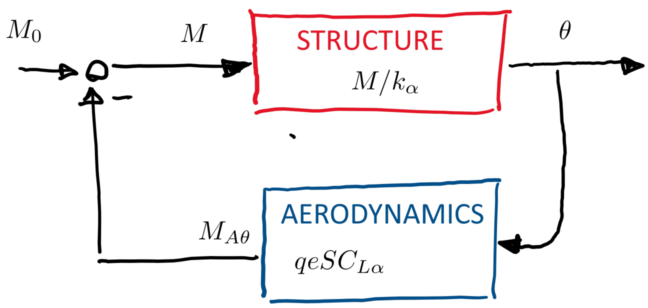

Figure 1.2: Feedback structure for the static aeroelastic torsional problem.

1.1.2 Analysis of the interative algorithm for the solution of the static aeroelastic problem

Given the feedback structure of the static aeroelastic torsional problem for the typical section, shown in Figure 1.2, it is possible to design an iterative algorithm to solve the problem and find the torsion angle θ. The iterative algorithm is

$M = M_0 = qS(eC_{L0} + cC_{m_{CA}})$
$\theta_0 = 0$
$\theta_1 = M/k_\alpha$
$i = 1$
While $|\theta_{i-1} - \theta_i > \text{Toll}$
    $i++$
    $M_{A\theta} = qk_A\theta_{i-1}$
    $M = M_0 + M_{A\theta}$
    $\theta_i = M/k_\alpha$
EndWhile

The iterative algorithm leads to a linear difference equation

$$k_\alpha\theta_{i+1} - qSeC_{L\alpha}\theta_i = M_0 \tag{1.9}$$

The solution to this equation is the composition of a general solution θ_g plus the solution to the forcing term θ_f

$$\theta = \theta_g + \theta_f \tag{1.10}$$

The general solution has the form $\theta_{gi} = A\rho^i$. To compute it we need to substitute into Eq.(1.9) without the forcing term

$$k_\alpha A\rho^{i+1} - qSeC_{L\alpha}A\rho^i = 0 \tag{1.11}$$
$$A\rho^i(k_\alpha\rho - qk_A) = 0 \tag{1.12}$$
$$\rho = q\frac{k_A}{k_\alpha} = \frac{q}{q_D} \tag{1.13}$$

The solution ot the forcing term i.e., the regime solution is obtained when $\theta_{i+1} = \theta_i$. So,

$$\theta_F = \frac{M_0}{k_\alpha - qk_A} \tag{1.14}$$

So the solution is

$$\theta_i = A\left(\frac{q}{q_D}\right)^i + \frac{M_0}{k_\alpha - qk_A}. \tag{1.15}$$

If we add the initial condition $\theta_0 = \tilde{\theta}$, it is possible to compute the constant A

$$\theta_0 = \tilde{\theta} = A + \frac{M_0}{k_\alpha - qk_A}. \tag{1.16}$$

$$A = \tilde{\theta} - \frac{M_0}{k_\alpha - qk_A}. \tag{1.17}$$

The complete solution is so equal to

$$\theta_i = \tilde{\theta}\left(\frac{q}{q_D}\right)^i + \left(1 - \left(\frac{q}{q_D}\right)^i\right)\frac{M_0}{k_\alpha - qk_A} \tag{1.18}$$

It is possible to see that the iterative solution converges to the exact solution for $i \to \infty$ when $q < q_D$ while in the other cases it does not converge. Additionally, the more q is close to q_D the slower is the convergence toward the solution.