as in the previous case and all of the constraints are satisfied. It's also interesting to note that MPC algorithm predict disturbance influence on the future process evolution and can take advantage of using accompanying forces.

6. CONCLUSIONS

In this paper two stage MPC control algorithm for DP problem was proposed. It was shown that MPC can be successfully implemented for DP and provide high-quality control subject to constraints and slowly-varying external disturbances. Nowadays MPC control can be implemented in real-time by means of modern computers in a lot of situations. So, it is necessary to extend the area of practical implementations of MPC approach for DP and other ship control problems.

REFERENCES

Camacho E.F. and Bordons C. (2004) Model Predictive Control. 2nd ed. Springer-Verlag, London. 405 p.

Fossen T. I. (2011) Handbook of Marine Craft Hydrodynamics and Motion Control. John Wiley & Sons, Ltd.

Fossen, T. I. and J. P. Strand. (1999) Passive Nonlinear Observer Design for Ships Using Lyapunov Methods: Experimental Results with a Supply Vessel. Automatica, Vol. (35), No. (1), pp. 3-16.

Loria A., T. I. Fossen, and E. Panteley. (2000) A Separation Principle for Dynamic Positioning of Ships: Theoretical and Experimental Results. IEEE Transactions of Control Systems Technology, Vol. 8, No. 2, pp. 332-343.

Maciejowski J.M. (2002) Predictive Control with Constraints. Prentice Hall, London. 331 p.

Mizuno N., Kakami H., Okazaki T. (2012) Parallel simulation based predictive control scheme with application to approaching control for automatic berthing // Proc. of 9th IFAC Conf. on Maneuvering and Control of Marine Craft. Arenzano, Italy, September 19–21.

Sørensen, A. J. (2005). Structural Issues in the Design and Operation of Marine Control Systems. IFAC Journal of Annual Reviews in Control, Vol. 29, Issue 1, pp. 125-149, Elsevier Ltd, ISSN: 1367-5788.

Sørensen, A. J. (2011) A survey of dynamic positioning control systems. Annual Reviews in Control, 35, pp. 23–136.

Veremey E.I. (2012) H_{∞}-Approach to Wave Disturbance Filtering for Marine Autopilots // Proceedings of 9th IFAC Conference on Maneuvering and Control of Marine Craft. Arenzano, Italy, September 19–21.