SAGNAC EFFECT

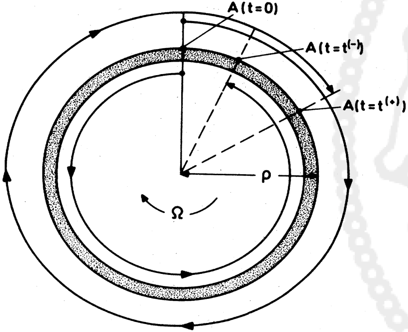
Alberto Rolando

Department of Aerospace Science and Technology
Politecnico di Milano

Version 0.6.d

SAGNAC EFFECT

Consider a ring interferometer like the one in figure.
Laser light enters the interferometer at point A and is split into ClockWise (CW) and Counter ClockWise (CCW) propagating beams by a beam splitter.



If the interferometer is not rotating, the CW and CCW propagating beams recombine at point A after a time given by

$$t = \frac{2\pi\rho}{c} \tag{1}$$

SAGNAC EFFECT

However, if the interferometer is rotating, with angular velocity Ω about an axis through the centre and perpendicular to the plane of the interferometer, then the beams re-encounter at different times because the CW (co-directional with Ω) propagating beam must traverse a slightly longer path in order to complete one round trip, since the interferometer rotates through a small angle during the round-trip transit time.
Similarly, the CCW propagating beam traverses a slightly shorter path.

The round-trip transit time of the CW beam t^+ is given by:

$$t^+ = \frac{2\pi\rho + \rho\Omega t^+}{c} \tag{2}$$

SAGNAC EFFECT

Similarly, the round-trip transit time of the CCW beam t^- is given by:

$$t^- = \frac{2\pi\rho - \rho\Omega t^-}{c} \tag{3}$$

Solving (2) and (3) for t^+ and t^- and then taking the difference gives

$$\Delta t = t^+ - t^- = \frac{4\pi\rho^2\Omega}{c^2 - \rho^2\Omega^2} \tag{4}$$

For real-world values of ρ and Ω, $(\rho\Omega)^2 \ll c^2$ so that

$$\Delta t \approxeq \frac{4\pi\rho^2\Omega}{c^2} \tag{5}$$

SAGNAC EFFECT

The round-trip optical path lenght difference ΔL is:

$$\Delta L = c\Delta t = \frac{4\pi\rho^2}{c}\Omega \tag{6}$$

From (6) we see that the round-trip optical path difference, according to this analysis, is directly proportional to the rotation rate of the interferometer.

It can be shown that (6) can be generalised to an arbitrary interferometer shape yielding:

$$\Delta L = \frac{4A}{c}\Omega \tag{7}$$

where A is the area enclosed by the light path.

SAGNAC EFFECT

From (7) we see that the phase difference, $\Delta\phi$ between the counter-propagating beams after one round trip is given by:

$$\Delta\phi = \Delta L \frac{2\pi}{\lambda} = \frac{4A}{c\lambda_r}\Omega \tag{8}$$

where A is the area enclosed by the light path and

$$\lambda_r = \frac{\lambda}{2\pi} \tag{9}$$

is the beam reduced wavelength.

For a Fiber Optic Gyro (FOG) with n turns, eq. (8) becomes

$$\Delta\phi = n\frac{4A}{c\lambda_r}\Omega \tag{10}$$

List of Acronyms

CCW	Counter ClockWise
CW	ClockWise
FOG	Fiber Optic Gyro