How to: Monetary Policy

1 Introduction

In general the setup is described somewhat like the following:

$$\pi_t = m_t + \nu_t \tag{1.1}$$
$$x_t = \theta_t + \pi_t - \pi_t^e - \epsilon_t \tag{1.2}$$
$$L = \sum_{t=1}^{\infty} \beta^t L_t(\pi_t, x_t) \tag{1.3}$$
$$L_t = \frac{1}{2}\left((\pi_t - \bar{\pi})^2 + \lambda\,(x_t - \bar{x})^2\right) \tag{1.4}$$

Equation (1.1) describes the demand side of the economy. For simplicity, we assume that inflation is determined by the monetary policy and some stochastic demand shock ν_t. Equation (1.2) describes a Phillips-curve relationship, describing that output/employment, x_t, is determined by two types of stochastic shocks as well as surprise inflation, $\pi_t - \pi_t^e$. Lastly, equation (1.3)-(1.4) describes how we can measure social loss as fluctuations from a target level of inflation and employment.[1]

We can extend our assumptions/setup in a number of ways, some of which being:

- Assume shocks are persistent, for instance that ϵ_t follows an AR(1) process:

$$\epsilon_t = \rho\epsilon_{t-1} + u_t, \qquad 0 < \rho < 1 \tag{1.5}$$

- Assume that inflation evolves according to the New Keynesian Phillips Curve (NKPC) instead of (1.2):

$$x_t = \theta_t + \kappa_1\pi_t - \kappa_2 E[\pi_{t+1}] - \epsilon_t \tag{1.6}$$

- Assume that private agents has a certain trigger-strategy, such as tit-for-tat:

$$\pi_t^e = \begin{Bmatrix} E[\pi_t^C], & \text{for } \pi_{t-1} = \pi_{t-1}^C \\ E[\pi_t^D], & \text{for } \pi_{t-1} \neq \pi_{t-1}^C \end{Bmatrix} \tag{1.7}$$

- Include foreign currency zone and allow for a pegged currency, for instance the euro zone. Assuming they have commitment policy this would imply:

$$\pi_t^{EU} = \frac{\lambda^{EU}}{1+\lambda^{EU}}\epsilon_t^{EU}, \qquad Corr(\epsilon_t, \epsilon_t^{EU}) = \alpha \tag{1.8}$$

- Assume different political parties have different preferences, λ^j.

[1] For a microeconomic foundation for the loss function, you can check out appendix 8.6.2 in Walsh (2010), `http://www2.um.edu.uy/dtrupkin/walsh.pdf`.

- Assume imperfect control over inflation, such that monetary policy affects output through a DIS curve:

$$x_t = E_t[x_{t+1}] - \sigma^{-1}(i_t - E_t[\pi_{t+1}]) + \eta_t \tag{1.9}$$

In the case of (1.5)-(1.6), see notes from assignment 4. In the case of the dynamic setup of (1.7) and trigger-strategies, see PS15. For different political parties and cooperation see PS16, and for the pegged currency case see PS17.[2] In the following sections, we will go through some standard results/cookbook approaches in the simple static cases and leave the more advanced dynamic problems for problem sets.

[2]For the case of imperfect control over inflation, forward guidance and other extensions, one source is Henrik Jensen's Monetary Policy course site, `http://www.econ.ku.dk/personal/Henrikj/monpol2016/slides.asp`.

2 A collection of cookbook approaches for finding equilibria

Beside the system of equations (1.1)-(1.4) we need to specify the timing in the model. In the following 'standard' sections, we will generally assume the timing:

1. The monetary authority announces their policy rule (only commitment case)
2. The demand shock, ν_t, and the θ_t shock are realized.
3. Private agents form expectations of inflation, knowing both ν_t and θ_t.
4. ϵ_t is realized
5. Monetary authority chooses their actual monetary policy, m_t.

In the last section we will briefly show how a different assumption of timing can affect our solution approach.

2.1 Commitment policy in equilibrium

The cookbook approach for commitment policy is the following:

- **Postulate a monetary policy rule:** It can be shown that optimal monetary policy with a quadratic loss function follows a linear rule. Thus in our case, we announce a rule of the type:

$$m_t = a_1 + a_2\nu_t + a_3\theta_t + a_4\epsilon_t \tag{2.1}$$

  where the parameters a_1 through a_4 are yet to be determined.

- **Impose credibility of the rule:** If we are in an equlibrium, private agents trust that we will in fact carry out monetary policy according to (2.1). With our assumed timing, this implies that expected inflation follows:

$$\pi_t^e = E[m_t + \nu_t|\theta_t, \nu_t] = a_1 + \nu_t(a_2 + 1) + a_3\theta_t \tag{2.2}$$

- **Minimize expected social loss:** Use the result in (2.1), (2.2) and the Phillips-curve to describe the expected loss

$$\begin{aligned} E[L_t] &= \frac{1}{2}E\left((\overbrace{m_t + \nu_t}^{=\pi_t} - \bar{\pi})^2 + \lambda(\overbrace{\theta_t + \pi_t - \pi_t^e - \epsilon_t}^{=x_t} - \bar{x})^2 \right) \\ &= \frac{1}{2}E\left((\overbrace{a_1 + \nu_t(a_2+1) + a_3\theta_t + a_4\epsilon_t}^{\text{Using (2.1)}} - \bar{\pi})^2 + \lambda(\overbrace{\theta_t + \epsilon_t(a_4 - 1)}^{\text{Using (2.1) and (2.2)}} - \bar{x})^2 \right) \end{aligned} \tag{2.3}$$

  Next we need to square the terms in (2.3) and take expectations, before minimizing the expression. We will use that (almost) all cross-products in expectation are zero.[3] We finally obtain the expected loss

$$E[L_t] = \frac{1}{2}\left(a_1^2 + (a_2+1)^2\sigma_\nu^2 + a_3^2\sigma_\theta^2 + a_4^2\sigma_\epsilon^2 + \bar{\pi}^2 - 2a_1\bar{\pi} + \lambda(\sigma_\theta^2 + (a_4-1)^2\sigma_\epsilon^2 + \bar{x}^2\right) \tag{2.4}$$

[3] See appendix for an elaboration on this part.

The expression in (2.4) is straightforwardly differentiable. Thus we will have the 4 first order conditions

$$\min_{a_1,a_2,a_3,a_4} E[L_t] \tag{2.5}$$
$$\frac{\partial E[L_t]}{\partial a_1} = a_1 - \bar{\pi} \qquad = 0 \tag{2.6}$$
$$\frac{\partial E[L_t]}{\partial a_2} = (a_2 + 1)\sigma_\nu^2 \qquad = 0 \tag{2.7}$$
$$\frac{\partial E[L_t]}{\partial a_3} = a_3\sigma_\theta^2 \qquad = 0 \tag{2.8}$$
$$\frac{\partial E[L_t]}{\partial a_4} = a_4\sigma_\epsilon^2 + \lambda(a_4 - 1)\sigma_\epsilon^2 \qquad = 0 \tag{2.9}$$

This solves for the optimal monetary policy, inflation and output. Firstly, monetary policy is given by plugging in optimal policy parameters from (2.6)-(2.9) into the policy rule in (2.1):

$$m_t^C = \bar{\pi} - \nu_t + \frac{\lambda}{1+\lambda}\epsilon_t \tag{2.10}$$

Next, plug in monetary policy in (2.10) into equation (1.1) to obtain equilibrium inflation:

$$\pi_t^C = \bar{\pi} + \frac{\lambda}{1+\lambda}\epsilon_t \tag{2.11}$$

Lastly, x_t^C is derived by using $\pi_t^C - \pi_t^e$ in the phillips-curve equation (1.2):

$$\begin{aligned} x_t^C &= \theta_t + \overbrace{\frac{\lambda}{1+\lambda}\epsilon_t}^{\pi_t - \pi_t^e} - \epsilon_t \\ &= \theta - \frac{1}{1+\lambda}\epsilon_t \end{aligned} \tag{2.12}$$

The key take-aways of commitment policy (with this particular timing at least) is:

- Inflation is stabilized around target level $\bar{\pi}$ (often assumed zero) perfectly offsetting demand shocks. Letting our policy parameter $a_2 = -1$ the demand shock ν_t will not affect inflation or output.
- We cannot stabilize output around the target level $\bar{x}$, as this would be anticipated and only create an inflation bias.
- We cannot stabilize θ_t shocks, as they are anticipated by private agents. This is due to our assumption of timing however.
- We can stabilize ϵ_t shocks partially (not fully as with demand shocks, ν_t). The optimal trade-off between inflation and output is given by the coefficients, $\lambda/(1+\lambda)$ and $-1/(1+\lambda)$.

2.2 Discretionary Policy

When solving for discretionary policy, we'll just start by simplifying the setup, and note that by picking m_t after the demand shock, ν_t, we have perfect control over inflation. Thus we will use π_t as control variable.

- **Minimize actual social loss**.[4] Plug in all equations in our actual loss function and minimize wrt. π_t directly:

$$\min_{\pi_t} L_t = \frac{1}{2}\left((\pi_t - \bar{\pi})^2 + \lambda(\theta_t + \pi_t - \pi_t^e - \epsilon_t - \bar{x})^2\right) \tag{2.13}$$

[4] Note that the crucial assumption for this first step, is that the monetary authority chooses their monetary policy after all other shocks.

From this we have the first order condition that

$$\frac{\partial L_t}{\partial \pi_t} = \pi_t - \bar{\pi} + \lambda(\theta_t + \pi_t - \pi_t^e - \epsilon_t - \bar{x}) = 0 \quad \Rightarrow \quad \pi_t = \frac{\lambda}{1+\lambda}\left(\bar{\pi}\lambda^{-1} + \pi_t^e + \bar{x} + \epsilon_t - \theta_t\right) \quad (2.14)$$

- **Compute rational expectations of inflation:** Use that private agents know that inflation will follow the rule (2.14), such that

$$\pi_t^e = E\left[\frac{\lambda}{1+\lambda}(\bar{\pi}\lambda^{-1} + \pi_t^e + \bar{x} + \epsilon_t - \theta_t)|\theta_t\right] = \frac{\bar{\pi} + \lambda(\pi_t^e + \bar{x} - \theta_t)}{1+\lambda} \quad (2.15)$$

  using that $E[\epsilon_t|\theta_t] = 0$. Solving for π_t^e in (2.15) we have the rational expectation of inflation given by

$$\pi_t^e = \bar{\pi} + \lambda(\bar{x} - \theta_t) \quad (2.16)$$

- **Combine results in equilibrium:** Substitute (2.16) into (2.14) to obtain equilibrium inflation

$$\pi_t^D = \bar{\pi} + \lambda(\bar{x} - \theta_t) + \frac{\lambda}{1+\lambda}\epsilon_t \quad (2.17)$$

  To arrive at the equilibrium for x_t^D, use (2.16)-(2.17) in the Phillips-curve to obtain

$$\begin{aligned} x_t^D &= \theta_t + \pi_t^D - \pi_t^e - \epsilon_t \\ &= \theta_t - \frac{1}{1+\lambda}\epsilon_t \end{aligned} \quad (2.18)$$

The main result compared to commitment policy is the inflation bias: Discretionary policy entails an inflation bias compared to commitment policy: The monetary authority has an incentive to stabilize output around the level $\bar{x} - \theta_t$. As private agents anticipate this, only inflation will increase, without having an effect on output.

2.3 Pegging the currency

The cookbook approach for finding the equilibrium outcome with a pegged currency is:

- **Assume pegged currency:** In our setup, this translates to adopting a monetary policy. Per construction the inflation level is thus given by foreign inflation rule:

$$\pi_t^P = \pi_t^{EU}, \quad (2.19)$$

  which may be further specified as following some sort of policy rule as in equation (1.8).

- **Impose credibility:** Assuming that we are in an equilibrium, private agents expect the rule in (2.19), thus we have

$$\pi_t^e = E\left[\pi_t^{EU}|\theta_t\right] \quad (2.20)$$

- **Combine results in equilibrium:** Use (2.19) and (2.20) in the Phillips-curve to obtain the output level

$$x_t^P = \theta_t + \pi_t - \pi_t^e - \epsilon_t = \theta_t - \epsilon_t + \pi_t^{EU} - E\left[\pi_t^{EU}|\theta_t\right] \quad (2.21)$$

Main results of pegged currency equilibrium:

- The main upside of the pegged currency is that compared to discretionary policy, we eliminate the inflation bias.

- The downside of the approach depends on how the adopted inflation policy correlates with domestic shocks. If domestic and foreign shocks are uncorrelated, the pegged currency will not be able to stabilize domestic shocks ϵ_t.

2.4 Optimal central banker solution

When choosing an optimal central banker, we assume that we have discretionary policy, but that we can hire a central banker with a different λ than our own true preference for output-stabilization. This means that we can choose the policy outcome

$$\begin{pmatrix} \pi_t^B \\ x_t^B \end{pmatrix} = \begin{pmatrix} \bar{\pi} + \lambda^B(\bar{x} - \theta_t) + \dfrac{\lambda^B}{1+\lambda^B}\epsilon_t \\ \theta_t - \dfrac{1}{1+\lambda^B}\epsilon_t \end{pmatrix} \tag{2.22}$$

where λ^B represents the relative preference for output-stabilization of our hired central banker. We solve the problem as follows:

- When choosing the optimal CB, we assume that we have to hire him before knowing the shocks. Thus we need to evaluate the expected loss of hiring λ^B:

$$\begin{aligned} E[L_t] &= \frac{1}{2}\left(E\left[(\pi_t^B - \bar{\pi})^2\right] + \lambda E\left[(x_t^B - \bar{x})^2\right]\right) \\ &= \frac{1}{2}\left(E\left[\left(\lambda^B(\bar{x} - \theta_t) + \frac{\lambda^B}{1+\lambda^B}\epsilon_t\right)^2\right] + \lambda E\left[\left(\theta_t - \frac{1}{1+\lambda^B}\epsilon_t - \bar{x}\right)^2\right]\right) \end{aligned} \tag{2.23}$$

  Before differentiating we start squaring the terms and take expectations. As in the case of commitment policy, we will use that all the cross-terms are zero in expectations.

$$E[L_t] = \frac{1}{2}\left((\lambda^B)^2(\bar{x}^2 + \sigma_\theta^2) + \left(\frac{\lambda^B}{1+\lambda^B}\right)^2\sigma_\epsilon^2 + \lambda\left(\sigma_\theta^2 + \bar{x}^2 + \frac{1}{(1+\lambda^B)^2}\sigma_\epsilon^2\right)\right) \tag{2.24}$$

- Ideally we would minmize the expected loss in (2.24) by differentiating with respect to λ^B setting it equal to zero and solving for λ^B. That is, solve the equation:

$$\frac{\partial E[L_t]}{\partial \lambda^B} = \lambda^B(\bar{x}^2 + \sigma_\theta^2) + \frac{\lambda^B}{(1+\lambda^B)^3}\sigma_\epsilon^2 - \frac{\lambda}{(1+\lambda^B)^3}\sigma_\epsilon^2 = 0 \tag{2.25}$$

  However, as we cannot solve this, we will instead charactize the optimal solution by evaluating derivatives in certain values of λ^B.

- **Is it optimal to choose** $\lambda^B = 0$?

$$\frac{\partial E[L_t]}{\partial \lambda^B}(\lambda^B = 0) = -\lambda\sigma_\epsilon^2 < 0, \qquad \lambda > 0 \tag{2.26}$$

  The derivative in (2.26) states that if we consider choosing a central banker with $\lambda^B = 0$, marginally increasing λ^B would lower the expected loss. Thus the optimal central banker is characterized by $(\lambda^B)^* > 0$.

- **Is it optimal to choose** $\lambda^B = \lambda$?

$$\frac{\partial E[L_t]}{\partial \lambda^B}(\lambda^B = \lambda) = \lambda(\bar{x}^2 + \sigma_\theta^2) > 0, \qquad \lambda > 0 \tag{2.27}$$

  The derivative in (2.27) states that if we consider hiring a central banker with the same preferences for output-stabilization as our own, $\lambda^B = \lambda$, we could lower the expected loss by decreasing λ^B. Thus the optimal central banker must have preferences such that $(\lambda^B)^* < \lambda$. Thus we know that $0 < (\lambda^B)^* < \lambda$.

The intuition behind choosing a central banker where $\lambda^B < \lambda$ comes from balancing the two effects:

- Choosing $\lambda^B < \lambda$ means that the loss from the inflation bias is lowered: $\lambda(\bar{x}^2 + \sigma_\theta^2)$.

- Choosing $\lambda^B < \lambda$ means that the stabilization of the ϵ_t shocks is not optimal anymore, as a suboptimal large part of the shock affects output.
- The optimal monetary policy weighs the two effects in optimum, thus $0 < (\lambda^B)^* < \lambda$. Lastly, note that choosing $\lambda^B = \lambda$ is equivalent to choosing our own discretionary policy, while $\lambda^B = 0$ is equivalent to choosing a pegged currency where foreign inflation is always zero. Thus the central banker solution is preferred to discretionary policy and pegged currency.

3 Solutions without the equilibrium assumption

When we deal with the dynamic setups we will often consider scenarios, where private agents intially expect the monetary authority to perform discretionary policy and we have to create credibility of an alternative policy such as commitment or pegged currency.

3.1 Obtaining credibility of pegged currency

Assume that private agents expect discretionary policy. In order to obtain credibility of our pegged currency policy, we need to perform the pegged currency policy even though it is not expected. Thus we find the outcome for creating credibility by:

- **Assume pegged currency:** The same first step as in the equilibrium approach. Thus we adopt the foreign inflation level:

$$\pi_t = \pi_t^{EU} \tag{3.1}$$

- **Impose no credibility:** Assuming that we are **not** in equilibrium implies that private agents expect the outcome of second step in discretionary policy, equation (2.16):

$$\pi_t^e = \bar{\pi} + \lambda(\bar{x} - \theta_t) \tag{3.2}$$

- **Combine results in Phillips-curve:**

$$\begin{aligned} x_t &= \theta_t + \pi_t^{EU} - (\bar{\pi} + \lambda(\bar{x} - \theta_t)) - \epsilon_t \\ &= (1+\lambda)\theta_t + \pi_t^{EU} - \bar{\pi} - \epsilon_t - \lambda\bar{x} \end{aligned} \tag{3.3}$$

  In order to create credibility we have to live with a relatively large shock to output/employment, as we play π_t^{EU}, but agents expect the inflation bias.

3.2 Obtaining credibility of commitment rule

The idea is basically the same. Monetary authority uses commitment rule, but private agents expect discretionary policy:

- **Use commitment policy rule from equilibrium approach:**

$$\pi_t = \bar{\pi} + \frac{\lambda}{1+\lambda}\epsilon_t \tag{3.4}$$

- **Private agents expect discretionary policy:**

$$\pi_t^e = \bar{\pi} + \lambda(\bar{x} - \theta_t) \tag{3.5}$$

- **Combine results in Phillips-curve:**

$$\begin{aligned} x_t &= \theta_t + \bar{\pi} + \frac{\lambda}{1+\lambda}\epsilon_t - (\bar{\pi} + \lambda(\bar{x} - \theta_t)) - \epsilon_t \\ &= (1+\lambda)\theta_t - \frac{1}{1+\lambda}\epsilon_t - \lambda\bar{x} \end{aligned} \tag{3.6}$$

  In order to obtain credibility of the policy rule, we have to live with a relatively large shock to employment. The intuition is that we will not have any inflation bias, but agents will expect it.

4 Comparing policy solutions

In general the upsides and downsides of the different policy approaches are summarized by:

Monetary policy	**Upsides**	**Downsides**	**Other comments**
Commitment	1) No inflation bias. 2) Optimal stabilization of ϵ_t shocks	1) No stabilization of the $\bar{x} - \theta_t$ part.	In a dynamic setup with credibility, we can deviate in one period and improve by stabilizing the $\bar{x} - \theta_t$ part.
Discretion	1) Optimal stabilization of ϵ_t shocks.	1) No stabilization of the $\bar{x} - \theta_t$ part. 2) Inflation bias: $\lambda(\bar{x} - \theta_t)$.	Nash-equilibrium solution in static game. Never preferred over CB.
Central banker	1) Eliminates part of inflation bias 2) Part of ϵ_t shocks stabilized	1) No stabilization of the $\bar{x} - \theta_t$ part. 2) Suboptimal stabilization of ϵ_t shocks. 3) Part of inflation bias not eliminated.	CB chooses optimal trade-off between inflation bias and ϵ_t shocks. Discretionary policy and pegged currency solution is nested in CB solution.
Pegged currency (with $\pi^{EU} = 0$)	1) No inflation bias.	1) No stabilization of the $\bar{x} - \theta_t$ part. 2) No stabilization of ϵ_t shocks.	Never preferred over CB.
Pegged currency (corr($\epsilon_t, \epsilon_t^{EU}$)$= \alpha > 0$)	1) No inflation bias. 2) When $\alpha > 0$ partial stabilization of ϵ_t shocks.	1) No stabilization of $\bar{x} - \theta_t$ shocks. 2) Suboptimal stabilization of ϵ_t shocks.	If $\alpha = 1$ solution can be equivalent to commitment policy $\rightarrow$ optimal.

5 Robustness of solution approaches, timing and specification

The solution approaches outlined in section 2 are relatively robust towards changes in specification and timing. Some changes in model specification, such as including a DIS curve (equation (1.9)), changes the fundamental approach.[5]

However, other changes such as those explained in equation (1.5)-(1.8) in the introduction does not change the fundamental steps of our solution approach, but alters how we perform each step. For instance, changing our model setup as in assignment 4 such that

$$\begin{aligned}
\epsilon_t &= \rho \epsilon_{t-1} + u_t \\
x_t &= \theta_t + \pi_t - E_t[\pi_{t+1}] - \epsilon_t,
\end{aligned}$$

we would still perform discretionary policy by the steps 1) Minimize actual social loss, 2) compute rational expectations and 3) combine results in equilibrium. However, the way we have changed our model implies that the second step, computing rational expectations, is a lot more involved than outlined in section 2.2. The same general comment can be tied to the assumption of timing: The general steps for solving the problem will often be the same, but often change how we carry out the specific steps.

[5] For the interested, this involves solving a system of expectational difference equations. This can be done by using a Bellman equation/value-function approach. Once again, you could check out the course site for Monetary Policy for inspiration. (or ask me)

5.1 An example of an alternative assumption of timing

As an example on how our solution changes with the timing of the model, assume that we have the following setup:

$$\pi_t = m_t \tag{5.1}$$
$$x_t = \theta_t + \pi_t - \pi_t^e - \epsilon_t \tag{5.2}$$
$$L = \sum_{t=1}^{\infty} \beta^t L_t(\pi_t, x_t) \tag{5.3}$$
$$L_t = \frac{1}{2}\left(\pi_t^2 + \lambda(x_t - \bar{x})^2\right), \tag{5.4}$$

where we assume that both θ_t and ϵ_t are IID errors with zero mean and constant standard deviations $\sigma_\theta, \sigma_\epsilon$. Furthermore, we assume that the timing of the model follows from:

1. Monetary policy rule is announced.
2. Private agents form expectations of inflation.
3. The supply shocks are realized, ϵ_t, θ_t.
4. Monetary policy is carried out.

Commitment policy with alternative timing:

We can carry out our 3 steps of commitment policy:

- **Postulate a policy rule** (recall that a linear rule is sufficient):

$$\pi_t = a_1 + a_2\theta_t + a_3\epsilon_t \tag{5.5}$$

- **Impose credibility of the rule:** Assuming that we are in an equilibrium, private agents expect monetary policy to follow the rule (5.5). Using the timing stated above, they form expectations not knowing ϵ_t and θ_t is, as they are realized after expectations are formed:

$$\pi_t^e = E\left[a_1 + a_2\theta_t + a_3\epsilon_t\right] = a_1, \tag{5.6}$$

  implying that surprise inflation will follow $\pi_t - \pi_t^e = a_2\theta_t + a_3\epsilon_t$.

- **Minimize expected social loss:** Use the results in (5.5) and (5.6) and the Phillips-curve in our expected loss function:

$$\begin{aligned} E[L_t] &= \frac{1}{2}E\left((\overbrace{a_1 + a_2\theta_t + a_3\epsilon_t}^{=\pi_t})^2 + \lambda(\overbrace{\theta_t + \pi_t - \pi_t^e - \epsilon_t}^{=x_t} - \bar{x})^2\right) \\ &= \frac{1}{2}E\left((a_1 + a_2\theta_t + a_3\epsilon_t)^2 + \lambda(\theta_t(1+a_2) + \epsilon_t(a_3 - 1) - \bar{x})^2\right) \end{aligned} \tag{5.7}$$

  In order for us to minimize this expression, we need to start by evaluate expectations first. Once again using that all cross-terms in expectations are zero, we only have squared terms left after squaring and taking expectations:

$$E[L_t] = \frac{1}{2}\left(a_1^2 + a_2^2\sigma_\theta^2 + a_3^2\sigma_\epsilon^2 + \lambda(\sigma_\theta^2(1+a_2)^2 + \sigma_\epsilon^2(a_3-1)^2 + \bar{x}^2)\right) \tag{5.8}$$

Mimizing this expression we differentiate with respect to our 3 policy parameters, a_1, a_2, a_3 and solves:

$$\begin{aligned}
\frac{\partial E[L_t]}{\partial a_1} =& a_1 = 0 && (5.9)\\
\frac{\partial E[L_t]}{\partial a_2} =& a_2\sigma_\theta^2 + \lambda(1+a_2)\sigma_\theta^2 = 0 && (5.10)\\
\frac{\partial E[L_t]}{\partial a_3} =& a_3\sigma_\epsilon^2 + \lambda(a_3-1)\sigma_\theta^2 = 0 && (5.11)
\end{aligned}$$

Solving this we get the inflation policy with commitment

$$\pi_t^C = \underbrace{0}_{=a_1} - \underbrace{\frac{\lambda}{1+\lambda}}_{=-a_2}\theta_t + \underbrace{\frac{\lambda}{1+\lambda}}_{=a_3}\epsilon_t. \tag{5.12}$$

Lastly, using the Phillips-curve we get the equilibrium employment:

$$x_t^C = (\theta_t - \epsilon_t)\frac{1}{1+\lambda}. \tag{5.13}$$

In general, the solution approach is not changed with the new timing. However, performing the steps of **imposing credibility of rule**, and **minimizing expected loss** involves some new information as we no longer condition inflation expectations on θ_t.

Discretionary policy with alternative timing:

We follow the same general solution approach as described in section 2.2:

- **Minimize actual social loss:** As monetary policy is carried out after observing all shocks and we do not need to announce a policy rule, as we do not have commitment, the objective of the monetary authority is to solve the problem:

$$\min_{\pi_t} L_t = \frac{1}{2}\left(\pi_t^2 + \lambda(\theta_t + \pi_t - \pi_t^e - \epsilon_t - \bar{x})^2\right). \tag{5.14}$$

  This yields the first order condition and solution that

$$\frac{\partial L_t}{\partial \pi_t} = \pi_t + \lambda(\theta_t + \pi_t - \pi_t^e - \epsilon_t - \bar{x}) = 0 \qquad \Rightarrow \qquad \pi_t = \frac{\lambda}{1+\lambda}(\bar{x} - \theta_t + \epsilon_t + \pi_t^e) \tag{5.15}$$

- **Compute rational expectations of inflation:** Assuming that we are in a discretionary equilibrium, private agents form expectations based on equation (5.15). With our assumed timing, private agents do not know θ_t or ϵ_t shocks, implying that they will expect inflation according to

$$\pi_t^e = E\left[\frac{\lambda}{1+\lambda}(\bar{x} - \theta_t + \epsilon_t + \pi_t^e)\right] = \frac{\lambda}{1+\lambda}(\bar{x} + \pi_t^e),$$

  as they expect both θ_t and ϵ_t to be zero. Solving for π_t^e this implies

$$\pi_t^e = \lambda\bar{x} \tag{5.16}$$

- **Combine results in equilibrium:** Lastly, we combine the result in (5.15) − (5.16) as well as the Phillips-curve to obtain equilibrium outcome:

$$\begin{aligned}
\pi_t^D =& \lambda\bar{x} + \frac{\lambda}{1+\lambda}(\epsilon_t - \theta_t) && (5.17)\\
x_t^D =& (\theta_t - \epsilon_t)\frac{1}{1+\lambda} && (5.18)
\end{aligned}$$

Thus once again our solution approach is the same, but the derivations, in particular how to compute rational expectations of inflation change slightly.

Appendices

A Expectations, squared terms and derivatives

When we solve for the optimal policy under commitment or central banker, we have to take into account that we are using expected losses. In section 2.1 we encountered the expected loss function:

$$E[L_t] = \frac{1}{2}E\left(\overbrace{(a_1 + \nu_t(a_2+1) + a_3\theta_t + a_4\epsilon_t - \bar{\pi})^2}^{K_1} + \lambda\overbrace{(\theta_t + \epsilon_t(a_4-1) - \bar{x})^2}^{K_2}\right) \tag{A.1}$$

In general, it turns out that it matters whether we start by taking derivatives or expectations.[6] Thus we would prefer to write out (A.1) before differentiating. In this case, we have assumed that $\nu_t, \theta_t, \epsilon_t$ are all mean zero stochastic shocks with zero covariance. If we were to write out the K_1 part we would obtain:

$$\begin{aligned} K_1 = &a_1^2 + (a_2+1)^2\nu_t^2 + a_3^2\theta_t^2 + a_4^2\epsilon_t^2 + (-\bar{\pi})^2 \\ &+2a_1\left((1+a_2)\nu_t + a_3\theta_t + a_4\epsilon_t - \bar{\pi}\right) \\ &+2(1+a_2)\nu_t\left(a_3\theta_t + a_4\epsilon_t - \bar{\pi}\right) \\ &+2a_3\theta_t\left(a_4\epsilon_t - \bar{\pi}\right) \\ &+2a_4\epsilon_t(-\bar{\pi}), \end{aligned} \tag{A.2}$$

where the first line contains all the squared terms, and the last 4 contains all the cross-products. Taking expectations now, we use that we assume that all shocks are mean zero and have zero covariance:

$$E[\nu_t] = E[\theta_t] = E[\epsilon_t] = 0 \tag{A.3}$$

$$E[\nu_t\theta_t] = E[\nu_t\epsilon_t] = E[\theta_t\epsilon_t] = 0. \tag{A.4}$$

Note that assumptions $(A.3) - (A.4)$ imply that almost all the cross-products in $(A.2)$ will be zero in expectation:

$$\begin{aligned} E[K_1] = &a_1^2 + (a_2+1)^2 E\left[\nu_t^2\right] + a_3^2 E\left[\theta_t^2\right] + a_4^2 E\left[\epsilon_t^2\right] + \bar{\pi}^2 \\ &-2a_1\bar{\pi} \end{aligned} \tag{A.5}$$

Lastly, note that with the assumption that the 3 shocks are mean zero variables implies that

$$\sigma_x^2 \equiv E\left[x^2\right] - \underbrace{E\left[x\right]^2}_{=0,\text{ when mean zero}} = E\left[x^2\right]. \tag{A.6}$$

Using this we conclude that

$$E[K_1] = a_1^2 + (a_2+1)^2\sigma_\nu^2 + a_3^2\sigma_\theta^2 + a_4^2\sigma_\epsilon + \bar{\pi}^2 - 2a_1\bar{\pi}. \tag{A.7}$$

You can verify by arguments $(A.3) - (A.4)$ and $(A.6)$ that the last part of $(A.1)$ is given by

$$E[K_2] = \sigma_\theta^2 + \sigma_\epsilon^2(a_4-1)^2 + \bar{x}^2 \tag{A.8}$$

[6] In our simple monetary policy setup, we generally do not need to take expectations first, as the **Lebesgue Dominated Convergence Theorem** holds. However, taking expectations first does not make our problem harder to solve, but it makes our approach more robust towards more 'funky' setups. Thus we prefer this approach.