1.1.1 Elastic twist of the typical section

Consider the typical section model swont in figure 1.1.

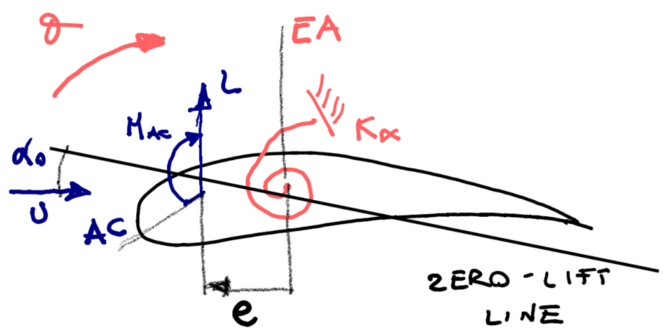

Figure 1.1: Typical section model

The action of aerodynamic pressures on the airfoil may be globally represented by a force called "Lift" acting through the Aerodynamic Center (AC), and a moment applied to the same point. The lift generate by the airfoil will be expressed, using a linear approximation, as

$$L = qSC_{L\alpha}\alpha \tag{1.1}$$

The moment will be independent of the angle of attack and expressed as

$$M_{AC} = qScC_{m_{ac}} \tag{1.2}$$

where c is the chord of the wing. Initially the direction of the flow is parallel to the direction of the zero-lift line. Then the airfoil is rotated by an angle α_0 as a constrained system to geenrate the lift required by the trim condition (e.g. the angle of attack necessary to generate a lift equal to the weight of the wing). Then the constraint is released an the airfoil is allowed to rotate elastically around the elastic axis EA through an additional twist angle θ. The total angle of attack will be so equal to

$$\alpha = \alpha_0 + \theta \tag{1.3}$$

To generalize, it will be possible to express the lift as

$$L = qSC_L(\alpha_0) + qSC_{L\alpha}\theta \tag{1.4}$$

where the lift is considered nonlinear, and the linearisation is applied only to the effect of the torsional deformation. Sometimes $C_L(\alpha_0)$ is also indicated as C_{L0} that is the lift coefficient necessary to the model to generate the required lift in the reference trim condition under consideration.

To identify the elastic deformation it is necessary to write the equilibrium equation about the EA considering the torsional sping k_α and the moment generated by aerodynamic forces

$$K_\alpha \theta = Le + M_{AC} \tag{1.5}$$

where e is the distance between the AC and the EA. Substituting the expression of the aerodynamic actions

$$K_\alpha \theta = qSeC_{L\alpha}(\alpha_0 + \theta) + qScC_{m_{ac}} \tag{1.6}$$

$$(K_\alpha - qSeC_{L\alpha})\theta = qSeC_{L\alpha}\alpha_0 + qScC_{m_{ac}} \tag{1.7}$$

Solving for θ we can see that the resulting torsional deformation of the TS will be

$$\theta = qS\frac{eC_{L\alpha}\alpha_0 + cC_{m_{ac}}}{K_\alpha - qSeC_{L\alpha}} \tag{1.8}$$

So in general we can expect that the larger is the dynamic pressure q the larger will be the torsional deformation obtained.