Project Assignment

ENG-270: Computational Methods and Tools

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1 Algal proliferation prediction model

1.1 Deviation from project proposal

After careful consideration and realizing the time required to develop functional code, we made several changes to our initial project proposal. First, we clearly defined our objective: to model algal growth and predict biomass in Lake Zug. In this model, we account for atmospheric CO_2 levels, surface and bottom temperatures, and Chlorophyll-a concentration, as these are the data provided by the Datalakes platform. We deviated from our initial idea by choosing to ignore lake stratification, vertical mixing, and the impact of algal growth on oxygen levels and therefore on biodiversity, as these processes would have made the project too complex given the time constraints. Although this model can be adapted to other lakes, we decided to focus on a single lake for better visual clarity and to graph our results. We believe that this approach is the most effective as graphs are simple tools that illustrate variations over time, making it easier to understand algal growth trends over decades.

1.2 Introduction to the problem

Algal growth in freshwater ecosystems is a critical process, influencing water quality, nutrient availability, oxygen levels, and overall ecosystem health. Excessive algal growth can severely impact lake ecosystems, causing oxygen depletion and harmful algal blooms. These events can have huge consequences, including public health risks, economic losses such as reduced tourism and health costs, and social impacts, such as ecosystem degradation, recreational limitations, and restrictions on water use (Hoagland & Scatasta, 2006).

Water temperature and atmospheric CO_2 levels, are expected to increase over time, possibly influencing algal growth and productivity. Therefore, it is imperative to create models that predict these changes, to facilitate interventions and understanding of this phenomenon.

This project aims to develop a predictive model for algal growth in Lake Zug using historical measurements of atmospheric CO_2, surface and bottom water temperatures, and chlorophyll-a concentrations. By simulating algal biomass at different depths over time, this model provides insight into how these environmental changes may influence algal dynamics. While simplified, this approach allows us to simulate algal growth and visualize trends over decades, providing a useful tool for both researchers and policymakers. Our model is based entirely on equations found in the literature.

1.3 Approach used

During this project, we decided to write our code in both `C` and MATLAB. MATLAB was used for regression analysis, data visualization, and graphing, while the `C` code, which executes significantly faster, was used to solve the differential equations and to implement the full mathematical model using predicted environmental data (atmospheric CO_2 from the NOAA Global Monitoring Laboratory (n.d.), and surface and bottom temperatures from Eawag (n.d.)).

Our first objective was to extrapolate these datasets in order to obtain values from 2025 to 2050 using MATLAB. For the temperature data, which exhibit a clear seasonal pattern, we performed a sinusoidal extrapolation, whereas a quadratic extrapolation was used for atmospheric CO_2. To facilitate our work, we considered only the data corresponding to the first day of each month, from November 2025 to November 2050.

Our next objective was to model the temperature at all depths of the lake. To do so, we used the equation proposed by Vercauteren et al. (2011), taking the surface and bottom temperatures as boundary conditions. We then simulated the temporal evolution of the temperature between these two boundaries. Since the bottom temperature is measured at a depth of 195 m, our diffusion model uses 195 spatial points, spaced 1 m apart (dz = 1 m). The diffusion process was run for 10 000 iterations with a time step of dt = 500 s, corresponding to a total simulation period of approximately 58 days. This time step was chosen after testing several values, in order to achieve a suitable compromise between computational cost and sufficient thermal diffusion.

$$\frac{\partial T}{\partial t} = \alpha \cdot \frac{\partial^2 T}{\partial z^2} \tag{1}$$

where the diffusivity coefficient $\alpha = 6.6 \cdot 10^{-5}\ m^2 s^{-1}$ also taken from Vercauteren et al. (2011).

To compute the temperature dependence of Henry's constant, we used the Van't Hoff relation :

$$K_H(T) = K_H(T_0) \cdot exp(-\frac{\Delta H}{R}(\frac{1}{T} - \frac{1}{T_0})) \tag{2}$$

where T_0 = 298 K, ΔH = - 2400 K and $K_s(T_0)$ = 1450 mg/L.

We made an approximation that our dissolved CO_2 concentration is the same at every depth. We then used the value of $K_H$to calculate our aqueous carbon dioxide $CO_2(aq)$ concentration (mg/L):

$$[CO_2(aq)] = K_H \cdot [CO_2(g)] \tag{3}$$

Light attenuation with depth (cal/$(cm^2 day^{-1})$ was described using Liu (2005):

$$I_z = I_0 \cdot e^{-kz} \tag{4}$$

where the light-attenuation coefficient k = 0.547, and I_0 = 1084 cal/$(cm^2 day^{-1})$

The $K_s(T)$ value calculated with Hossain et al. (2019):

$$K_s(T) = -0.2067 \cdot T^2 + 125.6 \cdot T - 19029 \tag{5}$$

where the temperature is given in Kelvin and the $K_s(T)$ is in mg/L.

We based our growth model on Hossain et al. (2019), which is an adapted Monod equation. In fact, we used CO_2 as the limitant factor and we set an irradiance fraction (irradiance at the depth we're interested in over irradiance at the surface, fraction between 0 and 1), which decreases with depth:

$$\mu = A\, e^{-E/(RT)} \, \frac{[CO_2]}{K_s(T) + [CO_2]} \cdot \frac{I_z}{I_0} \tag{6}$$

Here, E is the activation energy (2537 calmol^{-1}), A is a pre-exponential factor (0.0077 day^{-1}), R is the universal constant for ideal gases (8.314 J$mol^{-1}K^{-1}$), I(z) the irradiance at a depth z, I_0 the irradiance at surface level (=1084 cal$cm^{-2}day^{-1}$) (Confédération suisse, 2023), and $K_s(T)$ is the temperature-dependent half-saturation coefficient (mg/L) taken from Hossain et al. (2019). We projected growth until the year 2050 using MATLAB to model surface and bottom water temperatures through a sinusoidal representation of seasonal variability combined with predicted atmospheric CO_2 levels, calculated from existing measurements.

Because algal growth depends on light availability, we related algal biomass to the irradiance at depth the same way we did wih the growth rate :

$$B(z) = B_0 \cdot \frac{I_z}{I_0} \tag{7}$$

with B_0 and I_0 being the surface biomass and irradiance.

As chlorophyll-*a* concentration can be estimated to represent approximately 1–2 % of algal biomass (Environmental Sciences Section et al., 1991), we estimated the initial algal biomass using the chlorophyll-*a* concentration obtained from the Alplakes platform with 1.5 %. This assumption is a clear simplification: light penetration is not the only factor limiting algal growth, and chlorophyll-*a* concentration is only a rough indicator of biomass, which can vary depending on the location within the lake as well as other environmental factors.

Nevertheless, by combining these elements, we can approximate algal biomass at all depths at monthly time intervals up to 2050. Because our data was recorded on a monthly basis, we made the simplifying assumptions that each month consists of 30 days and that day-to-day variations are negligible. Under these assumptions, we obtain the following equation for the algal biomass after one month:

$$B_T(z) = B(z) + 30\mu B(z) \tag{8}$$

where μ is the growth rate, B(z) is the biomass at the start of the month and $B_T(z)$ is the biomass after 30 days.

1.4 Results

Using our code, we can generate two predictive graphs: one showing algal growth at different depths and another showing algal biomass at varying depths, projected until the year 2050.

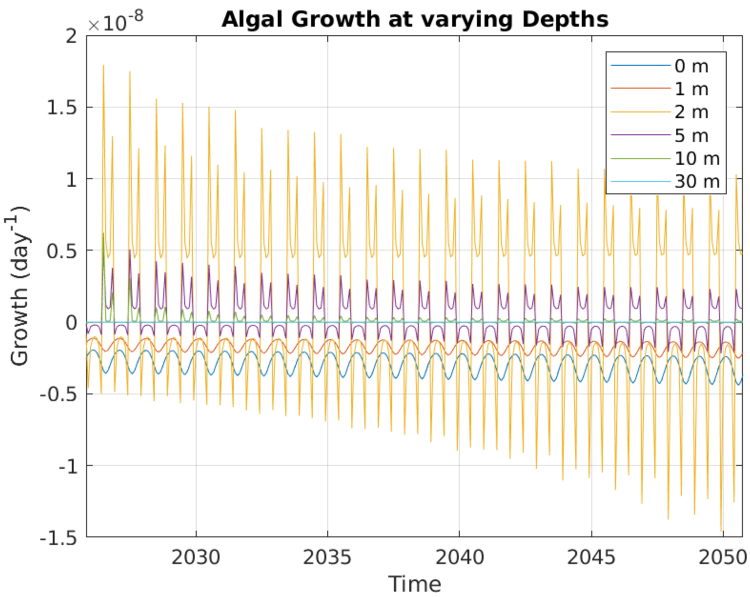

Figure 1: Algal Growth Prediction

From *Figure 1*, we can observe a pattern indicating that algal growth is strongly dependent on depth. By taking six different depths into account, we can see how much depth affects algal growth. The largest increase in growth can be seen between 2 and 5 m, whereas at 10 m and 30 m the growth variability is very small, the growth rate being negative at the surface level (0 to 1 m). All depths show a strong seasonal variation and an oscillating trend, with pronounced peaks in July and November. At shallow depths, the variations are much larger, as temperature changes are more drastic and light variations also play a much larger role. There also seems to be a decreasing trend overall in algal growth. We think that this may be due to temperature stress, as the surface temperature of the lake is rising with time, although in our model carbon dioxide levels in the water also rise, which should positively impact growth.

What may seem surprising is that algal growth at around 2 m depth is higher than right at the surface. This may be because conditions there are often more optimal overall, even though light is slightly reduced. At the surface, light intensity can exceed what algae can use efficiently, with very strong light possibly causing photoinhibition and damaging the photosynthetic machinery of the algae (Masojídek et al., 2021). At about 2 m depth, light is still abundant but less stressful, closer to the photosynthetic optimum of algae. A slightly greater depth also means that temperatures vary less between night and day, as well as seasonally, allowing for a more stable environment for growth. However, as our growth model only takes into account temperature, CO_2 availability and light intensity, these are the only conclusions that can be drawn from this graph.

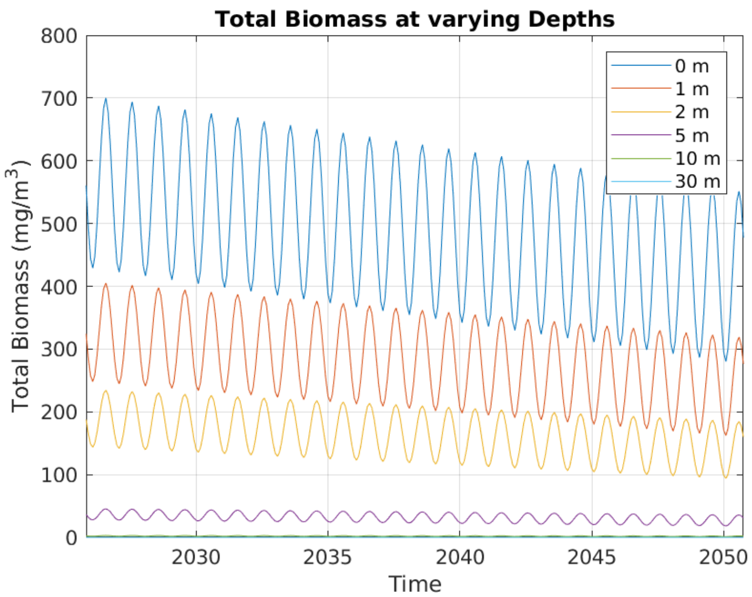

Figure 2: Biomass prediction

In *Figure 2*, we can see that algal biomass is higher near the surface and decreases with depth. This is due to the choice we made in our model, which only takes light intensity into account when scaling biomass with depth, as there was little information or available models in the literature, especially given our time constraints. As discussed previously, this limits our findings, since light intensity is not the only factor influencing growth and can even be detrimental if it is too strong. However, we can see that oscillations, much like in *Figure 1*, are much larger at the surface. These oscillations are negligible at greater depths (10 m and 30 m), where biomass remains much more stable. Furthermore, we observe a large decline in biomass, especially in the 0 to 2 m depth range, likely due to temperature stress factors we have mentioned previously. However, it is also worth noting that at the surface, as well as at 1 m depth, the growth rate seems to be always negative, all year round, which calls somewhat into question the validity of our model. However, Soranno et al. (2025) showed that although climate seems to have abrupt but temporary changes in algal biomass, only 13% of the 24 452 lakes showed a persistent regime shift across 34 years, with lakes impacted by anthropogenic activities least affected by changes in climate. As Zug is heavily urbanised, this could mean that the climate affects the algal growth and biomass less.

A decrease in algal biomass in freshwater systems can have big ecological consequences. Microorganisms like algae play a big part in primary production and conversion of CO_2 into O_2. A sustained decline would equally impact the whole food web, as there would be less oxygen production, also negatively impacting local food chains.

However, our model is far from being perfect. We have had to make many simplifications to make the problem easier to deal with, especially regarding nutrient availability including compounds such as nitrogen and phosphorus, essential for microorganisms like algae.

Although neglecting lake stratification simplifies our analysis, it is an important factor that should not be ignored, as it influences ecological aspects such as oxygen availability at greater depths and affects organisms like fish and other aquatic life. These omissions mean that while our predictions provide interesting and potentially useful insights, they should be interpreted with a pinch of salt, as real-world systems are far more complex and dynamic. Ignoring currents and assuming a homogeneous lake also severely affects our results.

1.5 Conclusion

Although we had to simplify our original proposal, we successfully managed in this project to model algal growth in Lake Zug at different depths, taking into account rising CO_2 levels and increasing water temperatures.

According to our model, there should be a drop-off in algal biomass and reduced growth at surface depths (0 to 1 m), while seasonal variations persist at lower depths (2 to 5 m). At greater depths (deeper than 10 m), where biomass is already low, little change is observed and levels remain essentially constant.

The use of graphical representations proved to be an effective way to highlight variations in algal populations over time and as a function of depth. Despite simplifying the input parameters, we were able to address the main objective of the project: developing a predictive model for algal growth in Lake Zug based on historical measurements.

It would be interesting to investigate how this model applies to other lakes. Since our code is easily transposable, this would allow us to assess whether a similar decrease in algal biomass is observed elsewhere.

1.6 References

1. Hossain, S. Z., Al-Bastaki, N., Alnoaimi, A. M. A., Ezuber, H., Razzak, S. A., & Hossain, M. M. (2019). *Mathematical modeling of temperature effect on algal growth for biodiesel application. Renewable Energy and Environmental Sustainability,* 4, 8.

2. NOAA Global Monitoring Laboratory. (n.d.). *Trends in CO2 - NOAA Global Monitoring Laboratory.* https://gml.noaa.gov/ccgg/trends/mlo.html

3. Hoagland, P., & Scatasta, S. (2006). *The economic effects of harmful algal blooms. In Ecological studies* (pp. 391–402). https://doi.org/10.1007/978-3-540-32210-8_30

4. Environmental Sciences Section, Inorganic Chemistry Unit, & Wisconsin State Lab of Hygiene. (1991). *ESS Method 150.1: Chlorophyll - spectrophotometric (pp. 3-359-3–362).* https://polk.wateratlas.usf.edu/upload/documents/methd150.pdf

5. Alplakes. (n.d.). https://www.alplakes.eawag.ch/map/zug?layers=satellite_chlorophyll

6. Eawag: Swiss Federal Institute of Aquatic Science and Technology. (n.d.). *Surface and bottom temperature data for Lake Zug* [Data set]. Datalakes. https://www.datalakes-eawag.ch/datadetail/1325 (accessed October, 2025)

7. Masojídek, J., Ranglová, K., Lakatos, G. E., Benavides, A. M. S., & Torzillo, G. (2021). *Variables governing photosynthesis and growth in microalgae mass cultures.* Processes, 9(5), 820. https://doi.org/10.3390/pr9050820

8. Conféderation suisse. (2023). *Hoch aufgelöste Einstrahlungsdaten für die ganze Schweiz.* In Bundesamt Für Energie BFE.Retrieved November 22, 2025, from https://www.bfe.admin.ch und-geoinformation/geoinformation/geodaten/solar/solare-einstrahlung.html

9. Vercauteren, N., Huwald, H., Bou-Zeid, E., Selker, J. S., Lemmin, U., Parlange, M. B., & Lunati, I. (2011). *Evolution of superficial lake water temperature profile under diurnal radiative forcing.* Water Resources Research, 47(9). https://doi.org/10.1029/2011wr010529

10. Soranno, P. A., Hanly, P. J., Webster, K. E., Wagner, T., McDonald, A., Shuvo, A., Schliep, E. M., Reinl, K. L., McCullough, I. M., Tan, P., Lottig, N. R., & Cheruvelil, K. S. (2025). *Abrupt changes in algal biomass of thousands of US lakes are related to climate and are more likely in low-disturbance watersheds.* Proceedings of the National Academy of Sciences, 122(9), e2416172122. https://doi.org/10.1073/pnas.2416172122

11. Liu, W. (2005). Water column light attenuation estimation to simulate phytoplankton population in tidal estuary. *Environmental Geology*, 49(2), 280-292. https://doi.org/10.1007/s00254-005-0087-y