Complex Numbers

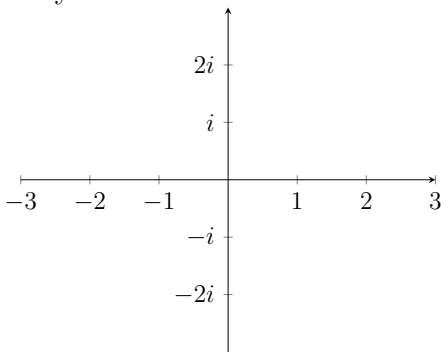
Lorenzo Bonanni

October 2021

Source: *Complex number fundamentals — Lockdown math ep. 3*

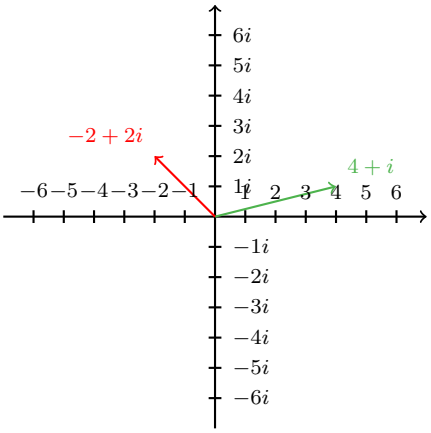
1 Assumptions

- There's a number i so that $i^2 = -1$
- i stays on a different number line that's perpendicular to Real Numbers



2 Operations

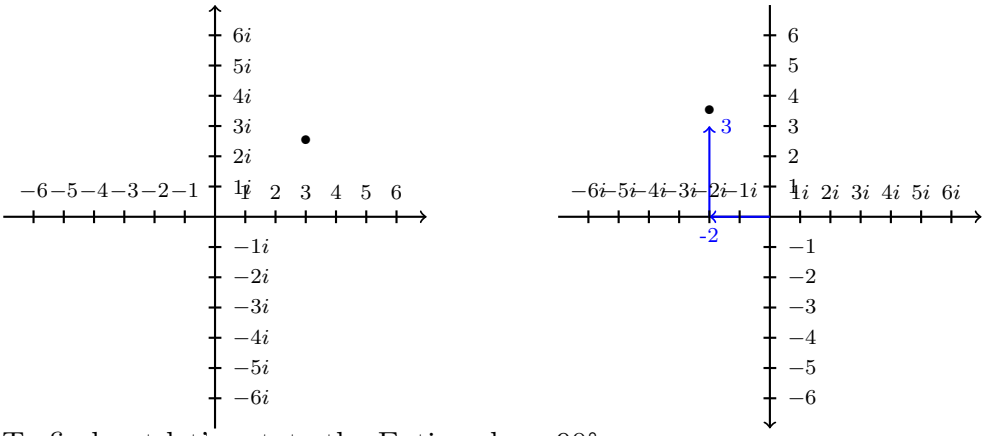
2.1 Sum



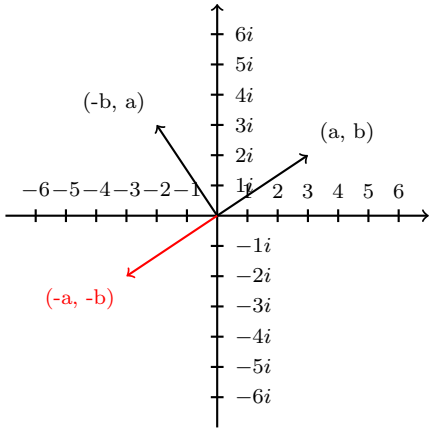
To sum those two vectors $-2+2i$ and $4+i$ we simply divide the imaginary part from the real one and then we sum those single components.
Real Part: $(4-2)$
Imaginary Part: $(1-2)i$
So the **result** is $2+3i$

2.2 Multiplication

Suppose we have the point (3,2) what is the 90° rotation of that point counterclockwise?



To find out let's rotate the Entire plane 90°
(3,2)-90°CW→(-2,3)
(a,b)-90°→(-b,a)-90°→(-a,-b)



Lets calculate the following equation $3 * (3 + 2i)$:
$3 * (3 + 2i) = 3i + 2i^2 = 3i + 2 * (-1) = -2 + 3i$
As we can see the result is like rotating $3 * (3 + 2i)$ by $90°$

2.2.1 3 Facts about Multiplications

1. $z * 1 = z$
2. $z * i = Rot90(z)$
3. $z * (c + di) = c * z + d * (zi)$

Let's solve $(2 + i)(2 - i)$
$(2 + i)(2 - i) =$
$2 * 2 + 2i - 2i - i^2 =$
$4 + 0i - (-1) =$
$5 + 0i = 5$

What is the complex number z so that multiplying by z has the effect of rotating 30°, or $\frac{\pi}{6}$ radians, counterclockwise?
$z = cos(\pi/6) + isin(\pi/6)$