1.1.5 Multi Degrees of Freedom systems: computation of equivalent springs

Computation of the stiffness of springs between two portions of the MDOF model of the wing. Lt's call k_{i+1} the stiffness of the spring between the two section characterized by the degrees of freedom θ_i and θ_{i+1}. The centre line of the first portion is positioned at a distance y_i from the wing root while the second one is at distance y_{i+1}. To compute the equivalent spring the VWP is used, but now the virtual work is defined as the product of the real deformation on the structure multiplied by a virtual load, an infinitesimal load compatible with the boundary conditions. Let's consider the case where we have a torsional moment M_{tR} applied at y_{i+1}, and as virtual load let's take a moment δM_t applied at the same position. Remembering that the torsional deformation for a beam is defined as M_t/GJ, the internal virtual work is

$$\delta W_i = \int_0^L \frac{M_{tR}}{GJ}\delta M_t \mathrm{d}\xi = \int_0^{y_{i+1}} \frac{M_{tR}}{GJ}\delta M_t \mathrm{d}\xi = \frac{M_{tR}y_{i+1}}{GJ}\delta M_t \tag{1.55}$$

The external work will be equal to the product of the real deformation at the point of application of the moment times the virtual loading applied there

$$\delta W_e = \theta_{i+1}\delta M_t \tag{1.56}$$

Putting all together we have

$$\left(\theta_{i+1} - \frac{M_{tR}y_{i+1}}{GJ}\right)\delta M_t = 0 \tag{1.57}$$

Given the arbitrariness of the virtual moment δM_t it is possible to compute the absolute rotation for the section at y_{i+1}. Considering now a virtual load applied at y_i we get

$$\left(\theta_i - \frac{M_{tR}y_i}{GJ}\right)\delta M_t = 0 \tag{1.58}$$

The equivalent stiffness k_i is defined as the ratio between the moment applied M_{tR} and the relative rotation between the two stations y_{i+1} and y_i, so

$$\Delta\theta = \theta_{i+1} - \theta_i = \frac{y_{i+1} - y_i}{GJ}M_{tR}. \tag{1.59}$$

Consequently,

$$k_i = \frac{GJ}{\Delta y} \tag{1.60}$$