POLITECNICO MILANO 1863

055738 – STRUCTURAL DYNAMICS AND AEROELASTICITY

# 08 Structural Dynamics: Modeling of structural damping

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# Material

Preumont Section 2.4

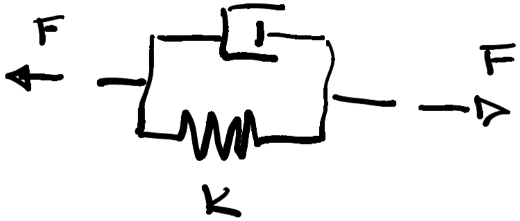
# Nature of dissipation forces

All structures have mechanism that leads to dissipation of energy. The dissipation is responsible of the decay of harmonic oscillations when set in motion.

There are several sources of dissipation:

- Dissipation associated with *straining of the material*, due to viscoelastic behavior of the constitutive law (very limited in metallic material, more relevant in composite structures due do matrix and to micro-sliding of fibers)
- Dissipation due to *friction for sliding* in inner joints, support connections etc. This is extremely important for complex structures made by several parts

# Visco-elastic behavior: Kelvin-Voigt



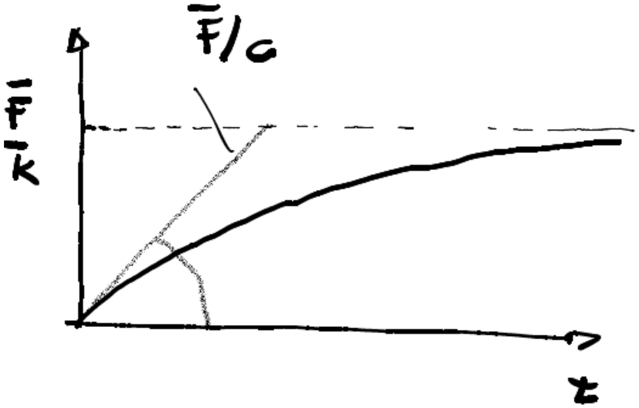

If $F = \bar{F} = \text{const.}$

$$u(t) = -\frac{\bar{F}}{k}\mathrm{e}^{-\frac{k}{c}t} + \frac{\bar{F}}{k}$$

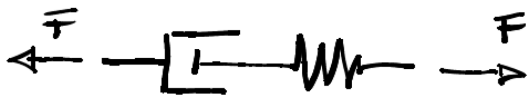
If $u = \bar{u} = \text{const.}$ then $F = k\bar{u}$.

$$\begin{cases} F &= F_e + F_d \qquad F_e = k u_e \\ U_e &= u_d = u \qquad F_d = c\dot{u}_d \end{cases}$$

$$F = ku + c\dot{u}$$



# Visco-elastic behavior: Maxwell

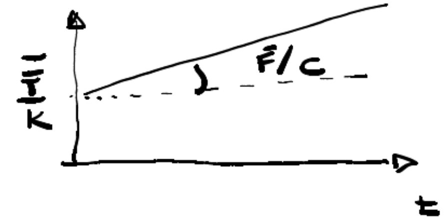



$$\begin{cases} F &= F_e = F_d \\ u &= u_e + u_d \end{cases} \qquad \begin{aligned} F_e &= k u_e \\ F_d &= c \dot{u}_d \end{aligned}$$

$$\dot{u} = \frac{\dot{F}}{k} + \frac{F}{c}$$


If $F = \bar{F} = \text{const.}$

$$\dot{u} = \frac{\bar{F}}{c}, \; u(0) = \frac{\bar{F}}{k}$$




$$u = \bar{F}\left(\frac{1}{k} + \frac{t}{c}\right)$$

If $u = \bar{u} = \text{const.}$

$$\begin{aligned} \dot{u} &= 0 \\ \frac{\dot{F}}{F} &= -\frac{k}{c}, \quad F(0) = k\bar{u} \end{aligned}$$

$$\log F = -\frac{kt}{c}, \rightarrow \; F = k\bar{u}e^{-\frac{kt}{c}}$$

# Equivalent damping

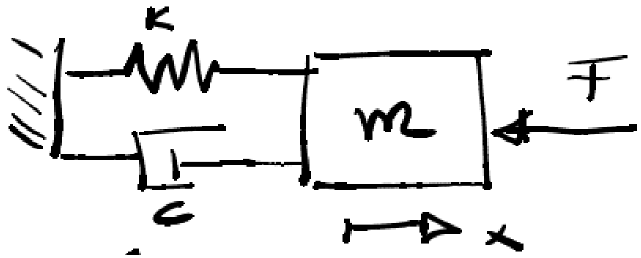
The behavior of this dissipation is complex, nonlinear, not-deterministic, characterized by a dependance on many parameters and operative conditions, and consequently it is *extremely difficult to be modelled* starting from first principles.

For small amplitude oscillations the mechanism can be represented through a linear contribution proportional to the time derivative of the free coordinates

$$F_d = \mathbf{C}\,\dot{u}$$

# Equivalent damping

Given a certain structural system it is possible to identify an equivalent damping exploiting a similitude with a harmonic damped oscillator. If $\xi \ll 1$ apply an harmonic excitation at the natural frequencies (i.e. eigenfrequencies) of the structure $\omega_n$




$$m\ddot{x} + c\dot{x} + kx = F$$
$$F = F_0 \sin \omega_n t$$

Making the hypotesis to be exactly in resonace condition

$$\rightarrow \ x = x_0 \cos \omega_n t$$

$$m\ddot{x} + 2\xi m \omega_n \dot{x} + \omega_n^2 x = F$$

Sustituting this expression in the dynamic equation

$$(-\omega_n^2 m + k) x_0 \cos \omega_n t - \omega_n c x_0 \sin \omega_n t = F_0 \sin \omega_n t$$

$$\Rightarrow x_0 = \frac{F_0}{\omega_n c} = \frac{F_0}{2k\xi}$$

# Equivalent damping

The work of viscous damping is proportional to the frequency: the viscous force grows with frequency

Reaction force on ground $R$

$$\text{quad}(z) = \begin{cases} 0 < z < \pi & 1 \\ \pi \leq z < 2\pi & -1 \end{cases}$$

$$\begin{aligned} R &= kx + c\dot{x} \\ R &= kx - c\omega_n x_0 \sin\omega_n t \\ R &= kx - c\omega_n x_0 \text{quad}(\omega_n t)\sqrt{1-\cos^2\omega_n t} \\ R &= kx - c\omega_n x_0 \text{quad}(\omega_n t)\sqrt{1-\left(\frac{x}{x_0}\right)^2} \end{aligned}$$

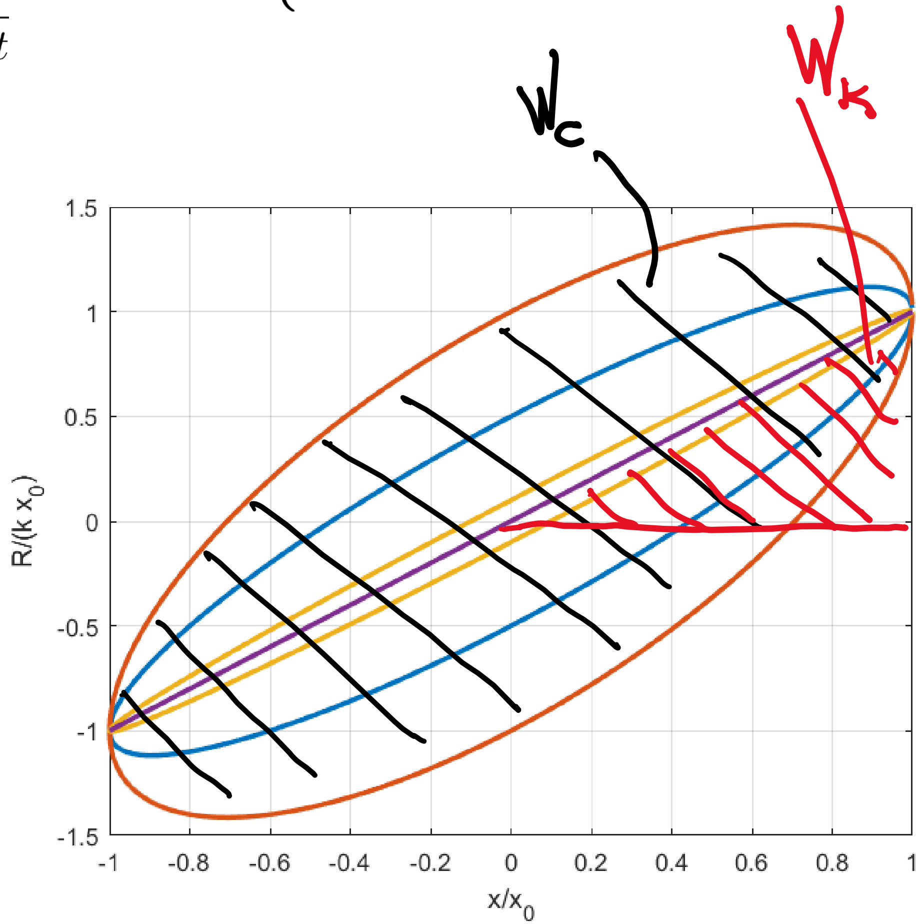
$$W_c = \oint R\mathrm{d}x = \oint kx\mathrm{d}x + \oint c\dot{x}\mathrm{d}x$$

$$\oint kx\mathrm{d}x = 0$$

$$W_c = 4\int_0^{x_0} cx_0 w_n \sqrt{1-\left(\frac{x}{x_0}\right)^2}\mathrm{d}x$$

$$W_c = \pi c x_0^2 \omega_n$$

$$W_k = \int_0^{x_0} kx\mathrm{d}x = \frac{1}{2}kx_0^2$$



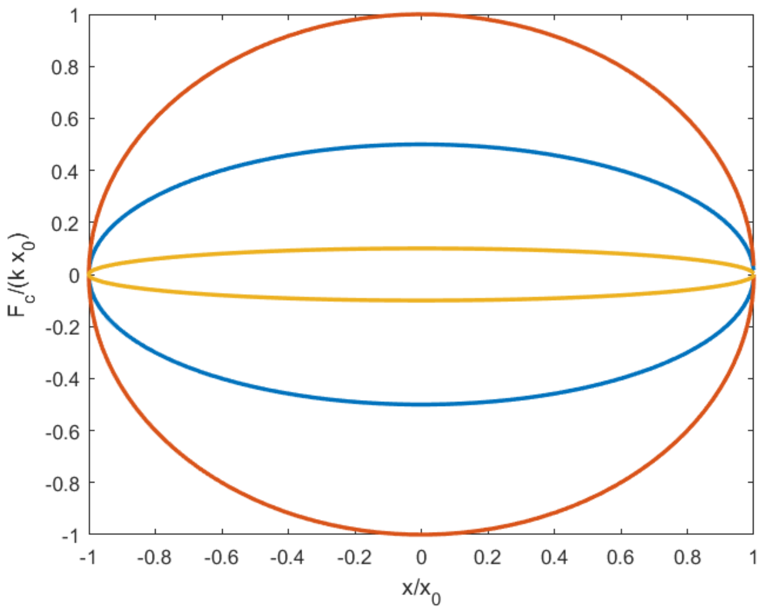
# Equivalent damping

Energy dissipation coefficient $e_D$

$$e_D = \frac{W_c}{2\pi W_k} = \frac{1}{2\pi}\frac{\pi c x_0^2 \omega_n}{\frac{1}{2}k x_0^2} = \frac{c\omega_n}{k} = 2\xi$$

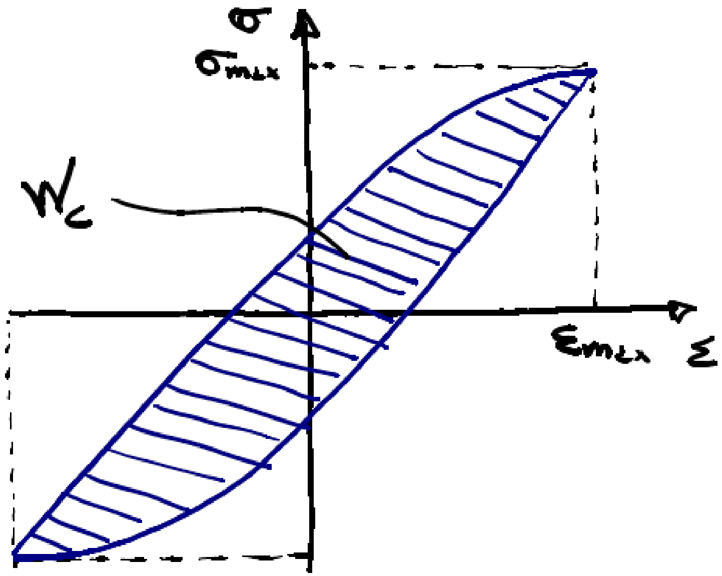
$$\xi = \frac{W_c}{4\pi W_k}$$

In this way it is possible to compute the "Equivalent damping"



# Equivalent damping

In principle it is reasonable to consider the possibility to evaluate the intrinsic material damping.




$$
\begin{aligned}
W_c &= \int_\varepsilon \sigma_{ij} \mathrm{d}\varepsilon_{ij} \\
\xi_{eq} &= \frac{W_c}{4\pi W_k} \\
W_k &= \frac{1}{2} E \varepsilon_{\max}^2
\end{aligned}
$$

# Modal Damping

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0}, \quad \mathbf{q} = \mathbf{\Phi}\mathbf{z}$$
$$\mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi}\ddot{\mathbf{z}} + \mathbf{\Phi}^T\mathbf{C}\mathbf{\Phi}\dot{\mathbf{z}} + \mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi}\mathbf{z} = \mathbf{0}$$

The <u>NORMAL DAMPING</u> is defined as a case where the modal damping matrix is diagonal i.e.,

$$\mathbf{\Phi}^T\mathbf{C}\mathbf{\Phi} = \begin{bmatrix} \ddots & & \\ & 2\xi_i\mu_i\omega_i & \\ & & \ddots \end{bmatrix}$$

with $\xi_i$ the midal damping ratio.

# Rayleigh damping

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$$
$$\rightarrow \xi_i = \frac{1}{2}\left(\frac{\alpha}{\omega_i} + \beta\omega_i\right)$$

The constants $\alpha$ and $\beta$ could be computed starting from the experimental evaluation of modal damping factors $\xi_i$

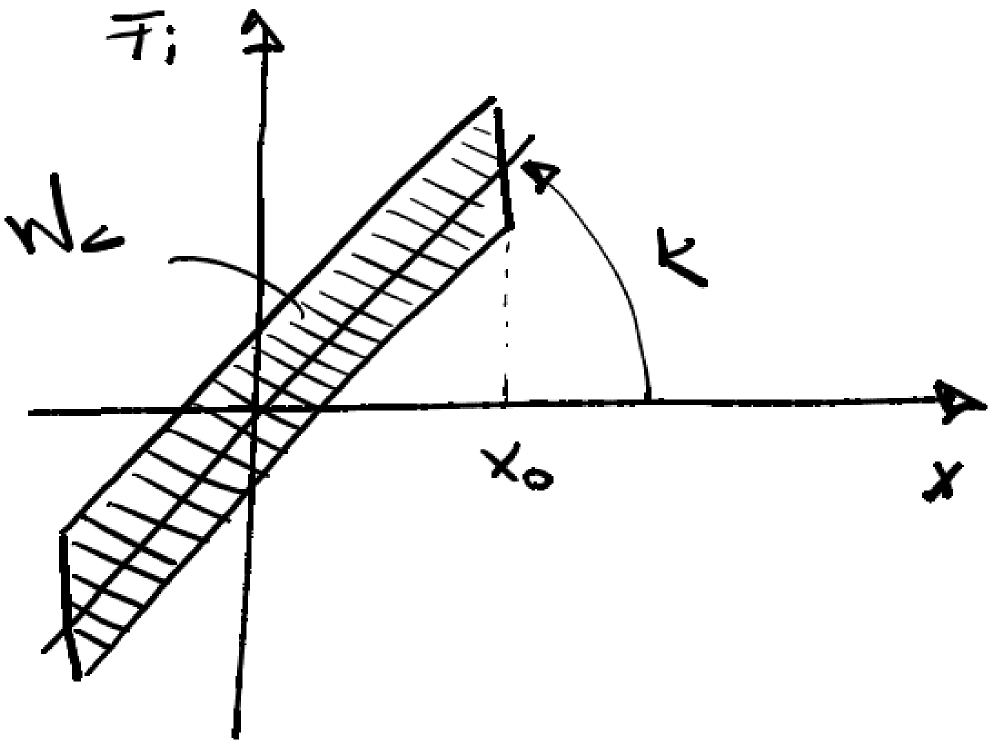
The two constants are sufficient to obtain a perfect modal diagonal damping only if two modes are considered, otherwise a “best fit” that minimize the error (eventually weighted to better approximate the most important modes) could be used

# Equivalent damping: Coulomb friction

$$F_d = \text{sign}(x)\mu N$$

with $N$ the normal reaction force
and $\mu$ the friction coefficient

$$\begin{aligned} W_k &= \frac{1}{2}kx_0^2 \\ W_c &= 4\mu N x_0 \end{aligned}$$




Consequently

$$\xi = \frac{2\mu N}{\pi k x_0}$$

# Hysteretic damping

The viscous damping does not simulate satisfactorily actual engineering materials. Many materials, when subjected to cyclic loading, exhibit a type of internal damping causing energy losses per cycle that are proportional to the square of the amplitude and independent of the frequency

If $W_c = \pi h x_0^2$

$$\xi = \frac{W_c}{4\pi W_k} = \frac{\pi h x_0^2}{4\pi \frac{1}{2} k x_0^2} = \frac{h}{2k}$$

$$F_c = 2m\xi\omega_n \dot{x} = \frac{1}{\omega_n^2} h\omega_n \dot{x}$$

$$F_c = \frac{h}{w_n}\dot{x}$$

Transforming in frequency domain $F_c = \mathrm{j}hx$. So, the equation of motion is

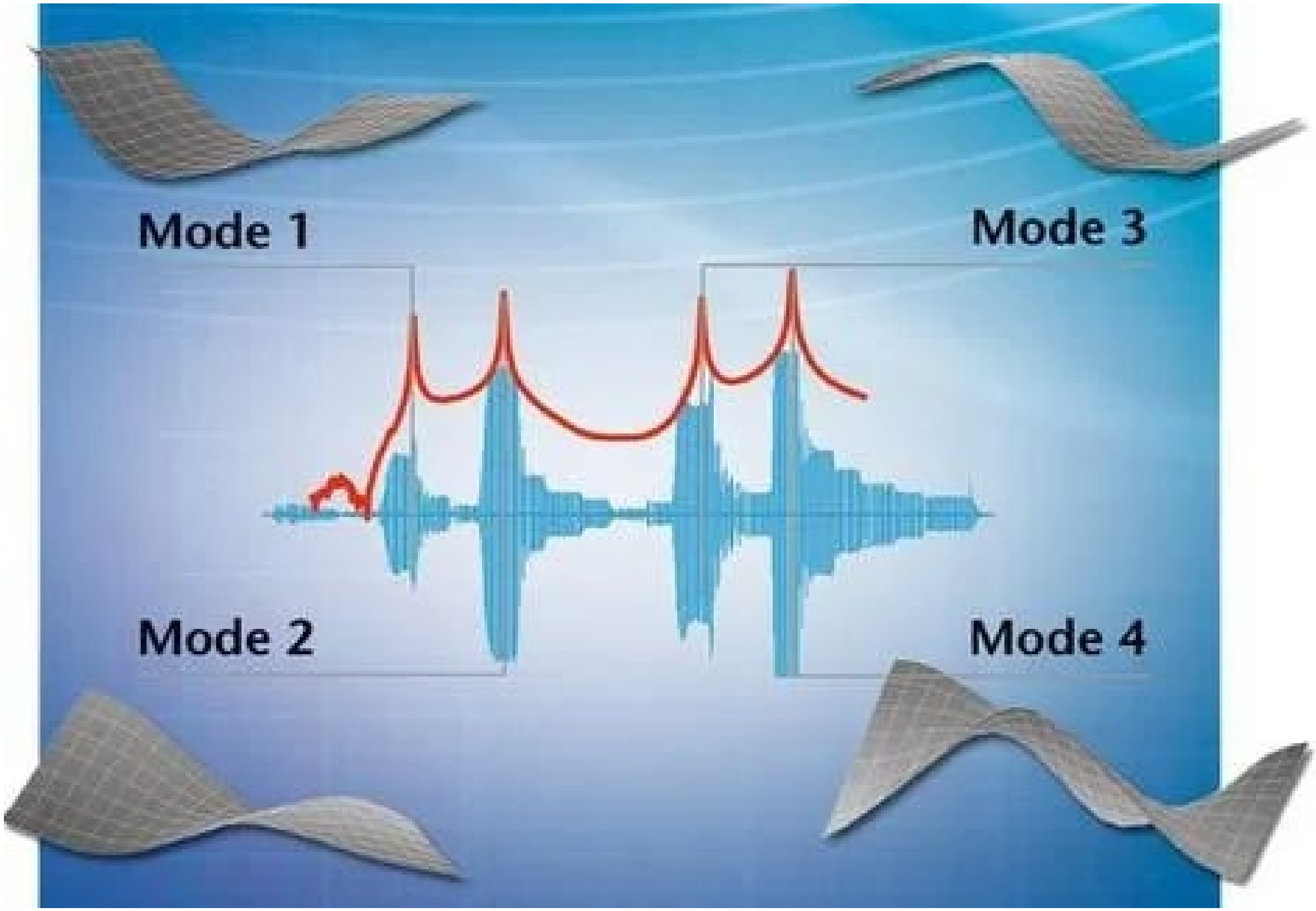
$$m\ddot{x} + k(1 + \mathrm{j}\eta)x = 0$$

$$\eta = \frac{\xi}{2}$$

The hysteretic damping is often represented as a complex stiffness (in frequency domain!!) showing a dependance on amplitude x and not on frequency

η **loss factor**

# Identification of modal forms



# Typical damping values

| | |
|---|---|
| Space Structures | 0.1 – 0.5 % |
| Aircraft structures with joints | 2.0 – 4.0 % |
| Civil structures | >= 5.0 % |