THE ATMOSPHERE

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Introduction

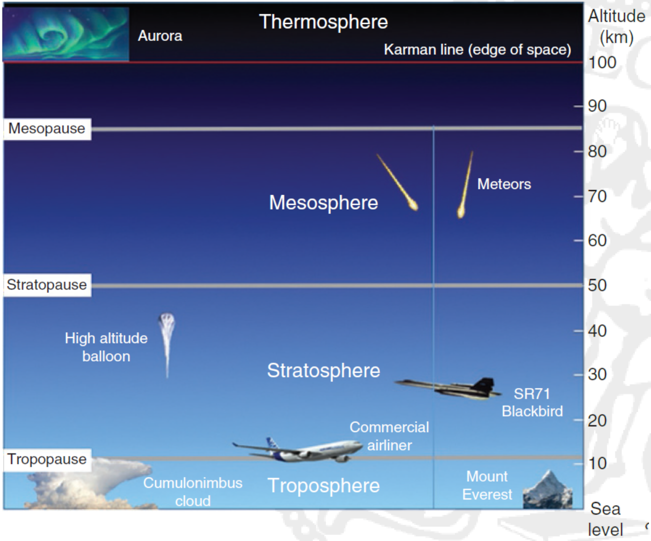
The knowledge of the value of a relevant number of quantities that characterise the physical and thermodynamic state of the air the aircraft is flying through and the relative motion of the latter with respect to the former is crucial for many aspects.

- Aerodynamic forces
- Safe flight envelope is given in term of air-related quantities (e.g. airspeed, pressure altitude, Mach number, etc.)
- Procedures and manoeuvres also rely on air-data
- thrust system(s) performance is affected by air data

International Standard Atmosphere (ISA)

- Developed by Air Research Development Command in 1959.
- Based on statistical observations carried out in North America.
- The only independent variable is altitude (i.e. no influence of latitude and longitude).
- The model is time-invariant.
- Only gravity acceleration acts on air (i.e. Coriolis acceleration is neglected).
- Atmosphere is at rest. No horizontal pressure gradients neither vertical movements are present.
- Atmosphere is fixed with Earth surface.
- No discontinuity is present.
- Simplified mathematical model describes thermodynamic behaviour

Layered model of atmosphere



ISA Equations: Perfect Gas

In ISA air is considered to obey to the perfect gas state equation, which can be rewritten as:

$$p = \rho R^* T \tag{1}$$

R^* is the dry air constant and is given by:

$$R^* = \frac{R}{M_{air}} = 287\,\mathrm{J\,kg^{-1}\,K^{-1}}$$

where:

- $R = 8.314\,51\,\mathrm{J\,mol^{-1}\,K^{-1}}$ is the universal gas constant
- $M_{air} = 28.97\,\mathrm{g\,mol^{-1}}$ is the molecular weight of dry air

ISA Equations: Hydrostatic balance

Hydrostatic balance can be expressed as:

$$dp = -\rho\, g\, dh \tag{2}$$

where:

- h is the altitude
- g is the gravitational acceleration, given by:

$$g = \frac{R_E^2}{(R_E + h)^2}\, g_0$$

  being

  - $R_E = 6.3567 \times 10^6$ m the Earth radius
  - $g_0 = 9.81\,\mathrm{m\,s^{-2}}$

For the purpose of the course we will assume

$$g \approx g_0 \tag{3}$$

ISA Equations: temperature gradient

In the troposphere (i.e. $h \leq 11 \times 10^3\,\mathrm{m}$) the variation of temperature T with altitude h is given by the relation:

$$dT = a\,dh \tag{4}$$

where the gradient a is

$$a = -6.5 \times 10^{-3}\,\mathrm{K\,m^{-1}}$$

In the stratosphere (up to $h = 20 \times 10^3\,\mathrm{m}$) it becomes:

$$T = const = T_{ST} \tag{5}$$

ISA Standard conditions

Since eq.(1), (2) and (4) are differential equations, an appropriate set of initial conditions is needed.

The ISA model sets p, ρ and T at sea level to the following values:

$$\begin{cases} p_0 = 101\,325\,\text{Pa} = 1013.25\,\text{hPa} = 29.92\,\text{in Hg} \\ \rho_0 = 1.225\,\text{kg}\,\text{m}^{-3} \\ T_0 = 288.15\,\text{K} = 15\,^\circ\text{C} \end{cases} \tag{6}$$

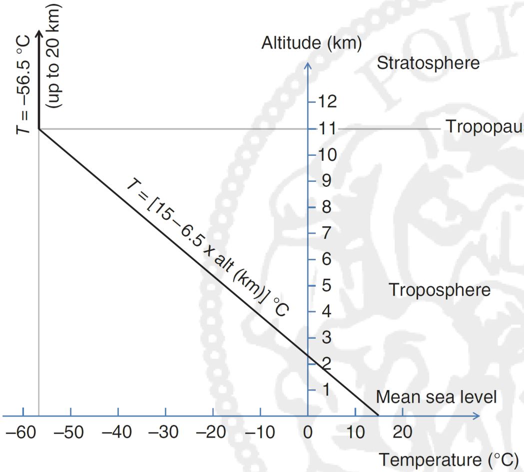
Using T_0 in (4) we obtain:

$$T_{ST} = 216.65\,\text{K} = -56.5\,^\circ\text{C}$$

NOTE

Values of p, ρ and T are not independent, since they must obey eq.(1). Given two, the value of the third is uniquely determined.

ISA Temperature vs altitude



Variation of T with altitude

Altitude-Static Pressure Relationship

Combining eq.(1) and (2) and remebering (3) yields:

$$-\frac{dp}{p} = \frac{g}{R^* T}\, dh \tag{7}$$

Two different cases are examined since the variation of T with h follows different models depending on altitude:

- Troposphere region where $T(h)$ is given by (4)
- Initial part of stratosphere, where $T(h)$ is given by (5)

Altitude-Static Pressure relationship in **troposphere**

Substituting for T in (7) and integrating both sides gives

$$-\int_{p_0}^{p} \frac{1}{p}\,dp = \frac{g_0}{R^*}\int_0^h \frac{1}{(T_0 - ah)}\,dh \tag{8}$$

from which

$$\log_e \frac{p}{p_0} = \frac{g_0}{a\,R^*}\log_e \frac{(T_0 - ah)}{T_0} \tag{9}$$

hence

$$p = p_0\left(1 - \frac{a}{T_0}\,h\right)^{\frac{g_0}{a\,R^*}} \tag{10}$$

and solving for h:

$$h = \frac{T_0}{a}\left[1 - \left(\frac{p}{p_0}\right)^{\frac{a\,R^*}{g_0}}\right] \tag{11}$$

Altitude-Static Pressure relationship in **stratosphere**

Substituting for T in (7) and integrating both sides gives

$$- \int_{p_T}^{p} \frac{1}{p}\, dp = \frac{g_0}{R^*\; TST} \int_{h_T}^{h} dh \tag{12}$$

where P_T is the value of pressure at tropopause altitude h_T.
Whence

$$p = p_T\; e^{\left(\frac{g_0}{a\, R^*}\right)(h - h_T)} \tag{13}$$

and solving for h:

$$h = h_T + \frac{R^*\; T_T}{g_0} \log_e \frac{P_T}{p} \tag{14}$$

Dependency of baro-altitude on T_0 and p_0

As it can be noted in eq (11), value of barometric altitude is dependent on the current values of temperature and pressure at sea level T_0 and p_0.

NOTE

Dependency of barometric altitude on T_0 is neglected, so in calibration formulas T_0 given by (6) is assumed. The error introduced with this assumption is considered to be acceptable.

However, errors due to the variation in the ground pressure from the assumed standard value must be accounted for, are taken out of the altimeter reading by setting a scale to a given pressure.

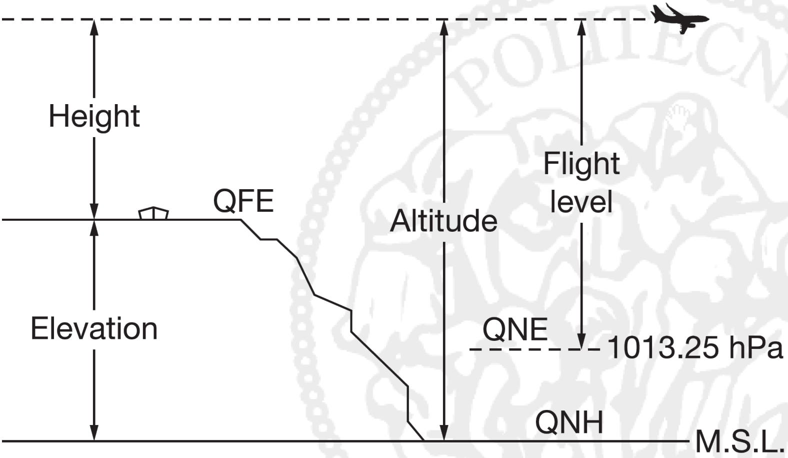
This action affects the zero point and so alters the altimeter reading over the whole of its range by a height corresponding to the pressure set as determined by the altitude-pressure law.

Altimeter setting

Three possible altimeter settings, labelled according to the so-called *Q-codes*, are in use for air navigation:

- **QNH**: p_0 is set to the current value of pressure at sea level. The value read on altimeter is defined *altitude*.
- **QFE**: p_0 is set to the current value of pressure at ground level - generally at aerodrome level. The value read on altimeter is defined *height*.
- **QNE**: p_0 is set to standard 1013.25 hPa. The value read on altimeter, divided by 100, is defined *flight level* (e.g. reading 37 000 ft becomes FL 370).

Altimeter setting



List of Acronyms

ISA International Standard Atmosphere