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Small-World Particle Swarm Optimization with Topology Adaptation

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ABSTRACT
Traditional particle swarm optimization (PSO) algorithms adopt completely regular network as topologies, which may encounter the problems of premature convergence and insufficient efficiency. In order to improve the performance of PSO, this paper proposes a novel topology based on small-world network. Each particle in the swarm interacts with its cohesive neighbors and by chance to communicate with some distant particles via small-world randomization. In order to improve search diversity, each dimension of the swarm is assigned with a specific network, and the particle is allowed to follow the historical information of different neighbors on different dimensions. Moreover, in the proposed small-world topology, the neighborhood size and the randomization probability are adaptively adjusted based on the convergence state of the swarm. By applying the topology adaptation mechanism, the particle swarm is able to balance its exploitation and exploration abilities during the search process. Experiments were conducted on a set of classical benchmark functions. The results verify the effectiveness and high efficiency of the proposed PSO algorithm with adaptive small-world topology when compared with some other PSO variants.

Categories and Subject Descriptors
I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search – *heuristic methods*.

General Terms
Algorithms, Performance, Experimentation.

Keywords
Global optimization, particle swarm optimization, small-world network, topology adaptation.

1. INTRODUCTION
Particle swarm optimization (PSO) has attracted significant attention since its introduction in 1995 [1]-[4]. As a representative Swarm Intelligence (SI) algorithm, PSO emulates the social behavior of birds flocking or fish schooling. A population of particles randomly scatters in the solution space and then cooperates to search for the global optimum. Each particle in the swarm is assigned with a velocity and a position. Then, the particle flies through the solution space with its velocity dynamically adjusted according to the historical best position information by itself (self-cognitive) and the other particles (social-influence). It has been proved that, in this way, particles have a tendency to fly towards better and better region in the search space. Owing to its conceptual simplicity and high efficiency, PSO algorithms have been widely used to optimize various problems in recent years [5]-[7].

It can be seen that the powerfulness of PSO for problem solving comes from the interaction of particles in the swarm. A particle by itself is almost incapable of solving any problem. Therefore, the particle swarm is not just a set of particles but an interactive population organized by some sort of social communication network or topology. The topology adopted by a PSO algorithm plays an important role in determining the search behavior of the algorithm.

The traditional PSO algorithms are either global or local versions. Both of them applied simplified social model, i.e., they adopted completely regular networks as the topologies of particles. It has been shown that the PSO algorithm using a global topology converges very fast, but may easily get trapped into local optima, whereas PSO with a local topology has more chances to find the global optimum with a slow convergence speed. Therefore, global and local versions of PSO algorithms, both have their strengths weaknesses, are suitable for tackling different kinds of problems.

Considering the above issues, this paper proposes an adaptive small-world topology of PSO in order to improve the general performance of the algorithm. In real world, most of the networks, such as biological, technological, and social networks, are not completely regular. Instead, the vertices in the networks frequently communicate with their close neighbors and also by chance to connect with some distant vertices. In this way, the networks have the small-world feature that any two nodes in the networks can be connected via a small number of hops. Inspired by this, in the proposed topology, each particle is immediately connected with its close neighbors and reconnected to some random particles far away with a certain probability. The small-world topology has both advantages of large clustering coefficient and small characteristic path lengths. Therefore, the corresponding PSO algorithm developed may possess good global search ability, like local versions of PSO, yet have fast convergence speed, like the global version of PSO.

Moreover, different from traditional PSO topologies that the whole swarm shares a single topology graph, the proposed topology is fined-grained that each dimension of the swarm is assigned with a specific particle network. The neighbors on different dimensions of a particle constitute a guidance vector, which is used in the velocity update of the particle. By using such a fine-grained topology, the search of the particle swarm is more diverse, and therefore the exploration ability of the proposed algorithm is enhanced.

Furthermore, in the proposed topology, the network structures are adaptively adjusted according to convergence state of the swarm. We define a stagnation coefficient of the entire swarm. Based on this coefficient, the vertex degree and small-world randomization probability are adjusted so as to balance the exploitation and exploration abilities of the particle population. By applying this topology adaptation, the search efficiency of the proposed algorithm can be further improved.

The rest of this paper is organized as follows. Section 2 presents the preliminaries of the small-world network and the PSO topology. Section 3 describes the implementation of the proposed PSO algorithm with adaptive small-world topology in detail. Experimental tests are carried out in Section 4, followed by conclusions drawn in Section 5.

2. PRELIMINARIES

2.1 Small-World Network

In the context of network theory, complex networks can be categorized into three classes. In the first class, networks are completely regular, such as the well-known complete graph, ring, wheel, star, grid, fractal graph, etc. In contrast, the second class is the completely random graph generated by some specific probabilistic model. However, researchers have found that many real-world networks such as biological, technological, and social networks do not belong to one of these two extremes. Instead, they lie between the regular and random networks and involve both regular and random features. These networks are generally classified into the third category, among which the small-world network has attracted lots of attention in recent years [1][2].

The small-world network is inspired by Milgram's "six degrees of separation" theory that two arbitrary people can be connected via six hops, i.e., the diameter of a social network is averagely not larger than six [8]. In 1998, Watts and Strogatz [9] modeled this small-world phenomenon and their proposed WS model is the first small-world network model. In WS model, regular networks are randomly rewired to introduce some amounts of disorder. Illustrated in Figure 1, starting with a ring of N vertices, each connected to its K nearest neighbors, with a probability P, each edge is reconnected to a vertex selected uniformly at random over the entire ring. This random rewiring procedure would not alter the number of vertices or edges in the network. However, the rewired edge is possibly to introduce some "long-range links" by which two vertices far away from each other can be connected. With only a small number of long-range links, the diameter of the graph can be far more reduced, while the clustering coefficient (representing the density of triangles in the network) stays large.

In the small-world network, the rewiring probability P is a very important parameter deciding the relative proportion of regularity and disorder. If $P = 0$, the network becomes a completely regular graph in which the diameter is proportional to the network size. On the other hand, $P = 1$ stands for sparse random graphs that have a vanishingly small clustering coefficient. As shown in Figure 1, a small-world network with $0 < P < 1$ lies between completely regular and random networks and owns a small diameter and a high clustering coefficient simultaneously. By tuning P, the small-world network can be fitted to many real-world networks such as social influence networks, telephone call graphs, voter networks, road maps, etc.

2.2 Population Topology of PSO Algorithm

The classical global version PSO algorithm adopts complete graph as its topology that any two particles are immediately connected and each particle is informed by the global best particle (gbest) of the entire swarm [10]. On the other hand, some other regular networks such as ring, wheel, grid, etc., are widely adopted by local version PSO algorithms in which each particle is informed by the local best particle (lbest) in its neighborhood [11]. Besides, some recently developed PSO variants, such as the dynamic multi-swarm PSO (DMS-PSO) [12] and the standard PSO 2011 (SPSO2011) [13], use random graphs.

However, as described in Subsection A), the real-world biological influence network is neither completely regular nor completely random. As PSO is a kind of bio-inspired algorithm simulating bird flocking or fish schooling, it is reasonable to develop small-world topologies for the algorithm. In [14], Kennedy conducted experiments to found that the small-world randomization on a certain number of links can produce a significant effect on the performance of PSO algorithm. Afterwards, a certain number of PSO variants based on small-networks have been proposed, including the network-structured PSO (NS-PSO) [15] and the small-world local PSO (SWLPSO) [16].

In this paper, we propose a novel adaptive small-world PSO algorithm (ASWPSO). The differences of ASWPSO with the previous small-world-based PSO algorithms are: a) the proposed algorithm uses a fine-grained topology on variable level that different dimensions of one particle can have different links, and b) the neighborhood size K and the disorder probability P are adaptively adjusted based on the convergence state of the swarm during the optimization process.

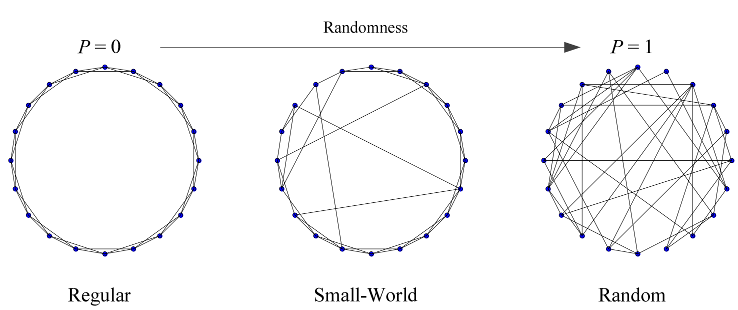


Figure 1. Small-world network: from regularity to disorder.

```
Procedure Topology Update of Particle i
  For each dimension j of particle i
    r = random (0, 1);
    If r < P then
      Randomly select a particle k from the entire swarm;
      nei_ij = k;
    Else
      Randomly select a particle k from particle i's K successor particles;
      If particle k is better than particle i then
        nei_ij = k;
      Else
        nei_ij = i;
      End if
    End if
  End for
End procedure
```

Figure 2. Pseudo code of the topology update process in ASWPSO.

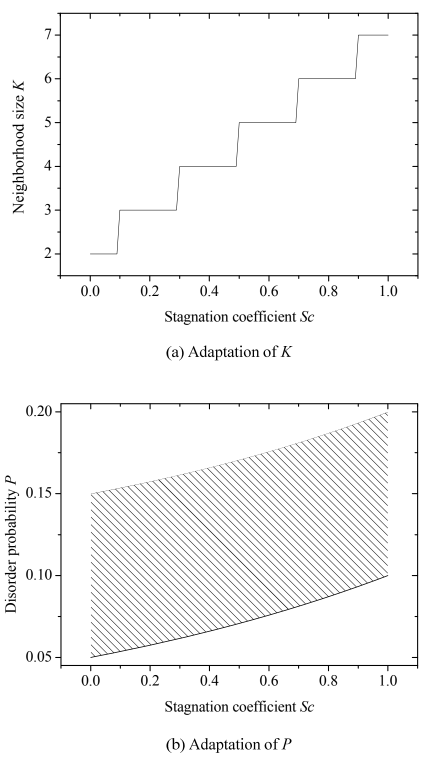


Figure 3. Curves of the adaptation of neighborhood size *K* and disorder probability *P* in the small-world topology.

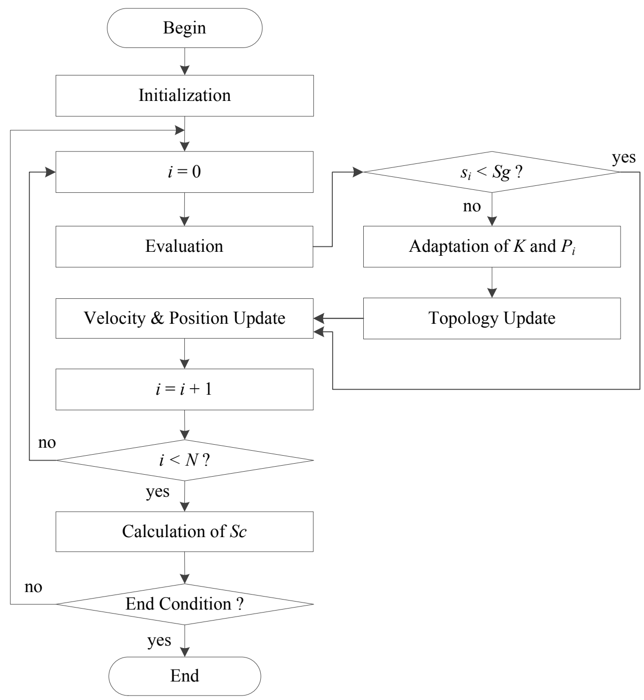


Figure 4. Flowchart of the proposed algorithm.

3. ADAPTIVE SMALL-WORLD PSO

3.1 The Proposed Small-World Topology

In this paper, a novel local topology based on the WS small-world network model for PSO algorithm is developed. At first, each particle i is connected with its K successor particles $(i+1)$, $(i+2)$, …, $(i+K)$. Then, for each link of particle i, we reconnect the edge to a random particle in the entire swarm with probability P. The novelty of the topology lies in that it is fine-grained and based on variable level. Particularly, for each dimension of the particle, the above small-world network generating process is carried out once, i.e., each dimension of PSO is assigned with a specific topology network.

Then, for each particle i, we generate a neighboring assembling vector $Nei_i = (nei_{i1}, nei_{i2}, \ldots, nei_{iD})$ with nei_{ij} being the index of the neighboring particle selected from the jth topology network. For simplicity, the small-world network building and the neighboring assembling vector generating processes are merged into a single process termed topology update. The pseudo code of the proposed topology update process is shown in Figure 2. It can be observed that, with probability P, nei_{ij} is a particle randomly selected from the entire swarm. Otherwise, nei_{ij} is either a particle in i's K successor particles (should be better than particle i) or particle i itself. This neighboring assembling vector will be used in the particle update process being described in the next subsection.

3.2 Particle Update

Suppose $V_i = (v_{i1}, v_{i2}, \ldots, v_{iD})$, $X_i = (x_{i1}, x_{i2}, \ldots, x_{iD})$, and $PBest_i = (p_{i1}, p_{i2}, \ldots, p_{iD})$ refer to the velocity, position, and personal best position vectors of the ith particle, respectively. The update equations for the jth dimension of particle i are defined as

$$v_{ij} = \omega \cdot v_{ij} + c \cdot random(0,1) \cdot (p_{kj} - x_{ij}) \tag{1}$$

$$x_{ij} = x_{ij} + v_{ij} \tag{2}$$

where ω and c are the inertia weight and accelerating coefficient, and $k = nei_{ij}$ representing the neighboring particle that informs the jth dimension of particle i.

Instead of using pbest and the gbest of the entire swarm (or lbest in the neighborhood) to update particle as traditional PSO algorithms, the proposed algorithm uses the neighboring assembling vector to guide the search. According to the producing process of the vector, it can be observed each dimension of the particle is guided by a relatively good particle (including itself) in its close neighborhood in most cases. At other times, the dimension is influenced by a random particle from the entire swarm with probability P. The benefits of using such an update mechanism are summarized as follows.

First, as particles are informed by their close neighbors with a great probability, parallel search is allowed. The particle swarm is able to explore diverse regions of the search space, which discourages premature convergence of PSO.

Second, by the small-world randomization on a certain number of links, the diameter of the topology graph is greatly reduced and the information propagation speed on the particle network is improved. The particle is essentially communicates with distant particles, in a way of "weak tie" [17]. Although particles interact frequently within cohesive neighborhood by strong ties, the shared information may be circumscribed or obsolete. In contrast, the weak ties can inject diverse information (with high entropy) into the small neighborhood, which is very likely to bring "innovation" of particles. This helps to enhance the global search ability of the proposed algorithm. Meanwhile, owing to the faster information propagation speed, the particle swarm can explore the search space with improved efficiency.

Third, the fine-grained topology allows particles to learn from different neighbors on different dimensions, which avoids the undue domination of a single particle on all dimensions. In this way, the search information of all particles in the entire swarm is used more sufficiently. Thus, compared to traditional coarse-grained PSO, the particle swarm in the proposed algorithm is less likely to get trapped in local optima,

3.3 Topology Adaptation

To enable a particle to steadily and smoothly search on a promising direction, we allow the particle to use a same neighboring assembling vector for a certain number of generations until it is stagnated. For particle i, only when it ceases improving for s_i generations larger than a threshold value Sg, it conducts the topology update procedure to generate a new neighboring assembling vector to guide search. Moreover, the neighborhood size K and the disorder probability P used in the topology update are adaptively adjusted based on the convergence state of the swarm. The proposed topology adaptation mechanism of ASWPSO is described as follows.

Define a stagnation coefficient Sc as

$$Sc = \frac{\sum_{i=1}^{N} s_i / N}{Sg} \tag{3}$$

where N is the population size and s_i is the number of stagnating generations for each particle. It can be observed that $Sc \in [0,1]$ reflects the stagnation state of the entire swarm. Based on the Sc

Table 1. Benchmark Functions

Function	Domain	Optimum	Name
$f_1(\boldsymbol{x}) = \sum_{i=1}^{D} x_i^2$	$[-100, 100]^D$	0	Sphere
$f_2(\boldsymbol{x}) = \sum_{i=1}^{D} \lvert x_i \rvert + \prod_{i=1}^{D} \lvert x_i \rvert$	$[-10, 10]^D$	0	Schwefel 2.22
$f_3(\boldsymbol{x}) = \sum_{i=1}^{D} (\sum_{j=1}^{i} x_j)^2$	$[-100, 100]^D$	0	Schwefel 1.2
$f_4(\boldsymbol{x}) = \max_i (\lvert x_i \rvert, 1 \le i \le D)$	$[-100, 100]^D$	0	Schwefel 2.21
$f_5(\boldsymbol{x}) = \sum_{i=1}^{D-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	$[-30, 30]^D$	0	Rosenbrock
$f_6(\boldsymbol{x}) = \sum_{i=1}^{D} (\lfloor x_i + 0.5 \rfloor)^2$	$[-100, 100]^D$	0	Step
$f_7(\boldsymbol{x}) = \sum_{i=1}^{D} ix^4 + random[0,1)$	$[-1.28, 1.28]^D$	0	Noise
$f_8(\boldsymbol{x}) = \sum_{i=1}^{D} -x_i \sin(\sqrt{x_i})$	$[-500, 500]^D$	-12569.5	Schwefel 2.26
$f_9(\boldsymbol{x}) = \sum_{i=1}^{D} [x_i^2 - 10\cos(2\pi x_i) + 10]$	$[-5.12, 5.12]^D$	0	Rastrigin
$f_{10}(\boldsymbol{x}) = -20\exp(-0.2\sqrt{1/D\sum_{i=1}^{D} x_i^2}) - \exp(1/D\sum_{i=1}^{D} \cos 2\pi x_i) + 20 + e$	$[-32, 32]^D$	0	Ackley
$f_{11}(\boldsymbol{x}) = 1/4000\sum_{i=1}^{D} x_i^2 - \prod_{i=1}^{D} \cos(x_i/\sqrt{i}) + 1$	$[-600, 600]^D$	0	Griewank
$f_{12}(\boldsymbol{x}) = \frac{\pi}{D}\{10\sin^2(\pi y_1) + \sum_{i=1}^{D-1} (y_i - 1)^2[1 + 10\sin^2(\pi y_{i+1})]$ $+(y_D - 1)^2\} + \sum_{i=1}^{D} u(x_i, a, k, m)$	$[-50, 50]^D$	0	Generalized Penalized Function
$f_{13}(\boldsymbol{x}) = \frac{1}{10}\{\sin^2(3\pi x_1) + \sum_{i=1}^{D-1} (x_i - 1)^2[1 + \sin^2(3\pi x_{i+1})]$ $+(x_D - 1)^2[1 + \sin^2(2\pi x_D)]\} + \sum_{i=1}^{D} u(x_i, a, k, m)$	$[-50, 50]^D$	0	

value in each generation, parameters K and P are adaptively adjusted according to Eqs. (4) and (5), whose curves are plotted in Figure 3.

$$K = \lfloor 2.5 + 5 \cdot Sc \rfloor \tag{4}$$

$$P_i = 0.05 \cdot 2^{Sc} + 0.1 \cdot i/N \tag{5}$$

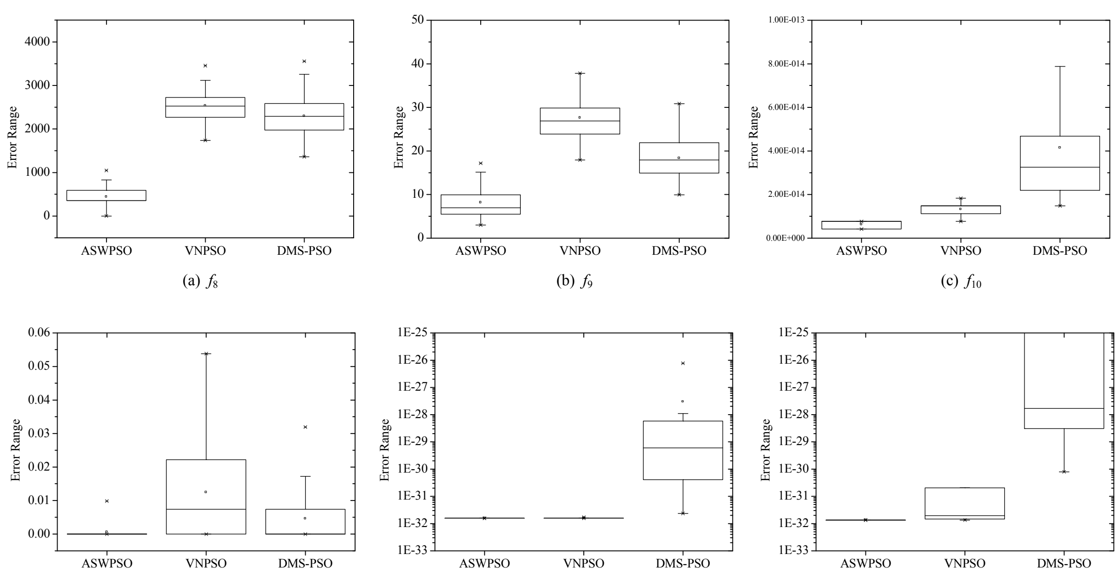
It can be observed in Figure 3(a) that K stepped from 2 to 7 with the increase of Sc. It means that, the more critical the particles are stagnated, the larger the neighborhood size is. By increasing K, the particle can have a broader field of view and hence be possible to jump out of the local optimum. According to Eq. (5), each particle is assigned with a specific disorder probability P_i, which diverse the search habits of different particles. As shown in Figure 3(b), each particle from 1 to N has a P_i ranging from 0.05 to 0.2 with the increase of Sc.

3.4 Overall Process

In this paper, we develop an ASWPSO algorithm with an adaptive small-world topology, whose flowchart is shown in Figure 4. In addition to the general process of traditional PSO, the topology update operation is executed once a particle ceases improving for Sg generations. In the operation, a neighboring assembling vector is built, which hereafter guides the search of the particle. Moreover, it is to be noticed that the parameters K and P in the small-world topology is adaptively adjusted according to stagnation coefficient Sc of the entire swarm. In the next section, the proposed algorithm will be experimentally studied.

Table 2. Statistical Results and Comparisons of the Three Algorithms

Function	VNPSO			DMS-PSO			ASWPSO		
	Mean	Best	Std	Mean	Best	Std	Mean	Best	Std
f_1	1.52E-40	3.90E-43	3.60E-40	1.17E-27	5.98E-32	4.40E-27	**8.07E-72**	**5.34E-73**	**1.43E-71**
f_2	1.29E-27	5.83E-30	5.56E-27	3.60E-09	7.97E-17	1.91E-08	**4.85E-43**	**1.24E-43**	**3.50E-43**
f_3	1.40E+00	3.32E-01	1.17E+00	1.83E+01	4.16E+00	1.39E+01	**2.68E-04**	**2.29E-05**	**1.71E-04**
f_4	7.37E-01	2.86E-01	3.11E-01	1.04E+00	3.81E-01	4.48E-01	**6.41E-04**	**6.46E-06**	**8.19E-04**
f_5	3.46E+01	4.64E-01	3.02E+01	3.49E+01	1.77E+01	2.63E+01	**2.22E+01**	**1.95E-01**	**1.73E+01**
f_6	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**
f_7	9.16E-03	5.42E-03	2.49E-03	1.19E-02	4.89E-03	4.85E-03	**1.33E-03**	**6.99E-04**	**7.57E-04**
f_8	2.53E+03	1.74E+03	3.66E+02	2.30E+03	1.36E+03	4.97E+02	**4.39E+02**	**3.82E-04**	**2.60E+02**
f_9	2.76E+01	1.79E+01	5.09E+00	1.83E+01	9.95E+00	5.64E+00	**8.17E+00**	**2.99E+00**	**3.68E+00**
f_{10}	1.33E-14	7.69E-15	3.05E-15	4.14E-14	1.48E-14	2.65E-14	**6.27E-15**	**4.14E-15**	**1.77E-15**
f_{11}	1.24E-02	**0.00E+00**	1.64E-02	4.60E-03	**0.00E+00**	7.96E-03	**5.75E-04**	**0.00E+00**	**2.21E-03**
f_{12}	**1.57E-32**	**1.57E-32**	2.36E-34	2.96E-28	2.34E-32	1.40E-27	**1.57E-32**	**1.57E-32**	**2.78E-48**
f_{13}	1.47E-03	**1.35E-32**	3.80E-03	3.27E-03	7.92E-31	5.78E-03	**1.35E-32**	**1.35E-32**	**5.57E-48**



(a) f_8 (b) f_9 (c) f_{10}

(e) f_{11} (f) f_{12} (g) f_{13}

Figure 5. Boxplots of the three algorithms on multimodal functions.

4. EXPERIMENTAL STUDIES
4.1 Experimental Setup
In the experiments, 13 benchmark functions with different features are used to test the performance of ASWPSO [19]. These functions are listed in Table I, where f_1 to f_5 are unimodal functions, f_6 is a step function, f_7 is a noisy quartic function, and f_8 to f_{13} are multimodal functions with a great number of local optima.

In ASWPSO, the inertia weight ω and accelerating coefficient c are setting to 0.7298 and 1.49618 respectively, and the stagnation threshold value is set as $Sg = 5$. The proposed algorithm is further compared with two existing PSO variants. Particularly, VNPSO is the PSO algorithm with a regular von Neumann topology [18], whereas DMS-PSO uses a completely random topology in the first 90% time and adopts the global topology for the final convergence [7]. The parameter settings of the two algorithms are according to the corresponding references.

All the algorithms are tested on 30 dimensions functions with population size 30 and function evaluations 300,000. For each function, 30 independent trials are carried out by applying VNPSO, DMS-PSO, and ASWPSO under the same circumstances. The statistical results are presented and compared in the following subsection.

4.2 Results and Comparisons
In Table II, the mean, best, and standard deviations of the error values achieved by VNPSO, DMS-PSO, and the proposed ASWPSO are listed. It can be observed that ASWPSO comprehensively outperforms VNPSO and DMS-PSO. On the one side, for unimodal functions, the proposed algorithm can achieve higher solution accuracy than VNPSO and DMS-PSO. On the other side, in optimizing multimodal functions, the proposed ASWPSO exhibits much stronger global search ability than the other two algorithms.

The performance of the three algorithms on multimodal functions f_8 to f_{13} are further compared by the box plots shown in Figure 5. In the figure, the minimum, lower quartile, median, upper quartile,

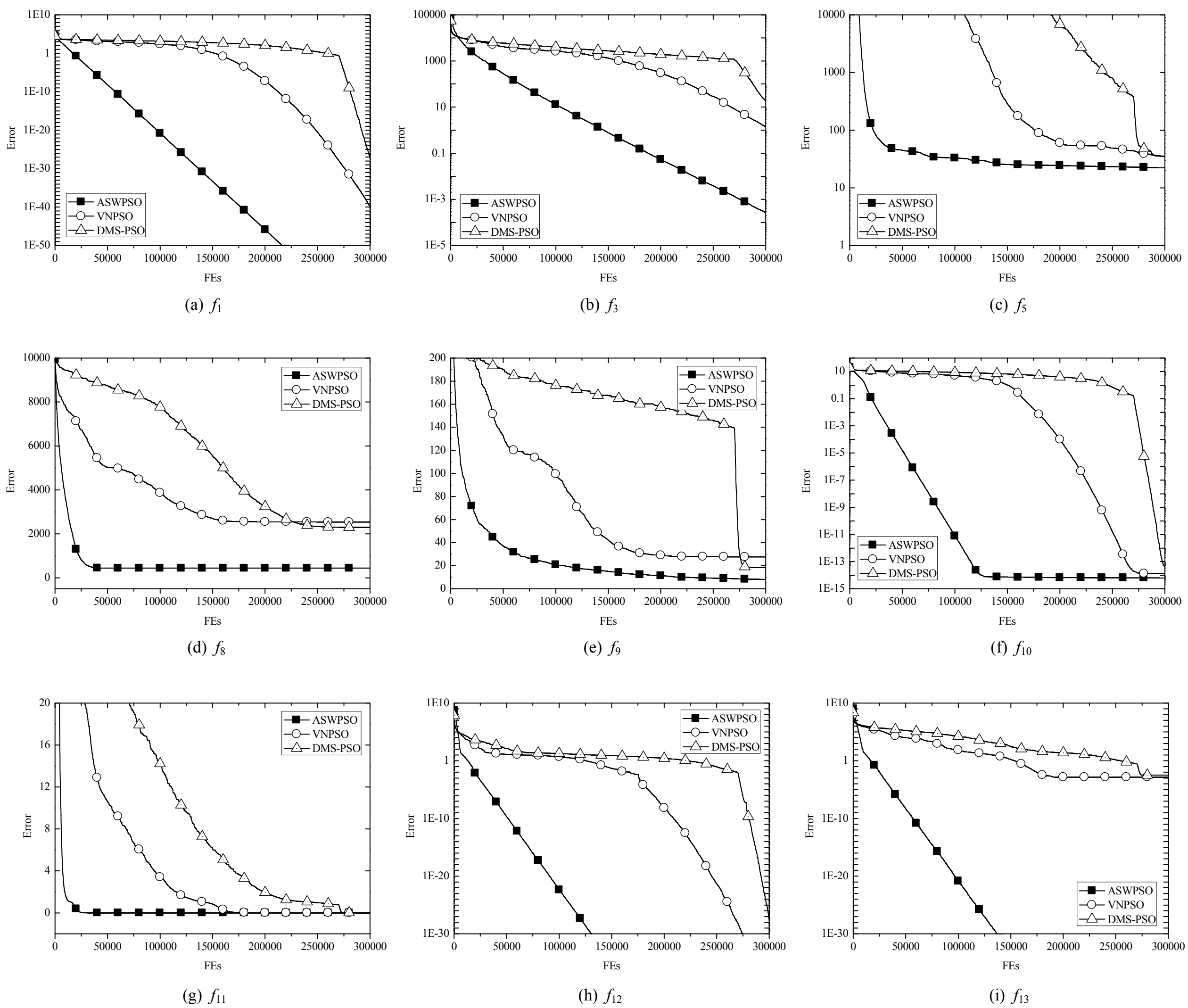


Figure 6. Convergence curves of the three algorithms.

maximum, and outliers of the results are graphically depicted. It can be seen that, for all the multimodal functions tested, the results of ASWPSO are better than those of VNPSO and DMS-PSO. Moreover, Figure 5 also shows the robustness of the proposed algorithm, for it can always obtain promising results in a narrow error range.

In addition, the convergence curves of the three algorithms in optimizing unimodal functions f_1, f_3, and f_5 and multimodal functions f_8 to f_{13} are plotted in Figure 6. The figure clearly shows that ASWPSO converges the fastest to achieve the highest solution accuracy among the three algorithms. To summarize, by using the adaptive small-world topology, the proposed ASWPSO is a very competitive PSO algorithm in terms of solution accuracy, search efficiency, and robustness.

5. CONCLUSIONS

In this paper, a novel PSO algorithm based on adaptive small-world topology (ASWPSO) is proposed. In ASWPSO, each particle is connected with several nearest neighbors, and, by the small-world randomization, it may be reconnected to some other particles from the entire swarm. The randomly built long-range links reduce the diameter of the topology and hence improve the convergence speed of the entire swarm. Moreover, we use a fine-grained topology based on variable level that allows particles adopt different small-world networks on different dimensions. This helps to diversify the search of particles so as to improve the global search ability of PSO. During the search process of the algorithm, the parameters in the small-world topology are adaptively adjusted based on the stagnation state of the entire swarm. In this way, the search effectiveness and efficiency of the swarm is further improved.

In the experiments, the proposed ASWPSO is tested on 13 benchmark functions, with performance compared with VNPSO using regular topology and DMS-PSO using random topology. Experimental results verify the high efficiency and robustness of using the proposed adaptive small-world topology.

6. ACKNOWLEDGMENTS

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