1.1.3 Twist due to flap rotation

Let us write the equilibrium about the elastic axis of the typical section with a movable surface, starting from the definition of the lift and moment considering also the effects generated by the movable surface

$$
\begin{aligned}
L &= qS(C_{L0} + C_{L\alpha}\theta + C_{L\beta}\beta) && (1.19)\\
M_{AC} &= qSc(C_{m_{CA}} + C_{m\beta}\beta) && (1.20)
\end{aligned}
$$

We will not consider any reference load in this case since we are interested in the variation with respect to the equilibrium condition. The equilibrium equation about the elastic axis is

$$
Le + M_{AC} = k_\alpha\theta \tag{1.21}
$$

Substituting the expression of lift and moment

$$
qSe(C_{L\alpha}\theta + C_{L\beta}\beta) + qScC_{m\beta}\beta = k_\alpha\theta \tag{1.22}
$$

Ordering on the left end side all terms related to θ

$$
(k_\alpha - qSeC_{L\alpha})\theta = qS(eC_{L\beta} + cC_{m\beta})\beta \tag{1.23}
$$

Consequently we can compute the elastic torsion that is generated by the rotation of the movable surface β

$$
\theta = qS\frac{eC_{L\beta} + cC_{m\beta}}{k_\alpha - qSeC_{L\alpha}}\beta \tag{1.24}
$$

The lift generated by the rotation of the movable surface is the sum of the lift generated by a rigid airfoil L_R plus the lift due to elastic torsion L_E

$$
L = L_R + L_E = qS\left(C_{L\beta}\beta + C_{L\alpha}qS\frac{eC_{L\beta} + cC_{m\beta}}{k_\alpha - qSeC_{L\alpha}}\beta\right) \tag{1.25}
$$

If we express the L though an elastic stability derivative $(C_{L\beta})_e$

$$
L = qS(C_{L\beta})_e\beta \tag{1.26}
$$

so

$$
(C_{L\beta})_e = \left(C_{L\beta} + C_{L\alpha}qS\frac{eC_{L\beta} + cC_{m\beta}}{k_\alpha - qSeC_{L\alpha}}\right) \tag{1.27}
$$

The control surface elastic efficiency E_c is the the ration between the elastic and the rigid stability derivatives

$$
\frac{(C_{L\beta})_e}{C_{L\beta}} = 1 + \frac{qSeC_{L\alpha} + qScC_{m\beta}\frac{C_{L\alpha}}{C_{L\beta}}}{k_\alpha - qSeC_{L\alpha}} \tag{1.28}
$$

Remembering that $k_A = SeC_{L\alpha}$ the expression can be written as

$$\frac{(C_{L\beta})_e}{C_{L\beta}} = 1 + \frac{qk_A + qScC_{m\beta}\frac{C_{L\alpha}}{C_{L\beta}}}{k_\alpha - qk_A} \tag{1.29}$$

$$\frac{(C_{L\beta})_e}{C_{L\beta}} = \frac{k_\alpha + qScC_{m\beta}\frac{C_{L\alpha}}{C_{L\beta}}}{k_\alpha - qk_A} \tag{1.30}$$

$$E_c = \frac{(C_{L\beta})_e}{C_{L\beta}} = \frac{1 + qSc\frac{C_{L\alpha}C_{m\beta}}{k_\alpha C_{L\beta}}}{1 - \frac{q}{q_D}} \tag{1.31}$$