POLITECNICO MILANO 1863

055738 – STRUCTURAL DYNAMICS AND AEROELASTICITY

16 The aeroelastic problem: Formulation of static and dynamic response problems

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Material

Masasati notes Chapter 8

Cooper Wright Chapters 11 & 14

Response problems

The analysis of the response is meaningful only for flight conditions where the aircraft is STABLE.

The types of input:

- Controls (i.e. movable surfaces)
- Gusts
- Exciters (vibrating masses, bonkers, etc…)

Types of analysis: Equilibrium maneuvers

The aircraft is in a steady maneuver, so it is balanced, or trimmed, and so no changes in times must be considered. Examples: steady level flight, pull-up push-down, banked turn, …

EASA CS-25 says “if deflection under load would significantly change the distribution of internal loads, this re-distribution must be kept into account”.

Consequently, the problem could be approached using static aeroelasticity methods

The problem could be approached

a) Reataining a rigid aircraft model and modifying the aerodynamics to keep into account the flexibility
b) Using a fully flexible model

Equilibrium maneuvers

For the aerodynamics it is possible to use the **quasi-steady** behavior since the frequency of the input forces is basically 0, which mean that the **structure responds instantaneously to the forcing** (if this is not the case then a dynamic maneuver approach should be followed for the evaluations)

$$M\ddot{q} + C\dot{q} + Kq - q_\infty \int_0^t h_{am}(t-\tau)\, q(\tau)\, d\tau = F_e$$

$$q_\infty \int_0^t h_{am}(t-\tau)\, q(\tau)\, d\tau = \frac{q_\infty}{U_\infty} C_a \dot{q} + q_\infty K_a q$$

$$M\ddot{q} + C\dot{q} + Kq - \frac{q_\infty}{U_\infty} C_a \dot{q} - q_\infty K_a q = F_e$$

Equilibrium maneuvers

$$M = \begin{bmatrix} M_{RR} & M_{RE} \\ M_{ER} & M_{EE} \end{bmatrix}$$

R RIGID
E ELASTIC

$$C = \begin{bmatrix} 0 & 0 \\ 0 & C_{EE} \end{bmatrix} \quad K = \begin{bmatrix} 0 & 0 \\ 0 & K_{EE} \end{bmatrix}$$

$$M_{RR}\,\ddot{q}_R + \cancel{M_{RE}\,\ddot{q}_E} - \frac{q_\infty}{V_\infty} C_{2RR}\,\dot{q}_R - \cancel{\frac{q_\infty}{V_\infty} C_{2RE}\,\dot{q}_E} +$$

$$- q_\infty K_{2RR}\, q_R - q_\infty K_{2RE}\, q_E = Fe_R$$

$$\ddot{q}_E \simeq 0 \quad \dot{q}_E \simeq 0$$

$$M_{ER}\,\ddot{q}_R + \cancel{M_{EE}\,\ddot{q}_E} + \cancel{C_{EE}\,\dot{q}_E} + K_{EE}\, q_E +$$

$$-\left(\frac{q_\infty}{V_\infty}\right) C_{2ER}\,\dot{q}_R - \cancel{\left(\frac{q_\infty}{V_\infty}\right) C_{2EE}\,\dot{q}_E} - q_\infty K_{2ER}\, q_R +$$

$$- q_\infty K_{2EE}\, q_E = Fe_E$$

Equilibrium maneuvers

$$\underbrace{(K_{EE} - q_\infty K_{a_{EE}})}_{K_{AE}} q_E = F_{e_E} - M_{ER}\ddot{q}_R + \left(\frac{q_\infty}{U_\infty}\right) C_{a_{ER}} \dot{q}_R + q_\infty K_{a_{ER}} q_R$$

K_{AE} is the aeroelastic stiffness (that is singular at the divergence dynamic pressure q_D)

$$\text{if } q_\infty << q_D \quad K_{AE} \cong K_{EE}$$

It is possible to see the deformation of the structure as the combined effect of external loads, inertia forces due to rigid movements and aerodynamic loads.

Equilibrium maneuvers

If the vehicle changes shape new terms proportional to dynamic pressure appear in the first rigid equation.

The equation has the same format of the rigid ones, but the matrixes contain new aerodynamic derivative terms (green).

If $M_{ER} = 0$ then there are no aerodynamic stability derivatives that are proportional to rigid accelerations (as usually in Flight Mechanics)

$$\left(M_{RR} + q_\infty K_{aRE} K_{AE}^{-1} M_{ER}\right) \ddot{q}_R +$$

$$- \left(\frac{q_\infty}{U_\infty}\right)\left(C_{aRR} - q_\infty K_{aRE} K_{AE}^{-1} C_{aER}\right) \dot{q}_R +$$

$$- q_\infty \left(K_{aRR} - q_\infty K_{aRE} K_{AE}^{-1} K_{aER}\right) q_R =$$

$$= F_{eR} + q_\infty K_{aRE} K_{AE}^{-1} F_{eE}$$

Note: if the proper orthogonal modes of a free-free structure are used, $M_{ER} = 0$ by default
Whenever the elastic modes used are orthogonal to rigid modes $M_{ER} = 0$

Dynamic maneuvers

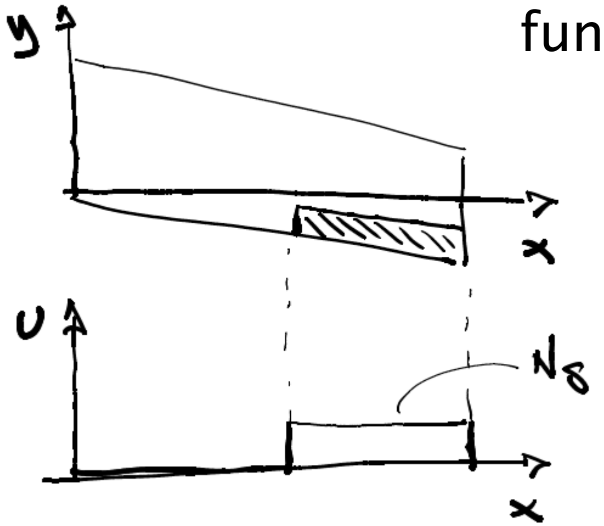
When we are interested in dynamic maneuvers, we will consider the application of a time history of a control surface movement to see what happens to loads.

It is essential to model the behavior of a control surface and its actuation behavior. Consider the case where a servo-system is applied. (we will consider the approach based on Laplace transform only for easiness of the exposition, but the same steps could be followed in any approach).

Modeling of a control surface

Specific shape function used to represent the rigid rotation of the movable surface will be added to the usual shape functions. In this way a specific degree of freedom associated with the rotation of the movable surface will be defined

The set of shape functions will be the union of blocked-surface shape functions + movable surface shape functions



$$v = N q + N_\delta \delta$$

Modeling of a control surface

If we consider an ideal servo actuator that can keep exactly any required position, independently from the load applied on it, it is possible to consider the control rotation δ as an input.

The second equation could be used to identify the load F_δ that must be applied by the actuator/pilot to keep the position δ. This load is in general function of time (so a control system is required to adapt it in order to keep delta constant).

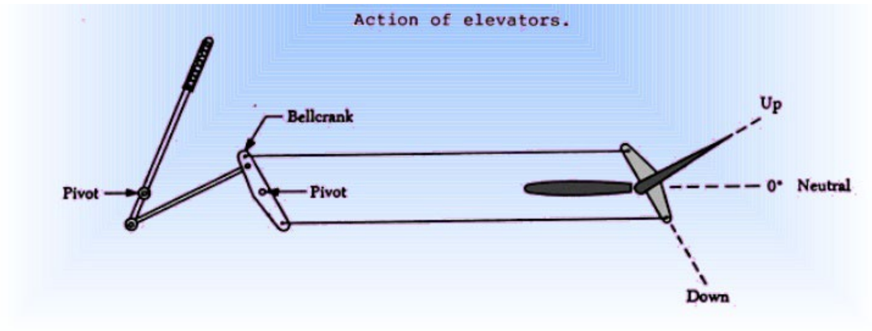
$$\underbrace{\left(s^2M + sC + K - q_\infty H_{am}(s, M)\right)}_{Z(s)} \begin{Bmatrix} q \\ \delta \end{Bmatrix} = \begin{Bmatrix} F_q \\ F_\delta \end{Bmatrix}$$

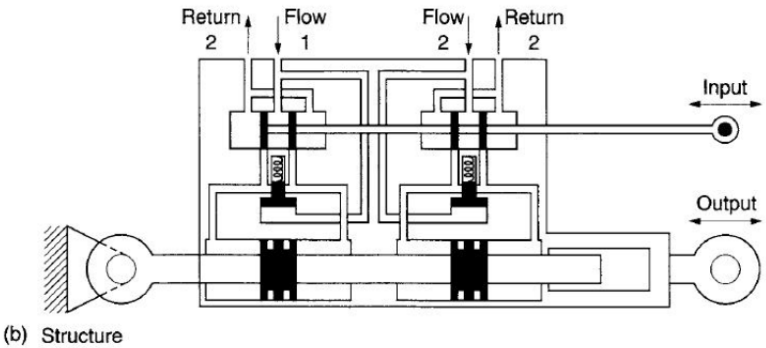
$$\begin{bmatrix} Z_{qq} & Z_{q\delta} \\ Z_{\delta q} & Z_{\delta\delta} \end{bmatrix} \begin{Bmatrix} q \\ \delta \end{Bmatrix} = \begin{Bmatrix} F_q \\ F_\delta \end{Bmatrix}$$

$$Z_{qq}\, q = F_q - Z_{q\delta}\, \delta \quad (1)$$

$$Z_{\delta q}\, q + Z_{\delta\delta}\, \delta = F_\delta$$

Control chain and servo-actuators





For light aircraft a direct mechanical control chain connects the actuators with the control sticks in the cockpit

For larger aircraft the forces to be sustained are too big, so a hydraulic system with irreversible actuators is installed.

The figure shows a duplex actuator where the two valves are supplied by different hydraulic circuits. If there is a pressure loss in one valve the other valve will keep on working without obstructing the

flow.

The pushrod from the pilot could be joined/replaced by an electrical signal that controls the hydraulic power.

Modeling of a servo-controlled surface

To improve the approximation, it is possible to include the dynamics of the actuator though a servo-system transfer function

Keep into account that this model is still ideal, because also in this case the force Fδ must change instantaneously to adapt to changes of q

Servo hydraulic actuator q = valve opening, Q flux, P and G gains

$$\delta = H_{servo}(s)\,\delta_c$$

$$Z_{qq}\,q = F_q - Z_{q\delta}\,H_s(s)\,\delta_c$$

$$Z_{\delta q}\,q + Z_{\delta\delta}\,\delta = F_\delta$$

is the force applied by the actuator/pilot

$$q = G(\delta_c - \delta)$$

q VALVE OPENING

$$\dot{\delta} = P q$$

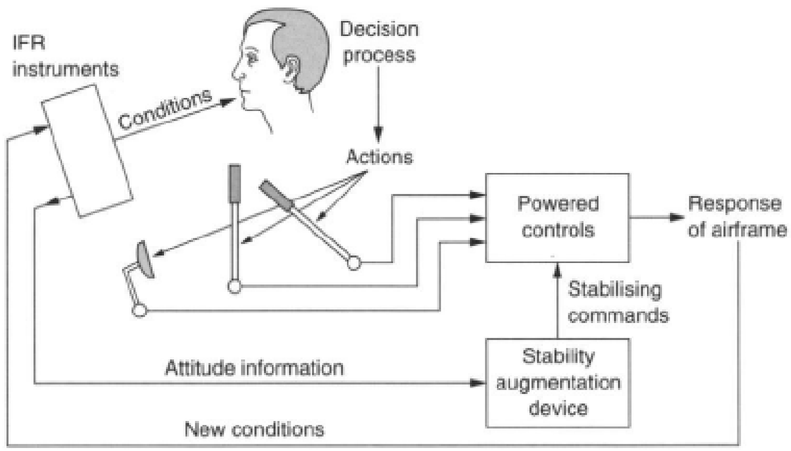
$$Q = A\dot{x}$$

FLUX

$$\dot{\delta} = PG\,\delta_c - PG\,\delta \;\rightarrow\; \delta = \frac{PG}{s+PG}\,\delta_c$$

$$(s^2 + 2\xi\omega_0 s + \omega_0^2)(s + PG)\,\delta = \delta_c$$

Modeling of a Flight Control System



(b)

$$\delta_c = H_p(s)\,\delta_{PILOT} + H_{FCS}(s)\,H_{SN}(s)\begin{Bmatrix} q \\ \dot{q} \\ \ddot{q} \end{Bmatrix}$$

$$\delta_c = H_{FCS}(s)\begin{Bmatrix} \delta_p \\ q \\ \dot{q} \\ \ddot{q} \end{Bmatrix}$$

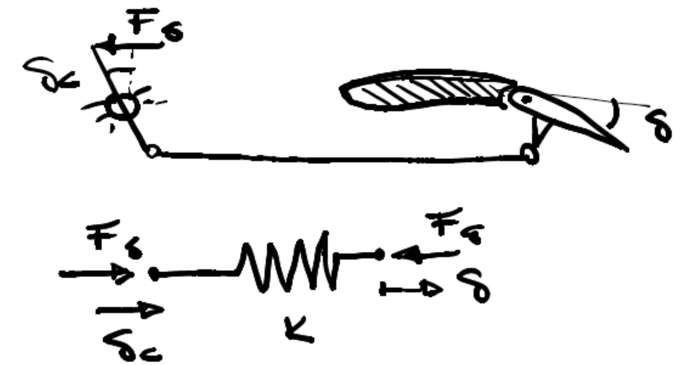
Once the powered control are available, it is a small step to make them accept a signal coming from an automatic system.

The transfer function of the pilot may be not so easy to identify
H_{SN} is the transfer function of sensors

Modeling of a dynamic compliance surface

In the case where there is no servo-actuator too (Hservo = 1) it is possible to have δ and δ_c different because the control chain is in reality an elastic system.

It is possible to see that $\delta = \delta_c$ only if $k = \infty$ or $F\delta = 0$
Both conditions are ideal so in practice it is necessary to consider also the dynamic admittance



$K \to \infty \qquad \delta \to \delta_c$

$K \to 0 \qquad \delta \to 0$

$F_\delta \to 0 \qquad \delta \to \delta_c \ \forall K$

$\frac{F_\delta}{K} = (\delta - \delta_c) \qquad \delta = \delta_c + \frac{1}{K} F_\delta$

$\delta = H_s(s)\,\delta_c + H_c(s)\,F_\delta$

H_c = ADMITTANCE OR DYNAMIC COMPLIANCE

Complete model of a servo-controlled surface

$$\begin{bmatrix} Z_{qq} & Z_{q\delta} & 0 \\ Z_{\delta q} & Z_{\delta\delta} & -1 \\ 0 & 1 & -H_c \end{bmatrix} \begin{Bmatrix} q \\ \delta \\ \bar{F}_\delta \end{Bmatrix} = \begin{Bmatrix} F_q \\ 0 \\ H_{SERVO}\,\delta_c \end{Bmatrix}$$

$$\begin{aligned} x &= \frac{b_{c3}}{s^3 + a_1 s^2 + a_2 s + a_3} x_c + \frac{b_{f2} s + b_{f3}}{s^3 + a_1 s^2 + a_2 s + a_3} f \\ &= h_c x_c + h_f f, \end{aligned}$$

Gust Response

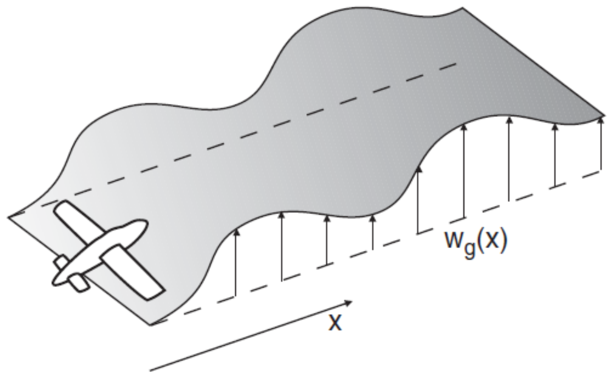


Figure 14.1 Aircraft encountering turbulence.

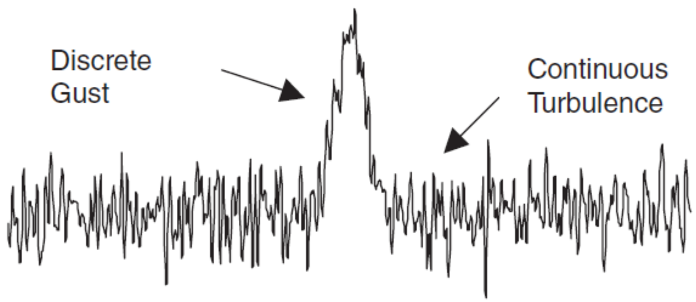
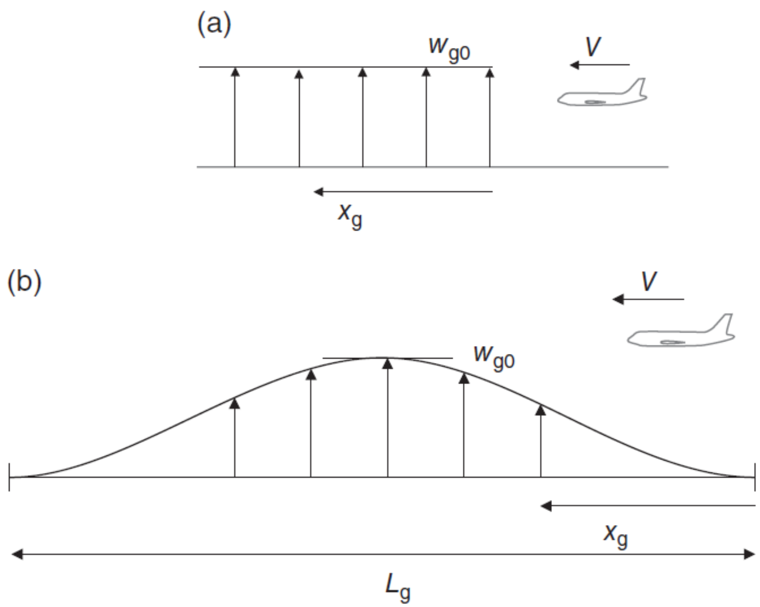


Figure 14.2 Continuous and discrete turbulence.

For design purposes atmospheric change of wind speed is divided in two ideal categories

1. *discrete gusts*, where the gust velocity varies in a deterministic manner, usually in the form of a '1-cosine' shape
2. *continuous turbulence*, where the gust velocity is assumed to vary in a random manner

Gust Response: sharp gust and "1-cosine" gust



$$w_g(x_g) = \begin{cases} 0 & x_g < 0 \\ w_{g0} & x_g \geq 0 \end{cases}$$

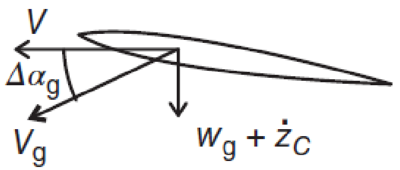
$$w_g(x_g) = \frac{w_{g0}}{2}\left(1 - \cos\frac{2\pi x_g}{L_g}\right) \qquad 0 \leq x_g \leq L_g$$

The intensity of gust and gust length to consider, are assigned by the certification standards CS-23 CS-25 as function of altitude and speed

$x_g = V\,t$

$$w_g(t) = \frac{w_{g0}}{2}\left(1 - \cos\frac{2\pi V}{L_g}t\right)$$

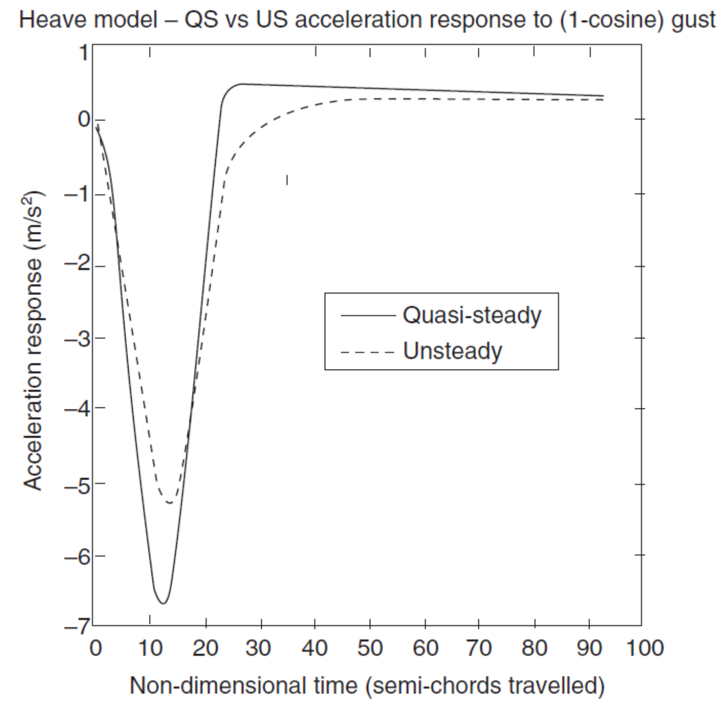
Gust Response: sharp gust and “1-cosine” gust



$$m\ddot{z}_C + \frac{1}{2}\rho V S_W a \dot{z}_C = -\frac{1}{2}\rho V S_W a w_g$$

$$m\left(\frac{2V}{c}\right)^2 z''_C + \frac{1}{2}\rho V S_W a \frac{2V}{c}\left\{z''_C * \Phi\right\} = -\frac{1}{2}\rho V S_W a \left\{w'_g * \Psi\right\}$$

Using Wagner and Sear functions instead of a quasi-steady model it leads to an alleviation to the maximum acceleration reached by the rigid airfoil



Gust penetration effect

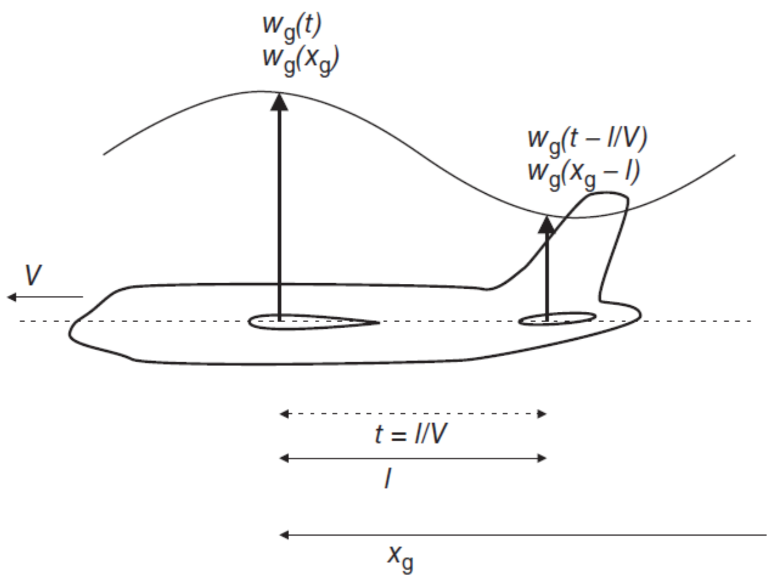
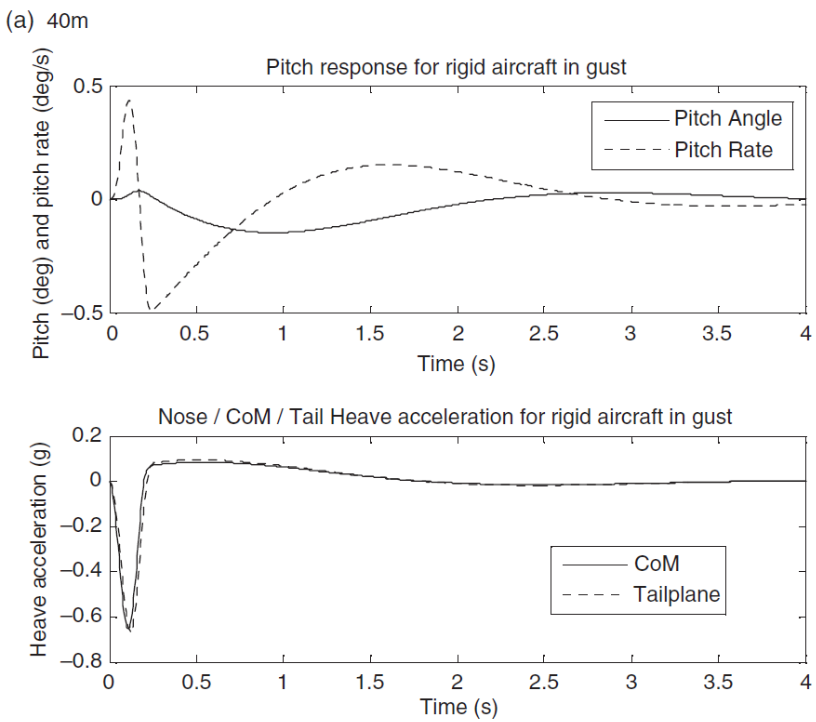


Figure 14.9 Gust penetration effect.



Gust: effects of flexibility

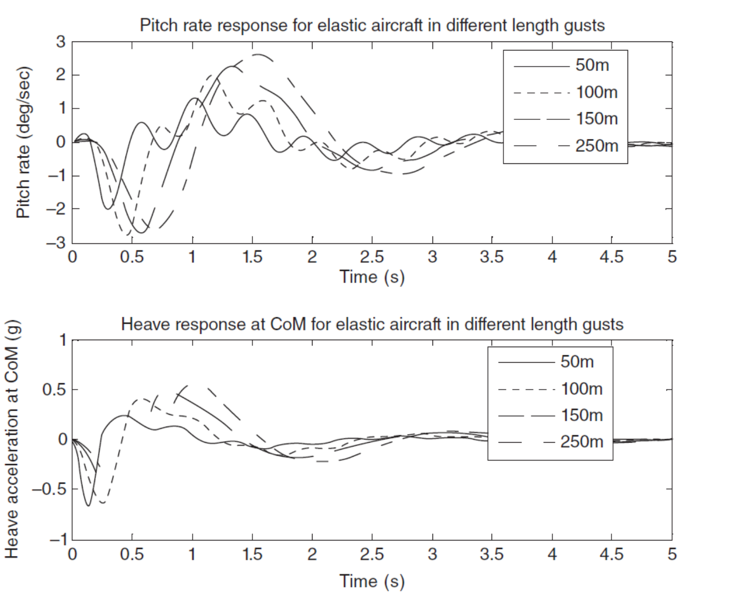


Figure 14.15 Response of a flexible aircraft to a '1-cosine' gusts of various/lengths – heave/pitch mode with fuselage bending mode (2 Hz/4%).

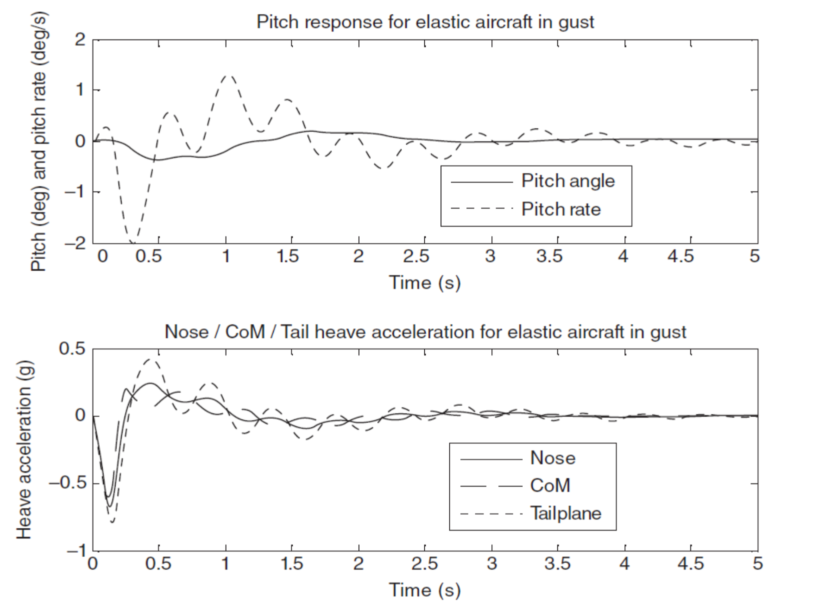
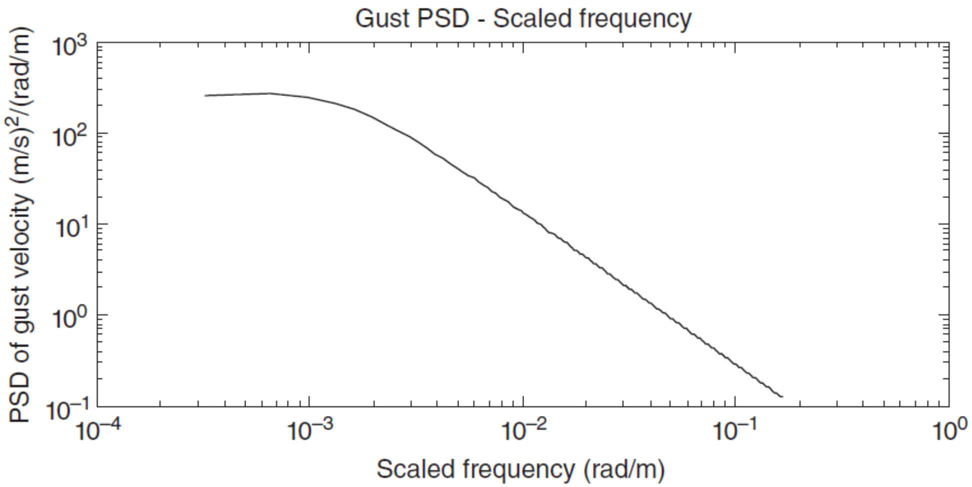
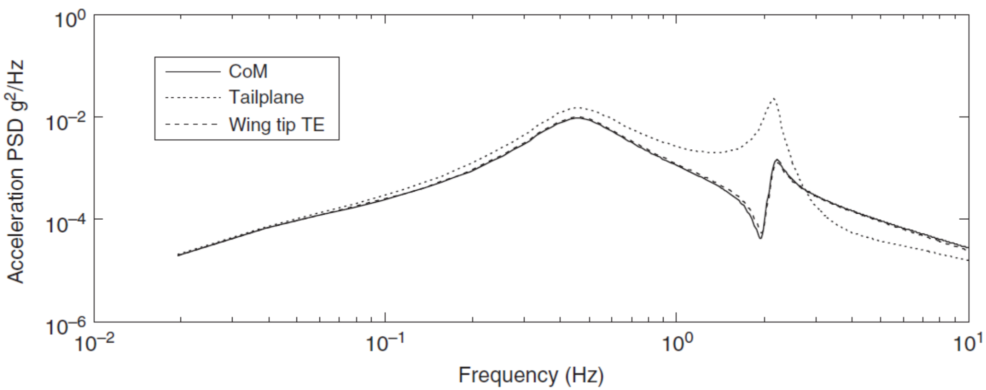


Figure 14.16 Response of a flexible aircraft to a tuned 50 m '1-cosine' gust – heave/pitch model with fuselage bending mode (2 Hz/4%).

Gust: stochastic analysis





$$\Phi_{gg}(\Omega) = \sigma_g^2 \frac{L}{\pi} \frac{1 + (8/3)(1.339\Omega L)^2}{\left[1 + (1.339\Omega L)^2\right]^{11/6}}$$

Von Karman PSD for turbulence:
Ω scaled frequency ω/V rad/m
σ_g RMS or gust intensity m/s
L Characteristic scale (762m, 2500 ft)

Aeroelastic Control

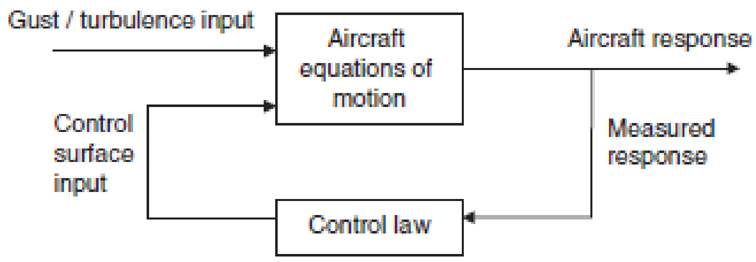


Figure 11.4 Block diagram for the aeroservoelastic system.

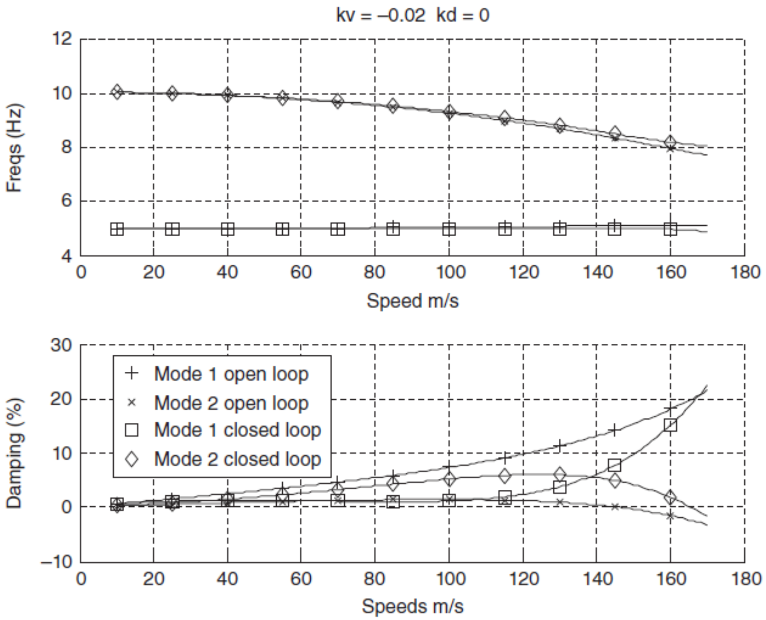


Figure 11.5 Open and closed loop frequency and damping ratio trends.

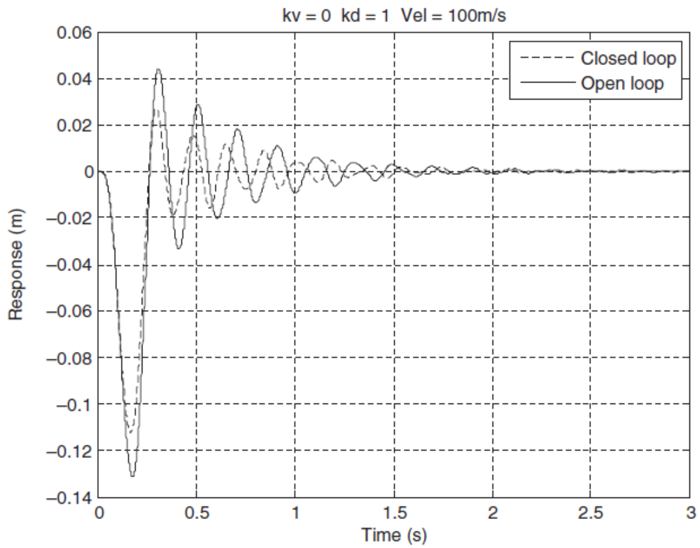
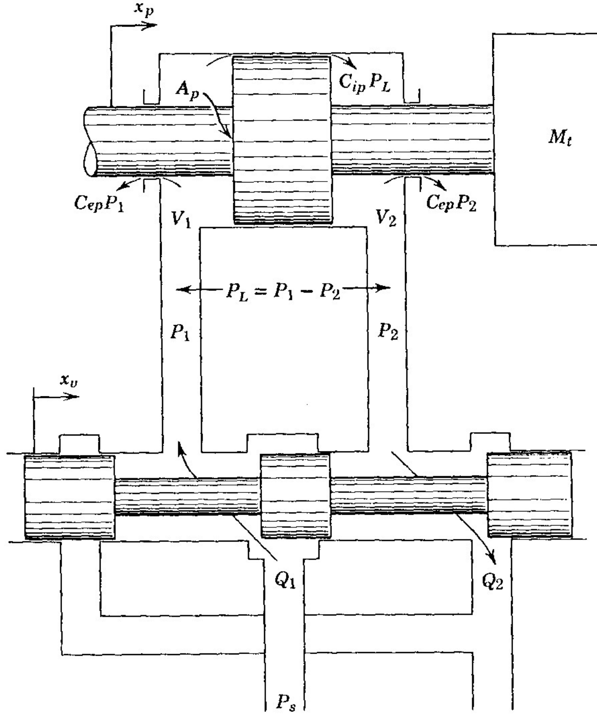


Figure 11.7 Leading edge tip response of wing to gust with/without the control law.

Objectives:

- Extension of flutter-free envelope
- Load alleviation
- Improvement of ride qualities

Appendix: hydraulic servo actuator



Servo-actuator with a spool servo-valve. Using mass conservation (V volume, p pressure, Q flux, C orifice loss factor

$$
\begin{aligned}
\frac{dV_1}{dt} + \frac{V_1}{\beta_e}\frac{dp_1}{dt} &= Q_1 - C_{ip}(p_1 - p_2) - C_{ep}p_1 \\
\frac{dV_2}{dt} + \frac{V_2}{\beta_e}\frac{dp_2}{dt} &= -Q_2 + C_{ip}(p_1 - p_2) - C_{ep}p_2 \\
V_1 &= V_0 + A_p x_p \\
V_2 &= V_0 - A_p x_p
\end{aligned}
$$

Appendix: hydraulic servo actuator

$$
\begin{aligned}
A_p \dot{x}_p + \frac{V_0 + A_p x_p}{\beta_e} \frac{dp_1}{dt} &= Q_1 - C_{ip}(p_1 - p_2) - C_{ep} p_1 \\
-A_p \dot{x}_p + \frac{V_0 - A_p x_p}{\beta_e} \frac{dp_2}{dt} &= -Q_2 + C_{ip}(p_1 - p_2) - C_{ep} p_2
\end{aligned}
$$

Calling $p_L = p_1 - p_2$ and $Q_L = 1/2(Q_1 + Q_2)$ and considering $V_1 \approx V_2 \approx V_0$ the difference of the two equations results in

$$
A_p \dot{x}_p + \frac{V_0}{2\beta_e} \frac{dp_L}{dt} = Q_L - \left(C_{ip} + \frac{C_{ep}}{2} \right) p_L \tag{25}
$$

Force equation

$$
A_p p_L = M_t \ddot{x}_p + F_L \tag{26}
$$

with M_t mass of the piston and F_L the load acting on the piston.

Appendix: hydraulic servo actuator

Servo-valve. significantly nonlinear element. Q_L is function of the spool position and of the pressure difference. It is possible to linearize using Taylor series

$$Q_L = \frac{\partial Q_L}{\partial x_v} x_v + \frac{\partial Q_L}{\partial p_L} p_L + \dots \quad (27)$$

$$Q_L = K_q x_v - K_c p_L \quad (28)$$

K_q flow gain
K_c flow pressure coefficient

$$A_p s x_p + \frac{V_0}{2\beta_e} s p_L = K_q x_v - K_{ce} p_L \quad (29)$$

$$A_p p_L = M_t s^2 x_p + F_L \quad (30)$$

Appendix: hydraulic servo actuator

$$\left(A_p s + \frac{M_t V_0}{2 A_p \beta_e} s^3 + \frac{M_t K_{ce}}{A_p} s^2\right) x_p = K_q x_v - \frac{K_{ce}}{A_p}\left(1 + \frac{V_0}{2 K_{ce} \beta_e} s\right) F_L$$

$$x_p = \frac{\frac{K_q}{A_p} x_v - \frac{K_{ce}}{A_p^2}\left(1 + \frac{V_0}{2 K_{ce} \beta_e} s\right) F_L}{\frac{M_t V_0}{2 A_p^2 \beta_e} s^3 + \frac{M_t K_{ce}}{A_p^2} s^2 + s}$$

It is often written as

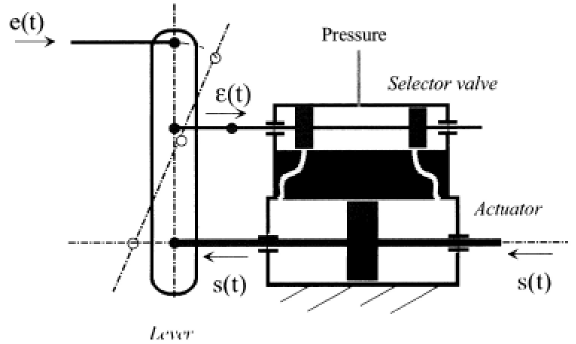
$$x_p = \frac{\frac{K_q}{A_p} x_v - \frac{K_{ce}}{A_p^2}\left(1 + \frac{V_0}{2 K_{ce} \beta_e} s\right) F_L}{s\left(\frac{1}{\omega_h^2} s^2 + \frac{2\delta_h}{\omega_h} s + 1\right)}$$

with the hydraulic natural frequency and damping ratio equal to

$$\omega_h = \sqrt{\frac{2 \beta_e A_p^2}{V_0 M_t}} \qquad \delta_h = \frac{K_{ce}}{A_p} \sqrt{\frac{\beta_e M_t}{2 V_0}}$$

Appendix: hydraulic servo actuator

If the actuator is used for position control than there must be a servo or at least the input position must be transmitted by an input rod to a link that connects the valve with the piston as in the figure.



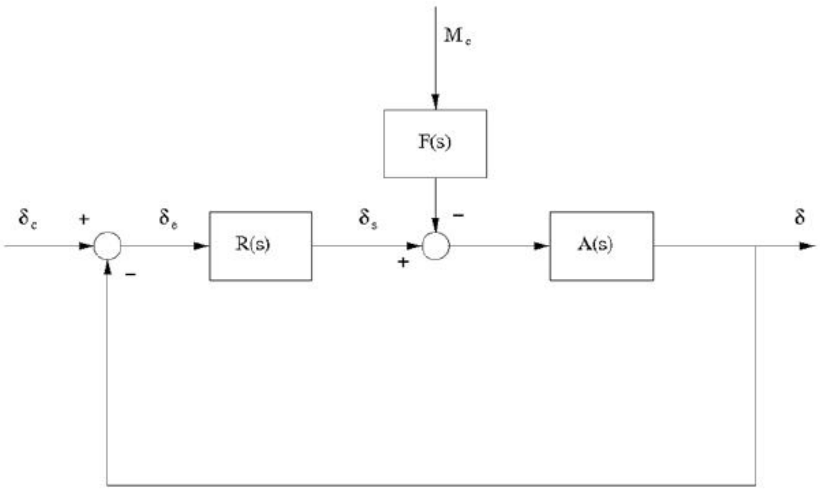
$$x_v = \frac{a_2 e - a_1 x_p}{a_1 + a_2} = \tau_e e - \tau_p x_p \tag{35}$$

So the closed loop transfer function is equal to

$$x_p = \frac{\tau_e \frac{K_q}{A_p} e - \frac{K_{ce}}{A_p^2}\left(1 + \frac{V_0}{2K_{ce}\beta_e} s\right) F_L}{\left(\frac{1}{\omega_h^2} s^3 + \frac{2\delta_h}{\omega_h} s^2 + s + \tau_p \frac{K_q}{A_p}\right)} \tag{36}$$

$$x_p = H_e(s) e + H_F(s) F_L \tag{37}$$

Appendix: hydraulic servo actuator



The three parts are

- the actuator-valve dynamics $A(s) = \dfrac{\frac{K_q}{A_p}}{s\left(\frac{1}{\omega_h^2}s^2+\frac{2\delta_h}{\omega_h}s+1\right)}$
- the servo dynamics $R(s)$
- the compliance with locked servo $F(s) = \frac{K_{ce}}{A_p^2}\left(1+\frac{V_0}{2K_{ce}\beta_e}s\right)$

Appendix: hydraulic servo actuator

There are two transfer function composition of three parts

- the closed loop servo-valve transfer function

$$H_e(s) = \frac{A(s)R(s)}{1 + A(s)R(s)} \tag{38}$$

- the dynamic compliance

$$H_F(s) = \frac{F(s)A(s)R(s)}{1 + A(s)R(s)} \tag{39}$$

Appendix: hydraulic servo actuator

For the servo stability it is necessary that the open loop transfer function of the position control loop is stable. The crossover frequency is $\omega_c \approx K_v = \tau_e \frac{K_q}{A_p}$

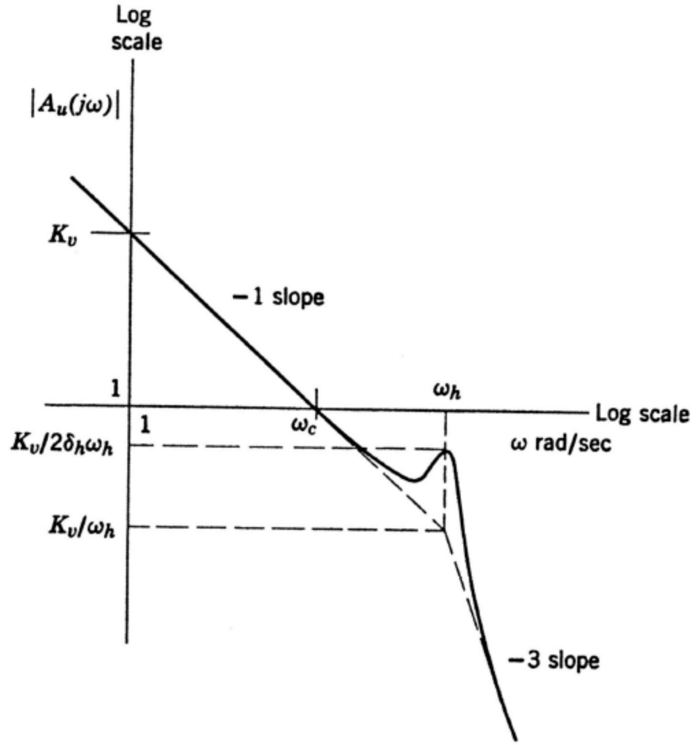
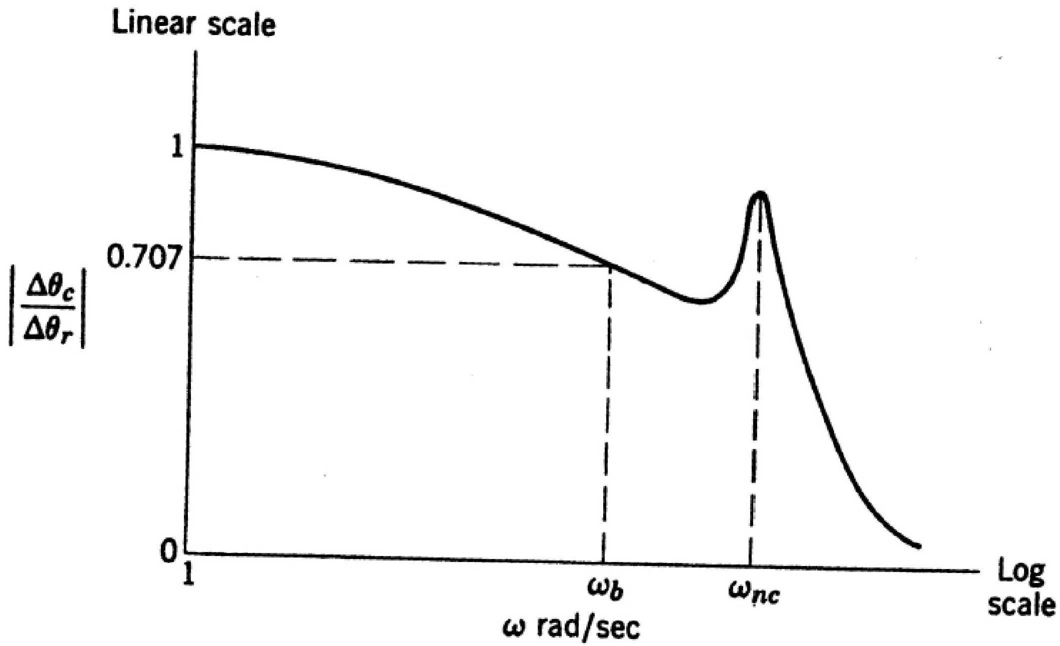


Figure 8-6 Bode diagram of position control loop.

Appendix: hydraulic servo actuator

The closed loop transfer function obtained is the following

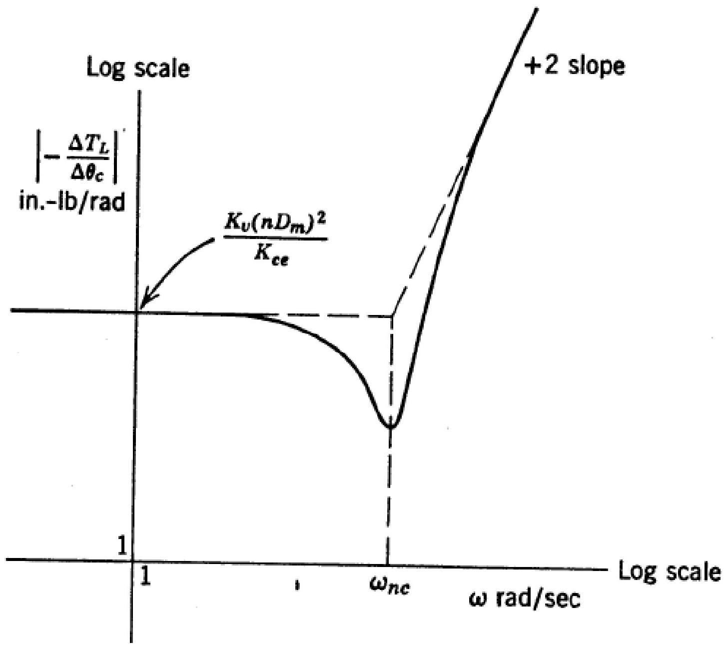


$\omega_{nc} \approx \omega_h$

$\omega_b \approx \omega_c$

Appendix: hydraulic servo actuator

The dynamic stiffness looks like this



Before reaching the actuator natural frequency the stiffness is reduced. The airworthiness norms require to consider this effect together coupled with the stiffness of the actuator attachments for aeroservoelastic stability.