hw6

March 29, 2020

1 Homework 6: Least Squares

1.1 Hovercraft rendezvous.

a).
The variables in this question are the Position(X), Velocity(V) and Thrust(U) of both Alice and Bob at each second.
Constraints:
$x_{t+1} = x_t + \frac{1}{3600}v_t$
$v_{t+1} = v_t + u_t$
$x_{Alice,t=1} = (0,0), x_{Bob,t=1} = (0.5,0)$
$v_{Alice,t=1} = (0,20), v_{Bob,t=1} = (30,0)$
$x_{Alice,t=60} = x_{Bob,t=60}$

Objective: $Minimize\{\sum_{t=1}^{60} ||u_t^A||^2 + ||u_t^B||^2\}$

[1]:
```
using JuMP, Gurobi;
```

As an explanation: the first two columns stand for the X,V and U of Alice, while the last two stand for these of Bob!

[2]:
```
m1a = Model((Gurobi.Optimizer));
set_silent(m1a);
X = @variable(m1a, [1:60, 1:4]);
V = @variable(m1a, [1:60, 1:4]);
U = @variable(m1a, [1:60, 1:4]);
@constraint(m1a, X[1,:] .== [0,0,0.5,0]);
@constraint(m1a, V[1,:] .== [0,20,30,0]);

for i = 2:60
    @constraint(m1a, X[i,:] .== X[i-1,:] + V[i-1,:] / 3600);
    @constraint(m1a, V[i,:].== V[i-1,:] + U[i-1,:]);
end

@constraint(m1a, X[60,1:2] .== X[60,3:4]);
@objective(m1a, Min, sum(U.^2));
optimize!(m1a);
```

```
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```
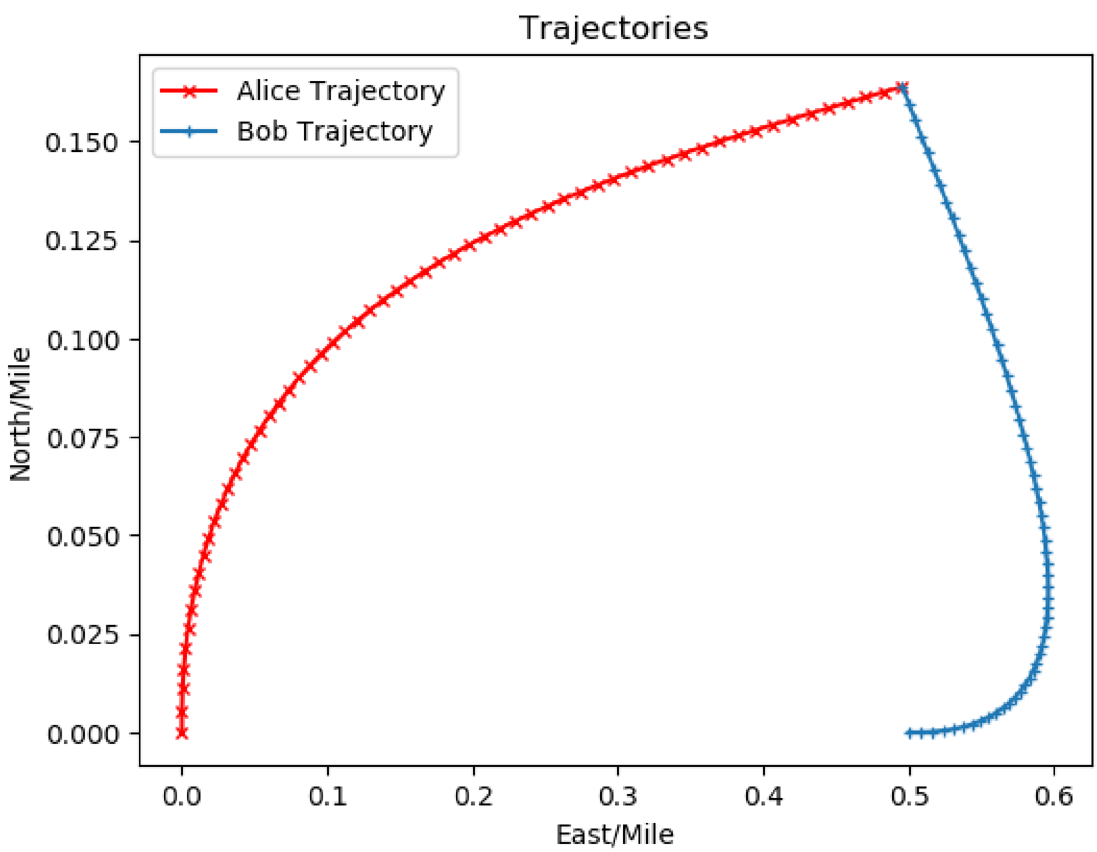

[3]:
```
using PyPlot;
figure();
plot(value.(X[:,1]), value.(X[:,2]), label = "Alice Trajectory", marker = "x",␣
 ↪ms = 5, "r");
plot(value.(X[:,3]), value.(X[:,4]), label = "Bob Trajectory", marker = "+", ms␣
 ↪= 5);
xlabel("East/Mile");
ylabel("North/Mile");
title("Trajectories");
legend();
```



[4]:
```
println("Final Location of Alice: ",round.(value.(X[60,1:2]),digits =6));
println("Final Location of Bob: ", round.(value.(X[60,3:4]),digits = 6));
println("Final Velocity of Alice: ",round.(value.(V[60,1:2]),digits =6));
println("Final Velocity of Bob: ", round.(value.(V[60,3:4]),digits = 6));
```

```
Final Location of Alice: [0.495833, 0.163889]
Final Location of Bob: [0.495833, 0.163889]
Final Velocity of Alice: [45.769231, 4.871795]
Final Velocity of Bob: [-15.769231, 15.128205]
```

Yes, they rendezvous at the 60th seconds.

b).
The constraints are same as those in a but with an additional one;
Additional constraint:
$v_{Bob,t=60} = v_{Alice,t=60}$

[5]:
```
m1b = Model((Gurobi.Optimizer));
set_silent(m1b);
Xb = @variable(m1b, [1:60, 1:4]);
Vb = @variable(m1b, [1:60, 1:4]);
Ub = @variable(m1b, [1:60, 1:4]);

@constraint(m1b, Xb[1,:] .== [0,0,0.5,0]);
@constraint(m1b, Vb[1,:] .== [0,20,30,0]);
@constraint(m1b, Vb[60,1:2] .== Vb[60, 3:4]);

for i = 2:60
    @constraint(m1b, Xb[i,:] .== Xb[i-1,:] + Vb[i-1,:] / 3600);
    @constraint(m1b, Vb[i,:].== Vb[i-1,:] + Ub[i-1,:]);
end

@constraint(m1b, Xb[60,1:2] .== Xb[60,3:4]);
@objective(m1b, Min, sum(Ub.^2));
optimize!(m1b);
```

```
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```
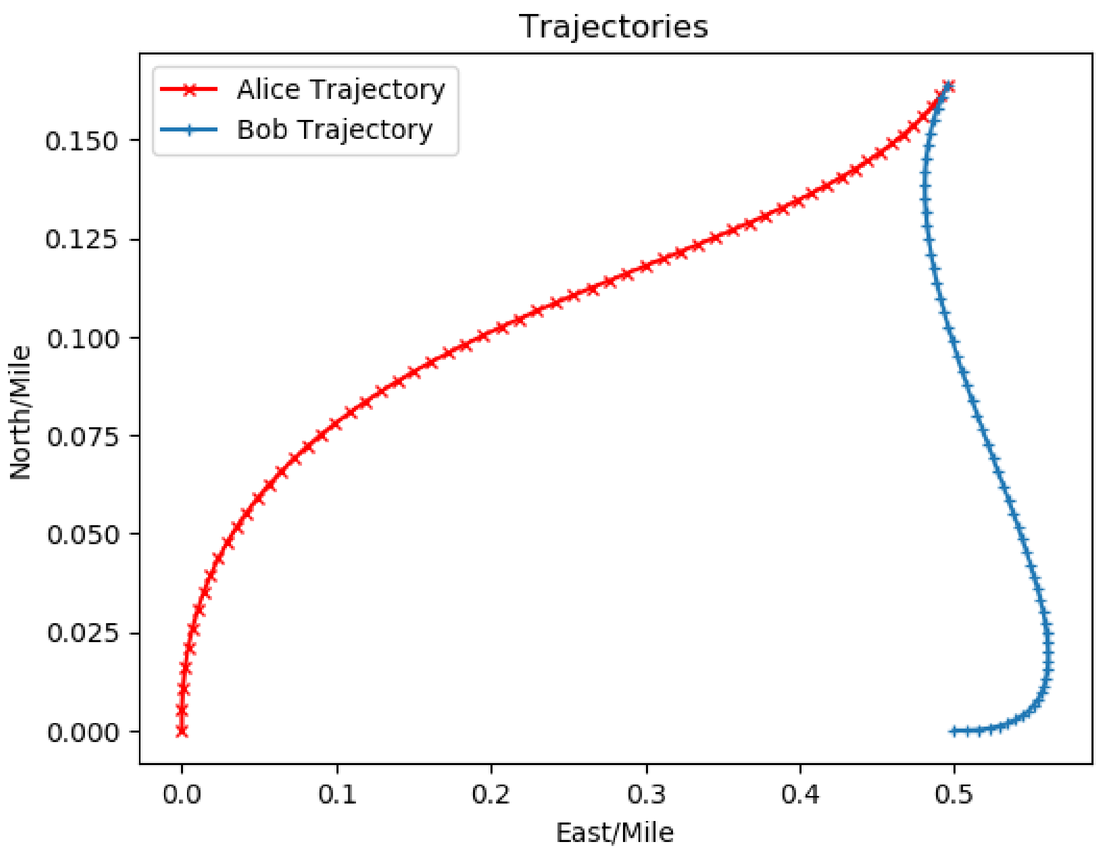

[6]:
```
using PyPlot;
figure();
plot(value.(Xb[:,1]), value.(Xb[:,2]), label = "Alice Trajectory", marker =␣
 ↪"x", ms = 5, c = "r");
plot(value.(Xb[:,3]), value.(Xb[:,4]), label = "Bob Trajectory",marker = "+",␣
 ↪ms = 5);
xlabel("East/Mile");
ylabel("North/Mile");
title("Trajectories");
legend();
```



The trajectories looks different from that in question a), as Alice and Bob are supposed to have the same velocity at the 60th second.

[7]:
```
println("Final Location of Alice: ",round.(value.(Xb[60,1:2]),digits =6));
println("Final Location of Bob: ", round.(value.(Xb[60,3:4]),digits = 6));
println("Final Velocity of Alice: ",round.(value.(Vb[60,1:2]),digits =6));
println("Final Velocity of Bob: ", round.(value.(Vb[60,3:4]),digits = 6));
```

```
Final Location of Alice: [0.495833, 0.163889]
Final Location of Bob: [0.495833, 0.163889]
Final Velocity of Alice: [15.0, 10.0]
Final Velocity of Bob: [15.0, 10.0]
```

[8]:
```
println("The meeting locations are ->");
println("In a). $(value.(X[60,1:2])); in b). $(value.(Xb[60,1:2]))")
println("\n\nIs their meeting location same as that in question a).?");
print(round.(value.(Xb[60,:]), digits = 6) == round.(value.(X[60,:]), digits =␣
↪6));
```

```
The meeting locations are ->
In a). [0.4958333333333333, 0.16388888888888892]; in b). [0.4958333333333335,
```

```
0.16388888888888883]
```

```
Is their meeting location same as that in question a).?
true
```

1.2 Quadratic form positivity.

a).The original constraint is
$2x^2 + 2y^2 + 9z^2 + 8xy - 6xz - 6yz \leq 1$
In the matrix form, we have:
$v^\top Qv \leq 1$, Q is symmetric;
$v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$
$Q = \begin{pmatrix} 2 & 4 & -3 \\ 4 & 2 & -3 \\ -3 & -3 & 9 \end{pmatrix}$

b).

[9]:
```
using LinearAlgebra;
Q = [2 4 -3; 4 2 -3; -3 -3 9];
(L,U) = eigen(Q);
## L is  eigen values
## U is orthogonal matrix
```

[10]:
```
(diagm(L))
```

[10]:
```
3×3 Array{Float64,2}:
 -2.0  0.0   0.0
  0.0  3.0   0.0
  0.0  0.0  12.0
```

Here we can see one of these eigen values is negative, so $Q > 0$ cannot be met. In other words, suppose we have a vector v, we can never ensure that ($v^\top Qv > 0$). But however, to have an ellipsoid, the constraint $Q \geq 0$ should always be satisfied. So this problem is not convex.

c). $||Av||^2 - ||Bv||^2 = v^\top A^\top Av - v^\top B^\top Bv = v^\top(A^\top A - B^\top B)v = v^\top Qv$
$Q = A^\top A - B^\top B$
$A^\top A$ and $B^\top B$ are both semi-positive-definite matrices.
Let's take a look at Q:
$Q = ULU^\top$
$Q = ULU^\top = U \left\{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 12 \end{pmatrix} - \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right\} U^\top$
Hence, we can just make $U \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 12 \end{pmatrix} U^\top = A^\top A$ and $U \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} U^\top = B^\top B$

$$A = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{12} \end{pmatrix} U^\top$$
$$B = U \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} U^\top$$

[11]:
```
(L,U) = eigen(Q);
L1 = Array(L);
L1[1] = 0;
L1 = L1 .^0.5;
L2 = zeros(3);
L2[1] = (-L[1])^0.5;
```

[12]:
```
A = U*diagm(L1)*U';
B = U*diagm(L2)*U';
```

[13]:
```
println("So the matrix A is ->")
for i = 1:3
    println(round.(A[i,:], digits = 6));
end
```

```
So the matrix A is ->
[1.154701, 1.154701, -0.57735]
[1.154701, 1.154701, -0.57735]
[-0.57735, -0.57735, 2.886751]
```

[14]:
```
println("So the matrix B is ->")
for i = 1:3
    println(round.(B[i,:], digits = 6));
end
```

```
So the matrix B is ->
[0.707107, -0.707107, 0.0]
[-0.707107, 0.707107, 0.0]
[0.0, -0.0, 0.0]
```

[15]:
```
Q_new = A'* A - B' * B
```

[15]:
```
3×3 Array{Float64,2}:
  2.0   4.0  -3.0
  4.0   2.0  -3.0
 -3.0  -3.0   9.0
```

[16]:
```
print("The bias between the original Q and reconstructed Q is ->", sum((Q -␣
↪Q_new).^2));
print("\nSo A'A - B'B is equavalent to Q.");
```

```
The bias between the original Q and reconstructed Q is ->8.38164711797325e-29
So A'A - B'B is equavalent to Q.
```

d).

Now we have L, the eigen values. If we can find a vector x, where $x^\top diag(L)x \leq 1$, no matter how large the x is, the constraint 1 can still be satisfied. (E.g. One x that meet this requirement is $(2\sqrt{3}, 2, 1)$).
$x = U^\top v$
$v = (U^\top)^{-1}x$
Then we can get the direction v. Let's talk on x instead of v first;
$-2x_1^2 + 3x_2^2 + 12x_3^2 \leq 1$
$x_1^2 \geq 1.5x_2^2 + 6x_3^2 - 0.5$
As long as we can find such x, we can get the v vector by the equation above;
Just use $x = (2\sqrt{3}, 2, 1)$ as an example;

[17]:
```
x = Array([2*3^0.5; 2; 1]);
v = inv(U') * x;
v /= norm(v)
```

[17]:
```
3-element Array{Float64,1}:
  0.21501775468273507
 -0.9731592968892739
 -0.08202650821026695
```

Now we have already get a direction that meet the question's requirement. Let us check if this corret:

[18]:
```
LogAmp = (-1:10);
for i = 1:length(LogAmp)
    Amp = 10.0^LogAmp[i];
    v_new = Amp*v;
    val = v'*Q*v;
    println("The length of the vector is $(Amp), and the value of v'Qv is␣
 ↪$(round(val, digits = 6)); \nConstraint_1 is $(val <= 1)\n");
end
```

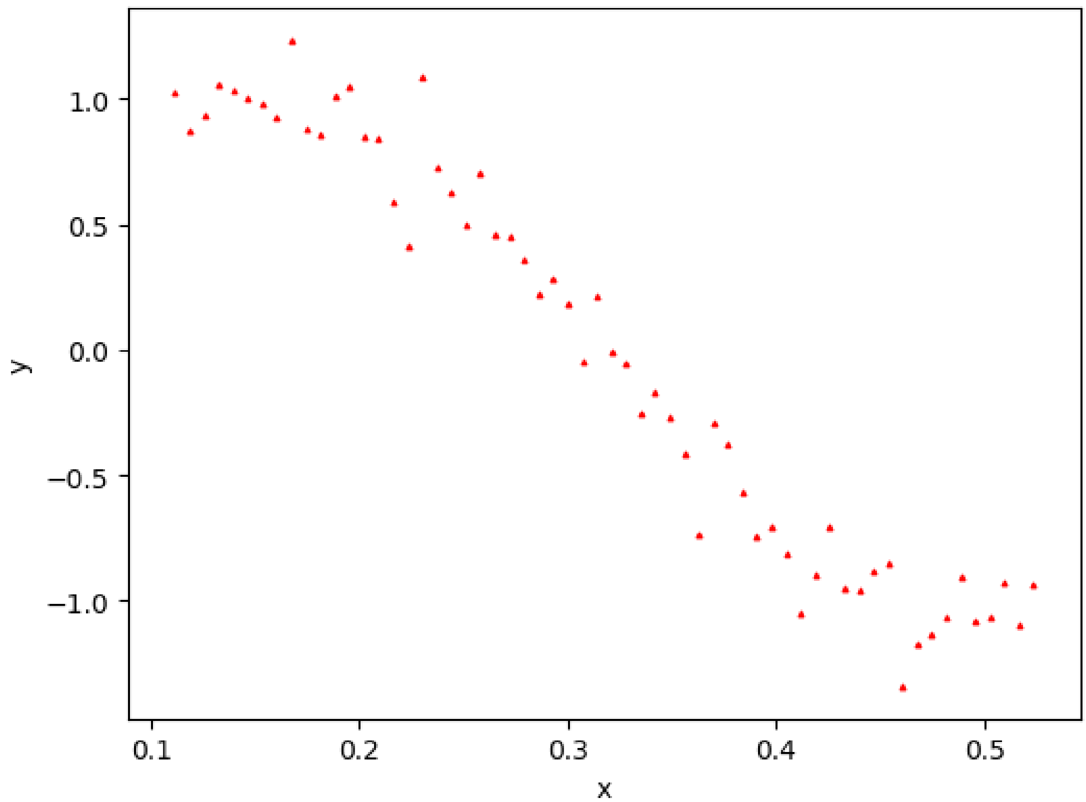
```
The length of the vector is 0.1, and the value of v'Qv is 0.0;
Constraint_1 is true

The length of the vector is 1.0, and the value of v'Qv is 0.0;
Constraint_1 is true

The length of the vector is 10.0, and the value of v'Qv is 0.0;
Constraint_1 is true

The length of the vector is 100.0, and the value of v'Qv is 0.0;
Constraint_1 is true
```

```
The length of the vector is 1000.0, and the value of v'Qv is 0.0;
Constraint_1 is true

The length of the vector is 10000.0, and the value of v'Qv is 0.0;
Constraint_1 is true

The length of the vector is 100000.0, and the value of v'Qv is 0.0;
Constraint_1 is true

The length of the vector is 1.0e6, and the value of v'Qv is 0.0;
Constraint_1 is true

The length of the vector is 1.0e7, and the value of v'Qv is 0.0;
Constraint_1 is true

The length of the vector is 1.0e8, and the value of v'Qv is 0.0;
Constraint_1 is true

The length of the vector is 1.0e9, and the value of v'Qv is 0.0;
Constraint_1 is true

The length of the vector is 1.0e10, and the value of v'Qv is 0.0;
Constraint_1 is true
```

So we can see this question became unbounded, and vector v can be arbitarily large;

1.3 Lasso regression.

[19]:
```
using CSV, PyPlot;
data = CSV.read("lasso_data.csv");

X = data[:,1];
Y = data[:,2];
figure();
scatter(X,Y, c = "r", marker = "^", s = 3);
xlabel("x");
ylabel("y");
```



[20]:
```
## making plot coordinates and match them with data X
N =50; ## number of points in the curve
X_plot = [i for i = 0:(N-1)] /(N-1) * 0.6 .+ 0.0;
#X_plot = [i for i = 0:(N-1)] /(N-1);
#X_plot *= maximum(X) - minimum(X);
#X_plot .+=  minimum(X);
```

a).

This question is an oridinary polynomial regression problem.
The objective is to minimize the mean square error between the data sets and the fitted curve. The variables are the polynomial coefficients, a;
$f(x) = \sum_{i=0}^{d} a_i x^i$
$Argmin(\sum_{j=1}^{N}(y_j - f(x_j))^2)$
x_j, y_j are the data. a_i is the i_{th} order polynomial coefficient.

[21]:
```
using JuMP, Ipopt,Gurobi;
```

[22]:
```
function getPoly(X,order)
    X_poly_matrix = zeros(length(X), order+1);
    for i = 1:order+1
        X_poly_matrix[:,i] = X.^(i-1);
```

```julia
    end
    return X_poly_matrix;
end
```

[22]: getPoly (generic function with 1 method)

[23]:
```julia
#m3a = Model(with_optimizer(Ipopt.Optimizer));
m3a = Model((Gurobi.Optimizer));
#m3a =Model(solver=GurobiSolver(OutputFlag=0))
#m3a = Model((Ipopt.Optimizer));
set_silent(m3a);

order_set = [5;15];
X = data[:,1];
Y = data[:,2];

Y_fitted_a = zeros(length(X_plot),length(order_set)); ## The first col is that␣
↪of d = 5 and the next  col is d = 15;
Y_cal_m = zeros(length(X),length(order_set));


for i = 1:length(order_set)


    order = order_set[i];

    X_poly = getPoly(X ,order);
    a = @variable(m3a, [1:(order+1)]);
    res = @variable(m3a, [1:length(X)]);
    @constraint(m3a, res .== (X_poly*a - Y));

    @objective(m3a, Min, sum((res).^2)/length(res));
    optimize!(m3a);

    println("\n\nThe polynomial coefficients are (from 0 order to higher order)␣
↪->\n",value.(a));
    println("The MSE of this fitting is -> ", JuMP.objective_value(m3a));
    X_plot_poly = getPoly(X_plot , order);
    Y_fitted_a[:,i] = X_plot_poly* value.(a);

end
```

```
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The polynomial coefficients are (from 0 order to higher order) ->
```
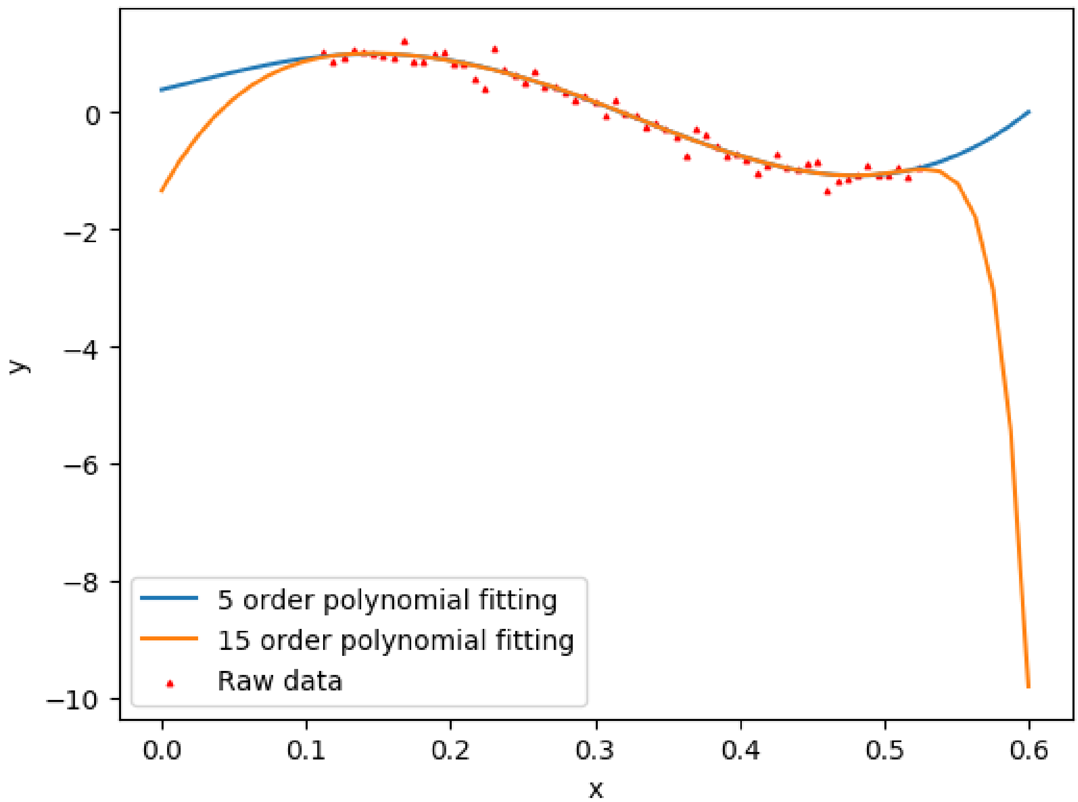

```
[0.3898950228186052, 5.73720250742401, 16.757282067608376, -254.1413141784733,
495.93298795464165, -247.3873075245843]
The MSE of this fitting is -> 0.01689988349314956


The polynomial coefficients are (from 0 order to higher order) ->
[-1.3371916887438686, 44.09937448383822, -287.2753991632128, 702.1306625632528,
43.02222722083481, -2474.6072012925233, -2081.9158690588442, 6513.438388120847,
19011.301841247314, 11806.302499387548, -50425.52808207874, -158788.32258178963,
-134174.10117745004, 446232.08246814396, 1.5000969599329894e6,
-2.1377115245777923e6]
The MSE of this fitting is -> 0.016843199493316422
```

Comments: For the 5 degree polynomial, that coefficients are larger than the first several coefficients which represent the lower orders. However, the corfficients of the higher orders in the 15 degree one, are extremely large.

[24]:
```
using PyPlot;
figure();
scatter(X,Y, c = "r", marker = "^", s = 3, label = "Raw data");
for i = 1:length(order_set)
    #plot(X, Y_cal_m[:,i], label = "£(order_set[i]) order polynomial fitting");
    plot(X_plot, Y_fitted_a[:,i], label = "$(order_set[i]) order polynomial␣
 ↪fitting");
end
xlabel("x");
ylabel("y");
legend();
```



b).

$f(x) = \sum_{i=0}^{d} a_i x^i$
$Argmin(\sum_{j=1}^{N}(y_j - f(x_j))^2 + \lambda||a||_2)$
x_j, y_j are the data. a_i is the i_{th} order polynomial coefficient.

[25]:
```
m3b = Model((Gurobi.Optimizer));
set_silent(m3b);
order_set = [15];
X = data[:,1];
Y = data[:,2];

Y_fitted_b = zeros(length(X_plot),length(order_set)); ## The first col is that␣
↪of d = 5 and the next  col is d = 15;
Y_cal_m = zeros(length(X),length(order_set));

for i = 1:length(order_set)
    order = order_set[i];
    lambda = 1e-6;
    X_poly = getPoly(X,order);

    a = @variable(m3b, [1:(order+1)]);
```

```julia
    res = @variable(m3b, [1:length(X)]);


    @constraint(m3b, res .== (X_poly*a - Y));

    #@constraint(m3a, a[1] == 510)


    @objective(m3b, Min, (sum(res.^2) .+ lambda* sum(a.^2))/length(res));
    optimize!(m3b);
    println("\n\nThe polynomial coefficients are (from 0 order to higher order)␣
↪->\n",value.(a));
    println("The MSE of this fitting is -> ", JuMP.objective_value(m3b));
    X_plot_poly = getPoly(X_plot, order);
    Y_fitted_b[:,i] =X_plot_poly* value.(a);
end
```

```
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The polynomial coefficients are (from 0 order to higher order) ->
[-0.06959121026110433, 15.382605672726102, -58.101276280265864,
15.600902791711446, 42.66479311077873, 32.187682625787325, 14.70853460939357,
2.6232773791303785, -3.0550949512270953, -4.621193261888788, -4.253387473429799,
-3.2668796167989376, -2.2752868059767515, -1.4894064090330597,
-0.9341257282504349, -0.5678886618431788]
The MSE of this fitting is -> 0.01703881579449471
```

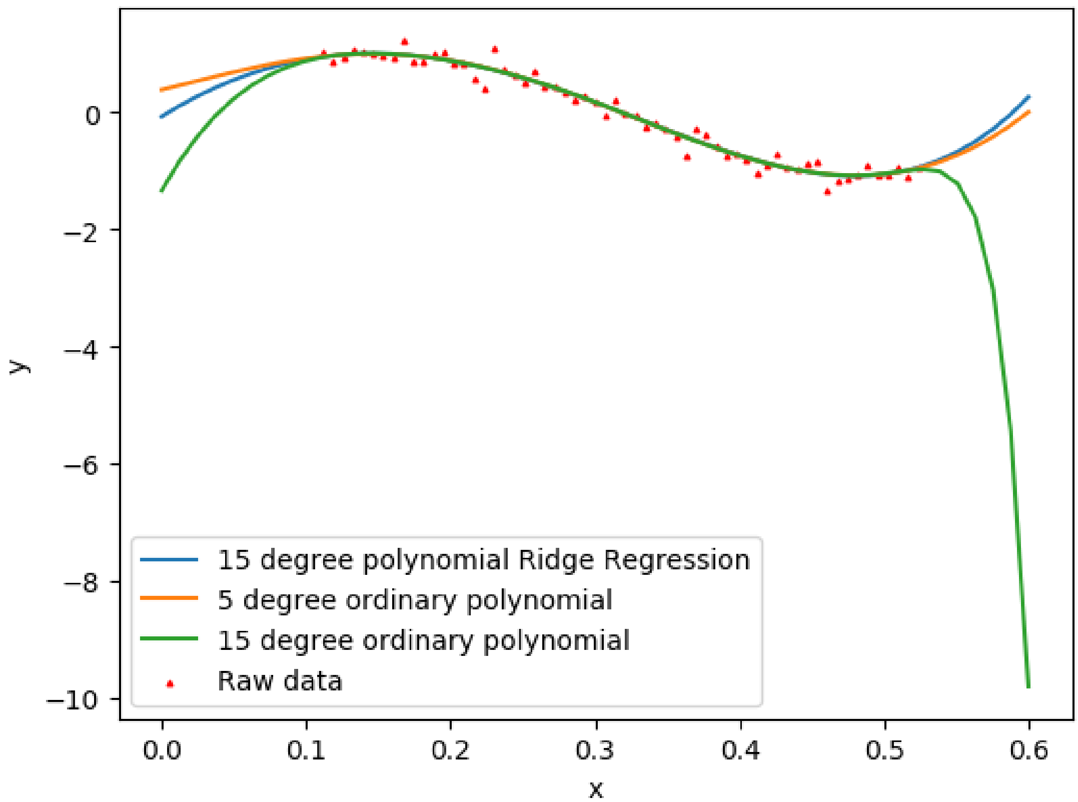
[ ]:

```

```

[26]:

```julia
figure();
scatter(X,Y, c = "r", marker = "^", s = 3, label = "Raw data");
for i = 1:length(order_set)
    #plot(X, Y_cal_m[:,i], label = "£(order_set[i]) order polynomial fitting");
    plot(X_plot, Y_fitted_b[:,i], label = "$(order_set[i]) degree polynomial␣
↪Ridge Regression");
end
plot(X_plot, Y_fitted_a[:,1], label = "5 degree ordinary polynomial");
plot(X_plot, Y_fitted_a[:,2], label = "15 degree ordinary polynomial");
xlabel("x");
ylabel("y");
legend();
```



The l_2 regularization makes the 15 degree polynimial fitting more similar to the 5 degree ordinary polynomial fitting. When comparing it with 15 degree ordinary polynomial fitting, its polynomial coefficients for higher orders become much smaller,but its Mean Square Error is greater.

c).

[27]:
```
m3c = Model((Gurobi.Optimizer));
set_silent(m3c);
order_set = [15];
X = data[:,1];
Y = data[:,2];

lambda_lasso = [10.0^i for i = 1:-1:-9];

Y_fitted_c = zeros(length(X_plot),length(lambda_lasso)); ## The first col is␣
↪that of d = 5 and the next  col is d = 15;
Y_cal_m = zeros(length(X),length(order_set));

N_non_zeros = zeros(length(lambda_lasso));
MSE = zeros(length(lambda_lasso));
```

```
for i = 1:length(lambda_lasso)
    order = 15;
    lambda = lambda_lasso[i];
    X_poly = getPoly(X,order);

    a = @variable(m3c, [1:order+1]);

    res = @variable(m3c, [1:length(X)]);


    @constraint(m3c, res .== X_poly*a - Y);

    abs_a = @variable(m3c, [1:order+1]);
    @constraint(m3c,abs_a .>= a);
    @constraint(m3c,abs_a .>= -a);

    @objective(m3c, Min, (sum(res.^2) + lambda* sum(abs_a))/length(res));
    optimize!(m3c);

    X_plot_poly = getPoly(X_plot, order);
    l1_a = value.(a);

    non_zeros = length(l1_a);
    for j = 1:length(l1_a)
        if abs(l1_a[j]) <= 1e-5
            l1_a[j] = 0.0;
            non_zeros -= 1;
        end
    end
    println("\n\nLasso lambda = $(round(lambda,digits = 9))\nThe polynomial␣
↪coefficients are (from 0 order to higher order)->\n",round.(l1_a,digits =␣
↪9));
    println("The MSE of this fitting is -> ", JuMP.objective_value(m3c));
    MSE[i] = JuMP.objective_value(m3c);
    N_non_zeros[i] = non_zeros;

    Y_fitted_c[:,i] = X_plot_poly* l1_a;
end
```

```
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Lasso lambda = 10.0
The polynomial coefficients are (from 0 order to higher order)->
[0.0, -0.145968309, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
0.0, 0.0]
```
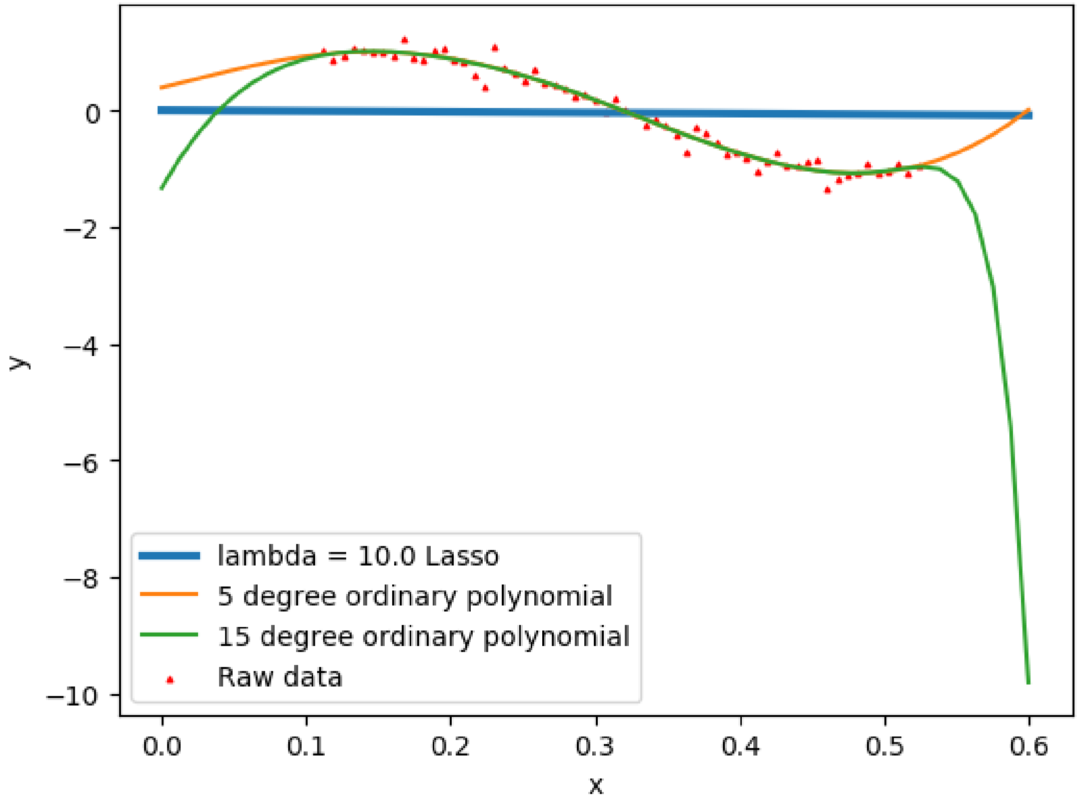

```
The MSE of this fitting is -> 0.6461937018639613


Lasso lambda = 1.0
The polynomial coefficients are (from 0 order to higher order)->
[1.77910438, -5.687325981, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
0.0, 0.0, 0.0, 0.0]
The MSE of this fitting is -> 0.1750848492565986


Lasso lambda = 0.1
The polynomial coefficients are (from 0 order to higher order)->
[2.001351189, -6.363357977, -4.5876e-5, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
0.0, 0.0, 0.0, 0.0, 0.0]
The MSE of this fitting is -> 0.05635106518216829


Lasso lambda = 0.01
The polynomial coefficients are (from 0 order to higher order)->
[1.228589368, 1.506285549, -18.581905803, 0.0, 0.0, 49.271865995, 0.0, 0.0, 0.0,
0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]
The MSE of this fitting is -> 0.03623730074137063


Lasso lambda = 0.001
The polynomial coefficients are (from 0 order to higher order)->
[0.095817626, 13.30548729, -49.42393207, 3.8661e-5, 54.305665788, 36.764597025,
0.000214016, 6.019e-5, 2.8426e-5, 1.4864e-5, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]
The MSE of this fitting is -> 0.019566792225665843


Lasso lambda = 0.0001
The polynomial coefficients are (from 0 order to higher order)->
[-0.085860817, 15.39380407, -55.794126453, 0.000264603, 76.913513744,
15.106904941, 0.0007809, 0.000134485, 5.5592e-5, 2.2125e-5, 0.0, 0.0, 0.0, 0.0,
0.0, 0.0]
The MSE of this fitting is -> 0.017189083104063682


Lasso lambda = 1.0e-5
The polynomial coefficients are (from 0 order to higher order)->
[-0.10576424, 15.624177419, -56.503813487, -0.001425287, 79.508387299,
12.584824833, 0.000658108, -0.000330838, -0.096677004, -0.001134989,
-0.000626957, -0.000413612, -0.000272123, -0.000173819, -0.000108522,
-6.6324e-5]
The MSE of this fitting is -> 0.016943387378790465
```

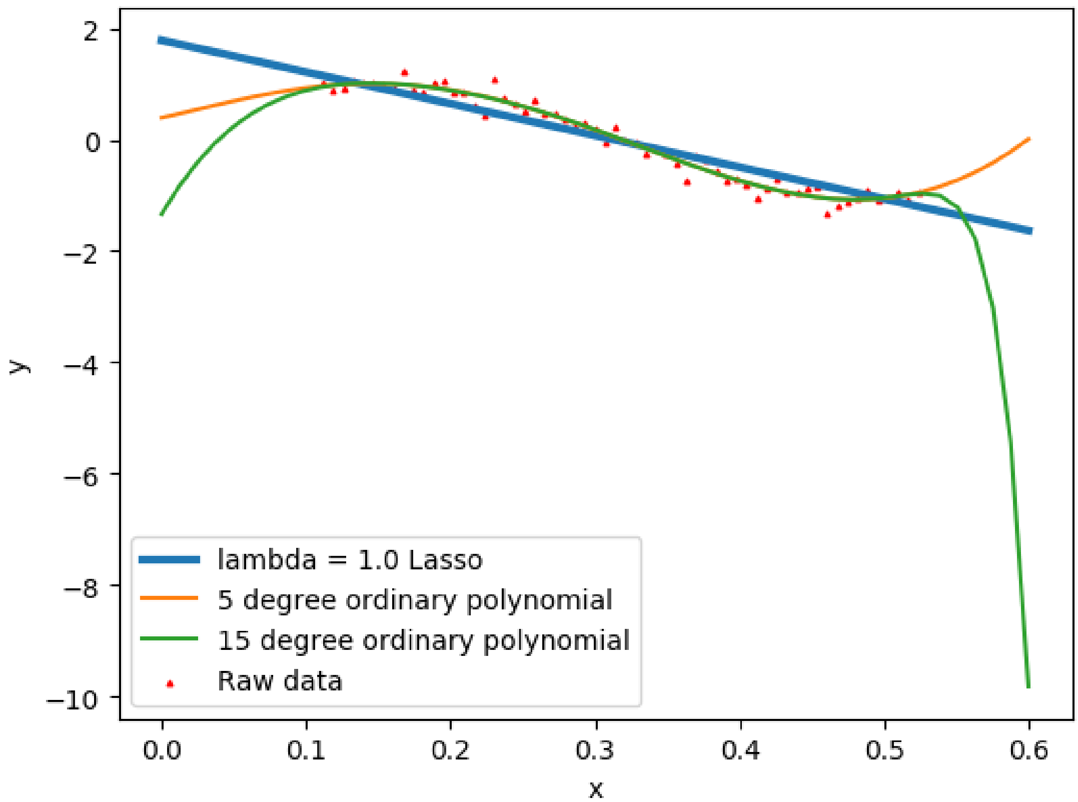
```
Lasso lambda = 1.0e-6
The polynomial coefficients are (from 0 order to higher order)->
[0.115208568, 12.086302237, -39.329510648, -15.388082896, 0.000133769,
161.957864946, 0.003771091, -0.000225535, -0.003973522, -248.89593618,
-31.120585457, -0.005725013, -0.002226523, -0.001182355, -0.000697489,
-0.000424763]
The MSE of this fitting is -> 0.016898595068379044


Lasso lambda = 1.0e-7
The polynomial coefficients are (from 0 order to higher order)->
[-0.26230595, 20.351586986, -106.134430321, 225.346531435, -323.980023393,
10.781323921, 565.990593782, 0.0181982, -0.133218792, -566.696306964,
-431.817879134, -0.231222556, -0.060180216, -0.025791767, -0.01491687,
-0.009288948]
The MSE of this fitting is -> 0.016885719036416044


Lasso lambda = 1.0e-8
The polynomial coefficients are (from 0 order to higher order)->
[-0.81919416, 34.08229199, -237.687932876, 843.48945186, -1757.583174917,
1249.415427324, 1079.546909052, -453.650123146, -1338.736452148, -583.668792675,
57.866475818, 505.681667532, 708.565610474, 566.3143814, 293.962532617,
35.708509161]
The MSE of this fitting is -> 0.016879928640059227


Lasso lambda = 1.0e-9
The polynomial coefficients are (from 0 order to higher order)->
[-1.337191689, 44.099374484, -287.275399163, 702.130662563, 43.022227221,
-2474.607201293, -2081.915869059, 6513.438388121, 19011.301841247,
11806.302499388, -50425.528082079, -158788.32258179, -134174.10117745,
446232.082468144, 1.50009695993299e6, -2.137711524577792e6]
The MSE of this fitting is -> 0.016843199493316422
```

[28]:
```
for i = 1:length(lambda_lasso)
    figure();

    #plot(X, Y_cal_m[:,i], label = "£(order_set[i]) order polynomial fitting");
    plot(X_plot, Y_fitted_c[:,i], label = "lambda =␣
↪$(round(lambda_lasso[i],digits = 10)) Lasso", linewidth = 3);
    plot(X_plot, Y_fitted_a[:,1], label = "5 degree ordinary polynomial");
    plot(X_plot, Y_fitted_a[:,2], label = "15 degree ordinary polynomial");

    scatter(X,Y, c = "r", marker = "^", s = 3, label = "Raw data");
    xlabel("x");
    ylabel("y");
```

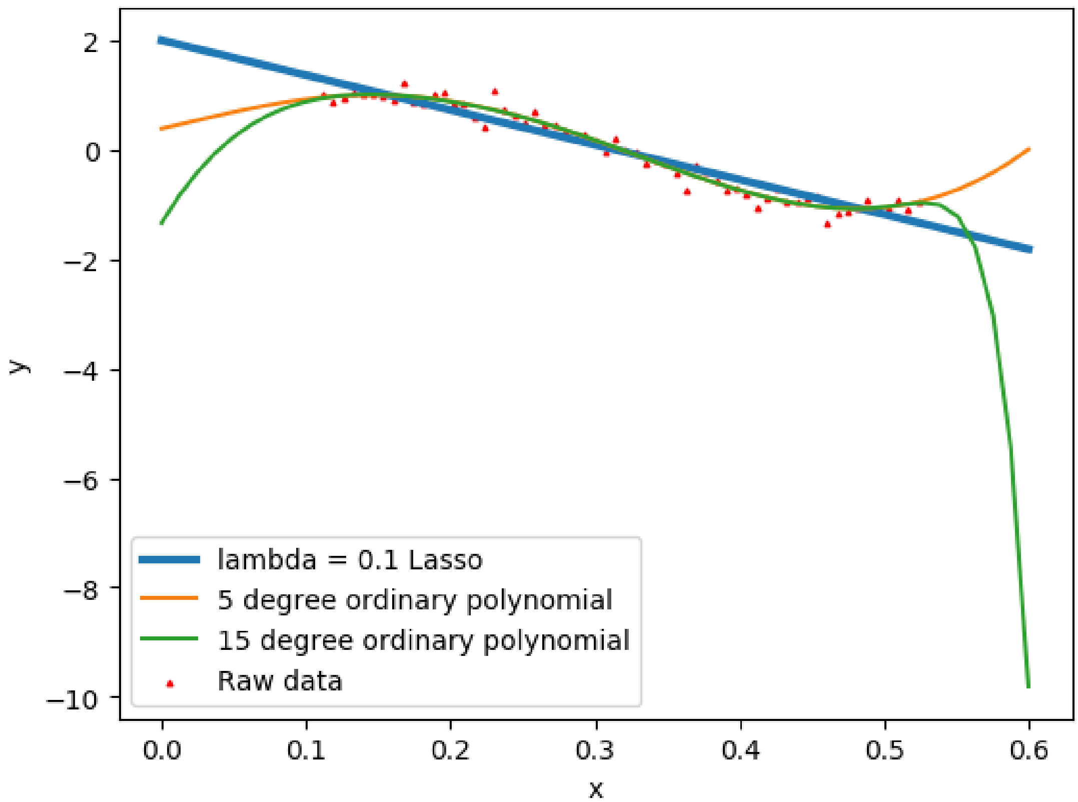
```
    legend();
end
```



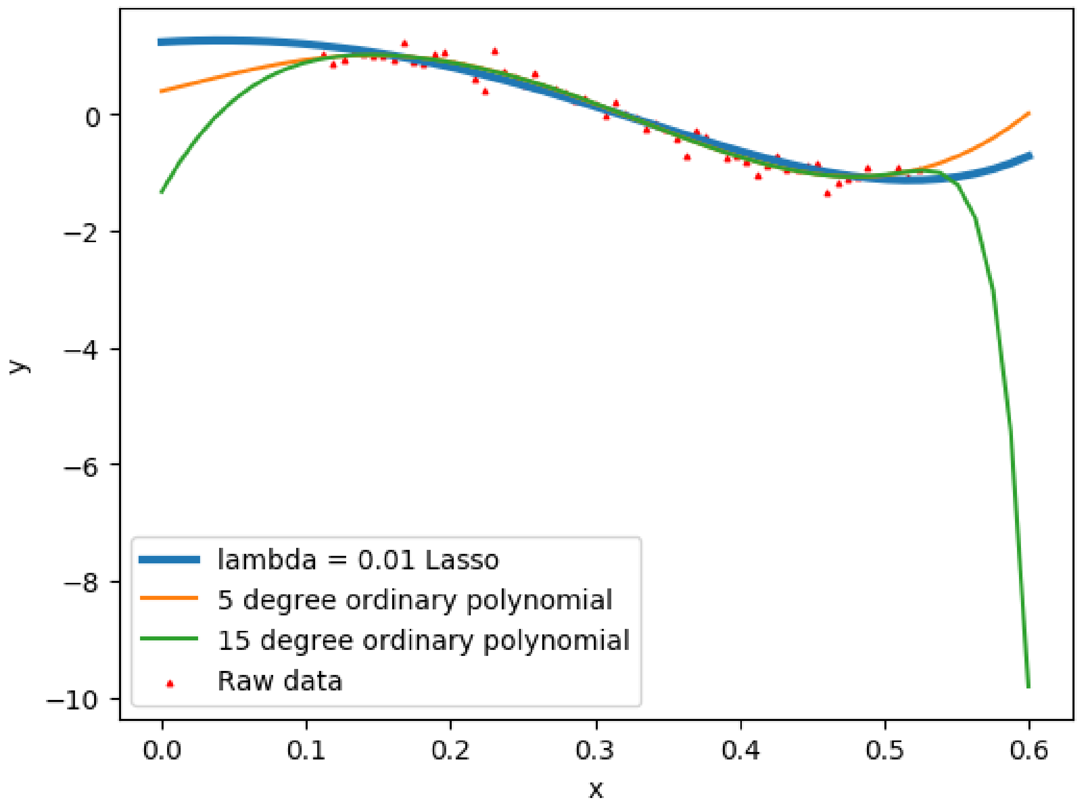
lambda = 10.0 Lasso
5 degree ordinary polynomial
15 degree ordinary polynomial
Raw data
0
−2
−4
−6
−8
−10
y
0.0
0.1
0.2
0.3
0.4
0.5
0.6
x



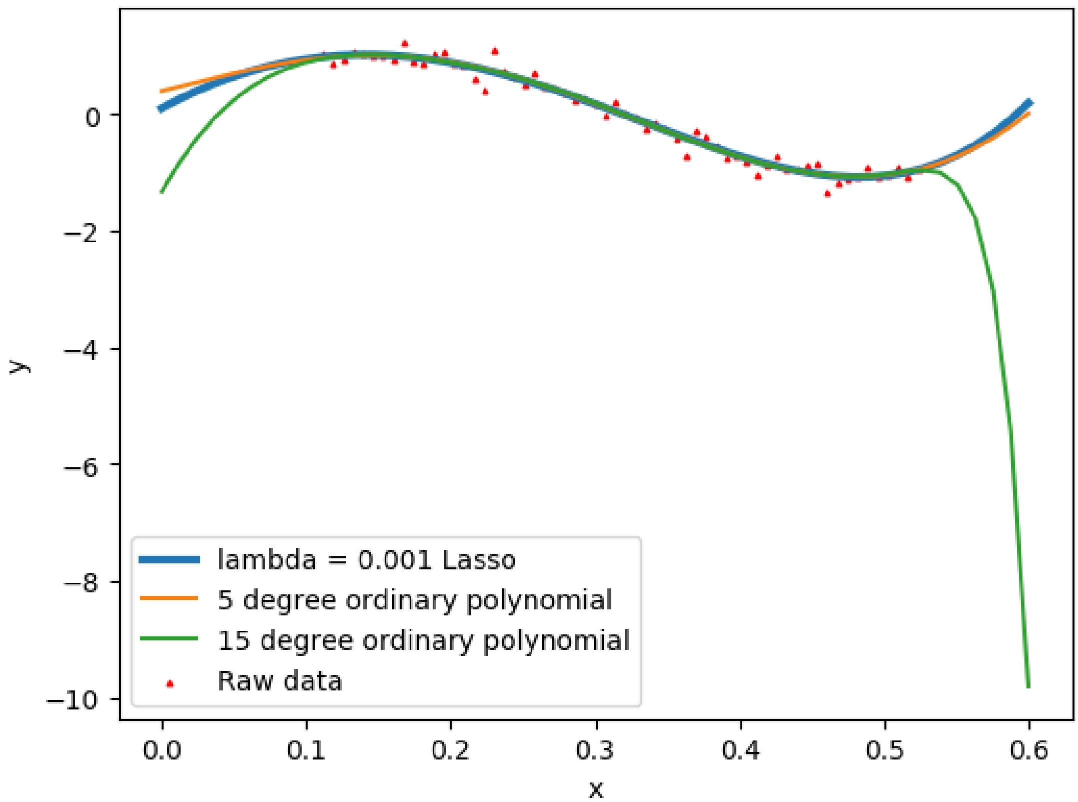
lambda = 1.0 Lasso
5 degree ordinary polynomial
15 degree ordinary polynomial
Raw data
y
x
2
0
−2
−4
−6
−8
−10
0.0
0.1
0.2
0.3
0.4
0.5
0.6

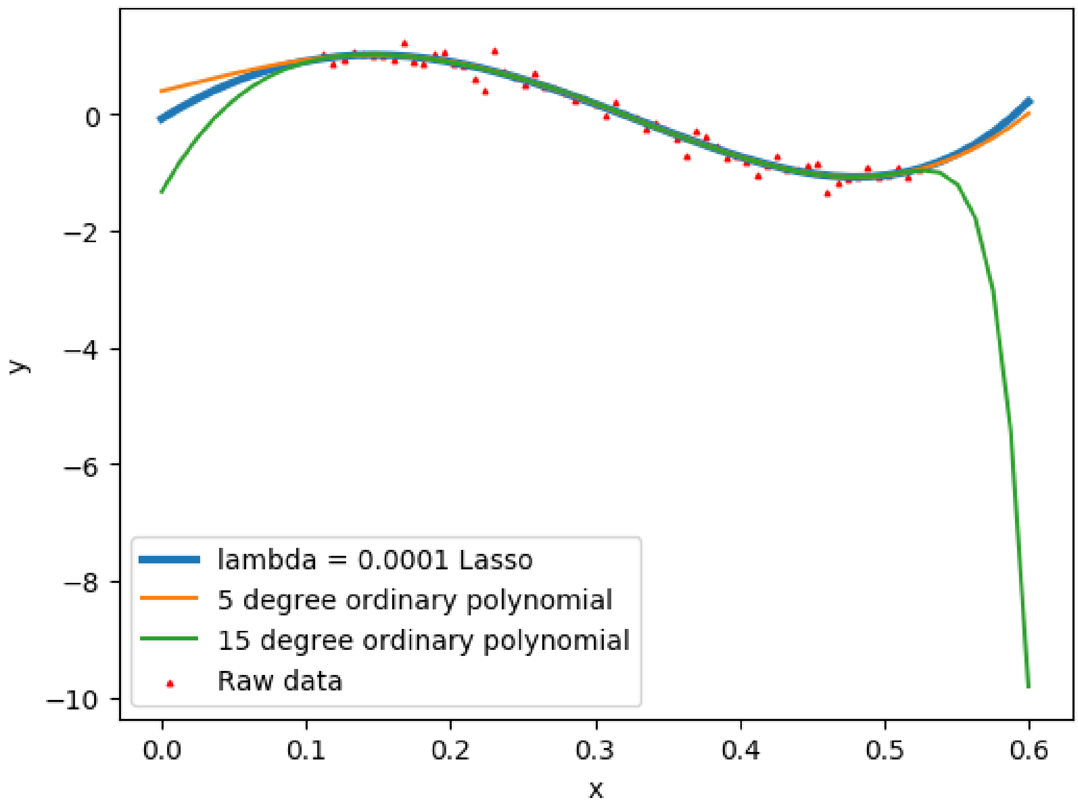


2
0
−2
−4
−6
−8
−10
y
0.0
0.1
0.2
0.3
0.4
0.5
0.6
x
lambda = 0.1 Lasso
5 degree ordinary polynomial
15 degree ordinary polynomial
Raw data

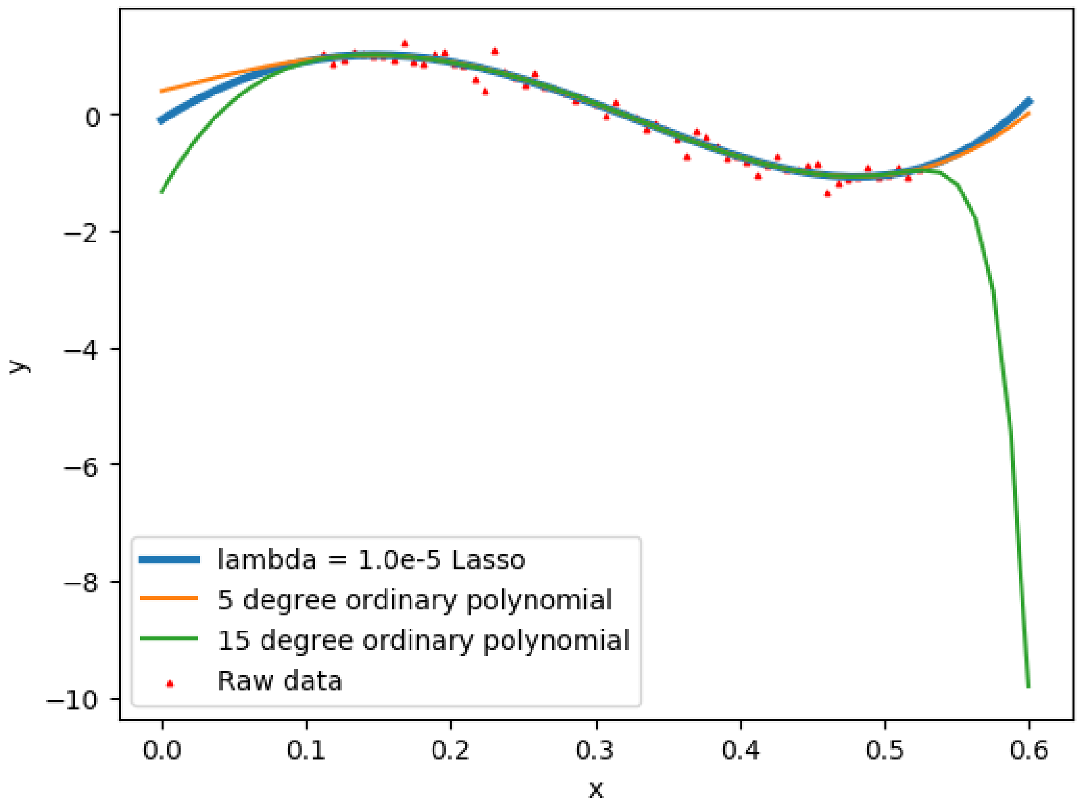


lambda = 0.01 Lasso
5 degree ordinary polynomial
15 degree ordinary polynomial
Raw data
y
x
0
−2
−4
−6
−8
−10
0.0
0.1
0.2
0.3
0.4
0.5
0.6

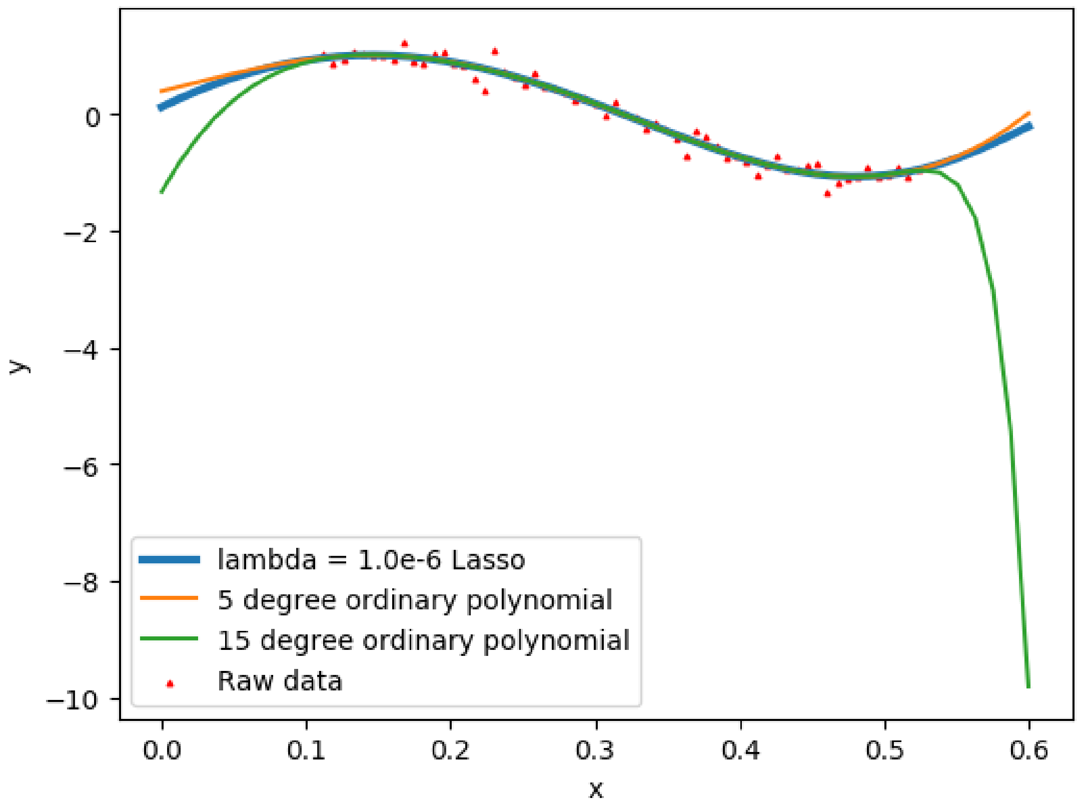

lambda = 0.001 Lasso
5 degree ordinary polynomial
15 degree ordinary polynomial
Raw data
0
−2
−4
−6
−8
−10
y
0.0
0.1
0.2
0.3
0.4
0.5
0.6
x



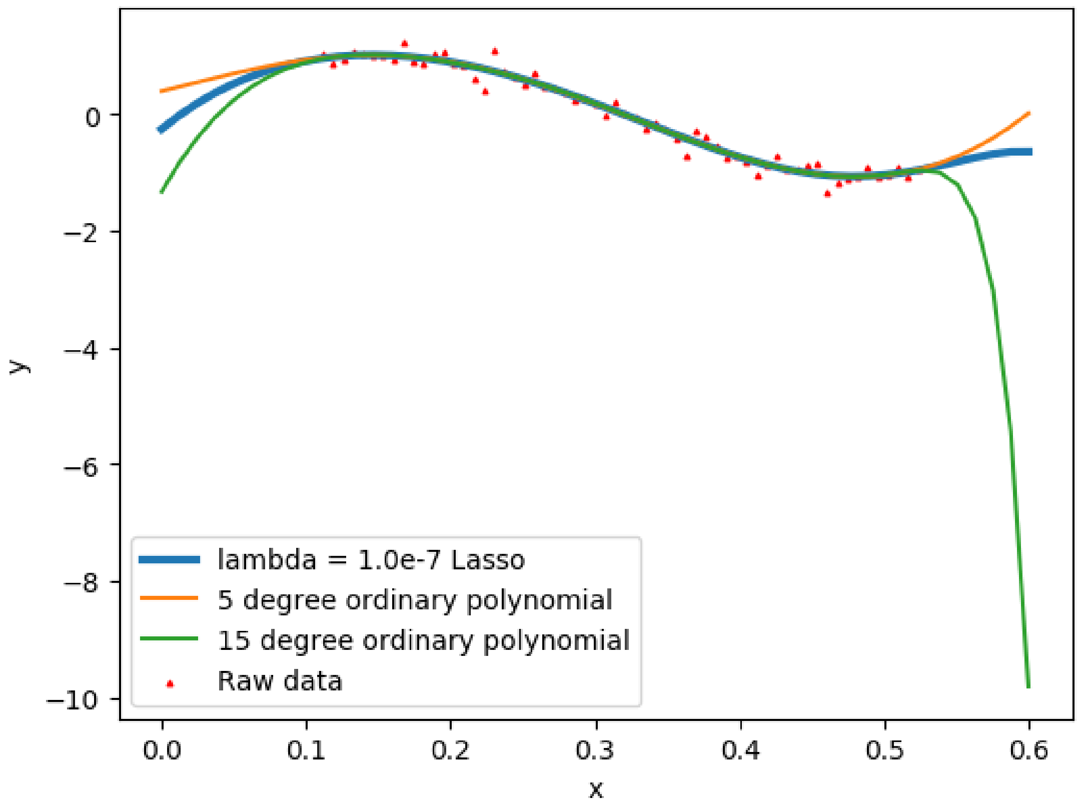
lambda = 0.0001 Lasso
5 degree ordinary polynomial
15 degree ordinary polynomial
Raw data
y
x
0
−2
−4
−6
−8
−10
0.0
0.1
0.2
0.3
0.4
0.5
0.6



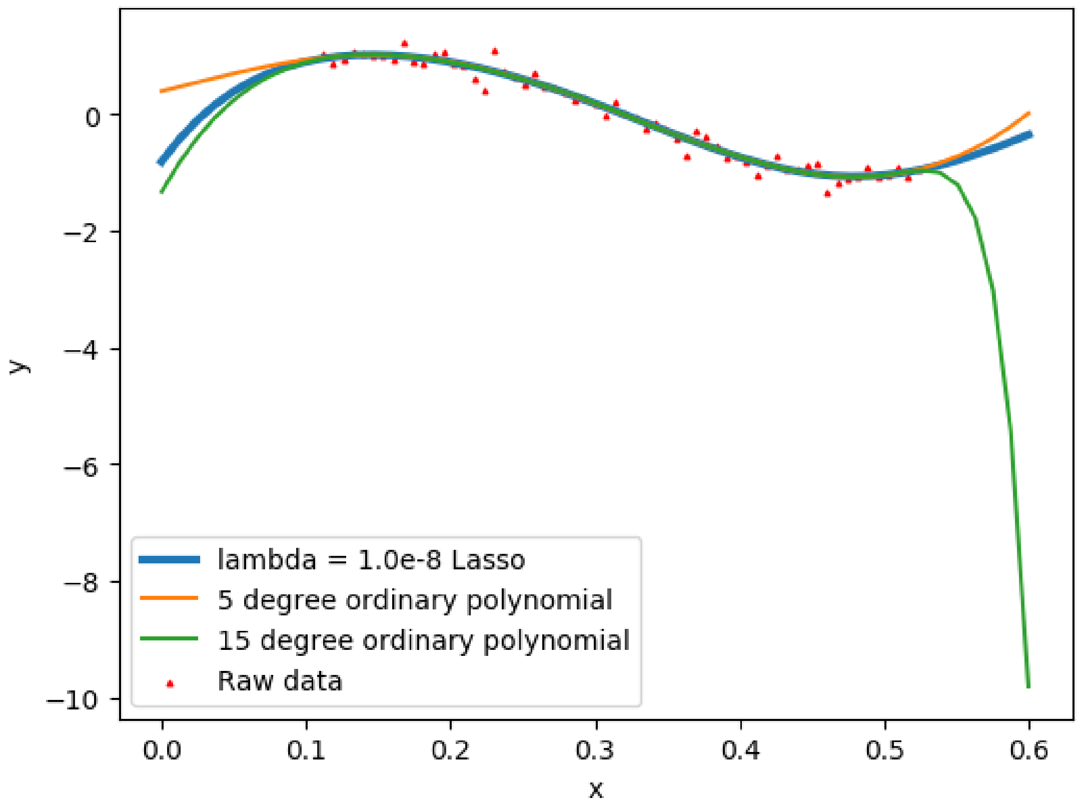
0
−2
−4
−6
−8
−10
y
0.0
0.1
0.2
0.3
0.4
0.5
0.6
x
lambda = 1.0e-5 Lasso
5 degree ordinary polynomial
15 degree ordinary polynomial
Raw data



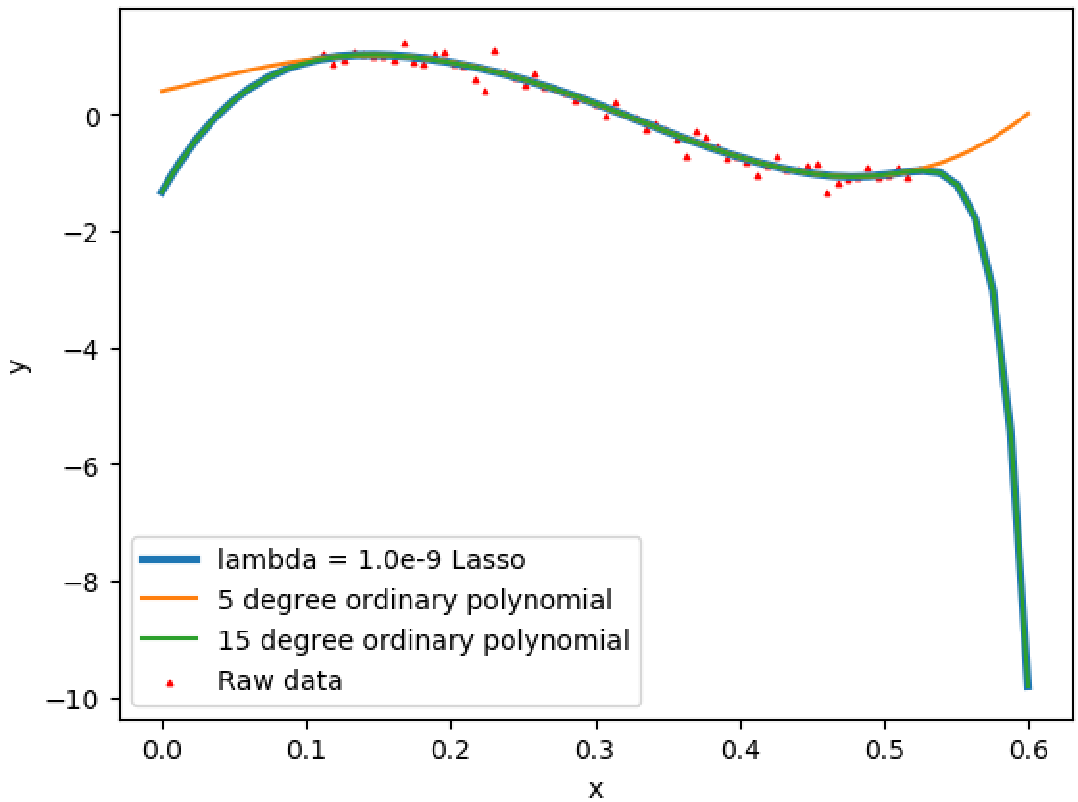
lambda = 1.0e-6 Lasso
5 degree ordinary polynomial
15 degree ordinary polynomial
Raw data
0
−2
−4
−6
−8
−10
y
0.0
0.1
0.2
0.3
0.4
0.5
0.6
x



0
−2
−4
−6
−8
−10
y
0.0
0.1
0.2
0.3
0.4
0.5
0.6
x
lambda = 1.0e-7 Lasso
5 degree ordinary polynomial
15 degree ordinary polynomial
Raw data

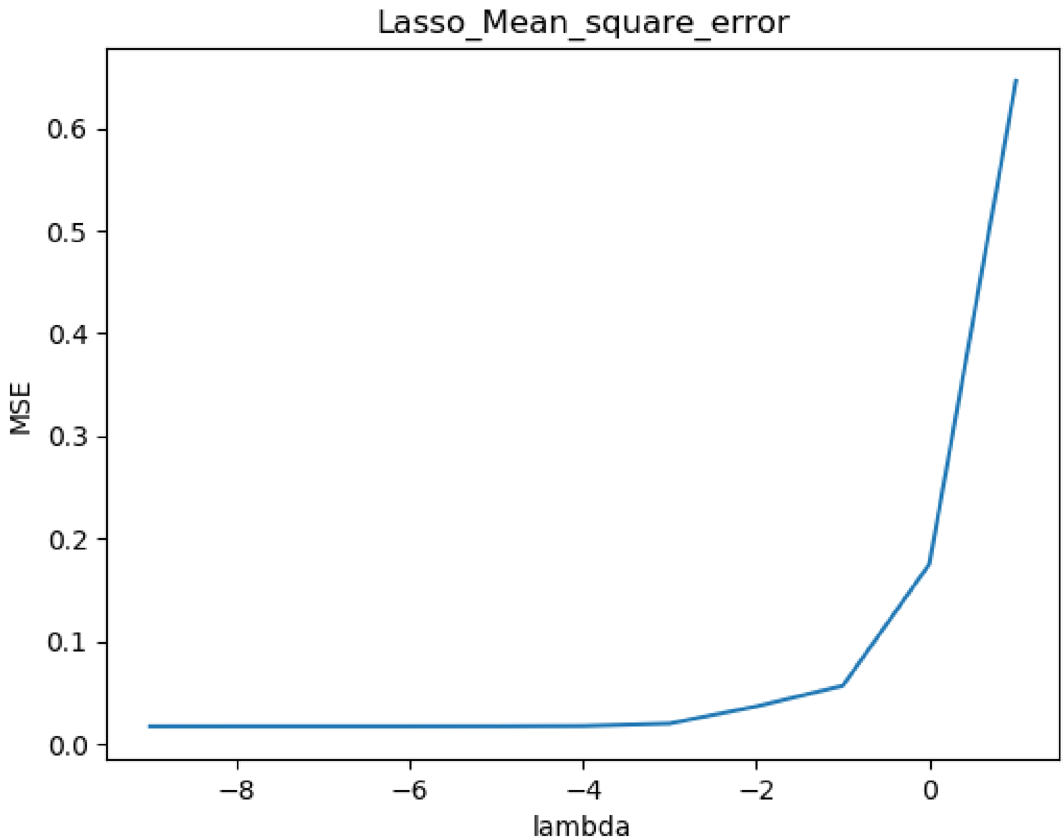


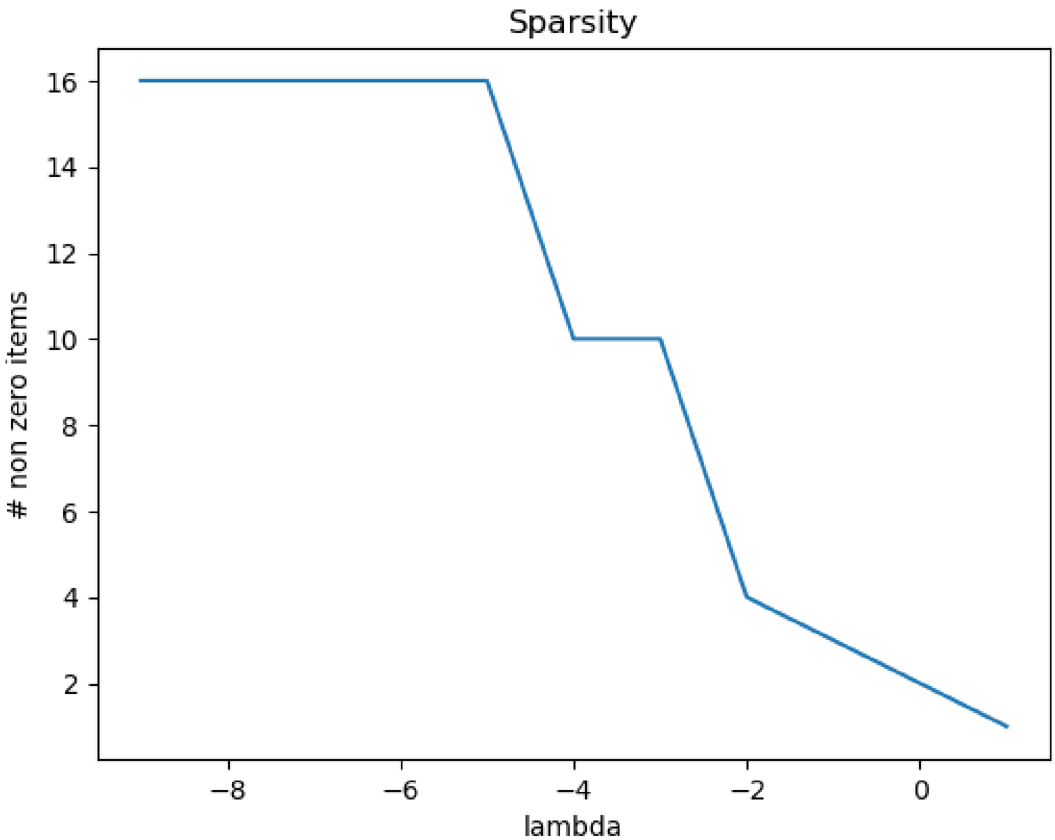
lambda = 1.0e-8 Lasso
5 degree ordinary polynomial
15 degree ordinary polynomial
Raw data
y
x
0
−2
−4
−6
−8
−10
0.0
0.1
0.2
0.3
0.4
0.5
0.6



From these plots we can see when λ is greater than 0.01, the fitting is not good. Most polynomial parameters are zeros, thus the lasso fitting curve looks very straight. Meanwhile, when λ is smaller than 10^{-7}, the fitting curve looks very similar to the ordinary 15 degree polynomial fitting, so it doesn't simplify the model.
A λ equal to 10^{-3} may be a good value. In this scenario, the lasso curve looks like 5 degree ordinary polynomial fitting, representing that this model effectively simplified our model to a lower degree one.

[29]:
```
figure();
plot(log10.(lambda_lasso), MSE);
xlabel("lambda");
ylabel("MSE");
title("Lasso_Mean_square_error")
figure();
plot(log10.(lambda_lasso), N_non_zeros);
xlabel("lambda");
ylabel("# non zero items");
title("Sparsity");
```


Lasso_Mean_square_error
MSE
0.0
0.1
0.2
0.3
0.4
0.5
0.6
−8
−6
−4
−2
0
lambda


Sparsity
non zero items
lambda
16
14
12
10
8
6
4
2
−8
−6
−4
−2
0

[ ]: