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THE EXPANDING DOMAIN OF AEROELASTICITY

by

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SUMMARY.

BY MEANS of a broad survey of the developments in aeroelastic science during the past ten years and of comparable developments in associated fields, in so far as they involve elastic deformation, the present paper attempts to show how the individual subjects concerned are developing strong interconnections. At one time the subjects were for the most part regarded as discrete entities; now they are developing into component parts of an integrated whole —the dynamics of a deformable aeroplane.

It is suggested that in the future it may be necessary to discontinue the present methods of treatment of the component subjects as separate, and to adopt a standard method of attack on the problem treated as a unified whole, especially when adequate computational aids become available. A modified approach, involving an arrangement of the subjects in a frequency spectrum, is also discussed. Some speculations on the elastic provisions which may be necessary in the future are included.

1. INTRODUCTION.

The branch of aeronautical science which has become known as aeroelasticity is of comparatively recent development. It was first studied as such rather more than a decade ago, when Roxbee Cox and Pugsley, following their pioneer work on loss and reversal of aileron control and on the rolling power of a monoplane, began a correlation of this work with the flutter investigations of Frazer, Duncan and Collar. This correlation led to the formulation and development of stiffness criteria for aircraft wings and components: criteria which have combined an admirable simplicity with a remarkable degree of success in the prevention of troubles involving elastic deformation of aircraft.

In 1937, Pugsley presented to the Royal Aeronautical Society a statement[1] of the progress of the subject to that date. Since then, no general review has been made, in spite of great developments and changes of emphasis, particularly during the past few years. The rapid advances in aeronautical science which have been made during the war have resulted in corresponding strides in aeroelasticity, and the subject has been expanded to such an extent that it is now encroaching on fields which have previously been regarded as the preserves of other subjects; and indeed, the subjects involved may now be regarded as beginning to merge into one another.

The implications of this trend of development have an obvious interest, and it is thought that a discussion of the existing state of affairs may be helpful at the present time. Accordingly, the following notes have been prepared to indicate how aeroelasticity is beginning to link up with other subjects, and what may be the future methods of dealing with this coalescence of subjects.

2. THE ORIGINAL SCOPE OF AEROELASTICITY.

Although Pugsley's paper[1] drew attention to the possible extension of aeroelastic studies to tailplanes, in fact the phenomena dealt with referred exclusively to wings: and indeed, at the time, little attention had been paid to aircraft components other than wings. Work on tail units had been carried out in flutter and buffeting investigations, but from the point of view of stiffness wings presented the main problem. Pugsley illustrated his subject by a stiffness diagram in which the ordinate was a modified (non-dimensional) wing torsional stiffness criterion and the abscissa, nominally, the wing density: in point of fact his independent variable might equally well have been taken as time, since there had been a steady increase in wing density through the years, and he was regarding the future as a period of very high wing density.

He showed how the criterion had changed with increasing wing density, and how it had been modified by the influence of various phenomena in turn: these phenomena, in chronological order of importance, were:—

(*a*) Wing-aileron flutter.
(*b*) Reversal of aileron control.
(*c*) Wing flexure-torsion flutter.
(*d*) Wing divergence.

In the case of wing-aileron flutter, aileron mass-balance had provided the cure; for the other three problems, however, wing torsional stiffness was shown to be the property of prime importance. Pugsley's paper concluded with some speculations as to which of the three phenomena, if any, would be the determining factor for the elastic properties of wings.

3. THE FORCES INVOLVED.

As a preliminary to the discussion of the expansion of the domain of aeroelasticity, it is convenient here to consider what forces are involved in the subject. The name itself defines by implication two of the forces, namely, aerodynamic forces and elastic forces; and these appear alone in studies of unaccelerated motion such as problems of steady rolling or aileron reversal. The flutter and buffeting fields, however, introduce a third and equally important type, namely, inertia forces. In addition, there may be certain other forces, for example, gravitational forces which make contributions to the total stiffnesses; but these are relatively unimportant, and we shall restrict ourselves to the consideration of aerodynamic, elastic and inertia forces.

4. THE "TRIANGLE OF FORCES" AND PROBLEMS AKIN TO AEROELASTICITY.

It is illuminating to consider some of the problems governed by the three types of force mentioned above; to do this, we may make use of the diagram shown in Fig. 1.

The three types of force are placed separately at the vertices of the equilateral "triangle of forces" AEI, where the initials indicate the force concerned. We now write down the subjects originally within the domain of aeroelasticity, bonding them to the appropriate vertices as shown, and placing them within the triangle when they are

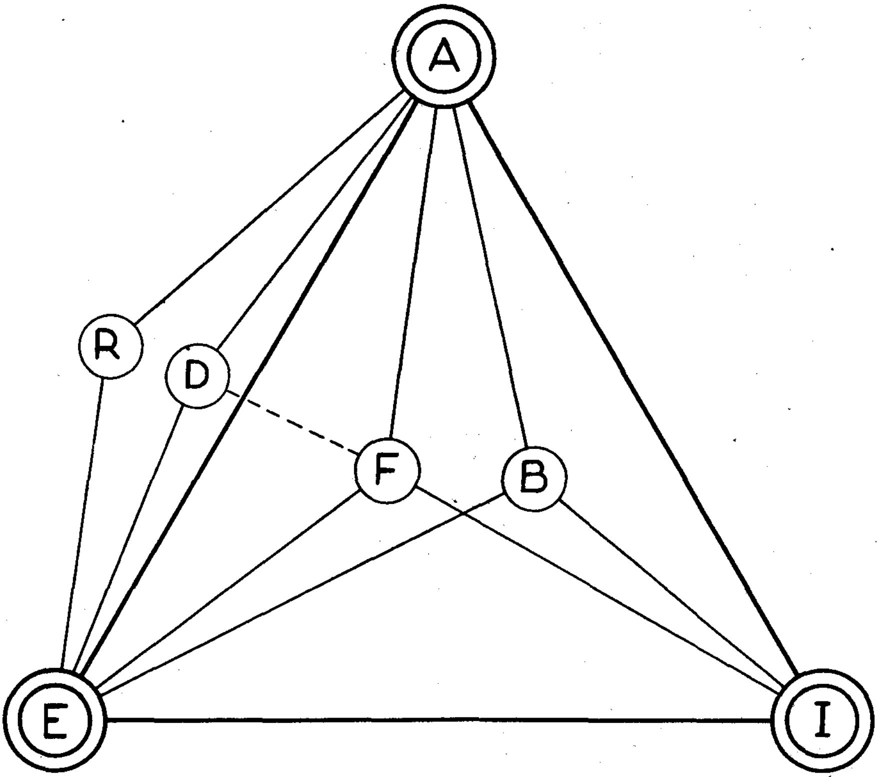

Fig. 1

A : aerodynamic forces
E : elastic forces
I : inertia forces

F : flutter
B : buffeting
D : divergence
R : reversal of control

bonded to all three vertices. Thus flutter (F) and buffeting (B) are located within the triangle. On the other hand, reversal of control (R) and divergence (D) are outside the triangle, since they are unconnected with inertia forces;* though we may draw a bond between divergence and flutter.

Inspection of Fig. 1, however, leads naturally to the question: in view of their obvious connections with aeroelasticity, what of the subjects in the aerodynamic-inertia force field and in the inertia-elastic force field? We may in fact add to our diagram, as in Fig. 2, a number of subjects originally belonging to these fields. In the first field is the vast subject of rigid aeroplane dynamics, of which a major component is the subject of stability and control (S); again, most loading or strength problems (L) of past years, including gust problems (G), have been treated on the basis of interaction of aerodynamic and inertia forces. In the inertia-elastic field is a subject which, at the time of Pugsley's review[1] was rapidly growing in importance—that of mechanical vibration (V); and entering this field we find the problem of the stresses induced by impact loads (Z) which, originally treated as an inertia problem only, was acquiring a connection with elastic forces.

Fig. 2 shows clearly that the no-man's land surrounding the territory of aeroelasticity was of very small extent; and any attempt at expansion must involve intrusion into other fields of work.

* By divergence we imply here an infinitely slow divergence, such as occurs at a critical divergence speed.

In the next paragraph we attempt to define briefly the scope of the problems shown in Fig. 2 at the beginning of the past decade of development.

5. DELIMITATION OF THE ORIGINAL SCOPE OF THE PROBLEMS.

If we were to attempt here to define any precise limits to the subjects with which we are concerned, we should have to include in a single paragraph a very large part of the whole science of aerodynamics. No apology is needed therefore if we deal with the subjects in a very broad and general way, and if in so doing we find that in our statement considerable omissions and lacunæ appear.

5·1. Aeroelasticity.

We have already indicated, by reference to Pugsley's work, what was the general scope of this subject. It related almost exclusively to aeroplane wings, and laid down, on a theoretical-cum-statistical basis, standards of torsional stiffness for wings: standards of flexural stiffness were not found to be

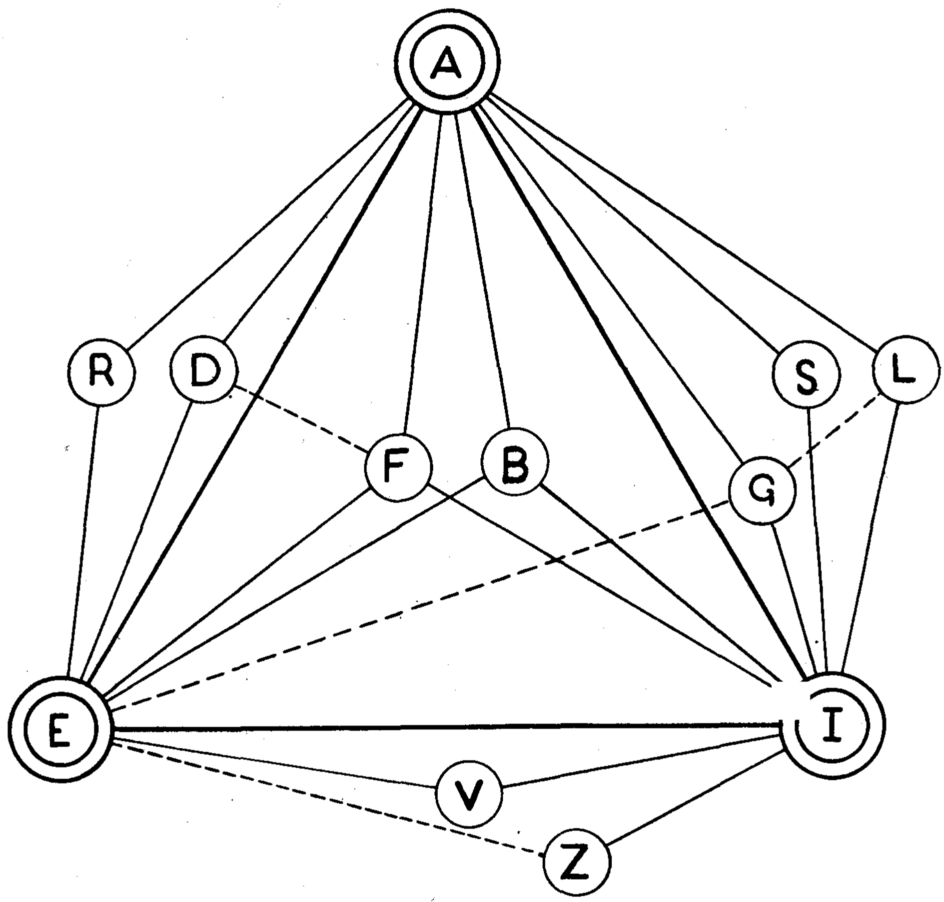

Fig. 2

A : aerodynamic forces
E : elastic forces
I : inertia forces

F : flutter
B : buffeting
S : stability and control
D : divergence
R : reversal of control
G : gusts
L : loading problems
V : mechanical vibration
Z : impacts

required. It is true that standards of torsional stiffness for ailerons had been tentatively proposed also; but the number of criteria was very small. In addition, the bases on which the component subjects were built up were quite narrowly limited in the manner indicated below.

5.1.1. *Flutter.*

The degree of complexity of the theoretical treatment of any subject by any one method increases very rapidly with the number of forces involved and particularly with the number of degrees of freedom which must be introduced. In consequence, flutter, which from the first involved all three types of force, and which was treated by the classical method of examination of the Routh stability criteria, had of necessity to be restricted to problems involving very few degrees of freedom: the maximum number was usually three. In addition, very considerable simplifications had to be introduced: of these, the most important was the concept of semi-rigidity, in accordance with which the elastic body with its infinite number of degrees of freedom is replaced by a body having only a finite number. For wings, these degrees of freedom were usually a single mode of displacement in torsion, another in flexure, and (rigid) aileron rotation.

On this basis, it was found that, up to ten years ago, most flutter problems could be adequately dealt with.[2] It thus appeared that it was unnecessary to introduce bodily freedoms of the aeroplane as a whole: this result was largely inferred from the satisfactory solutions obtained on the restrictive assumptions made, though one early research of Frazer and Duncan[3] had examined the effect of rolling freedom on antisymmetrical flexure-aileron flutter and had shown that little modification was required to the anti-flutter recommendations.

Broadly speaking, therefore, it was found to be possible to regard the fuselage—or at least the forward part—as rigid and infinitely massive, so that in effect it had no bodily freedoms; and the components whose flutter properties were to be studied were regarded as having only a small number of degrees of freedom and as being encastré in the fixed fuselage.

Even with the above simplifications, theoretical treatment was very complicated, and further assumptions were necessary, particularly for the formulation of a simple stiffness criterion. The aerodynamic actions, for example, were treated by strip theory, making use of a fundamental set of constant* derivative coefficients proposed by Duncan and Collar in a research on airscrew blade flutter.[5]

In the formulation of the stiffness criterion it was also necessary to treat of an " average " wing, in which the distribution of chord with span, the distribution of mass in the chord, and the position of the flexural axis, were all maintained constant: although in various researches the effects of these parameters were examined. Lastly, no distinction was made between the stiffness appropriate to the semi-rigid representation of a wing and the stiffness of the actual wing as found by test.

5.1.2. *Divergence.*

The problem of divergence can be treated either as a special case of the general flutter investigation or as a relatively simple problem *per se*. The remarks on flutter therefore apply also to divergence. There is, however, one additional limitation: a wing approaching its divergence speed, if encastré in such a manner that its root incidence were maintained constant, would change its lift with increasing rapidity. Strictly, therefore, a specification of the lift conditions, implying pitching of the fuselage, should be introduced; but this was generally neglected.

* It was of course realised that the aerodynamic forces are strictly functions of the frequency parameter : this dependence, for translational and pitching derivatives, had been investigated by Duncan and Collar.[4]

Some attention had been given to wing-aileron divergence; but the pilot's reaction to this phenomenon was generally ignored. At the wing-aileron divergence speed, both wing and aileron are neutrally stable, and the latter condition implies zero stick force. Thus a pilot will have warning, through the progressive lightening of his ailerons, of the approach of this phenomenon, and its critical speed can be controlled by alteration of the aileron hinge moment characteristics. Moreover, since aileron travel is limited, the corresponding wing twist is also limited: usually to an amount which would not imply danger to the structure. In view of these considerations, it is not surprising that wing-aileron divergence has not presented a problem of urgency in practice.

5.1.3. *Loss and Reversal of Control.*

Ten years ago this subject was restricted to loss and reversal of aileron control.[6, 7] Here also the remarks given above on flutter apply almost in their entirety; although some investigations into methods of dealing with aerodynamic loading[8] and into the case of an elastic (as distinct from semi-rigid) wing[9] were made.

In view of the relatively simple nature of the calculations involved, the results were not normally presented in the form of a stiffness criterion, although a torsional stiffness criterion of the normal form was involved; and hence parameters such as wing taper were taken into account in the calculations made.

5.1.4. *Buffeting.*

The subject of tail buffeting falls within the domain of aeroelasticity, but is in many ways a subject apart from the others. It is concerned[10] with the transient vibrations resulting in the tail unit from aerodynamic impulses due to the eddying wing wake. Since these impulses are quite random in character, there is no analytical theory associated with the phenomenon. Its cure is usually of an *ad hoc* nature; and by proper positioning of the tail organs and by clean design the phenomenon can be avoided. Since the whole trend of modern development has been towards cleaner designs, cases of occurrence of buffeting are now rare; and we shall not refer to the subject again.

5.2. Subjects Associated with Aeroelasticity in the "Triangle of Forces."

The subjects of 5.1 have been treated very broadly, and with an almost complete lack of detail. Even more broad and less detailed must be our attempt to delimit the scope, ten years ago, of the subjects with which we are now concerned. They had been studied extensively for many years, and were in consequence much more developed than aeroelasticity; we shall therefore only attempt to consider how they stood in relation to aeroelasticity.

5.2.1. *Dynamics of a Rigid Aeroplane—Stability and Control.*

The structure of this subject had been erected on a foundation of aerodynamic and inertia forces only: it had not been found necessary to introduce elastic deformations. It is true that the work of Gates showed an awareness of the possible importance of elasticity—he had at an early date been concerned with flutter as a problem in stability—but with the exception perhaps of control circuits, elasticity effects were generally ignored. The formal treatment of stability and control, including the separation of longitudinal stability from other aspects, was of course well established; and attention was therefore mainly directed to the important component details of the subject, and in particular to control surface hinge moments.

The development of aeroelasticity re-emphasised the question of elastic deformation; and in their work on the effect of response on aileron hinge moments, Gates and Irving[11] introduced the appropriate aileron reversal speed correction.

It may be remarked that the exclusion of elastic forces was in fact quite natural, for two reasons: first, the frequencies of oscillation occurring in stability investigations were on the average much lower than the gravest natural frequency of elastic oscillation; secondly, there were very little data on the elastic properties of aircraft. Such information as existed was mainly of an *ad hoc* nature, and Roxbee Cox's statistical surveys were only just reaching a point at which it was possible, through the medium of criteria, to fix the desirable elastic properties of aircraft and to give guidance to the research worker.

5.2.2. *Loading Problems.*

In the determination of the aeroplane strength needed to resist the aerodynamic and inertia loads applied, elastic deformations were almost invariably ignored. For one thing, the number of loading cases to be covered, and the lack of precise information on the aerodynamic loads, implied margins of safety which were more than adequate to deal with variations in load due to elastic deformations. Again, for the main components affected, strength was largely determined by spar design, while stiffness provision was made (if the intrinsic spar stiffness was insufficient) by the addition of skin of such a gauge that modification to the calculation of the strength characteristics was unnecessary.

Thus, although it had been foreseen by Pugsley and Roxbee Cox[12] that ultimately there must be some inter-relation between strength and stiffness, that stage had not been reached; although in one field—that of gust effects—it was just being realised that structural elasticity must appreciably affect the magnitude of the loads due to gusts.

5.2.3. *Gust Problems.*

At the period we are considering, studies of gust effects were not very advanced, having been severely hampered by an almost complete lack of knowledge of the structure of gusts. But investigations of the loads due to gusts of arbitrary structure were being made under the guidance of Bryant[13] (for wing loads) and Williams[14] (principally for tail loads).

In the main these investigations related to rigid aeroplanes, but in each case some attention was given to the effect of bending freedom of the wing. Thus, although gust problems belonged principally to the aerodynamic-inertia force field, elastic effects were beginning to appear in the theory: in Fig. 2 this is indicated by a tentative (dotted) bond to the elastic force vertex.

5.2.4. *Mechanical Vibration.*

Ten years ago this subject was still in its infancy. In the early days of flying, mechanical vibration does not appear to have presented a serious problem. Probably this was due to the small powers involved (which imply also fairly small amplitudes of impressed force) and to the relatively high damping of the wooden construction then common—particularly since this involved many bracing wires, joints, and so forth.

With the coming of cantilever wings, with the replacement of wood by metal, and with steadily improving methods leading to a more integrated form of construction, the internal damping available had steadily decreased. At the same time, power plants had developed greatly; so that, given a coincidence between an airframe frequency and a powerful engine harmonic, unpleasant vibration was almost bound to develop.

At the time we are considering, Constant's work[15] on thresholds of unpleasantness in vibration was underlining the existence of the problem, while various facets were attracting the attention of Carter and of Morris. But theoretical solutions of the problem were very restricted. They usually involved some far-reaching assumptions—*e.g.*, in engine-airscrew vibration, that the engine bearers were attached to a rigid body and that the airscrew could be regarded as a rigid disc—which rendered the solution of doubtful general applicability.

At the same time, even these solutions were almost entirely restricted to the determination of natural frequencies of vibration, for the very good reason that there was available very little reliable information on the forcing torque impulses or on the damping; with the result that no serious attempts to forecast amplitudes of response were possible. In view of these considerations it was obviously not expedient to introduce air forces into the problem, and no attempt to do this had been made.

5.2.5. *Impact Problems.*

Up to the time we are considering, landing impacts had been treated on the basis of inertia loads only: that is to say, the aircraft (apart from its undercarriage) was treated as rigid, while aerodynamic forces were ignored in view of the low landing speeds involved. However, it was realised that while these assumptions might be justifiable so long as wings were light and possessed a fairly high flexural frequency, the increasing weight of wings required a reconsideration of the picture. A paper by Fairthorne[16] drew attention to this and made a first examination of the effects of wing flexibility on landing impacts. Aerodynamic forces, however, still played no part in the theory. The bond to inertia forces, and the tentative bond to elastic forces, are indicated in Fig. 2.

6. DEVELOPMENTS IN THE SUBJECTS DURING THE PAST DECADE.

We must here content ourselves with only a brief summary of the main developments which have occurred during the past decade, in so far as such developments are related to the science of aeroelasticity. While war conditions provided the impetus which resulted in the developments concerned, these same conditions have prevented the publication of most of the researches—indeed, many have not been fully reported in any form; and this adds considerable difficulty to the task of a reviewer. However, as we are concerned in the main with general trends, we must regard the subjects touched on as illustrative only, and not as component parts of a detailed picture.

We may naturally enquire at this point what are the physical reasons underlying the developments in our field. In a word, the answer is speed. The remarkable developments in power plants—first in reciprocating engines, then in jet propulsion units—coupled with continued efforts in the direction of drag reduction, have resulted during the past decade in the first real excursion of aircraft into the much discussed but highly speculative realm of compressibility effects.

At the same time, the strength questions posed by this speed increase have been solved, partly by material developments, partly by improved constructional methods, but in the main by a steady movement towards the ideal of the flying wing: *i.e.*, by a steady process of transference of mass from the rest of the aircraft to the wing, in order to obtain the maximum relief from inertia loads. As a result, the elastic properties of aircraft, at one time a secondary or even a last-minute design consideration, have in many cases come to the forefront as one of the principal factors in structural design.

This growing importance of aerodynamic and elastic forces, implied in the design requirements for aeroelasticity,[17] has been strongly underlined, not only by the developments of aeroelastic theory itself, but also by the way in which the implications of elastic deformation (which introduces both aerodynamic and elastic forces) have obtruded themselves on the notice of those concerned with branches of theory in which such deformations could previously be ignored.

In the next paragraphs (6.1 and 6.2) we shall try to show how aeroelastic theory is expanding into other fields, and how the subjects in these fields are expanding into aeroelasticity: we shall find there is some overlapping of our discussion, as there is in the subjects concerned.

6.1. Expansion in the Aeroelastic Field.

In the main, the expansion of aeroelasticity in recent years has been in the nature of the growth of existing subjects rather than of the birth of new subjects. The majority of the subjects can still be comprehended within the fields of flutter, divergence, and loss of control: only one new field of major importance, that of effects on aeroplane stability (which is, of course, closely bound up with control effects) has emerged. In consequence, the developments have been largely in the nature of studies of new methods for dealing with the more complicated problems posed by the rapidly accelerating aircraft of the period.

6.1.1. *Flutter.*

We have already said that, in classical methods of studying flutter, it was necessary to limit the number of degrees of freedom to about three. However, one study had been made in 1936 by Duncan, Collar and Lyon[18] of a problem in which six degrees of freedom had been treated.

It is not easy to say precisely how complexity increases with number of degrees of freedom: probably it is not far wrong to suggest that the complexity is roughly proportional to the factorial of the number of degrees of freedom, if any one method is adhered to throughout. Thus an increase from three to six degrees of freedom necessitated new methods, and these were found in the paper considered by the adoption of inverse methods of solution.

Broadly speaking, the change from the classical method is as follows: instead of laying down the physical parameters and the stability condition, and then solving (by means of Routh's criteria or their equivalents) for the corresponding critical speeds and frequencies, one leaves free certain of the parameters and fixes the critical speed and frequency; the corresponding values of the parameters are then found (a process usually involving only the solution of algebraic equations). In this way a curve of critical speed against the values of the parameters may be found, and the appropriate critical speed corresponding to the correct value of the parameter is thus obtained.

This inverse method of solution has now become standard procedure for problems involving many degrees of freedom, and has indirectly been the means of widening greatly the understanding of the flutter problem. Under the next few headings some of the problems which have been treated in recent years are described shortly. We shall not refer here to flutter of wings not carrying engines, since this is dealt with later (6.1.1.5) in relation to stiffness criteria; nor shall we refer to propeller flutter. The latter subject has been extensively studied experimentally, but no adequate general theories have yet been proposed; and indeed the origin of flutter of metal blades remains in doubt.

6.1.1.1. *Elevator Flutter.*

We shall here restrict our discussion to symmetrical elevator flutter. Purely as a matter of interest, however, it may be noted that aeroelastic science had its beginnings in a study of antisymmetrical elevator flutter, made in 1916—thirty years ago—by Bairstow and Fage.[19] Following the particular occurrence they studied, the port and starboard halves of elevators were given stiff interconnections, and cases of antisymmetrical flutter have since been very rare.

Originally, studies of symmetrical elevator flutter involved two degrees of freedom only: fuselage bending and elevator rotation.

Calculations on this basis had, however, always given numerical results of the wrong order, even allowing for the lack of knowledge of the appropriate aerodynamic forces. In a study of the problem made under the direction of Jahn[20] at the R.A.E., therefore, the following additional degrees of freedom were successively introduced: pitching of the aircraft, vertical translation of the aircraft, and wing bending. (In this particular case two further freedoms, involving elasticities present in the circuit, were also added.) Parallel with this investigation was an experimental research conducted at the N.P.L. by Scruton.[21]

The results obtained in this way were very satisfactory, and design recommendations based on the studies were found to be beneficial. From the motions occurring in the critical condition, it was also found that the problem could be treated rather more simply by considering the motion to be built up from the normal modes of vibration of the aircraft as found in a resonance test.

The implications of this latter finding are of great interest and importance. It will be seen that the degrees of freedom involved are not only the normal modes of vibration—it may be noted here that Duncan[22] had suggested the use of normal modes of vibration as an approach to complex problems—but also the rigid body freedoms (which may in a sense be regarded as normal modes of zero frequency).

Thus the investigation pointed to the fact that an instability having its origin in the tail organs, involving as it does appreciable wing bending and bodily movement, can no longer be treated as a local affair, but must be regarded as " elevator-aeroplane " flutter. Or, in more general terms still, the phenomenon must be regarded as an aspect of the stability of a deformable aeroplane.

6.1.1.2. *Flutter of Wings Carrying Engines.*

This subject has always held considerable interest for flutter investigators. It was realised that the " mass-balancing " effect of engines, as normally disposed, must be considerable; but no quantitative estimates of the effects had been made. Research into this question has been vigorously prosecuted both at the N.P.L.[23] and at the R.A.E.,[24] as well as by certain aircraft firms. The problem is as yet far from final solution; but it is already apparent that a large number of normal modes must be introduced if an adequate solution is to be obtained.

6.1.1.3. *Antisymmetrical Wing Flexure-Torsion Flutter.*

In this phenomenon, as in others, it has been found that it is necessary to introduce not only the antisymmetrical normal vibration modes, but also the relevant rigid body freedoms—in this case freedom in roll.

6.1.1.4. *Mass-Balancing and Flutter Prevention Devices.*

In addition to the studies of elevator flutter referred to in 6.1.1.1, cases of flutter of ailerons, rudders and tabs have also been extensively studied. These studies, while underlining the necessity of treating deformation of the aircraft as a whole as a constituent motion, have also yielded much valuable information on mass-balance as a cure for control surface and tab flutter.

It has been shown, for example, that the virtual inertia of the air[25] must be taken into consideration in the determination of the appropriate inertia characteristics. Another very important result relates to the positioning of mass-balance weights. If it is desired to achieve mass-balance by the use of masses not attached to the control surface, but operated through a linkage, then the position of the mass in relation to the nodes in the vibration modes of the aircraft must be accurately known.[26]

For a balance mass operates as such in virtue of the accelerations imposed on it by the vibration of the aircraft, and if it is positioned near a node this acceleration will be small; the mass is then practically inoperative as a balance. In fact, if it is on the wrong side of the node it becomes an anti-balance mass. The node may be regarded as a virtual hinge; and if the balance mass is to play its proper part, it must be appropriately positioned in relation to this virtual hinge and the control surface hinge.

In the study of mass balancing of elevators and rudders in particular, it has been necessary to consider as part of the practical problem the effects of the balance masses on longitudinal and directional stability (in the classical sense). Thus there is, in a way, a dual linkage with stability and control matters; not only is the flutter problem becoming a problem in the stability of a deformable aeroplane, but the curative measures adopted must be considered in relation to the stability of the rigid aeroplane.

Considerations similar to those mentioned above apply also to the mass-balance of spring tabs. In this case the problem is complicated by the appearance of elastic couplings, and if the effects of these are to be overcome, the tab must be treated as rotating about its own hinge and a virtual hinge fixed by the geometry of the system. Here again, the balance mass must be correctly positioned in relation to these two hinges; this consideration has led to the definition of a " limiting length of balance arm "[27] and to great clarification of the problem of spring tab flutter.[28]

Studies of methods of prevention of flutter other than mass balance have also been prosecuted, but without very promising results. The introduction of motional forces, or damping, does not seem likely to provide a practical solution for a variety of reasons: the damping required varies acutely with many of the parameters involved, such as height and control surface inertia, and in any case would require prohibitive effort from the pilot if applied directly to the control surface. Irreversible and quasi-irreversible units still have promise,[29] and with careful attention to details of design may in time be adopted as alternatives to mass-balance.

6.1.1.5. *Researches on the Aerodynamic Forces of Flutter.*

While, as indicated above, methods of dealing with the flutter problem have steadily improved and extended, studies of the relevant aerodynamic forces have also been prosecuted. At the N.P.L. a number of investigations have been made into the validity of the constant strip theory derivative coefficients by observations of the flutter characteristics of a series of straight-tapered wings; and some modifications have been proposed.[30] At the same time Jones[31] has made a number of detailed theoretical investigations of the aerodynamic characteristics of oscillating wings of finite aspect ratio and varying planforms.

Some attention has also been given to compressibility effects. In the subsonic field investigations by Frazer[32] and by Jahn have indicated how increasing Mach number may affect critical flutter speeds. In the supersonic field, the aerodynamic forces have been studied by Collar[33] and by Temple and Jahn,[34] and wing flutter calculations for supersonic conditions have been made.

6.1.2. *Divergence.*

There has been no evidence that divergence represents a problem of acute practical importance, and in consequence relatively little attention has been paid to this phenomenon. It has not been entirely forgotten, since there has been a general tendency for wing flexural axes to move aft, and divergence speeds have therefore tended to become lower. In particular, studies have been made of the divergence speed of an elastic wing by iterative methods.[35, 36] The result still emerges, however, that divergence speeds are sufficiently high not to present practical problems.

If divergence does become a problem of

practical importance then, as remarked in 5.1.2, it will become necessary to introduce either a specification of the lift conditions or —since the varying lift will probably be unknown—to treat the response of the aircraft as an integral part of the problem. In this case inertia forces, usually excluded in studies of divergence (since the inertia forces corresponding to the deformation are negligible) will reappear in the problem.

Wing-aileron divergence has also been studied;[37] but here the remarks in 5.1.2 still apply.

Tailplane-elevator divergence speeds have received attention for the first time, in a study of loss of elevator control: this is dealt with later in 6.1.3.2.

6.1.3. *Loss and Reversal of Control.*

It is convenient to divide this topic under two headings: loss of aileron control and loss of elevator control.

6.1.3.1. *Loss of Aileron Control.*

The question of loss of aileron control has had to be kept continuously under review throughout the past decade, since the wing stiffness determined by the associated requirement has often been greater than that required by the wing stiffness criterion (which has assumed more and more the role of a flutter preventive).

Up to a certain stage the work on aileron reversal was ably summarised by Victory.[38] Since that time the increasing importance of compressibility effects, and the possibilities of flight at supersonic speeds, rendered a new investigation desirable; more particularly since wing tapers and stiffness distributions were becoming very different from those of the orthodox wing of ten years ago.

Accordingly, a study of the rolling power of an elastic wing was undertaken by Collar and Broadbent,[39] and the results were applied to flight at subsonic and supersonic speeds:[40] the aerodynamic forces being those given by Glauert[41, 42] in the subsonic range and by Collar[43] in the supersonic range.

The results indicate the very great importance of torsional stiffness for aircraft flying in the neighbourhood of the sonic speed, and show further how inadequate may be the stiffness found by the conventional test method as a measure of the effective stiffness of the elastic wing; since the deformation modes under a concentrated torque and under aerodynamic loading may be widely different.

While, therefore, it is apparent that the subject of aileron reversal and loss of aileron control has progressed considerably, at the same time it has been restricted to unaccelerated conditions; and inertia forces have been excluded from the researches. This has largely been due to the importance assigned to a high steady rate of roll as a criterion of aileron performance.

Nowadays rates of roll are becoming so high that there are signs of a change in this respect; and it is probable that aileron performance will in future be judged by the rapidity with which a vertical bank can be reached, or by some similar criterion. Such a manœuvre will involve acceleration and deceleration in roll; and studies of loss of control due to wing deformation may well have to include these inertia forces in the future.

6.1.3.2. *Loss of Elevator Control.*

With the increasing speeds of ten years ago, and in the absence of requirements for the stiffnesses of fuselage and empennage, it was to be expected that aeroelastic effects on the tail organs would make themselves felt. The first evidence of this was through effects on longitudinal stability, and though no cases of actual reversal of elevator control have been reported, observed tail deformations and a crop of associated troubles rendered an investigation of those effects necessary.

The investigation, made by Collar and Grinsted,[44] treated tailplane twist as the main variable, but examined also the effects of elevator twist and of fuselage flexibility under shear and moment loads. It was shown that severe structural distortion might result unless the elevator reversal speed were of the order of twice the maximum speed, and that in consequence high standards of stiffness for the fuselage and empennage were very desirable. The tailplane-elevator divergence speed was also studied—a phenomenon of a somewhat different character from wing-aileron divergence.

Finally, the investigation included a treatment of the effect of tailplane twist on longitudinal static stability, which showed how if the tailplane stiffness was inadequate a stable aeroplane might at high speeds become more stable or might develop instability, depending on tail setting, wing pitching moment and so forth.

This investigation was followed by a second study in the same field by Collar and Victory.[45] The object here was to define, in the form of stiffness criteria, desirable standards of structural stiffness for tailplanes, elevators and fuselages, and to examine how far these stiffnesses were interdependent in their effects on elevator control.

The question was approached theoretically by a consideration of loss of control and more particularly with a view to limitation within reasonable bounds of the structural distortions due to application of elevator. By this means appropriate forms for the criteria were derived, and the required order of magnitude indicated.

To supplement this a statistical survey* was also made of the experimental evidence on all these stiffnesses—evidence accumulated in a somewhat desultory way over a number of years: and appropriate criteria were defined and proposed for adoption.

* The word "statistical" is used somewhat loosely here and elsewhere. In nearly all such surveys of stiffness criteria the data are so varied in nature and origin, and so scanty, that it is not possible to obtain frequency distributions in the strict statistical sense: the best that can be done is to weigh the data on a basis of experience, common sense, and a knowledge of the origins of each item, and to take an arithmetic mean. To be honest, it is often difficult to avoid an element of casuistry in the process, since the data are often being used to test a theoretical result.

6.1.4. *Elastic Effects on Stability.*

It has been indicated above that this problem has appeared for the first time during the last decade. It might have been thought that the principal practical problem would have been one of lateral stability, resulting from the changes in effective dihedral produced by wing bending; but in fact the effects of elastic distortion on longitudinal stability have proved much more serious.

The previous problem had been envisaged by Pugsley, and Bryant and Pugsley[46] demonstrated how, with increasing wing loadings, the values of wing dihedral would have to be carefully watched if lateral instability were to be avoided. But further expansion of aeroelastic theory in this field does not appear to have been required by experience.

Regarding longitudinal stability, an early examination of the effects of wing torsion was made by Pugsley.[47]

Reference has already been made to the extension of the aeroelastic field to include loss of elevator control and to the study by Collar and Grinsted of the effects of tail and fuselage distortion on longitudinal stability. Further studies in this area are largely due to Gates, Lyon, and their collaborators, and are more in the nature of introduction of elastic effects into general stability studies; and, as such, will be dealt with later.

6.1.5. *Developments in Stiffness Criteria.*

We have already remarked that, when Pugsley reviewed the subject of aeroelasticity, there was virtually only one stiffness criterion: and even that criterion had not been accepted for current use except as recommended practice. This criterion laid down, in very simple terms, the wing symmetrical torsional stiffness, measured between the wing root and the mid-aileron section.

During the period under review, some changes have developed in this criterion and new proposals have been made. At the same time, new criteria have appeared; it is convenient to discuss these aspects under separate headings.

6.1.5.1. *The Wing Stiffness Criteria.*

The developments in the mid-aileron torsional stiffness criterion have recently been reviewed,[48] and new proposals made; we shall here make only a brief précis of this work.

First let us note that for conventional wings it has not been necessary to lay down any standard for bending stiffness: this in spite of the fact that, partly as a result, the ratio of bending to torsional stiffness has decreased on the average to about one-fifth or less of the value it had ten years ago. It would seem probable that the design of the wing for strength leads almost automatically to adequate bending stiffness. This view is supported by a survey of bending stiffnesses, which shows smaller variations in a bending stiffness criterion, from one aeroplane to another, than might have been expected if there were no controlling factor.

On the other hand, two criteria for wing torsional stiffness have been current for some years, having been made mandatory shortly before the war: these are the " mid-aileron " and the " equivalent tip " criteria. Only the first of these has been important in practice: satisfaction of the mid-aileron criterion has almost invariably implied satisfaction of the equivalent tip criterion by a wide margin.

Regarding the mid-aileron criterion, we may revert to Pugsley's speculation, referred to in 2, as to whether this would be defined by reversal of aileron control, wing flexure-torsion flutter, or wing divergence. The answer, so far as the past ten years are concerned, is wing flexure-torsion flutter.

Reversal of aileron control has had its own criterion, in the sense that it is a requirement that the aileron reversal speed shall exceed the maximum diving speed by an adequate margin; while it has not been possible to raise the flexure-torsion flutter speed economically by any means except that of increasing the wing torsional stiffness. It follows that the divergence speed has in practice remained at a higher level than the flexure-torsion flutter speed, and the latter has therefore defined the criterion.

Thus the mid-aileron wing torsional stiffness criterion has assumed almost exclusively the role of a preventive of flexure-torsion flutter. This criterion is remarkably simple in form, since it involves as parameters only the maximum equivalent airspeed, the mean chord and span, and the torsional stiffness. No account is taken of parameters which are in fact known to affect the flutter speed appreciably, such as wing taper, bending stiffness or position of the flexural axis. Even the most important parameter—inertia distribution—is left out of account except in that different values of the criterion are laid down for wings with and without wing engines. Finally, compressibility effects are ignored.

The great simplicity of the criterion, and its uniform success in preventing trouble, rendered any modification undesirable so long as no great weight penalty was incurred in its satisfaction (it must be remembered that a certain standard of stiffness, of the same order, is in any case required by aileron reversal considerations). This fortunate state of affairs has continued until quite recently.

In the past two or three years, however, an increasing tendency towards strong wing taper has produced wings in which the natural tendency is for the local wing torsional stiffness to fall off very rapidly towards the tip; and this and other factors demanded that the form of the criterion be reconsidered. To this end a number of investigations into the effects of the parameters concerned have been made.

It has been demonstrated that the optimum reference section for the measurement of the torsional stiffness is a function both of the modes of distortion assumed and of the taper, but that a section at about 0.7 span can safely be used. Investigations have also been made of the variation of critical flutter speed with taper, with assumed modes of twist and of bending, with wing density and inertia axis position, and with wing bending stiffness and flexural axis position. Account has also been taken of the difference between dynamic and static stiffness.

All these investigations have recently been reviewed,[46] and the results have been extended and incorporated in a proposed new criterion. This new proposal is an elaboration of the existing criterion, additional terms introducing the more important parameters such as taper, inertia axis position and Mach number.

At present, a survey of values of the proposed new criterion achieved by existing aircraft is in progress.

6.1.5.2. *Additional Criteria.*

Requirements now exist,[17] in the form of criteria, for torsional stiffness of ailerons, elevators and rudders. These criteria are all similar in form: this form being that derived from the theoretical studies of loss of elevator control mentioned earlier.[44, 45] The numerical values to be achieved are, however, different, being based on practical results rather than on theoretical predictions.

In the case of ailerons two values are defined, one for ailerons with distributed mass balance and the second for other ailerons; similarly, different values are required for elevators with and without horn balances.

In addition there is a requirement for the flexural stiffness of the overhang of an elevator beyond its outermost hinges.

With the exception of wings probably the most important stiffnesses are those of the tailplane in twist and of the fuselage in vertical flexure: and criteria have been defined for these stiffnesses. Although the wing and tailplane torsional stiffness criteria are identical in form, the tailplane criterion is not determined by flexure-torsion flutter considerations, while the wing criterion is. The tailplane criterion is fixed largely by considerations of longitudinal stability and control; it is noteworthy that the two criteria have, however, nearly the same numerical values. On the whole, the tailplane criterion is slightly more severe than the wing criterion, and it is probably this fact, together with the simpler structural problem, which has resulted in the practical freedom of tailplanes from flexure-torsion flutter.

Two tailplane criteria are defined, one for tailplanes without outboard fins and rudders and a rather more severe criterion for tailplanes carrying outboard fins and rudders.

The fuselage stiffness in vertical flexure is also determined from considerations of longitudinal stability and control.

Corresponding to vertical fuselage flexure and tailplane twist are lateral fuselage flexure and fin twist. No theoretical investigations into desirable values for the stiffnesses in this second case have been made; but a lateral stiffness criterion for fuselages has been defined, by analogy with the vertical stiffness criterion, and a value assigned to it on the basis of practical measurements.

It has not been found practicable to define a criterion for fins, in view of their very wide variations in shape and attachment—particularly in the case of outboard fins.

The last major structural criterion is that for torsional stiffness of fuselages: it is this criterion which would enter into calculations of antisymmetrical elevator flutter.

Finally, standards for the stiffnesses of control circuits have been laid down. These take the form of a requirement that the stretch produced by a reasonably big effort on the part of the pilot shall not exceed more than a given percentage of the total available movement. There is in addition a stiffness requirement for the aileron balance circuit, designed to ensure that serious upfloat of the ailerons shall not occur.

6.2. Expansion in Fields Allied to Aeroelasticity.

In 6.1 we indicated how aeroelasticity has been, in effect, expanding into other fields. In the present paragraph we shall attempt to show something of the reverse process.

6.2.1. *Elasticity in Problems of Stability and Control.*

We have already shown how elastic effects enter into both lateral and longitudinal control. In lateral stability, however, elastic effects do not appear to have presented a serious problem: such effects undoubtedly exist, since, for example, under normal acceleration the wing dihedral must change very appreciably. These problems of lateral stability have been kept in mind; but it is in the field of longitudinal stability that the effects of elastic deformation are most pronounced and can have serious consequences.

The treatment of the problem of longitudinal stability has been greatly simplified by the expositions by Gates of the fundamentals of longitudinal stability[49] and of the idea of manœuvrability.[50] By studying independently the static stability, stick fixed and free, and the manœuvrability, stick fixed and free, the important aspects of longitudinal stability of an aircraft are all covered. Without this subdivision, the introduction of elastic effects into longitudinal stability investigations would have involved much more arduous calculations and made the interpretation of the results much more difficult.

As it is, Gates and Lyon[51] and their collaborators have now proposed a theory in which the effects on all aspects of longitudinal stability of elastic deformations in the fuselage and empennage are taken into account;[52] and following the earlier precedent[44] have absorbed into stability theory the appropriate stiffness criteria and the elevator reversal speed.

It is in the field of longitudinal stability that another important deformation effect has first become serious: that of panel distortion under load.[53] In elevators, the distortion of the panels of the structure under aerodynamic loads can produce sufficient change in the effective camber to result in quite abnormal hinge moments; and the stick free stability can be adversely affected to a serious extent. To a lesser extent, panel deformation in ailerons can also be serious.

It may be remarked that the deflection of a panel is obviously not proportional to the applied load; and thus panel deflections have for the first time introduced serious elastic non-linearities into aeroelastic problems, and we can no longer talk with accuracy in terms of stiffnesses or stiffness criteria. How this difficulty is to be met is a matter still under consideration.

6.2.2. *Elastic Effects in Loading Problems.*

In 5.2.2. we stated that up to ten years ago loading problems were treated on the aerodynamic-inertia force basis, and that elastic deformations were in the main ignored. In this area, the position has not changed radically, and we are not yet being driven to a correlation between strength and stiffness. Nevertheless, there are indications that we are moving towards such a correlation, and we shall try here to indicate the roads by which this movement is taking place.

In the first place, both aeroelastic requirements and aerodynamic requirements for smoothness of profile are demanding much greater skin thicknesses than were common ten years ago. In consequence, the skin is rapidly becoming a main stress-bearing part of the structure, and it is not impossible that we shall soon see spars disappearing from wings in favour of all-skin designs. In this event the strength and stiffness will be much more closely tied together than is at present the case.

Again, there is an increasing realisation that the structural strength of an aircraft can no longer be adequately treated on the basis of static loads alone. Much attention has been given, and more is required, to the problem of fatigue of built-up structures; and the oscillatory loads involved may be of aerodynamic origin as well as due to mechanical vibration.

In this connection it may be remarked that proposals are in hand for strength testing an airframe under conditions designed to simulate those which would obtain during an occurrence of flutter. In this field of non-static loads we may also note the work which has been done on the strength of structures under repeated slow loading; although this has not resulted directly from elastic effects, it has an obvious bearing on strength under oscillatory loads of aero-elastic origin.

6.2.3. *Gusts.*

Knowledge of the structure of gusts has not increased to any appreciable extent during the period under review, and in consequence calculations still have to be made for somewhat arbitrary gust cases. But the methods of dealing with these cases have considerably developed.

This is best illustrated by reference to a calculation made by the Bristol Aeroplane Company in relation to one of their aeroplanes; in this calculation, the growth of the air forces according to Wagner's theory, the response of the aeroplane, and a large number of elastic freedoms for the structure, were all taken into account in some detail—a complete aerodynamic-elastic-inertia force problem.

At the same time, since gust cases are often severe, attention is being devoted by many investigators to the problem of cheating the gust. One way of doing this is to employ the elastic deformation of the wing to reduce the adverse loadings. For example, if it were possible by a structural artifice to obtain an effective flexural centre near to or ahead of the leading edge, then a gust load would cause a change in wing incidence near the tips in the sense required to reduce the applied load there; and other examples might be given.

Here, then, is a problem in which design for strength may be appreciably affected by the design for stiffness; and, it must be added, such a design would have to be watched carefully from other aeroelastic viewpoints, particularly those of flutter and of stability.

6.2.4. *Mechanical Vibration.*

In this field considerable advances, which owe much to the energetic researches directed by Morris,[54] have been made in the past ten years. We cannot enlarge on these advances here, since up to the present time the researches have been confined to the inertia-elastic field. We may, however, note that the stage has now been reached when the determination of the natural frequencies of a complete aircraft, and particularly those frequencies which seem likely to be important from the vibration viewpoint, can be predicted with fair confidence.

Thus the investigators have reached a milestone in the road of their research; and the next stage must be the determination of amplitudes of response by the inclusion in the equations of motion of the forcing impulses arising in the engine and of the air forces; since the latter provide both damping forces and modifications to the elastic stiffnesses.

That mechanical vibration and flutter are beginning to overlap is illustrated by the following occurrence. On a particular aircraft the elevator trim tabs were not mass-balanced, since the backlash in the operating system was very small. On one mark of the aircraft, which had one particular type of engine, no trouble occurred. But on a second mark, with a different engine, very unpleasant vibration developed, and was cured by reducing the backlash in the tab to a much smaller amount than had previously been thought necessary. It was evident, therefore, that the different forcing impulses from the second engine were producing a tab flutter, within the limits of the tab backlash, where previously none had existed.

6.2·5. *Impact Effects.*

As in the case of mechanical vibration, the researches on this subject have been confined, up to the present time, to the inertia-elastic field;[55, 56] and some important effects, particularly on the increase in local stress above that given by the rigid aeroplane case, have been observed.

Although increasing attention is being paid to this subject it has still not yet reached the stage where even on the restricted inertia-elastic force basis, accurate quantitative predictions of the important local stress variations resulting from a landing impact can be made. When this stage is reached it will be expedient to introduce the air forces also; for, as speeds at touch down steadily increase, these forces are becoming correspondingly more important, and may provide a significant correction to the stress calculations.

7. THE FUTURE.

We have seen in 6 how the individual studies with which we are concerned are expanding; and we may revert to our " triangle of forces " diagram to examine what is happening to the subjects there described. We may perhaps take the subject of aeroplane stability as an example.

In Fig. 2 this subject is bonded to aerodynamic and inertia forces only, and is unconnected to the third vertex of the triangle—elastic forces. In consequence, it lies outside the aeroelastic triangle. But, at the present day, it has acquired a bond to the third vertex—not perhaps so strong as its connection with aerodynamic and inertia forces, but nevertheless sufficient to draw the subject within the general domain of aeroelasticity. We may, in fact, re-draw our triangle as shown in Fig. 3.

In this triangle we see how most of the subjects have now become bonded—although at present the bonds are often tentative—to a third vertex, where previously only two such bonds existed; moreover, certain of the subjects are becoming bonded together by obvious interconnections. Thus, for example, loss and reversal of elevator control is obviously directly connected with longitudinal stability; while the study of gusts has much in common with both flutter studies and investigations of landing impacts, and so on.

In short, it is evident that we are no longer dealing with a series of subjects, each in its own watertight compartment: there is a definite coalescence of the subjects into an integrated whole, which may be defined as the dynamics of a deformable aeroplane. And we are faced with the question: what are to be our methods of treatment of this unified problem ? It is obviously not possible to be dogmatic on this important question; but the writer would suggest the following possible answers.

7.1. Treatment of the problem.

To the mathematician the description of the subject—dynamics of a deformable aeroplane—suggests the formal solution at once. We select as generalised co-ordinates quantities defining the translational and angular freedoms of the aeroplane as a rigid body; we add co-ordinates representing the control surface angles, tab angles, stick and pedal movements, and with these we include co-ordinates to represent the freedoms due to automatic controls; and finally we introduce a number of co-ordinates sufficient to describe the modes—normal or otherwise—of elastic deformation of the aircraft. We then derive, in terms of the inertia and elastic properties of the aircraft and the air forces, the complete Lagrangian equations of motion, including in the appropriate equations forcing terms representing mechanical

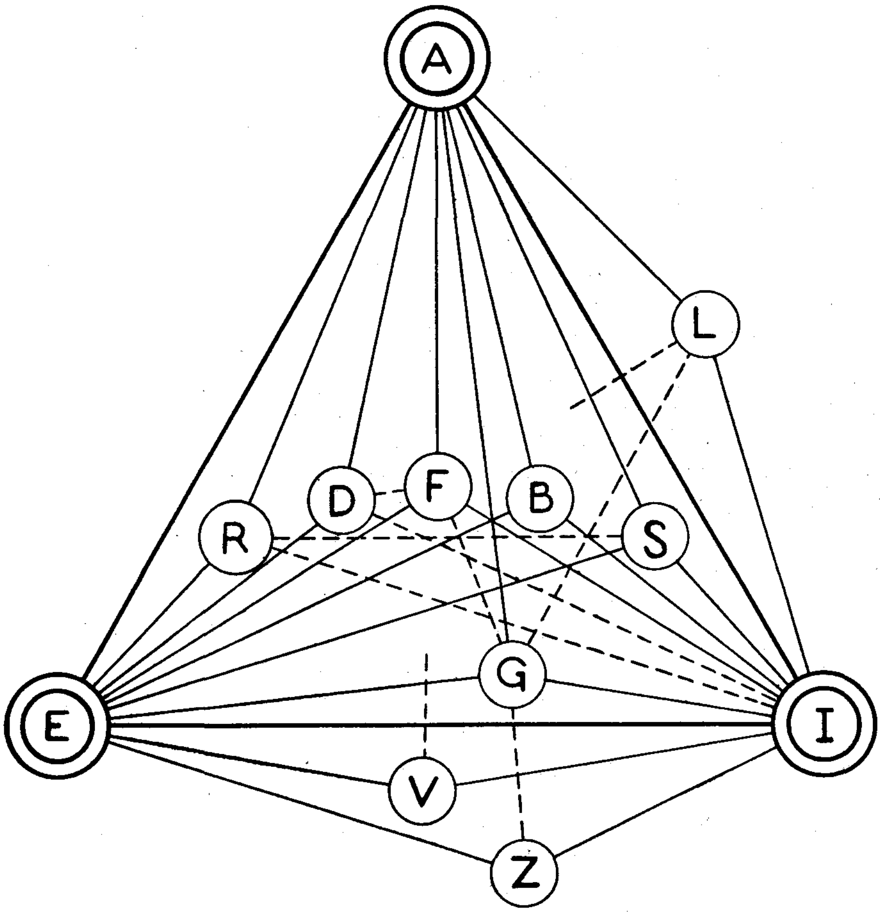

Fig. 3

A : aerodynamic forces
E : elastic forces
I : inertia forces

F : flutter
B : buffeting
S : stability and control
D : divergence
R : reversal of control
G : gusts
L : loading problems
V : mechanical vibration
Z : impacts

vibration impulses, gusts, landing impacts, pilot's efforts, and so forth.

In this way, we obtain a set of simultaneous differential equations, probably in twenty to thirty degrees of freedom, which describe completely the motion of the aircraft and which can tell us whether serious trouble is likely to arise or not.

From the practical viewpoint, of course, the labour of dealing with such a set of equations is prohibitive at present. But we must not dismiss out of hand the possibility of solving such a set of equations, at least so long as they are linear in the unknown co-ordinates or approximations to linearity can be safely made. For we have already remarked that inverse methods of solution have remarkable power; and by adoption of such methods of dealing with the equations we may well be able to reduce the problem to that of evaluation of numbers of numerical determinants of large order.

At present this process would be far too arduous and would offer such great possibilities—even probabilities—of error that no attempt would be worth while. But in the last few years considerable strides have been made in dealing with large numbers of simultaneous equations by the use of electrical, mechanical, and electro-mechanical machines;[57, 58] while the growing use of matrix algebra[59] makes the formulation of the problem, as well as its solution, much easier.

It is thus not outside the bounds of possibility that within a period of years we may have available computational aids of such power that the solution of equations of the kind we are considering may be a routine process.

In the writer's view it is more probable that the chief difficulty will be, not in the solution of the differential equations, but in the derivation of the equations themselves from the basic data: even at the present time, in advanced flutter calculations, the solution of the equations is the simpler half of the work. However, the formulation of the equations may not be so much more complex than at present, since in the majority of cases the number of coupling terms will probably be relatively small.

Meanwhile, we shall obviously continue to discuss our subjects as separate entities. The fact that the coupling terms in the equations of motion are few (or weak) has permitted us to regard the subjects as separate in past years, and there is no reason why we should not continue to take advantage of this fact (provided that we are careful to assess the magnitude of those couplings we ignore) for some time to come. But it may well be that a reconsideration of programmes and methods at this stage is worth while.

We may, for example, take the subjects we have discussed and, since they are becoming unified, try to relate them according to some particular characteristic: one such characteristic that suggests itself is frequency of oscillation. On this basis we could prepare a table with a spectrum of frequencies such as follows:—

SUBJECT	FREQUENCY
Reversal of control Divergence	Zero
Loading problems Stability	Zero or low
Gusts	Low or medium
Impacts Flutter	Medium or high
Vibration	High

On inspection of this table we see that it may be possible to solve our unified problem by unified treatment, but that it is probably not necessary to take the whole group at once. Thus the connection between the first and last subjects is negligible: we need not consider vibration questions in dealing with reversal of control, and *vice versa*. But in dealing with reversal of control we have a strong connection with stability studies: in stability studies we have a growing connection with flutter; and flutter and mechanical vibration are becoming closer: so that the tie between first and last exists.

Thus in the future, given our computational aids, it may well be that we shall write down for each subject a sufficient number of equations of motion to cover appropriate neighbouring subjects in the spectrum. In studying reversal of control we may include divergence and stability: in studying stability we may include reversal of control and flutter: in studying flutter we may include stability and mechanical vibration.

Viewed from this angle, it appears that the unification of the subject does not necessarily involve a hopeless complexity but offers rather the advantage that we may study the various aspects in relation to their associated subjects by a standard method of attack.

7.2. STANDARDS OF ELASTICITY.

We have speculated in the last paragraph on how our subject may be treated in the future; we may now perhaps be permitted to speculate further on the methods of curing trouble as it arises. And since the aerodynamic and inertia loads are likely still to be dictated mainly by other considerations, the principal control on aeroelastic and associated phenomena will continue to be exercised through stiffness provisions. We may make more use of inertia distributions than in the past, but except for mass-balance of control surfaces to prevent flutter, variation in inertia distribution is likely to be a supplementary measure to the main process of stiffness variation.

In the first place, it would appear that the simple stiffness criteria which have served us so well in the past are likely to require elaboration. We have remarked that this elaboration has already been proposed for the wing torsional stiffness criterion; although it does not at present amount to more than saying that the familiar criterion is no longer a constant, but is a function of all the important relevant parameters.

On a point of detail, it may be found to be desirable to square the criteria in their present form and to introduce the standard air density to render the criteria non-dimensional; if this were done the criteria would be in the nature of force or moment coefficients corresponding to unit increments of displacement, and would define the required stiffnesses explicitly.

But we may lose even this elaborated form as we modify our designs to penetrate further and further into the sonic regions and beyond. For not only must we introduce different functions for Mach number corrections at subsonic and supersonic speeds; we are also likely to find that new criteria are involved and that there is such an essential interdependence between the criteria that we cannot define them separately.

We are beginning to realise this in studies of sweptback wings for high speeds:[60] wing bending stiffness becomes increasingly more important, as sweepback increases, in such matters as aileron reversal, longitudinal stability and flutter. And the bending and torsional stiffnesses are almost indissolubly mixed in the theories—it is in fact no easy matter even to formulate unambiguous definitions of bending and torsional stiffness for cranked wings.

If, therefore, we find our simple stiffness criteria unsuitable or insufficient for design purposes we may have recourse to a process of " stiffness stressing "—a study by stress offices of deformations under various loading conditions, parallel to the studies of strength now made. In this way we may find a new correlation between strength and stiffness coming about. But in any case, it seems evident that for very high speed aircraft the utmost efforts should be made to ensure that elastic deformations are minimised; and, *ceteris paribus*, those aircraft having high structural stiffness are likely to be most successful.

8. CONCLUSIONS.

In what has gone before we have attempted to review in a very small compass the progress in the past decade of aero-elasticity and its allied subjects. The writer is aware that, in attempting this task, he has laid himself open to a charge of insularity; and, indeed, some may regard the reference list as parochial.

As regards the latter charge we may, perhaps, be allowed to remark that during the past ten years the number of papers presented to the Oscillation Sub-Committee of the A.R.C. alone is nearly six hundred; and almost without exception these papers have been related to aeroelasticity in the wider sense implied in the present paper.

The number of relevant communications to other bodies is not known. But it is apparent that in any attempt to survey the general trend of work in all areas, it is necessary to make a very restricted choice of references; in the present case it is hoped that those quoted will themselves give the interested reader more extended references in their own areas.

In the flutter field, a bibliography of British work containing about two hundred and fifty references was recently prepared by Graham;[61] other similar bibliographies in all fields would be very helpful.

The charge of insularity is well sustained and cannot be refuted. In explanation, however, we must point out that, except in the flutter field, countries other than Great Britain have given little attention to aeroelastic matters until the past few years. Only recently has it become the practice to examine on a more or less routine basis the effects of distortion on lateral and longitudinal control and stability.

Indeed, we have recently learned[62] that German military aircraft suffered heavily at one period during the war as a result of the lack of attention paid to aeroelastic phenomena; and much concentrated effort had to be applied to the task of finding out what was going wrong and into the redesign which was found to be necessary in many cases.

Moreover, nowhere else has theory or experiment thrown up a method of dealing with aeroelastic phenomena of a simplicity comparable with that of the stiffness criteria used in Great Britain.

Thus it may fairly be claimed that in co-ordinated aeroelastic research, the most consistent developments have taken place in Great Britain. This is not to say, of course, that in individual areas we have not learned much from other countries: the reverse is the case. To take only one example, in the flutter field the present emphasis on normal modes of vibration as found from resonance tests owes much to American influence. But co-ordinated aeroelastic research belongs peculiarly to Great Britain; and, as the writer has attempted to show, a larger co-ordination may very soon be necessary.

The writer wishes to express his thanks to the Ministry of Supply and Aircraft Production for permission to refer to many Reports which are as yet unpublished and to work for the Ministry on which he was personally engaged during the war years.

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NOTE.—It must be clearly realised that the list of references given here is by no means complete; indeed, it is no more than a small sample of the papers which might be consulted. However, in most cases the works quoted themselves contain reference lists covering their own areas reasonably fully.

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