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***Abstract*—MPC online control problem enhanced in CasADi, a framework written by Andersson et al. [1].**

I. Introduction

II. Proposed approach

A. Vehicle model

B. MPC Model

In order to build an MPC internal model with its own dynamics, and which is as simple as possible, a point mass model is chosen. This model has 5 DOF and encloses in itself the fundamental parameters of the driven vehicle as mass, aerodynamic coefficients, grip and power limits for braking and traction phases. With this MPC internal model, the optimal control problem is formulated in order to find the optimal control sequence necessary to minimize the travel time of the next N meters of the track. In particular the control inputs of the point mass model are the total longitudinal force F_x, and the angular acceleration along the z-axes $r_p = dr/dt$, where r is the yaw rate. The point mass model dynamics is then formulated in spatial domain instead of the time one, and the *direct collocation* is used to transform the OCP into an NLP. The NLP is coded in a scripting environment using the Matlab interface to the open-source CasADi framework [1], which provides building blocks to efficiently formulate and solve large-scale optimization problems, and solved through IPOPT **cita ipopt**.

Once the problem is solved an optimal control sequence for the steering angle has to be extracted from the NLP solution. In order to do that we use the following formula

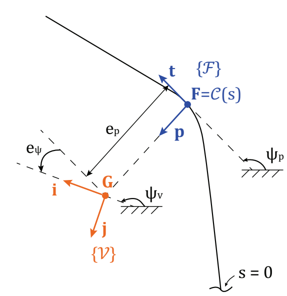
$$\delta = \alpha_1 + \beta_1 = Fy_{11}/C_f + (v + ra_1)/u \tag{1}$$

where C_f is the cornering stiffness of the front tire, a_1 is the distance between the CoM and the front axle, and we assume that the tire slip angles and the vehicle slip angles are equal for the wheels of the same axle, hence $\alpha_{11} = \alpha_{12} = \alpha_1$ and $\beta_{11} = \beta_{12} = \beta_1$. Furthermore, the lateral force of the front wheels are extimated with a steady state assumption in a way that $F_{y_{11}} = F_{y_{12}} = F_y a_2/(2l)$, where F_y is the total lateral force acting on the point mass model, available from the NLP solution, a_2 is the distance between the CoM and the rear axle and l is the wheelbase.

For what concern the race track, it is assumed planar and modelled through the parametric 2D curve

$$\mathcal{C}(\alpha) = \{\boldsymbol{x}(\alpha) = [x(\alpha), y(\alpha)]^T \in \mathbb{R}^2 : \alpha \in [\alpha_0, \alpha_f]\} \tag{2}$$

that identifies the road centerline, and the 1D curve $\mathcal{W}(\alpha)$ that specifies the track width. With reference to Figure 1a, the *curve parameter* α uniquely selects a point $\boldsymbol{F} = \boldsymbol{x}(\alpha)$ that defines the origin of the *Frenet-Serret frame* $\mathcal{F} = \{\boldsymbol{F}, (\boldsymbol{t}, \boldsymbol{p})\}$ whose unit vectors are, respectively, the tangent $\boldsymbol{t}$ and the normal $\boldsymbol{p}$ of the curve $\mathcal{C}$ in the point $\boldsymbol{F}$. The vehicle reference system $\mathcal{V} = \{\boldsymbol{G}, (\boldsymbol{i}, \boldsymbol{j})\}$ can be expressed in terms of the moving frame $\mathcal{F}$ with a *lateral displacement* e_p along the track normal direction $\boldsymbol{p}$ and the *heading error* e_ψ. In order to maintain $\mathcal{F}$ side-by-side with $\mathcal{V}$, the Frenet-Serret system has to proceed together with the vehicle: this leads to a relation between vehicle and Frenet-Serret velocities that ultimately imposes a bound between time and α increments.



(a)

Fig. 1. (**a**) vehicle pose respect to the Frenet-Serret reference system identified on the track curve.

C. Offset-free MPC

D. Spatial formulation

III. Preliminary results

IV. Conclusion

References

[1] Joel A.E. Andersson, Joris Gillis, Greg Horn, James B. Rawlings, and Moritz Diehl. CasADi: a software framework for nonlinear optimization and optimal control. *Math. Program. Comput.*, 11(1):1–36, mar 2019. ISSN 18672957. doi: 10.1007/s12532-018-0139-4. URL https://link.springer.com/article/10.1007/s12532-018-0139-4.