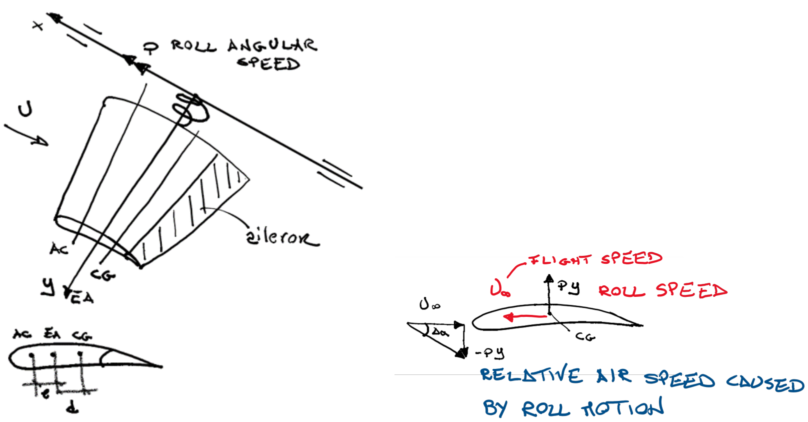
1.1.4 Static aeroelastic roll problem

Consider the case were a maneuver of the aircraft is considered, as the roll maneuver, including the effect of flexibility. Consider to apply this idea to the case of a Typical section wing allowed to rotate about the x axis (see Figure **??**. In this case the wing is allowed to perform a rigid movement, wing roll, and an elastic movement, the twis of the wing about the lumped sping k_α.



(a) Typical section model. (b) Change of angle of attack.

Figure 1.3: Rigid roll about the longitudinal axis of a flexible typical section.

In this case the deformation of the structure will be considered to happen at a much faster speed than the change of roll angle. Consequently the deformation of the structure will be considered *instantaneous*. So due to a change of forces a new twist angle will appear at the same instant without a dynamic evolution, behaving like a static system. It is said that we use a *static approximation of the structural deformable dynamics*. Also for aerodynamic forces, it can be said that their dynamic variation is so fast that they adapt instantaneously to the change of boundary conditions (in this case the shape of the wing) so that it can be considered to behave statically, using a *static approximation of aerodynamics*. The only elements for which the unsteadiness will be completely represented will be those related to the rigid body dynamics. This type of approximation is often employed in static aeroelasticity to identify the effect of manoeuvrers on the aircraft.

The rolling angular speed will generate on each section of the wing an airflow in the opposite direction $v_n = -py$ that causes a change of the angle of attach of each wing section

$$\Delta\alpha = -\tan\frac{py}{U_\infty} \approx -\frac{py}{U_\infty} \tag{1.32}$$

The rigid equation of motion for the roll of the wing could be written as:

$$I_{xx}\dot{p} = L_R \tag{1.33}$$

where I_{xx} is the moment of inertia of the wing about the roll axis and L_R is the roll aerodynamic moment. This aerodynamic moment is the result of the aerodynamic forces generated by: the aerodynamic force generated at the rigid angle of attack α_0 by each wing section, the change of aerodynamic force generated by the rotation of the movable surface β and the change of angle of attack generated by the roll movement $\frac{py}{U_\infty}$. This dependence is typically represented by expressing the roll moment L_R as the sum of three aerodynamic moment terms, that are the product of the dynamic pressure q, the surface of the wing S and the reference length represented by the wing span b, times a non dimensional coefficient i.e.,

$$I_{xx}\dot{p} = L_R = qSb\left(C_{\ell 0} - C_{\ell p}\frac{pb}{U_\infty} + C_{\ell\beta}\beta\right) \tag{1.34}$$

The first term of the roll aerodynamic moment will be here neglected because we are interested in the variations with respect tot the equilibrium conditions. So the equation considered by us will be

$$I_{xx}\dot{p} = qSb\left(-C_{\ell p}\frac{pb}{U_\infty} + C_{\ell\beta}\beta\right) \tag{1.35}$$

It is interesting to see how this equation is modified when we consider the presence of elastic deformation. For each wing section the following aerodynamic actions will be considered

$$L = qc(y)\left(C_{L\alpha}\left(\theta - \frac{py}{U_\infty}\right) + C_{L\beta}\beta\right) \tag{1.36}$$

$$M_{AC} = qc^2(y)C_{m\beta}\beta \tag{1.37}$$

Together with aerodynamic forces there is an additional forcing term due to inertia forces generated by the rigid roll movement

$$F_i = -m\dot{p}y \tag{1.38}$$

where m is the mass per unit span measured in kg/m. This force generates a twisting moment about the EA equal to

$$M_i = md\dot{p}y \tag{1.39}$$

Using all these distributed loads it is possible to compute the equilibrium equation about the twist axis as

$$k_\alpha\theta = \int_0^b (L(y)e(y) + M_{AC}(y))\,\mathrm{d}y + \int_0^b m(y)d(y)\dot{p}y\mathrm{d}y \tag{1.40}$$

Considering all properties as uniform along the span

$$k_\alpha\theta = qS\left(eC_{L\beta}\beta + eC_{L\alpha}\left(\theta - \frac{pb}{2U_\infty}\right) + cC_{m\beta}\beta\right) + \frac{mb^2}{2}d\dot{p} \tag{1.41}$$

The quantity $mb = m_T$ the total mass of the wing.

For the rigid roll equilibrium equation it will result

$$-\int_0^b m(y)y^2\mathrm{d}y\,\dot{p} + \int_0^b Ly\mathrm{d}y = 0 \tag{1.42}$$

The first integral on the left is equal to the moment of inertial of the wing with respect to the roll axis I_{xx}. So,

$$-I_{xx}\dot{p} + qc\int_0^b \left(C_{L\alpha}\left(\theta - \frac{py}{U_\infty}\right) + C_{L\beta}\beta\right) y\mathrm{d}y = 0 \tag{1.43}$$

Considering all properties as uniform

$$I_{xx}\dot{p} = qSb\left(\frac{C_{L\alpha}}{2}\theta - \frac{C_{L\alpha}}{3}\frac{pb}{U_\infty} + \frac{C_{L\beta}}{2}\beta\right) \tag{1.44}$$

Here we can recognize the rigid aerodynamic derivative coefficients i.e.,

$$C_{\ell p} = \frac{C_{L\alpha}}{3} \tag{1.45}$$

$$C_{\ell\beta} = \frac{C_{L\beta}}{2} \tag{1.46}$$

However, there is an additional term related to the twist deformation. Using the equilibrium equation about the EA it is possible to compute θ and then substitute it in this expression to return to a rigid roll expression that is only function of p and β.

Alternatively it is possible to write the problem in matrix from as

$$\begin{bmatrix} I_{xx} & -qSb\frac{C_{L\alpha}}{2} \\ -m_T\frac{db}{2} & (k_\alpha - qk_A)\end{bmatrix}\begin{Bmatrix}\dot{p} \\ \theta\end{Bmatrix} = qS\begin{Bmatrix} bC_{\ell\beta} \\ eC_{L\beta} + cC_{m\beta}\end{Bmatrix}\beta - qS\begin{Bmatrix} bC_{\ell p} \\ \frac{eC_{L\alpha}}{2}\end{Bmatrix}\frac{pb}{U_\infty} \tag{1.47}$$

Inverting the coefficient matrix it is possible to identify the roll acceleration and the wing twist generated. More simply, when $d = 0$, which mean when the center of gravity is coincident with the EA it is possible to use the second equation to compute the twist angle duet to β and p

$$\theta = qS\frac{(eC_{L\beta} + cC_{m\beta})}{k_\alpha - qk_A}\beta - qS\frac{eC_{L\alpha}}{2(k_\alpha - qk_A)}\frac{pb}{U_\infty} \tag{1.48}$$

This expression can be substituted in the equilibrium equation about the roll axis that becomes

$$\begin{aligned} I_{xx}\dot{p} = &- qSb\left(C_{\ell p} + qS\frac{eC_{L\alpha}^2}{4(k_\alpha - qk_A)}\right)\frac{pb}{U_\infty} + \\ &+ qS\left(C_{\ell\beta} + qS\frac{C_{L\alpha}}{2}\frac{(eC_{L\beta} + cC_{m\beta})}{k_\alpha - qk_A}\right)\beta \end{aligned} \tag{1.49}$$

So, the roll equation for the elastic wing could be written as

$$I_{xx}\dot{p} = qSb\left(-(C_{\ell p})_e\frac{pb}{U_\infty} + (C_{\ell\beta})_e\beta\right) \tag{1.50}$$

with the obvious meaning of $(\cdot)_e$ terms as elastic terms (to be compared with rigid terms). UIt is possible to say that by taking into consideration elasticity the aerodynamic stability derivatives of the roll problem are modified. Of course it will be possible to define for every derivative an elastic efficiency ratio as a ration between the elastic derivative and the rigid derivative. For example

$$E_c = \frac{(C_{\ell\beta})_e}{(C_{\ell\beta})_{\text{rigid}}}, \tag{1.51}$$

or similarly for $(C_{\ell p})_e$. [1]

If we focus on the $(C_{\ell\beta})_e$ derivative, it is possible to see the *control reversal problem* as the value of the dynamic pressure that makes the derivative equal to zero i.e.,

$$\text{find } q = q_R \text{ for which } (C_{\ell\beta})_e = 0 \tag{1.52}$$

This formulation could be also used to identify several *consistent performance problems* i.e., problems who allows to identify a performance index in a mathematically determinate framework. In this case the following consistent problems could be considered

1. Compute the angular speed generated by a rotation of the movable surface β at regime $\overline{p}$ i.e., when $\dot{p} = 0$.

$$\frac{\overline{p}b}{U_\infty} = \frac{(C_{\ell\beta})_e}{(C_{\ell p})_e}\beta \tag{1.53}$$

2. Compute the initial acceleration $\dot{p}_0$ generated by the rotation of the movable surface β. At the first instant $p = 0$ so

$$\dot{p}_0 = \frac{qSb(C_{\ell\beta})_e}{I_{xx}}\beta \tag{1.54}$$

[1] In case you consider to include the effect of $d \neq 0$ the result will be

$$\theta = qS\frac{(eC_{L\beta} + cC_{m\beta})}{k_\alpha - qk_A}\beta - qS\frac{eC_{L\alpha}}{2(k_\alpha - qk_A)}\frac{pb}{U_\infty} + \frac{m_T db}{2(k_\alpha - qk_A)}\dot{p}$$

This it turn will lead to a roll equilibrium equation about the elastic wing of this type

$$\left(I_{xx} + qSb\frac{C_{L\alpha}}{2}\frac{m_T db}{2(k_\alpha - qk_A)}\right)\dot{p} = qSb\left(-(C_{\ell p})_e\frac{pb}{U_\infty} + (C_{\ell\beta})_e\beta\right)$$

with the necessity to introduce a modification of the terms proportional to the acceleration that is not simply I_{xx} any more. This is often considered not acceptable and so in many cases reference systems where $m_T d$ is equal to zero are often chosen. The subject is discussed in more detail in the section where consistent static aeroelastic problems are described.