

Homework 9: More integer programs

Due date: Friday April 24, 2020

See the course website for instructions and submission details.

A Note on Course Evaluation: The course evaluation is released on Friday, April 17 and closes on Friday, May 1. Everyone in the class will receive an additional bonus homework point (not belonging to any specific homework) if more than 70% of the students in the class fill in the online evaluation.

I would like to take a moment to explain why your feedback is so important to me. I am continuously working to make the class better and my teaching more engaging, and your feedback is an essential part of this work. In this spirit, I promise that your feedback will be read carefully and taken into consideration in future iterations of this class. Thank you for helping me becoming a better instructor!

1. **Voting.** Governor Blue of the state of Berry is attempting to get the state legislator to gerrymander Berry's congressional districts. The state consists of ten cities, and the numbers of registered Republicans and Democrats (in thousands) in each city are shown below

City	Republicans	Democrats
1	80	34
2	60	44
3	40	44
4	20	24
5	40	114
6	40	64
7	70	14
8	50	44
9	70	54
10	70	64

Berry has five congressional representatives. To form the five congressional districts, cities must be grouped together according to the following restrictions:

- Districts cannot subdivide cities; all voters in a city must be in the same district.
- Each district must contain between 150,000 and 250,000 voters (there are no independent voters).

Governor Blue is a Democrat. Assume 100% voter turnout and that each voter always votes according to their registered party. Formulate and solve an optimization problem to help Governor Blue maximize the number of congressional districts that have a Democratic majority.

2. **The Queens problem.** You are given a standard 8×8 chess board. The following problems involve placing queens on the board such that certain constraints are satisfied. For each of the following problems, model the optimization task as an integer program, solve it, and show what an optimal placement of queens on the board looks like.

- a) Find a way to place 8 queens on the board so that no two queens threaten each other. We say that two queens *threaten* each other if they occupy the same row, column, or diagonal. Show what this placement looks like.
- b) Repeat part (a) but this time find a placement of the 8 queens that has point symmetry. In other words, find a placement that looks the same if you rotate the board 180° .

- c) What is the smallest number of queens that we can place on the board so that each empty cell is threatened by at least one queen? Show a possible optimal placement.
- d) Repeat part (c) but this time find a placement of the queens that also has point symmetry. Does the minimum number of queens required change? Show a possible optimal placement.

3. Relay Race. A new running team is planning for a 5×400 meters relay race. The team coach Fannie has decided that the following five people will run the race: Alice, Bob, Cindy, David and Elisa. However, Fannie has not yet decided the order in which they will run. The average time for each of them to finish running 400 metres are $82.5s, 77.1s, 81.3s, 74.9s, 80.6s$, respectively. In the taking-over zone, the runner who just finished must pass the baton to the next runner. The time it takes to pass the baton depends on the runners, and are listed for each combination of runners in seconds in the table below (the column gives the name of the runner who is passing the baton, and the row gives the name of the runner who receives the baton).

Taking-over time (seconds)	Alice	Bob	Cindy	David	Elisa
Alice	0	1.1	1.3	1.9	2.1
Bob	1.2	0	1.7	1.0	1.8
Cindy	1.7	1.4	0	0.9	1.7
David	2.1	0.8	1.6	0	2.4
Elisa	1.5	1.2	1.9	2.3	0

The team is using a special baton to track their performance during the race. This baton is quite expensive, and Fannie is worried that one of the runners might loose it. Therefore, Fannie takes care of bringing the baton to the race. She gives the baton to the first runner before the start of the race and gets it back from the last runner when the race is over (the time it takes for Fannie to give and receive the baton is not counted as part of the racing time).

Please help Fannie decide the order of the five runners. What is the optimal runner sequence? What is the optimal total time for this race?