# FE515 Assignment 3

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**Question 1**:

## 1.1

Download option prices of ticker VIX for all expiration dates and name it VIX.options

```
VIX.options <- getOptionChain("^VIX",NULL)
names(VIX.options)
```

```
##  [1] "nov.22.2023" "nov.29.2023" "dic.06.2023" "dic.13.2023" "dic.20.2023"
##  [6] "gen.17.2024" "feb.14.2024" "mar.20.2024" "apr.17.2024" "mag.22.2024"
## [11] "giu.18.2024" "lug.17.2024"
```

## 1.2

Download the current price (last quote price) for VIX

```
last_p <- getQuote("^VIX")[['Last']]
getQuote("^VIX")
```

```
##               Trade Time  Last     Change  % Change  Open  High   Low Volume
## ^VIX 2023-11-20 15:15:01 13.41 -0.3900003 -2.826089 14.26 14.31 13.39      0
```

## 1.3

For calls and puts of VIX.options at each expiration calculate the average of Bid and Ask. Create a new column named 'Price' to contain the result.

```
VIX.options$nov.22.2023$calls$Price <- 0.5*(VIX.options$nov.22.2023$calls$Bid
                                            + VIX.options$nov.22.2023$calls$Ask)
VIX.options$nov.29.2023$calls$Price <- 0.5*(VIX.options$nov.29.2023$calls$Bid
                                            + VIX.options$nov.29.2023$calls$Ask)
VIX.options$dic.06.2023$calls$Price <- 0.5*(VIX.options$dic.06.2023$calls$Bid
                                            + VIX.options$dic.06.2023$calls$Ask)
VIX.options$dic.13.2023$calls$Price <- 0.5*(VIX.options$dic.13.2023$calls$Bid
                                            + VIX.options$dic.13.2023$calls$Ask)
VIX.options$dic.20.2023$calls$Price <- 0.5*(VIX.options$dic.20.2023$calls$Bid
                                            + VIX.options$dic.20.2023$calls$Ask)
```

```
VIX.options$gen.17.2024$calls$Price <- 0.5*(VIX.options$gen.17.2024$calls$Bid
                                            + VIX.options$gen.17.2024$calls$Ask)
VIX.options$feb.14.2024$calls$Price <- 0.5*(VIX.options$feb.14.2024$calls$Bid
                                            + VIX.options$feb.14.2024$calls$Ask)
VIX.options$mar.20.2024$calls$Price <- 0.5*(VIX.options$mar.20.2024$calls$Bid
                                            + VIX.options$mar.20.2024$calls$Ask)
VIX.options$apr.17.2024$calls$Price <- 0.5*(VIX.options$apr.17.2024$calls$Bid
                                            + VIX.options$apr.17.2024$calls$Ask)
VIX.options$mag.22.2024$calls$Price <- 0.5*(VIX.options$mag.22.2024$calls$Bid
                                            + VIX.options$mag.22.2024$calls$Ask)
VIX.options$giu.18.2024$calls$Price <- 0.5*(VIX.options$giu.18.2024$calls$Bid
                                            + VIX.options$giu.18.2024$calls$Ask)
VIX.options$lug.17.2024$calls$Price <- 0.5*(VIX.options$lug.17.2024$calls$Bid
                                            + VIX.options$lug.17.2024$calls$Ask)

VIX.options$nov.22.2023$puts$Price <- 0.5*(VIX.options$nov.22.2023$puts$Bid
                                              + VIX.options$nov.22.2023$puts$Ask)
VIX.options$nov.29.2023$puts$Price <- 0.5*(VIX.options$nov.29.2023$puts$Bid
                                            + VIX.options$nov.29.2023$puts$Ask)
VIX.options$dic.06.2023$puts$Price <- 0.5*(VIX.options$dic.06.2023$puts$Bid
                                            + VIX.options$dic.06.2023$puts$Ask)
VIX.options$dic.13.2023$puts$Price <- 0.5*(VIX.options$dic.13.2023$puts$Bid
                                            + VIX.options$dic.13.2023$puts$Ask)
VIX.options$dic.20.2023$puts$Price <- 0.5*(VIX.options$dic.20.2023$puts$Bid
                                            + VIX.options$dic.20.2023$puts$Ask)
VIX.options$gen.17.2024$puts$Price <- 0.5*(VIX.options$gen.17.2024$puts$Bid
                                            + VIX.options$gen.17.2024$puts$Ask)
VIX.options$feb.14.2024$puts$Price <- 0.5*(VIX.options$feb.14.2024$puts$Bid
                                            + VIX.options$feb.14.2024$puts$Ask)
VIX.options$mar.20.2024$puts$Price <- 0.5*(VIX.options$mar.20.2024$puts$Bid
                                            + VIX.options$mar.20.2024$puts$Ask)
VIX.options$apr.17.2024$puts$Price <- 0.5*(VIX.options$apr.17.2024$puts$Bid
                                            + VIX.options$apr.17.2024$puts$Ask)
VIX.options$mag.22.2024$puts$Price <- 0.5*(VIX.options$mag.22.2024$puts$Bid
                                            + VIX.options$mag.22.2024$puts$Ask)
VIX.options$giu.18.2024$puts$Price <- 0.5*(VIX.options$giu.18.2024$puts$Bid
                                            + VIX.options$giu.18.2024$puts$Ask)
VIX.options$lug.17.2024$puts$Price <- 0.5*(VIX.options$lug.17.2024$puts$Bid
                                            + VIX.options$lug.17.2024$puts$Ask)
```

Showing one of the results above as proof.

```
head(VIX.options$dic.20.2023$puts[,c('ContractID','Expiration','Strike','Bid','Ask','Price')])
```

```
##           ContractID Expiration Strike Bid Ask Price
## 1 VIX231220P00010000 2023-12-20   10.0   0   0     0
## 2 VIX231220P00010500 2023-12-20   10.5   0   0     0
## 3 VIX231220P00011000 2023-12-20   11.0   0   0     0
## 4 VIX231220P00011500 2023-12-20   11.5   0   0     0
## 5 VIX231220P00012000 2023-12-20   12.0   0   0     0
## 6 VIX231220P00012500 2023-12-20   12.5   0   0     0
```

## 1.4

For calls and puts of VIX.options at each expiration, add a column of InTheMoney, which takes value TRUE when it is in-the-money, and FALSE otherwise. Compare it to ITM column to check your results. (Hint. A call option is in-the-money when its strike is less than the current price of underlying. A put option is in-the-money if its strike is greater than the current price of underlying. And the current price of underlying is the last quote price from 1.2)

```
VIX.options$nov.22.2023$puts$InTheMoney <- VIX.options$nov.22.2023$puts$Strike > last_p
VIX.options$nov.29.2023$puts$InTheMoney <- VIX.options$nov.29.2023$puts$Strike > last_p
VIX.options$dic.06.2023$puts$InTheMoney <- VIX.options$dic.06.2023$puts$Strike > last_p
VIX.options$dic.13.2023$puts$InTheMoney <- VIX.options$dic.13.2023$puts$Strike > last_p
VIX.options$dic.20.2023$puts$InTheMoney <- VIX.options$dic.20.2023$puts$Strike > last_p
VIX.options$gen.17.2024$puts$InTheMoney <- VIX.options$gen.17.2024$puts$Strike > last_p
VIX.options$feb.14.2024$puts$InTheMoney <- VIX.options$feb.14.2024$puts$Strike > last_p
VIX.options$mar.20.2024$puts$InTheMoney <- VIX.options$mar.20.2024$puts$Strike > last_p
VIX.options$apr.17.2024$puts$InTheMoney <- VIX.options$apr.17.2024$puts$Strike > last_p
VIX.options$mag.22.2024$puts$InTheMoney <- VIX.options$mag.22.2024$puts$Strike > last_p
VIX.options$giu.18.2024$puts$InTheMoney <- VIX.options$giu.18.2024$puts$Strike > last_p
VIX.options$lug.17.2024$puts$InTheMoney <- VIX.options$lug.17.2024$puts$Strike > last_p

VIX.options$nov.22.2023$calls$InTheMoney <- VIX.options$nov.22.2023$calls$Strike < last_p
VIX.options$nov.29.2023$calls$InTheMoney <- VIX.options$nov.29.2023$calls$Strike < last_p
VIX.options$dic.06.2023$calls$InTheMoney <- VIX.options$dic.06.2023$calls$Strike < last_p
VIX.options$dic.13.2023$calls$InTheMoney <- VIX.options$dic.13.2023$calls$Strike < last_p
VIX.options$dic.20.2023$calls$InTheMoney <- VIX.options$dic.20.2023$calls$Strike < last_p
VIX.options$gen.17.2024$calls$InTheMoney <- VIX.options$gen.17.2024$calls$Strike < last_p
VIX.options$feb.14.2024$calls$InTheMoney <- VIX.options$feb.14.2024$calls$Strike < last_p
VIX.options$mar.20.2024$calls$InTheMoney <- VIX.options$mar.20.2024$calls$Strike < last_p
VIX.options$apr.17.2024$calls$InTheMoney <- VIX.options$apr.17.2024$calls$Strike < last_p
VIX.options$mag.22.2024$calls$InTheMoney <- VIX.options$mag.22.2024$calls$Strike < last_p
VIX.options$giu.18.2024$calls$InTheMoney <- VIX.options$giu.18.2024$calls$Strike < last_p
VIX.options$lug.17.2024$calls$InTheMoney <- VIX.options$lug.17.2024$calls$Strike < last_p
```

Showing one of the results above as proof.

```
head(VIX.options$lug.17.2024$calls[,c('ITM','InTheMoney')])
```

```
##     ITM InTheMoney
## 1  TRUE       TRUE
## 2  TRUE       TRUE
## 3  TRUE       TRUE
## 4 FALSE      FALSE
## 5 FALSE      FALSE
## 6 FALSE      FALSE
```

## 1.5

For calls and puts of VIX at each expiration, delete all the fields except Strike, Bid, Ask, Price, and In-The-Money, and save them in .csv files with the format "VIXdata2021-09-26Exp2021-10-08puts.csv", here 2021-09-26 should be replaced by the date you download the data, and 2021-10-08 should be replaced by the date of expiration. (Hint. You may generate many .csv files, besides your .pdf report, please submit one of the .csv file.)

```r
selected_columns <- c("Strike", "Bid", "Ask", "Price", "InTheMoney")

file.path <- "VIXdata2023_11_19Exp2023_11_22calls.csv"
write.csv(VIX.options$nov.22.2023$calls[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2023_11_29calls.csv"
write.csv(VIX.options$nov.29.2023$calls[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2023_12_06calls.csv"
write.csv(VIX.options$dic.06.2023$calls[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2023_12_13calls.csv"
write.csv(VIX.options$dic.13.2023$calls[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2023_12_20calls.csv"
write.csv(VIX.options$dic.20.2023$calls[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2024_01_17calls.csv"
write.csv(VIX.options$gen.17.2024$calls[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2024_02_14calls.csv"
write.csv(VIX.options$feb.14.2024$calls[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2024_03_20calls.csv"
write.csv(VIX.options$mar.20.2024$calls[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2024_04_17calls.csv"
write.csv(VIX.options$apr.17.2024$calls[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2024_05_22calls.csv"
write.csv(VIX.options$mag.22.2024$calls[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2024_06_18calls.csv"
write.csv(VIX.options$giu.18.2024$calls[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2024_07_17calls.csv"
write.csv(VIX.options$lug.17.2024$calls[, selected_columns], file.path, row.names = FALSE)


file.path <- "VIXdata2023_11_19Exp2023_11_22puts.csv"
write.csv(VIX.options$nov.22.2023$calls[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2023_11_29puts.csv"
write.csv(VIX.options$nov.29.2023$puts[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2023_12_06puts.csv"
write.csv(VIX.options$dic.06.2023$puts[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2023_12_13puts.csv"
write.csv(VIX.options$dic.13.2023$puts[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2023_12_20puts.csv"
write.csv(VIX.options$dic.20.2023$puts[, selected_columns], file.path, row.names = FALSE)
```

```r
file.path <- "VIXdata2023_11_19Exp2024_01_17puts.csv"
write.csv(VIX.options$gen.17.2024$puts[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2024_02_14puts.csv"
write.csv(VIX.options$feb.14.2024$puts[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2024_03_20puts.csv"
write.csv(VIX.options$mar.20.2024$puts[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2024_04_17puts.csv"
write.csv(VIX.options$apr.17.2024$puts[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2024_05_22puts.csv"
write.csv(VIX.options$mag.22.2024$puts[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2024_06_18puts.csv"
write.csv(VIX.options$giu.18.2024$puts[, selected_columns], file.path, row.names = FALSE)

file.path <- "VIXdata2023_11_19Exp2024_07_17puts.csv"
write.csv(VIX.options$lug.17.2024$puts[, selected_columns], file.path, row.names = FALSE)
```

**Question 2**

## 2.1

Using Monte-Carlo Simulation to estimate the put option price using S0 = 100, K = 100, T = 1, sigma = 0.2, r = 0.05, you can use number of steps n = 252 and number of paths m = 10000

```r
S0 <- 100
K <- 100
T1 <- 1
sigma <- 0.2
r <- 0.05
n <- 252
h <- T1/n
S <- c()
Z <- c()
S[1] <- S0
m <- 10000
S.mat <- NULL

for(j in 1:m){
  for (i in 1:n) {
    Z[i] <- rnorm(1)
    S[i+1] <- S[i] + r*S[i]*h + sigma*S[i]*Z[i]*sqrt(h)
  }
  S.mat <- cbind(S.mat,S)
}

# calculate put option price
ST <- S.mat[n+1,]
```

```r
put_option_price <- exp(-r*T1)*mean(pmax(100 - ST, 0))
put_option_price
```

```
## [1] 5.652778
```

## 2.2

Implement Black-Scholes formula for pricing the put option Check the difference between the Black-Scholes price and the Monte-Carlo price.

```r
bs.put <- function(S0, K, T1, sigma, r){
d1 <- (log(S0/K) + (r+0.5*sigma^2)*T1)/(sigma*sqrt(T1))
d2 <- d1 - sigma*sqrt(T1)
return(-S0*pnorm(-d1) + exp(-r*T1)*K*pnorm(-d2))
}

bs.put(100,100,1,0.2,0.05)
```

```
## [1] 5.573526
```