Tutorium 8th Session

Long lived comet:

Hammersley and Handscomb (1964) presents a Monte Carlo calculation for the lifetime of a long lived comet. In their model, a comet with energy $x < 0$ takes time $(-x)^{-\frac{3}{2}}$ to complete one orbit, while one with $x > 0$ just leaves the solar system. Due to gravitational interaction with planets, the energy level changes randomly. Their model is that x changes to $x + Z$, where $Z \sim \mathcal{N}(0, \sigma^2)$. By choosing the units of energy appropriately, they can use $\sigma = 1$. A comet that starts with energy x_0 will stay in the solar system for time

$$T = \sum_{j=0}^{m-1} (-x_j)^{-\frac{3}{2}},$$

where $x_{j+1} = x_j + z_j$ and $m = \min\{j | x_j > 0\}$ indexes the first orbit to obtain a positive energy. Because the number of m of orbits to count is itself random it is hard to study the distribution of T analytically. However, the quantity T is well suited to Monte Carlo sampling and we can estimate $\mathbb{P}(T < t)$, i.e. the cumulative distribution function.

Consider a sample of $N = 10'000$ comets starting with $x_0 = -1$ and continuing until they either get positive energy (leaving the solar system) or complete 10^5 orbits (to make the problem computationally tractable).

- Compute the mean and the standard deviation of the sample. Repeat the experiments more times. What do you observe?
- Plot the survival distribution function, i.e. $\mathbb{P}(T > t)$.
- Compute the Chebyshev confidence interval for the probability that a comet will make more than 10^k, for $k \in \{1, 2, 3, 4\}$.
- For those comets observed to complete more that 10^4 orbits, show the histogram of the number of orbits observed.
- For those comets observed to leave the solar system, show the histogram of the number of orbits observed.

Hammersley, J. M. (1961). *On the statistical loss of long period comets from the solar system, ii. In Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability*, volume 3, pages 17–78.

Hammersley, J. M. and Handscomb, D. C. (1964). *Monte Carlo methods.* Methuen, London.