POLITECNICO MILANO 1863

055738 – STRUCTURAL DYNAMICS AND AEROELASTICITY

02 Static Aeroelasticity: Control surface effectiveness

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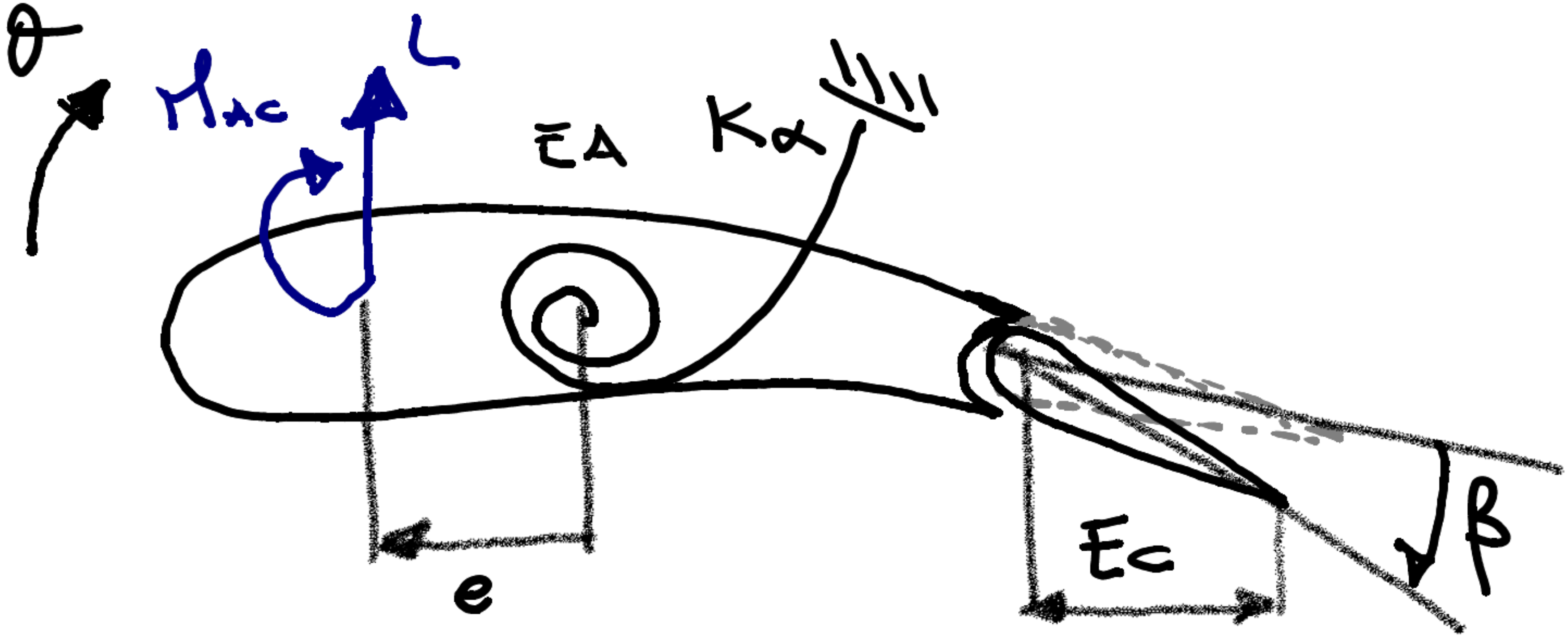
Material

1) Dowell: Chapter 2 Static Aeroelasticity Sections 2.1.1 and 2.3
2) BAH Chapter 8 Sections 8.1 and 8.2
3) Masarati DCFA Chapter 8.1 and 8.1.5
4) Fung Chapter 4.1

Typical section with flap



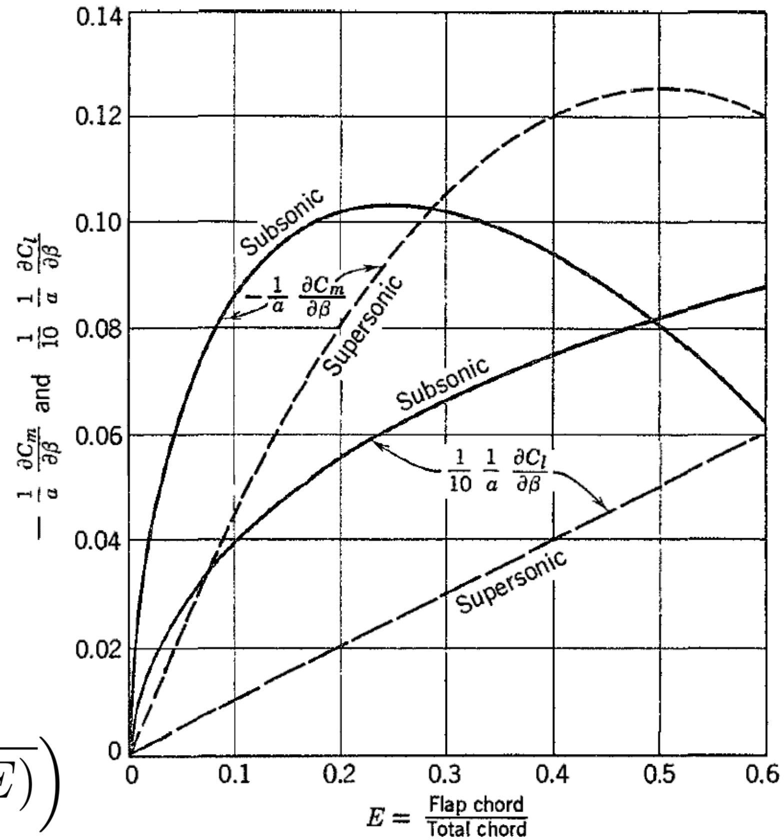
Since effects who contribute to trim elastic condition, i. e. initial angle of attack, camber etc. are additive linearly (<u>superimposition of linear effects</u>), here only perturbation caused by flap will be discussed. <u>θ is the perturbation with respect to trim condition.</u>

Aerodynamics

$$
\begin{aligned}
L &= qS(C_{L0} + C_{L\alpha}\theta + C_{L\beta}\beta) \\
M_{AC} &= qSc(C_{m_C A} + C_{m\beta}\beta)
\end{aligned}
$$

Warning: typically $C_{m\beta} < 0$

$$
\begin{aligned}
\frac{C_{L\beta}}{C_{L\alpha}} &= \frac{1}{\pi}\left(\cos^{-1}(1-2E) + 2\sqrt{E(1-E)}\right) \\
\frac{C_{m\beta}}{C_{L\alpha}} &= -\frac{1}{\pi}(1-E)\sqrt{E(1-E)}
\end{aligned}
$$



Compute the twist due to flap rotation

$$\theta = qS\frac{eC_{L\beta} + cC_{m\beta}}{k_\alpha - qk_A}\beta$$

Elastic stability derivative

Compute the elastic stability derivative $C_{L\beta}$ and compare it to the rigid one.

$$E_c = \frac{(C_{L\beta})_e}{C_{L\beta}} = \frac{1 + qSc\frac{C_{L\alpha}C_{m\beta}}{k_\alpha C_{L\beta}}}{1 - \frac{q}{q_D}}$$

This quantity is also called control elastic efficiency could be positive or negative

Reversal of aileron control

Usually $\frac{C_{L\alpha}C_{m\beta}}{C_{L\beta}} < 0$ the control efficiency reduces while q is increased until it becomes Ec = 0.

This condition is called <u>REVERSAL</u>

$$1 + q_R S c \frac{C_{L\alpha}C_{m\beta}}{C_{L\beta}} = 0$$

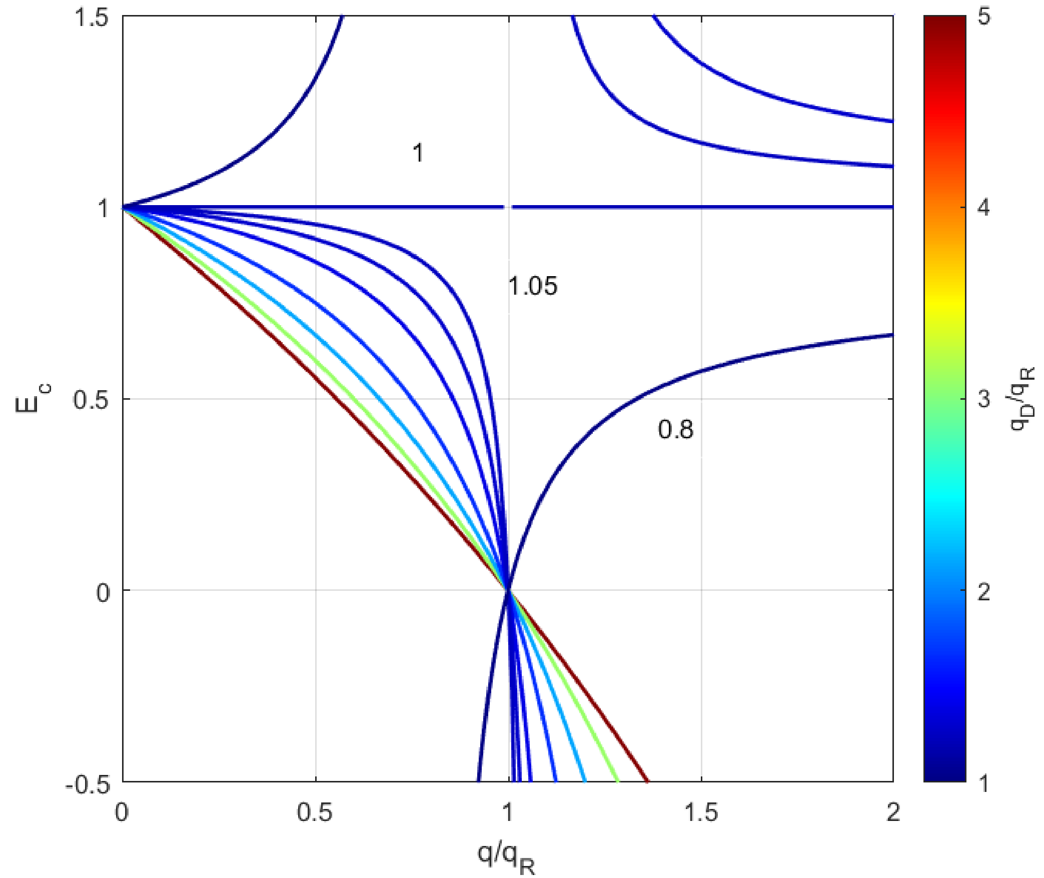
$$\boxed{q_R = -\frac{k_\alpha C_{L\beta}}{S c C_{L\alpha} C_{m\beta}}}$$

NB It does not depend on **e**

<u>The condition is critical when flying close to q_R because the control surface becomes ineffective, compromising aircraft controllability.</u>

Plot control surface elastic efficiency

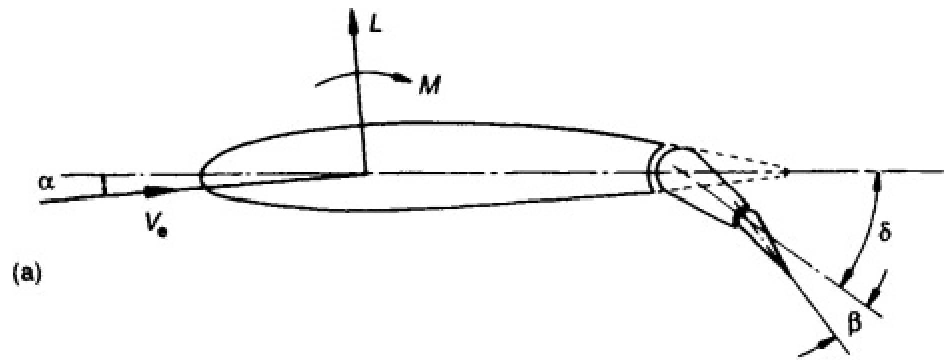
$$E_C = \frac{1 - \frac{q}{q_R}}{1 - \frac{q}{q_D}} = \frac{1 - \frac{q}{q_R}}{1 - \frac{q}{q_R}\frac{q_R}{q_D}}$$

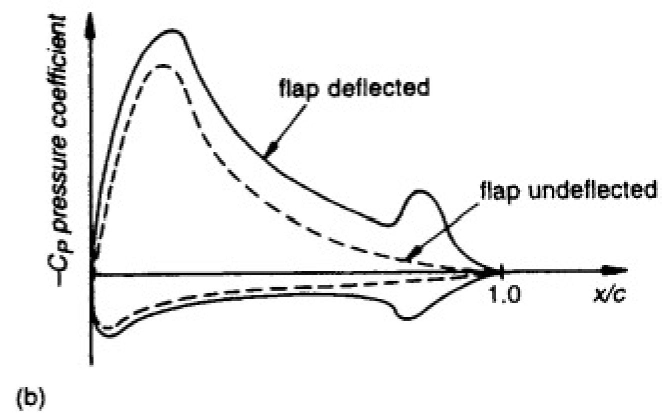


Control surface load distribution

When the aileron is activated the center of pressure is moved backward (toward the trailing edge) causing a nose-down pitch moment.

The elastic twist reduce the incidence and so loads and pitch moment. So, the effect in terms of loading is positive because there is a favorable redistribution of loads.





Matrix form of control reversal problem

It is possible to formulate the control reversal problem in another way. Let's write together the moment equilibrium about the elastic axis and the equation for lift

$$\begin{cases} (k_\alpha - qk_A)\theta = qS(eC_{L\beta} + cC_{m\beta})\beta \\ L = qS(C_{L\alpha}\theta + C_{L\beta}\beta) \end{cases} \tag{1}$$

Remember that θ and L presented here are only the *variations with respect to the trim values.*

$$\begin{bmatrix} 0 & k_\alpha - qk_A \\ 1 & -qSC_{L\alpha} \end{bmatrix} \begin{Bmatrix} L \\ \theta \end{Bmatrix} = qS \begin{Bmatrix} eC_{L\beta} + cC_{m\beta} \\ C_{L\beta} \end{Bmatrix} \beta$$

Matrix approach: case #1 Lift Increment

1. What is the lift increment and twist due to the deflection β of the control, for an assigned value of dynamic pressure q

$$\begin{Bmatrix} L \\ \theta \end{Bmatrix} = qS \begin{bmatrix} 0 & k_\alpha - qk_A \\ 1 & -qSC_{L\alpha} \end{bmatrix}^{-1} \begin{Bmatrix} eC_{L\beta} + cC_{m\beta} \\ C_{L\beta} \end{Bmatrix} \beta$$

2. To identify the control reversal condition q_R it is necessary to identify the case for which given any β the lift obtained is L= 0. Use the first equation to identify η and the second to compute q_R

Matrix approach: case #2 eigenvalue problem

3. Given a target variation of lift L compute the required rotation of the control surface and the resulting twist

$$\begin{bmatrix} -qS(eC_{L\beta} + cC_{m\beta}) & k_\alpha - qk_A \\ -qSC_{L\beta} & -qSC_{L\alpha} \end{bmatrix} \begin{Bmatrix} \beta \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ -1 \end{Bmatrix} L$$

This problem is solvable if the coefficient matrix is not singular

$$\left(\begin{bmatrix} 0 & k_\alpha \\ 0 & 0 \end{bmatrix} - q \begin{bmatrix} -S(eC_{L\beta} + cC_{m\beta}) & -k_A \\ -SC_{L\beta} & -SC_{L\alpha} \end{bmatrix} \right) \begin{Bmatrix} \beta \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ -1 \end{Bmatrix} L$$

$$\mathbf{A} \begin{Bmatrix} \beta \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ -1 \end{Bmatrix} L \qquad (1)$$

Matrix approach: case #2 eigenvalue problem

When L = 0, the problem is transformed in an EIGENVALUE problem. The eigenvalues are the values of q_i for which a nonzero control rotation and twist are possible even if L = 0. Those are the condition for whom the matrix **A** is singular

$$\mathbf{A}\begin{Bmatrix}\beta\\ \theta\end{Bmatrix} = (\mathbf{K}_s - q\mathbf{K}_{AE})\begin{Bmatrix}\beta\\ \theta\end{Bmatrix} = 0$$

Find the values of q for which

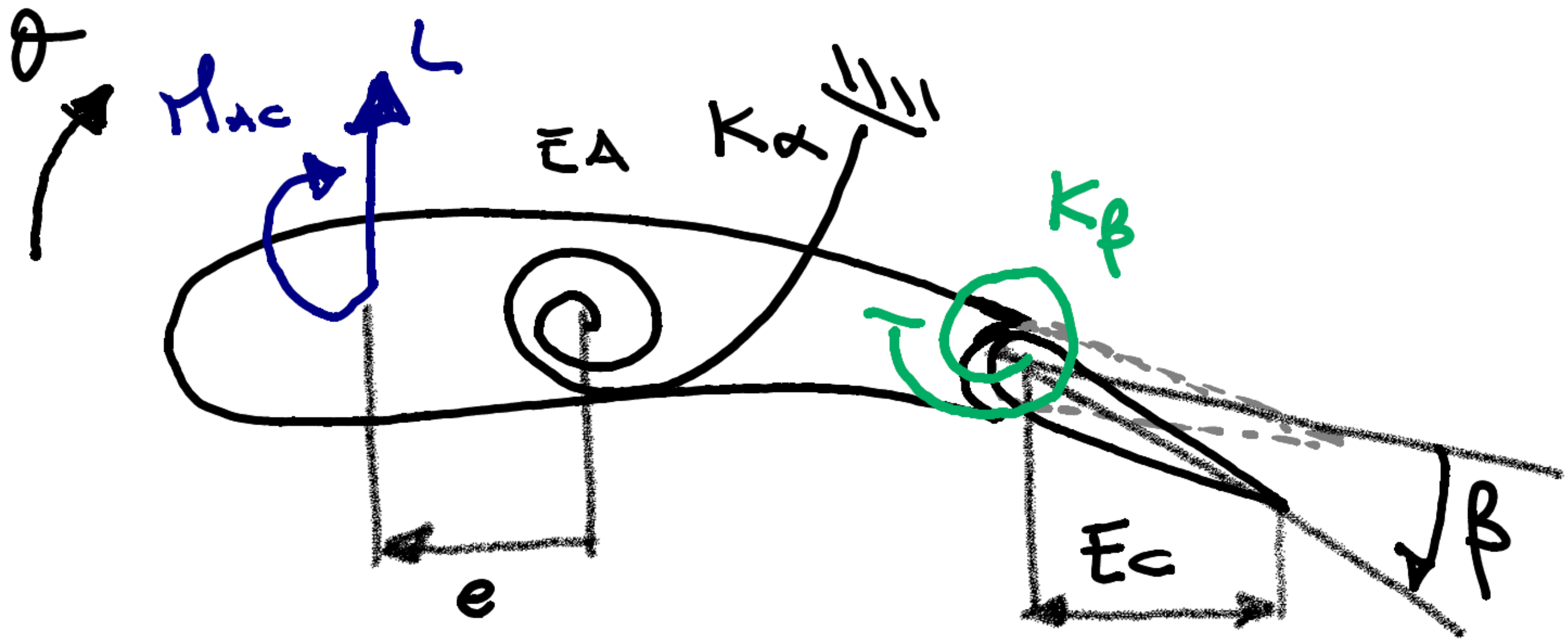
$$\det \mathbf{A} = 0 \qquad q_i^2 S^2 (eC_{L\beta} + cC_{m\beta})C_{L\alpha} + q_i S C_{L\beta}(k_\alpha - q_i k_A) = 0$$

$$\det \mathbf{K}_s - q\mathbf{K}_A = 0 \quad q_i\left(q_i S^2(eC_{L\beta}C_{L\alpha} + cC_{m\beta}C_{L\alpha} - eC_{L\beta}C_{L\alpha}) + SC_{L\beta}k_\alpha\right) = 0$$

$$\begin{cases} q_i = 0 \\ q_i = -\frac{k_\alpha C_{L\beta}}{ScC_{m\beta}C_{L\alpha}} \end{cases}$$

At zero dynamic pressure any rotation β produce no change of lift. So, it is a control inversion condition

Addition of control surface stiffness



H aerodynamic hinge moment that depend on the flap surface and flap chord

$$H = qS_f c_f (C_{H\alpha}\alpha + C_{H\beta}\beta)$$

Addition of control surface stiffness

Equilibium about the EA and the flap hinge

$$
\begin{aligned}
qSe(C_{L\alpha}\theta + C_{L\beta}\beta) + qScC_{m\beta}\beta - k_\alpha\theta &= 0 \\
qS_f c_f(C_{H\alpha}\theta + C_{H\beta}\beta) - k_\beta(\beta - \beta_0) = 0
\end{aligned}
$$

β_0 is the position for the control surface required by the control chain (i.e. the pilot, the actuator, the FCS)

$$
\left(\begin{bmatrix} k_\alpha & 0 \\ 0 & k_\beta \end{bmatrix} - q \begin{bmatrix} SeC_{L\alpha} & SeC_{L\beta} + ScC_{m\beta} \\ S_f c_f C_{H\alpha} & S_f c_f C_{H\beta} \end{bmatrix} \right) \begin{Bmatrix} \theta \\ \beta \end{Bmatrix} = \begin{Bmatrix} 0 \\ k_\beta \end{Bmatrix} \beta_0
$$

$$
\begin{aligned}
\mathbf{A}\mathbf{x} = \mathbf{F} \\
\mathbf{x} = \mathbf{A}^{-1}\mathbf{F}
\end{aligned}
$$

Forcing term
(external input)

Addition of control surface stiffness

$$\mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

$$\begin{aligned}
\det \mathbf{A} &= (k_\alpha - qSeC_{L\alpha})(k_\beta - qS_f c_f C_{H\beta} - q^2 S S_f c_f C_{H\alpha}(eC_{L\beta} + cC_{m\beta}) \\
\det \mathbf{A} &= k_\alpha k_\beta \left(\left(1 - q\frac{k_A}{k_\alpha}\right) \left(1 - q\frac{S_f c_f C_{H\beta}}{k_\beta}\right) - \frac{qSe}{k_\alpha} \frac{qS_f c_f}{k_\beta} C_{H\alpha} \left(C_{L\beta} + \frac{c}{e} C_{m\beta}\right) \right)
\end{aligned}$$

$$\begin{aligned}
\theta &= qS\frac{k_\beta}{\det \mathbf{A}}(eC_{L\beta} + cC_{m\beta})\beta_0 \\
\beta &= \frac{k_\beta}{\det \mathbf{A}}(k_\alpha + qSeC_{L\alpha})\beta_0
\end{aligned}$$

Addition of control surface stiffness

$$L = qS(C_{L\alpha}\theta + C_{L\beta}\beta)$$

$$L = qS\frac{k_\beta}{\det \mathbf{A}}\left(C_{L\alpha}qS(\cancel{eC_{L\beta}} + cC_{m\beta}) + C_{L\beta}k_\alpha \cancel{- C_{L\beta}qSeC_{L\alpha}}\right)\beta_0$$

$$L = qS\frac{k_\beta k_\alpha}{\det \mathbf{A}}\left(C_{L\beta} + \frac{qSc}{k_\alpha}C_{m\beta}C_{L\alpha}\right)\beta_0$$

if $L_R = qSC_{L\beta}\beta_0$

$$\frac{L}{L_R} = \frac{k_\beta k_\alpha}{\det \mathbf{A}}\left(1 + \frac{qSc}{k_\alpha}\frac{C_{m\beta}C_{L\alpha}}{C_{L\beta}}\right)$$

Addition of control surface stiffness

if $L_R = qSC_{L\beta}\beta_0$

$$\frac{L}{L_R} = \frac{k_\beta k_\alpha}{\det \mathbf{A}}\left(1 + \frac{qSc}{k_\alpha}\frac{C_{m\beta}C_{L\alpha}}{C_{L\beta}}\right)$$

1. Reversal is possible also in this case and q_R is the same as for infinite flap hinge stiffness

2. If we define

$$q_{D0} = \frac{k_\alpha}{SeC_{L\alpha}} \quad \text{and} \quad q_{Df} = -\frac{k_\beta}{S_f c_f C_{H\beta}}$$

$$x = \frac{C_{H\alpha}}{C_{H\beta}}\left(\frac{C_{L\beta}}{C_{L\alpha}} + \frac{c}{e}\frac{C_{m\beta}}{C_{L\alpha}}\right)$$

There is a change in the divergence speed and not on the control reversal speed

Then

$$\frac{k_\alpha k_\beta}{\det \mathbf{A}} = \frac{1}{\left(1 - \frac{q}{q_{D0}}\right)\left(1 + \frac{q}{q_{Df}}\right) + x\frac{q}{q_{D0}}\frac{q}{q_{Df}}}$$

Divergence happen when the denominator goes to zero.

Addition of control surface stiffness

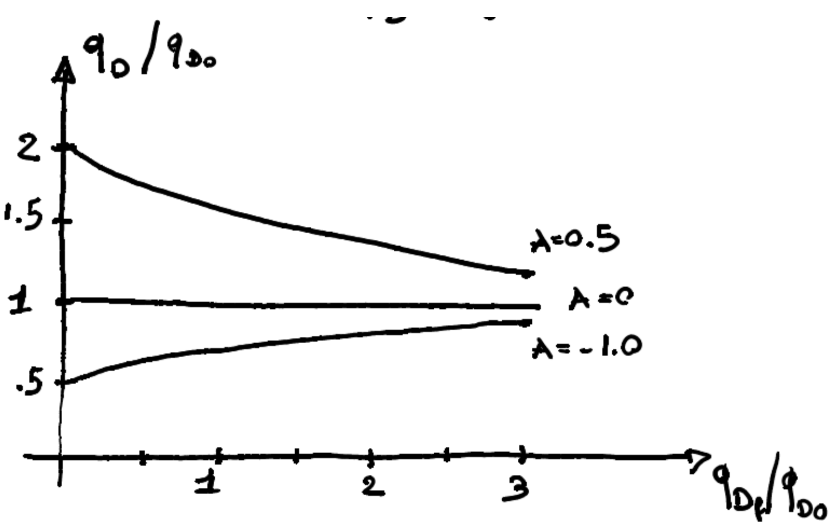
Solution for the denominator equal to zero

$$q_{D0}\frac{\left(1-\frac{q_{Df}}{q_{D0}}\right)+\sqrt{\left(1-\frac{q_{Df}}{q_{D0}}\right)^2+4(1-x)\frac{q_{Df}}{q_{D0}}}}{2(1-x)}$$

a) If the flap is unrestrained (null hinge stiffness)
Then
$$q_D=\frac{q_{D0}}{(1-x)}$$

b) If x = 0

$$q_D=q_{D0}$$

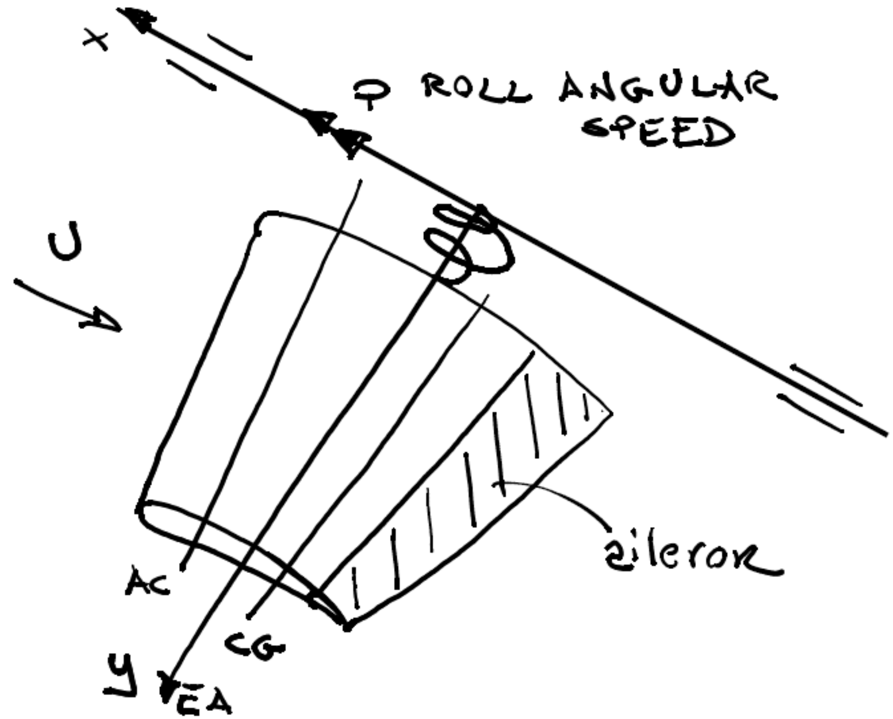


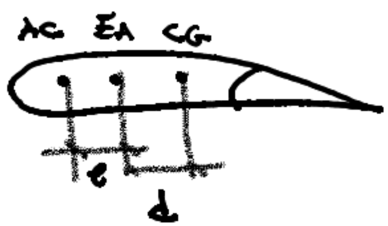
Rolling of a straight wing (typical section model)

We would like to represent a dynamic maneuver of the aircraft considering the effect of flexibility.

We allow the simple typical section model to rotate freely about the roll axis.

The proposed model allows the dynamics of the rigid roll movement caused by a rotation of the aileron to be investigated.





Rolling of a straight wing (typical section model)

Hypothesis:

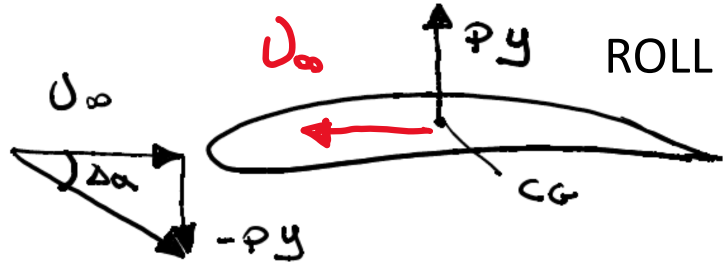
1) The motion is considered the sum of a <u>rigid movement</u> plus the <u>deformation of the wing structure</u>
2) It is supposed that the structure deformation behaves statically, i.e.
    a) the characteristic time required by the deformable structure to adapt to a change of loading is much smaller than the characteristic time of the rigid movement
    b) The structure is deformed as if the load is applied statically, i.e. the different loading conditions during the maneuver are equivalent to a sequence of static deformations
3) Aerodynamics too behaves statically

This model is still considered part of static aeroelasticity. The only inertia forces considered are those caused by rigid motion

Rolling of a straight wing (typical section model)

Aerodynamics of the wing section

FLIGHT SPEED



ROLL SPEED

RELATIVE AIR SPEED CAUSED BY ROLL MOTION

ROLL AERODYNAMIC MOMENT

$$\Delta\alpha = -\tan\left(\frac{py}{U_\infty}\right)$$

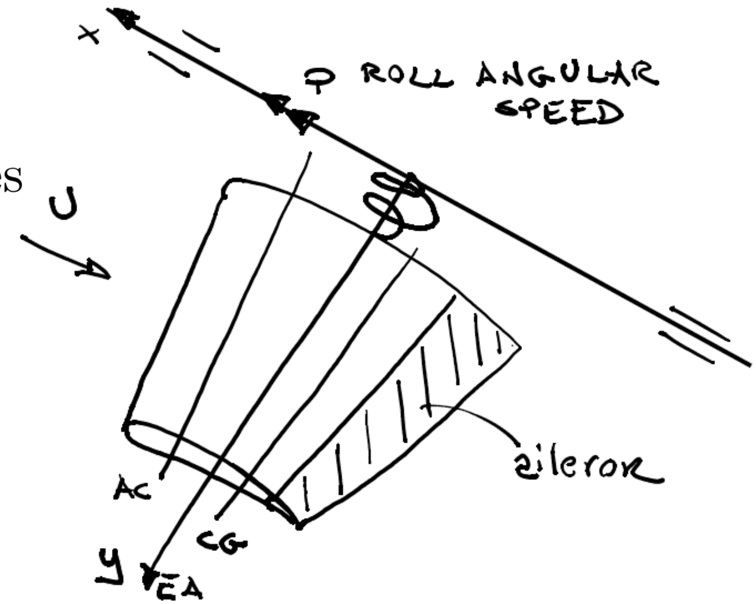
$$\Delta\alpha \approx -\frac{py}{U_\infty}$$

$$I_{xx}\dot{p} = L_R$$

$$I_{xx}\dot{p} = qSb\left(\cancel{C_{\ell 0}} - C_{\ell p}\frac{pb}{U_\infty} + C_{\ell\beta}\beta\right)$$

Rolling of a straight wing (typical section model)

$$
\begin{aligned}
L &= qc(y)\left(C_{L\alpha}\left(\theta - \frac{py}{U_\infty}\right) + C_{L\beta}\beta\right) \\
M_{AC} &= qc^2(y)C_{m\beta}\beta
\end{aligned}
$$

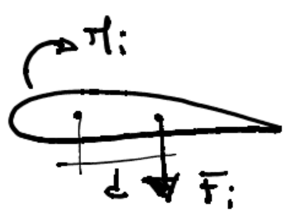


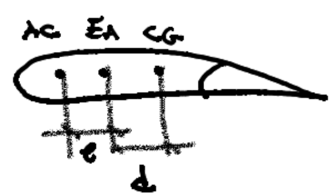
Together with aerodynamic forces there is an additional forcing term due to intertia forces generated by the rigid roll movement

$$F_i = -m\dot{p}y$$

where m is the mass per unit span [kg/m]
This force generates a twiting moment about the EA equal to

$$M_i = md\dot{p}y$$





Rolling of a straight wing (typical section model)

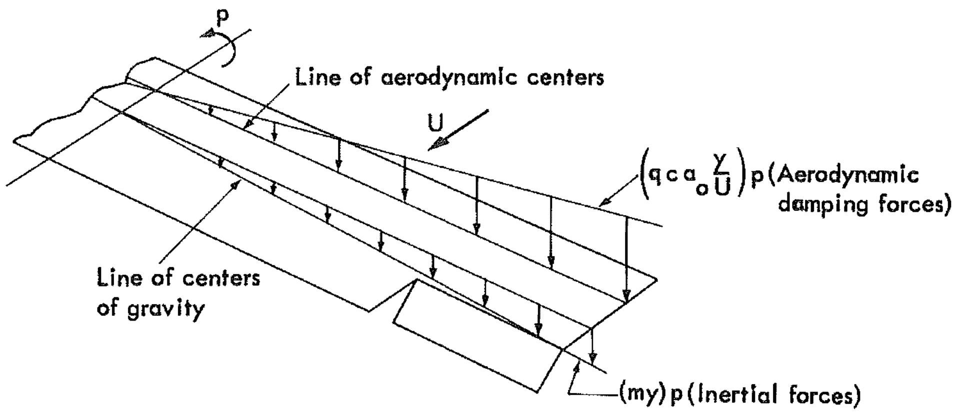


Fig. 8–13. Damping and inertial forces on rolling wing.

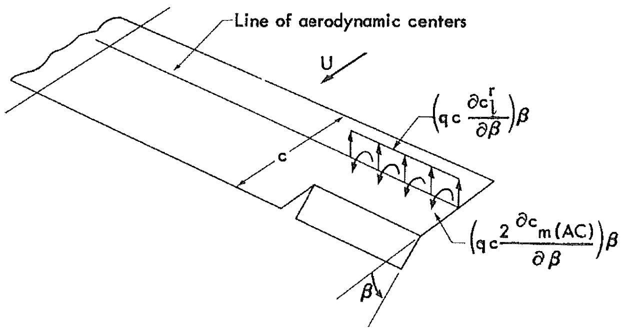


Fig. 8–12. Forces and moments due to displaced aileron.

Rolling of a straight wing (typical section model)

Write the two equations i.e.

1) Typical section pitch equation
2) Roll equation

Rolling of a straight wing (typical section model)

Matrix form of the problem

$$\begin{bmatrix} I_{xx} & -qSb\frac{C_{L\alpha}}{2} \\ -m_T\frac{db}{2} & (k_\alpha - qk_A) \end{bmatrix} \begin{Bmatrix} \dot{p} \\ \theta \end{Bmatrix} = qS \begin{Bmatrix} bC_{\ell\beta} \\ eC_{L\beta} + cC_{m\beta} \end{Bmatrix} \beta - qS \begin{Bmatrix} bC_{\ell p} \\ \frac{eC_{L\alpha}}{2} \end{Bmatrix} \frac{pb}{U_\infty}$$

If d = 0

$$\theta = qS\frac{(eC_{L\beta} + cC_{m\beta})}{k_\alpha - qk_A}\beta - qS\frac{eC_{L\alpha}}{2(k_\alpha - qk_A)}\frac{pb}{U_\infty}$$

Otherwise

$$\theta = qS\frac{(eC_{L\beta} + cC_{m\beta})}{k_\alpha - qk_A}\beta - qS\frac{eC_{L\alpha}}{2(k_\alpha - qk_A)}\frac{pb}{U_\infty} + \frac{m_T db}{2(k_\alpha - qk_A)}\dot{p}$$

Rolling of a straight wing (typical section model)

Solutions

$$I_{xx}\dot{p} = -\,qSb\left(C_{\ell p} + qS\frac{eC_{L\alpha}^2}{4(k_\alpha - qk_A)}\right)\frac{pb}{U_\infty} + \\ + qS\left(C_{\ell\beta} + qS\frac{C_{L\alpha}}{2}\frac{(eC_{L\beta} + cC_{m\beta})}{k_\alpha - qk_A}\right)\beta$$

$$I_{xx}\dot{p} = qSb\left(-(C_{\ell p})_e\frac{pb}{U_\infty} + (C_{\ell\beta})_e\beta\right)$$

Control reversal problem

$$\text{find } q = q_R \text{ for which } (C_{\ell\beta})_e = 0$$

Compare with the previous problem to see if the reversal dynamic pressure is the same

Rolling of a straight wing (typical section model) Consistent performance problems

This formulation could be also used to identify several *consistent performance problems* i.e., problems who allows to identify a performance index in a mathematically determinate framework. In this case the following consistent problems could be considered

1. Compute the angular speed generated by a rotation of the movable surface β at regime $\overline{p}$ i.e., when $\dot{p} = 0$.

$$\frac{\overline{p}b}{U_\infty} = \frac{(C_{\ell\beta})_e}{(C_{\ell p})_e}\beta$$

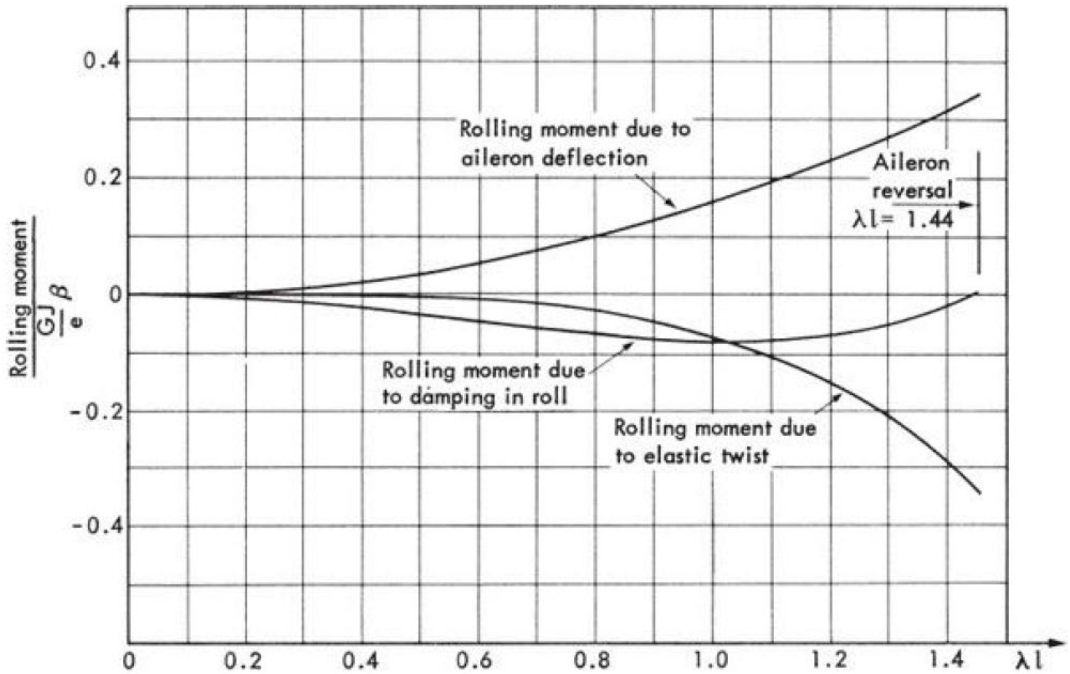
2. Compute the initial acceleration $\dot{p}_0$ generated by the rotation of the movable surface β. At the first instant $p = 0$ so

$$\dot{p}_0 = \frac{qSb(C_{\ell\beta})_e}{I_{xx}}\beta$$

Rolling of a straight wing: roll acceleration null, regime roll speed (and moments)

$$qSb(C_{\ell p})_e \frac{\overline{p}b}{U_\infty} = qSb\left(C_{\ell\beta}\beta + \boxed{((C_{\ell\beta})_e - C_{\ell\beta})}\right)\beta$$

$$\frac{qSC_{L\alpha}}{2}\,\frac{eC_{L\beta} + cC_{m\beta}}{k_\alpha - qk_A}$$



Certification Standards

The flexibility can affect the loads distribution. CS-25 states: **"If deflections under load would significantly change the distribution of internal or external loads, this redistribution must be taken into account"**.