LAB 3

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```r
library(zoo)
```

```
##
## Caricamento pacchetto: 'zoo'

## I seguenti oggetti sono mascherati da 'package:base':
##
##     as.Date, as.Date.numeric
```

```r
library(quantmod)
```

```
## Caricamento del pacchetto richiesto: xts

## Caricamento del pacchetto richiesto: TTR

## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```r
library(xts)
getSymbols('SPY', from = '1993-12-31', to = '2022-12-31')
```

```
## [1] "SPY"
```

```r
head(SPY)
```

```
##            SPY.Open SPY.High  SPY.Low SPY.Close SPY.Volume SPY.Adjusted
## 1993-12-31 46.93750 47.00000 46.56250  46.59375     312900     27.11358
## 1994-01-03 46.59375 46.65625 46.40625  46.46875     960900     27.04084
## 1994-01-04 46.53125 46.65625 46.46875  46.65625     164300     27.14994
## 1994-01-05 46.71875 46.78125 46.53125  46.75000     710900     27.20450
## 1994-01-06 46.81250 46.84375 46.68750  46.75000     201000     27.20450
## 1994-01-07 46.84375 47.06250 46.71875  47.03125     775500     27.36817
```

DAILY DATA

```r
#daily returns
returns_daily <- na.omit(log(SPY$SPY.Adjusted/lag(SPY$SPY.Adjusted)))
head(returns_daily)
```

```
##            SPY.Adjusted
## 1994-01-03 -0.002686203
## 1994-01-04  0.004026603
## 1994-01-05  0.002007483
## 1994-01-06  0.000000000
## 1994-01-07  0.005998243
## 1994-01-10  0.011888790
```

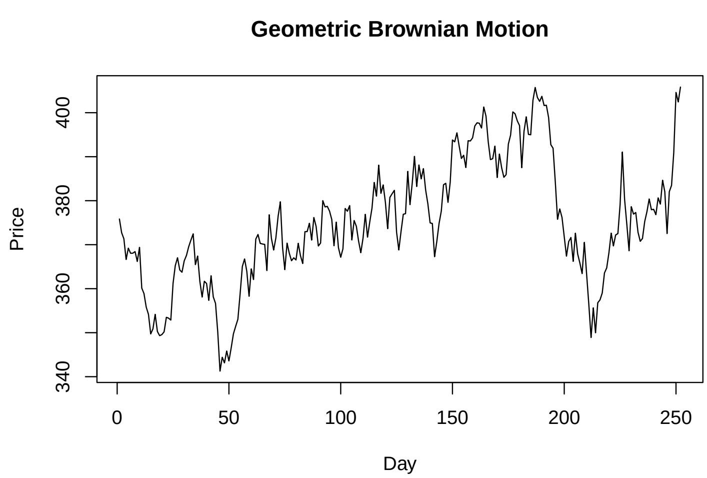
```r
#calibrate
sigma_hat <- sqrt(252)*sd(returns_daily)
sigma_hat
```

```
## [1] 0.1917567
```

```r
mu_hat <- 252*mean(returns_daily)+sigma_hat^2/2
mu_hat
```

```
## [1] 0.1093384
```

```r
#simulations
s0 <- as.numeric(SPY$SPY.Adjusted[length(SPY$SPY.Adjusted)])
dt<-1/252
GBM_t <- function(n){
  dRt_seq<-rnorm(1/dt,(mu_hat - sigma_hat^2/2)*dt,sigma_hat*sqrt(dt))
  St<-s0*exp(cumsum(dRt_seq))
  return(St)
}
s_mat<-sapply(1:10^5,GBM_t)
plot(s_mat[,1], type="l",xlab="Day",ylab="Price",main="Geometric Brownian Motion" )
```



```r
s1_sim<-s_mat[252,]
head(s1_sim)
```

```
## [1] 405.8246 467.5771 444.0709 332.9978 482.6264 402.8037
```

```r
F_bar<-mean(s1_sim)
F_bar
```

```
## [1] 421.8807
```

```r
s1_exp<-s0*exp(mu_hat)
s1_exp
```

```
## [1] 421.9328
```

```r
s1_sig<-sd(s1_sim)
s1_sig
```

```
## [1] 81.8056
```

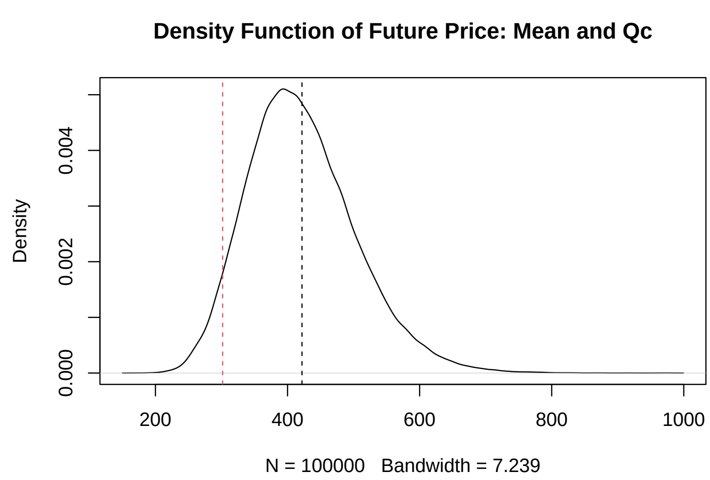
```r
#VaR
Qc <- quantile(s1_sim,0.05)
Qc
```

```
##       5%
## 301.7309
```

```r
VaR <- F_bar - Qc
VaR
```

```
##       5%
## 120.1498
```

```r
plot(density(s1_sim),main = "Density Function of Future Price: Mean and Qc" )
abline(v = F_bar, lty = 2)
abline(v = Qc, lty = 2, col = 2)
```



MONTHLY DATA

```r
#monthly returns
SPY.monthly<-to.monthly(SPY)
head(SPY.monthly)
```

```
##          SPY.Open SPY.High  SPY.Low SPY.Close SPY.Volume SPY.Adjusted
## dic 1993 46.93750 47.00000 46.56250  46.59375     312900     27.11358
## gen 1994 46.59375 48.31250 46.40625  48.21875    6837100     28.05918
## feb 1994 48.15625 48.28125 46.56250  46.81250   10974400     27.24087
## mar 1994 46.81250 47.31250 43.53125  44.59375   14705500     26.09944
## apr 1994 43.34375 45.35938 43.34375  45.09375   11429000     26.39205
## mag 1994 45.09375 45.93750 44.17188  45.81250    8545100     26.81276
```

```r
returns_monthly <- na.omit(log(SPY.monthly$SPY.Adjusted/lag(SPY.monthly$SPY.Adjusted)))
head(returns_monthly)
```

```
##          SPY.Adjusted
## gen 1994   0.03428137
## feb 1994  -0.02959770
## mar 1994  -0.04280427
## apr 1994   0.01114890
## mag 1994   0.01581505
## giu 1994  -0.02318162
```

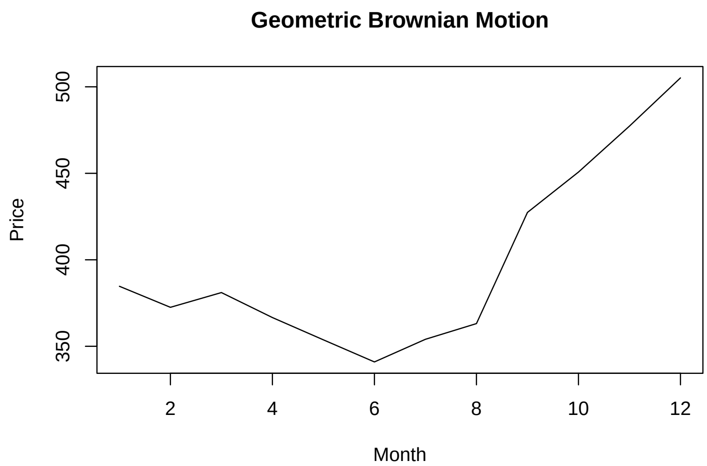
```r
#calibrate
sigma_hat <- sqrt(12)*sd(returns_monthly)
sigma_hat
```

```
## [1] 0.1526582
```

```r
mu_hat <- 12*mean(returns_monthly)+sigma_hat^2/2
mu_hat
```

```
## [1] 0.1025306
```

```r
#simulations
s0 <- as.numeric(SPY.monthly$SPY.Adjusted[length(SPY.monthly$SPY.Adjusted)])
dt<-1/12
GBM_t <- function(n){
  dRt_seq<-rnorm(1/dt,(mu_hat - sigma_hat^2/2)*dt,sigma_hat*sqrt(dt))
  St<-s0*exp(cumsum(dRt_seq))
  return(St)
}
s_mat<-sapply(1:10^5,GBM_t)
plot(s_mat[,1], type="l",xlab="Month",ylab="Price",main="Geometric Brownian Motion" )
```




```
s1_sim<-s_mat[12,]
head(s1_sim)
```

```
## [1] 505.1467 471.6962 477.7425 527.9735 437.7246 362.4981
```

```
F_bar<-mean(s1_sim)
F_bar
```

```
## [1] 419.0371
```

```
s1_exp<-s0*exp(mu_hat)
s1_exp
```

```
## [1] 419.0701
```

```
s1_sig<-sd(s1_sim)
s1_sig
```

```
## [1] 64.2664
```

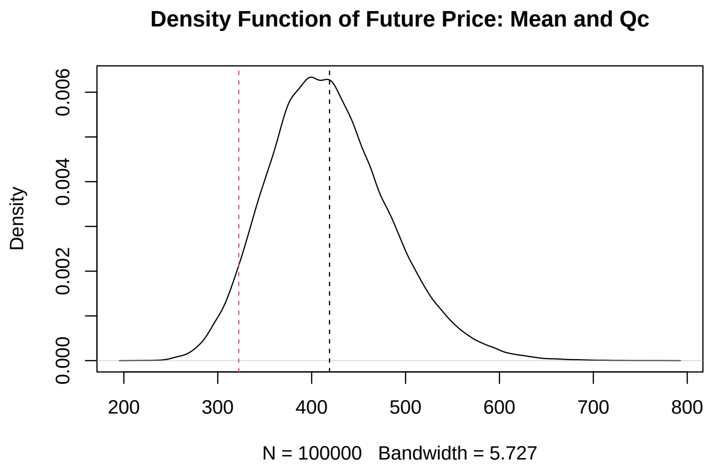
```
#VaR
Qc <- quantile(s1_sim,0.05)
Qc
```

```
##       5%
## 322.3587
```

```
VaR <- F_bar - Qc
VaR
```

```
##       5%
## 96.67835
```

```
plot(density(s1_sim),main = "Density Function of Future Price: Mean and Qc" )
abline(v = F_bar, lty = 2)
abline(v = Qc, lty = 2, col = 2)
```



VaR computed wth monthly data and VaR computed with daily data are different. The explanation is the assumption of independence. In fact, using daily data, we computed sigma hat scaling standard deviation of daily returns (sd * sqrt(252)) with the purpose of generating a GBM. Then, using monthly data, we computed sigma hat scaling standard deviation of monthly returns (sd * sqrt(12)). However, in order to get the same results for the VaR, the values of yearly returns volatility (sigma hat), used to generate the processes, should have been the same. But for variance, the IID assumption ignores potential correlation among daily returns over time. Specifically, looking at daily returns over this period, we note that returns do exhibit serial correlation

```
cor(returns_daily,lag(returns_daily),use = 'pairwise')
```

```
##              SPY.Adjusted
## SPY.Adjusted  -0.08309205
```

VOLATILITY INCREASE BY 5% (MONTHLY DATA)

```
#calibrate
sigma_hat <- sqrt(12)*(sd(returns_monthly)*1.05)
sigma_hat
```

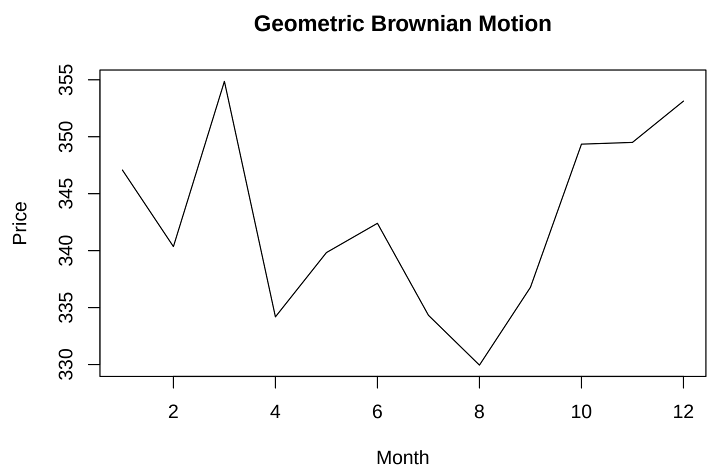
```
## [1] 0.1602911
```

```
mu_hat <- 12*mean(returns_monthly)+sigma_hat^2/2
mu_hat
```

```
## [1] 0.103725
```

```
#simulations
s0 <- as.numeric(SPY.monthly$SPY.Adjusted[length(SPY.monthly$SPY.Adjusted)])
dt<-1/12
```

```
GBM_t <- function(n){
  dRt_seq<-rnorm(1/dt,(mu_hat - sigma_hat^2/2)*dt,sigma_hat*sqrt(dt))
  St<-s0*exp(cumsum(dRt_seq))
  return(St)
}
s_mat<-sapply(1:10^5,GBM_t)
plot(s_mat[,1], type="l",xlab="Month",ylab="Price",main="Geometric Brownian Motion" )
```




```
s1_sim<-s_mat[12,]
head(s1_sim)
```

```
## [1] 353.1288 428.4098 348.7038 331.2383 481.7734 375.8973
```

```
F_bar<-mean(s1_sim)
F_bar
```

```
## [1] 419.3212
```

```
s1_exp<-s0*exp(mu_hat)
s1_exp
```

```
## [1] 419.5709
```

```
s1_sig<-sd(s1_sim)
s1_sig
```

```
## [1] 67.82515
```

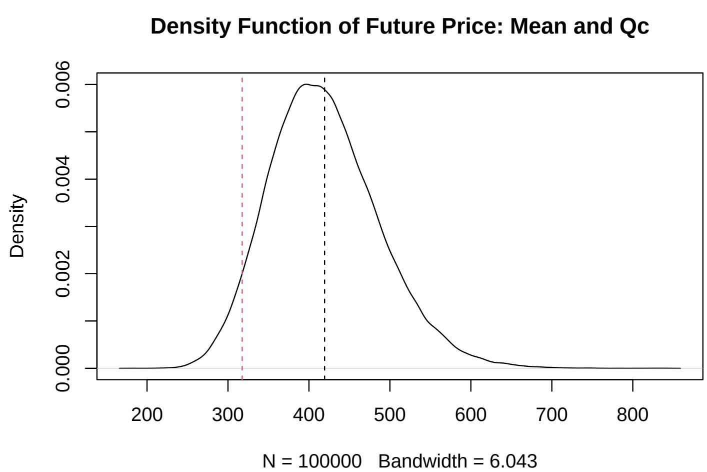
```
#VaR
Qc <- quantile(s1_sim,0.05)
Qc
```

```
##       5%
## 317.5259
```

```r
VaR <- F_bar - Qc
VaR
```

```
##       5%
## 101.7954
```

```r
plot(density(s1_sim),main = "Density Function of Future Price: Mean and Qc" )
abline(v = F_bar, lty = 2)
abline(v = Qc, lty = 2, col = 2)
```



Is the relationship between one-year VaR(0.05) and the annual volatility linear? VaR depends on the distribution of returns. We assume that Log-returns follow a normal distribution. For a normal distribution, the relationship between VaR and volatility is linear because it's based on standard deviations. Higher volatility generally implies a higher VaR. Under normal distribution the c quantile of returns Rd is:

Q(Rd , c) = mu + sigma*Zc

and

VaR(Rd , c) = mu - [mu + sigma*Zc] = -sigma*Zc = sigma*Z(1-c)

Therefore there is a linear relationship between VaR and standard deviation.

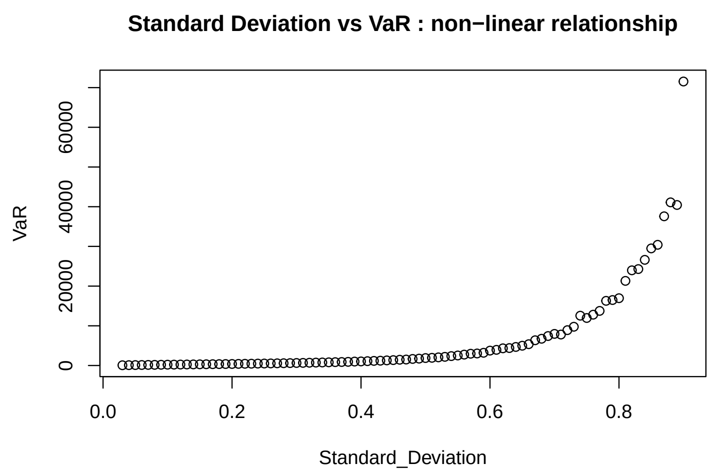
But if we assume that Log-returns follow a normal distributions, portfolio values follow a log-normal distribution, and in this case " Q(Fd , c) = mu + sigma*Zc " is not valid, and the relationship between VaR and standard deviation is not linear. Below a graphic representation.

```r
#relationship between VaR and standard deviation

Standard_Deviation=c()
VaR=c()
for (i in seq(0.03,0.9,0.01)){
  sd=i
  sigma_hat <- sqrt(12)*(sd)
  mu_hat <- 12*mean(returns_monthly)+sigma_hat^2/2
```

```
  s0 <- as.numeric(SPY.monthly$SPY.Adjusted[length(SPY.monthly$SPY.Adjusted)])
  s1_sim<-s0*exp(mu_hat-sigma_hat^2/2+sigma_hat*rnorm(10^5,0,1))
  F_bar<-mean(s1_sim)
  Standard_Deviation<-append(Standard_Deviation,sd)
  Qc<-quantile(s1_sim,0.05)
  VaR_value=F_bar - Qc
  VaR<- append(VaR,VaR_value)
}

plot(Standard_Deviation,VaR, main="Standard Deviation vs VaR : non-linear relationship")
```

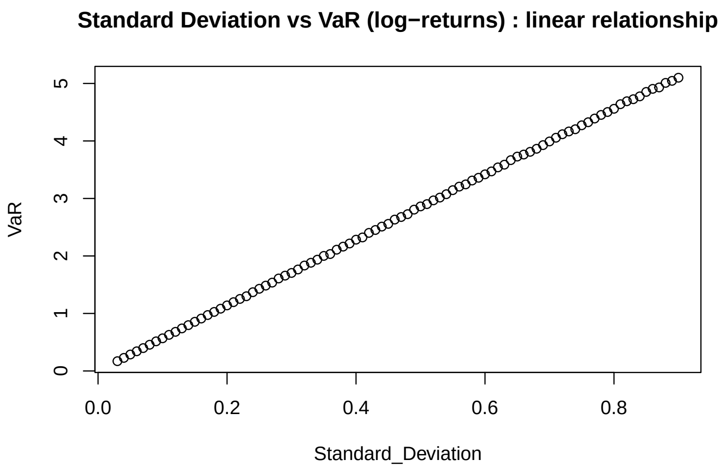


but for log-returns

```
#relationship between VaR (in terms of log-returns) and standard deviation

Standard_Deviation=c()
VaR=c()
for (i in seq(0.03,0.9,0.01)){
  sd=i
  sigma_hat <- sqrt(12)*(sd)
  mu_hat <- 12*mean(returns_monthly)+sigma_hat^2/2
  s0 <- as.numeric(SPY.monthly$SPY.Adjusted[length(SPY.monthly$SPY.Adjusted)])
  r1_sim<-mu_hat-sigma_hat^2/2+sigma_hat*rnorm(10^5,0,1)
  F_bar<-mean(r1_sim)
  Standard_Deviation<-append(Standard_Deviation,sd)
  Qc<-quantile(r1_sim,0.05)
  VaR_value=F_bar - Qc
  VaR<- append(VaR,VaR_value)
}

plot(Standard_Deviation,VaR, main="Standard Deviation vs VaR (log-returns) : linear relationship")
```


Standard Deviation vs VaR (log−returns) : linear relationship
VaR
0
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8
Standard_Deviation