Lab 4

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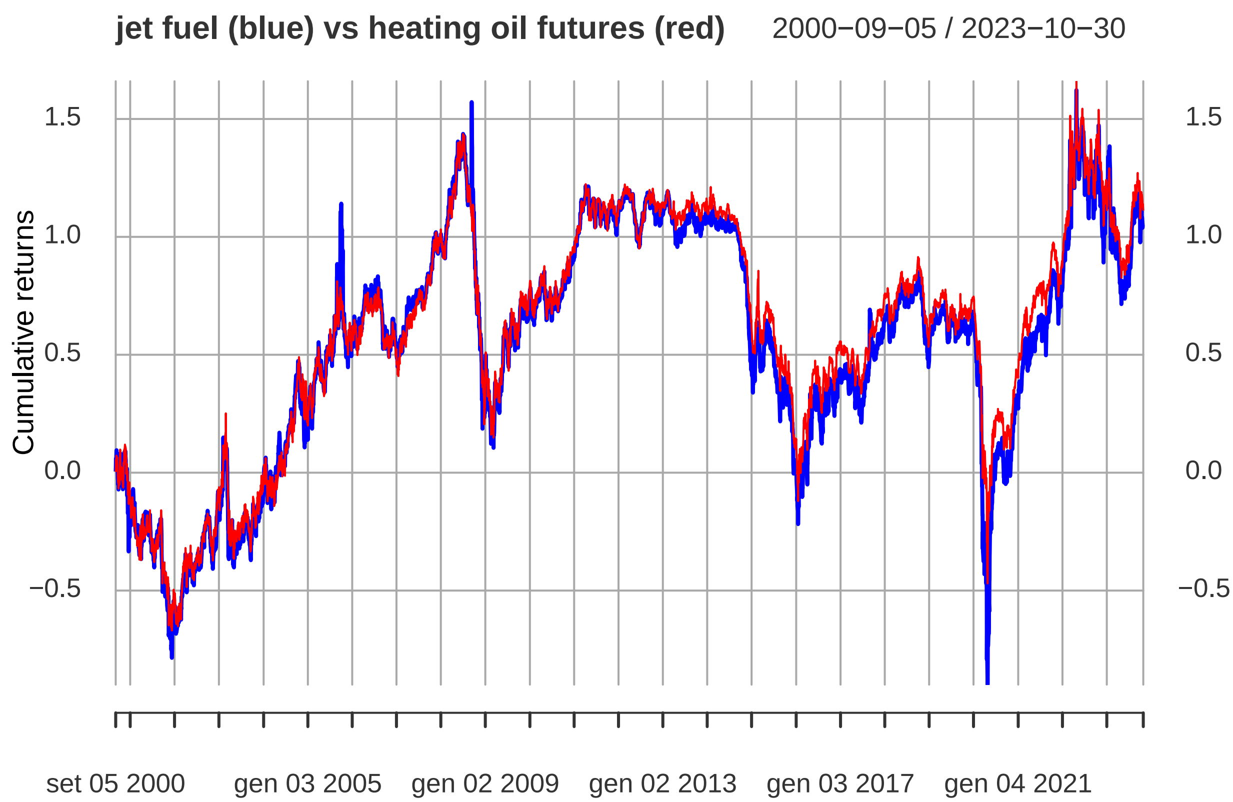
```r
#download the closing prices for heating oil futures
oil.futures <- getSymbols('HO=F',auto.assign = F,
                          from = as.Date("2000-09-01"), to = as.Date("2023-10-31"))
#download data for jet fuel prices
jet.fuel <- getSymbols('DJFUELUSGULF', src = 'FRED', auto.assign = F,
                       from = as.Date("2000-09-01"), to = as.Date("2023-10-31"))

#Merge the two time series altogether and compute the return for each
oil.futures <- na.omit(oil.futures)
jet.fuel <- na.omit(jet.fuel)

#plot the cumulative return of both time series in a single plot.
dataset <- merge(jet.fuel, oil.futures$`HO=F.Adjusted`, join = "inner")

returns<- na.omit(log(dataset/lag(dataset)))

{plot(cumsum(returns$DJFUELUSGULF), type = "l", col = "blue",
      xlab="Date",ylab="Cumulative returns", main = "jet fuel (blue) vs heating oil futures (red)")
lines(cumsum(returns$HO.F.Adjusted), col = "red")}
```




```r
#annual volatility for each. Comparison with that of the SPY over the same period
jf.sigma <- sd(returns$DJFUELUSGULF)*sqrt(252)
knitr::kable(jf.sigma, caption = "Jet Fuel annual volatility")
```

Table 1: Jet Fuel annual volatility

x
0.438699

```r
oilf.sigma <- sd(returns$HO.F.Adjusted)*sqrt(252)
knitr::kable(oilf.sigma, caption = "Heating Oil Futures annual volatility")
```

Table 2: Heating Oil Futures annual volatility

x
0.3735578

```r
SPY <- getSymbols('SPY',from = as.Date("2000-09-01"), to = as.Date("2023-10-31"), auto.assign = F)
SPY <- na.omit(log(SPY$SPY.Adjusted/lag(SPY$SPY.Adjusted)))
spy.sigma <- sd(SPY)*sqrt(252)
knitr::kable(spy.sigma, caption = "SPY annual volatility")
```

Table 3: SPY annual volatility

x
0.1955244

```r
#Regress the return on the jet fuel using the return on the futures contract
model <- lm(DJFUELUSGULF~HO.F.Adjusted, data = returns)
summary(model)
```

```
##
## Call:
## lm(formula = DJFUELUSGULF ~ HO.F.Adjusted, data = returns)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.31357 -0.00444  0.00000  0.00457  0.32213
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -1.670e-06  2.123e-04  -0.008    0.994
## HO.F.Adjusted  9.525e-01  9.024e-03 105.557   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01617 on 5796 degrees of freedom
## Multiple R-squared:  0.6578, Adjusted R-squared:  0.6578
## F-statistic: 1.114e+04 on 1 and 5796 DF,  p-value: < 2.2e-16
```

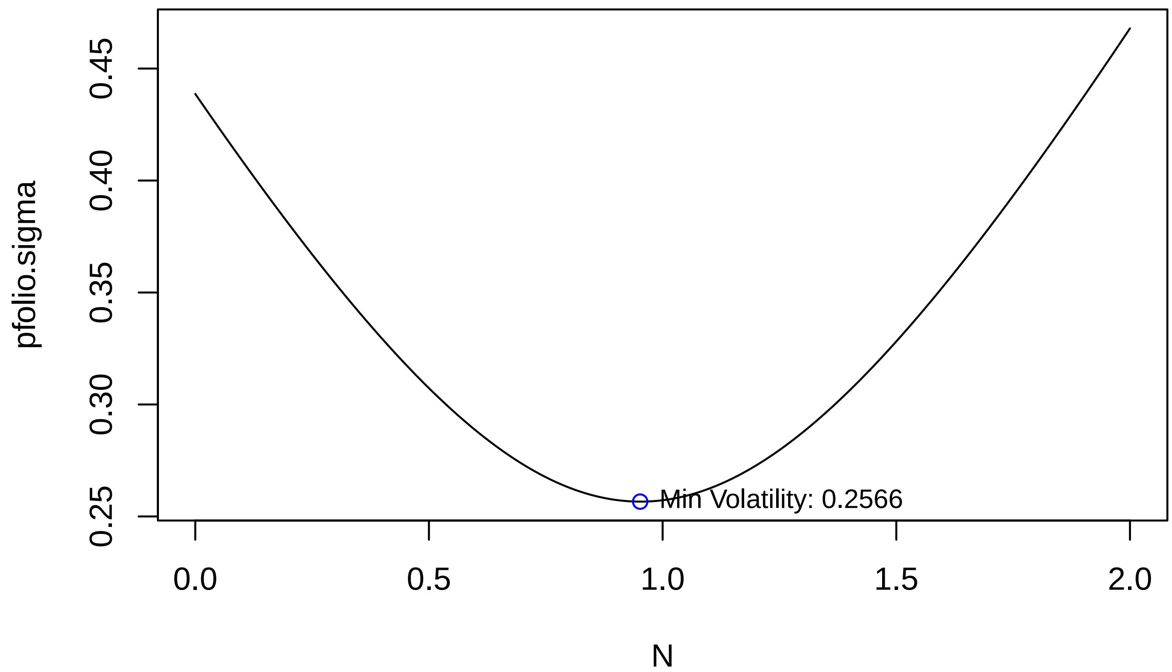
```r
knitr::kable(model$coefficients[2], caption = "Beta")
```

Table 4: Beta

	x
HO.F.Adjusted	0.9524904

```r
#hedged pfolio and sd for different N
N <- seq(0,2,0.001)
pfolio.sigma <-c()
for (n in N){
  pfolio <- returns$DJFUELUSGULF - n*returns$HO.F.Adjusted
  pfolio.sigma <-c(pfolio.sigma, sd(pfolio)*sqrt(252))}

min.sigma <- min(pfolio.sigma)
min.N <- N[which.min(pfolio.sigma)]
{plot(N, pfolio.sigma, type = 'l', pch = 19)
points(min.N, min.sigma, col="blue")
text(min.N, min.sigma, paste("Min Volatility:", round(min.sigma, 4)), pos = 4, cex = 0.8)}
```




```
#compare with beta
knitr::kable(N[which.min(pfolio.sigma)], caption = "N of contracts to short to have minimum volatility h
```

Table 5: N of contracts to short to have minimum volatility hedged pfolio

x
0.952

```
knitr::kable(model$coefficients[2], caption = "Beta of returns of jet fuel on returns of future contrac
```

Table 6: Beta of returns of jet fuel on returns of future contract

	x
HO.F.Adjusted	0.9524904

COMMENTS

Point 3: The plot show a close alignment between cumulative returns of jet fuel and cumulative returns of heating oil futures. Jet fuel and heating oil are closely related energy products, and their prices often exhibit high correlation or similarity in movements. Moreover, current and future prices (same asset) exhibit close correlation due to market efficiency and expectations. As a results of these two statements, cumulative returns of jet fuel and cumulative returns of heating oil future are highly correlated.

Point 4: SPY show a much lower volatility with respect to heating oil futures and jet fuel.

Point 5: Regressing the return on the jet fuel using the return on the heating oil futures contract we obtain a Beta equal to 0.9525 adn R squared of 0.6578. This implies that, using heating oil futures contract to hedge position in jet fuel, the hedging will be effective in reducing the original position's volatility by almost 41,5%, where $1 - \text{sqrt}(1 - \text{R2}) = 41{,}5\%$. Moreover, R squared equal to 0.6578 implies a correlation between jet fuel and heating oil future equal to 81,10%, that is sqrt(R squared).

Point 7: The beta of the regression in point 5 is equal to the number of contracts to short (computed in point 7) in order to achieve the optimal hedge. The optimal hedge aims to create a portfolio with minimized variance. It's derived by calculating the ratio of the covariance between the asset and the futures contract to the variance of the futures contract (the beta of the regression). This ratio indicates the precise number of futures contracts needed to achieve the minimum variance hedge. Mathematically, the beta signifies the precise relationship between their price movements. Therefore, using this parameter to decide the number of futures contract to short, allows to compensate the movement of the jet fuel price with the movement of the futures contract. N.B. : the volatility of the hedged portfolio returns is equal to 0.2566, that is 0.438699 x (1-0.415) –> volatility jet fuel returns x sqrt(1-R2).