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055738 – STRUCTURAL DYNAMICS AND AEROELASTICITY

Static Aeroelasticity of Typical Section

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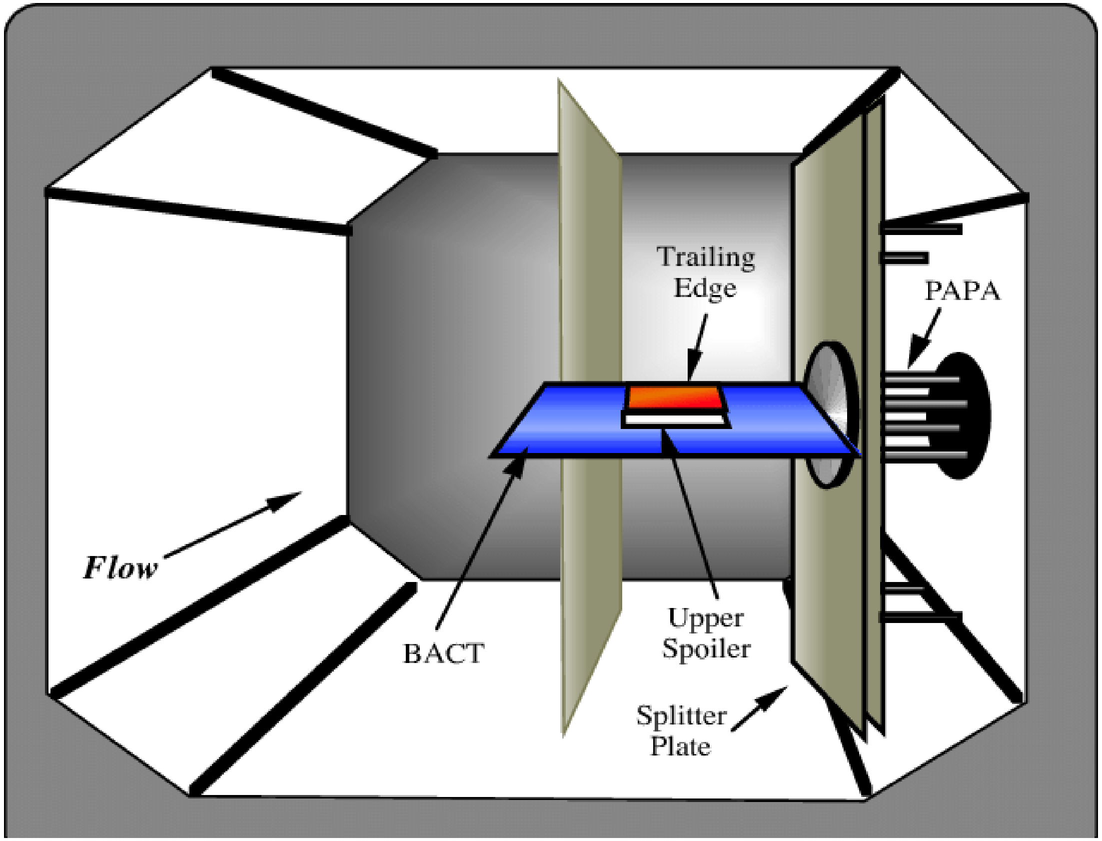
Material

1) Dowell: Chapter 2 Static Aeroelasticity Sections 2.1
2) BAH Chapter 8 Sections 8.1 and 8.2
3) Masarati DCFA Chapter 8.1
4) Fung Chapter 3

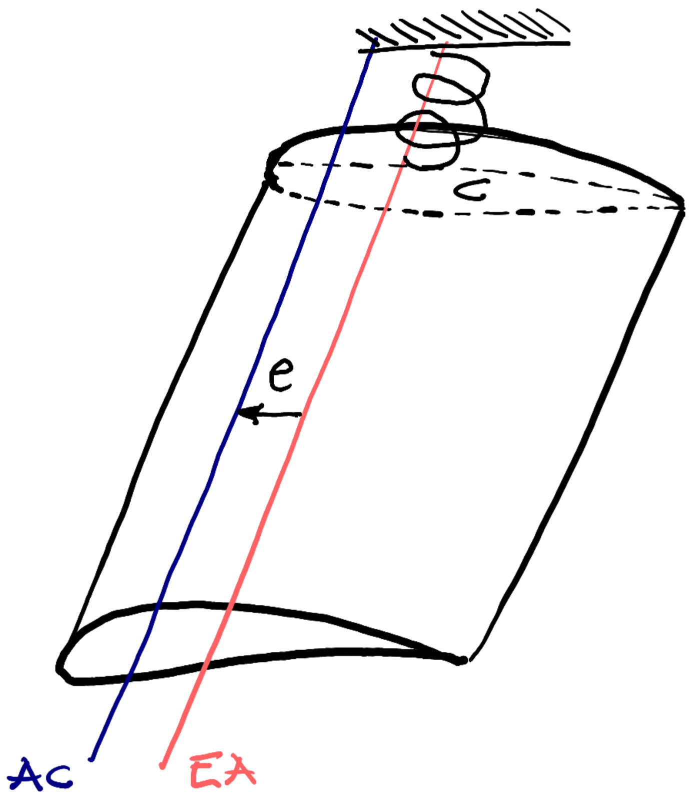
Typical section

Simplest aeroelastic system developed by Theodorsen and Garrick during 1930’s to have an experimental and mathematical model for elementary examination of flutter.



Typical section

RIGID WING MOUNTED ON AN ELASTIC LUMPED SUPPORT



Wing Structure:

- **Rigid** with a constant section.
- Span perpendicular to wind speed.
- **Deformability concentrate in a torsional spring at root**.

Aerodynamics:

- **2D strip theory**
- linear behavior since only low amplitude pitch oscillations are considered.

Typical section

Torsion is the "twisting" of one section of an object with respect to another.

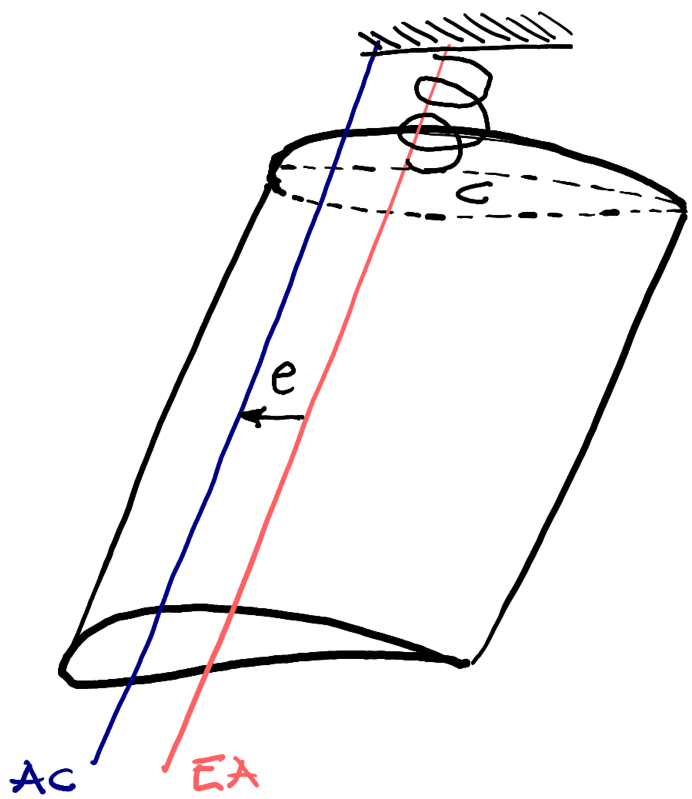
SHEAR CENTER (SC)

it is a point of a 2D section where if you apply a transverse shear load it produces zero change (i.e. rate) of twist of the section. It is a property of the cross-section

ELASTIC AXIS (EA)

It is the locus of shear centers along the span of the wing (may not be a straight line).

It is typically between 35-45% of the chord for semi-monocoque structures (with no balancing ballast).



Flexural center

Flexural Center (FC or center of twist): the point where the shear load P applied to cantilever where it does not cause torsion (i.e., rotation) of the section ACB, but not necessarily elsewhere in the wing.

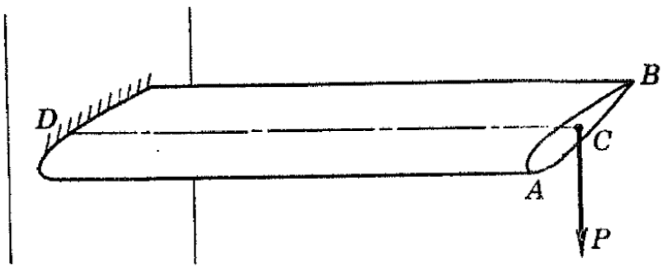
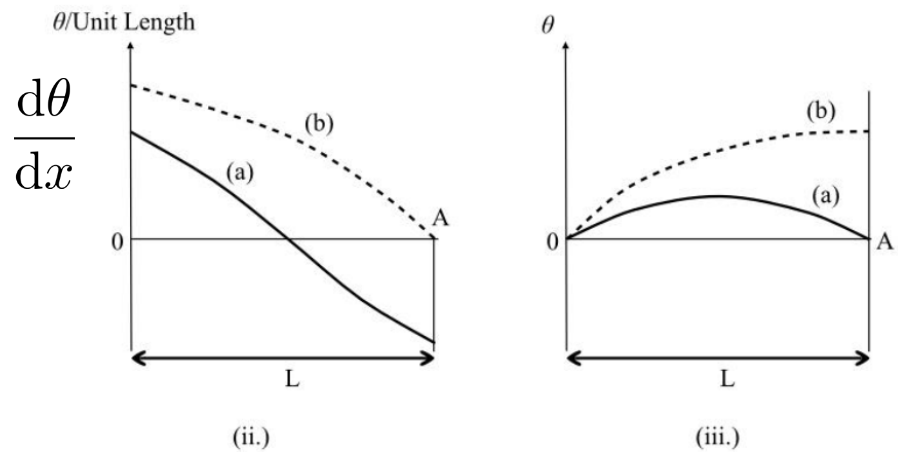


Fig. 1.7. Flexural center of a cantilever beam.



In general, the flexural center depend on the load distribution along the beam. The location of the flexural axis may be important for the assessment of bending-twisting coupling of structures.

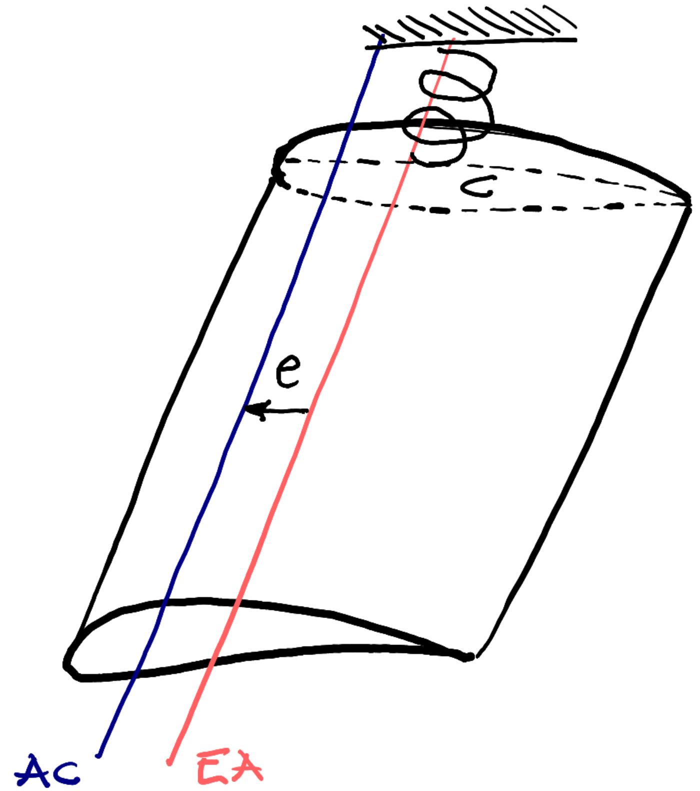
Exercise: Show that when the EA is straight the FC is coincident to SC.

Typical section

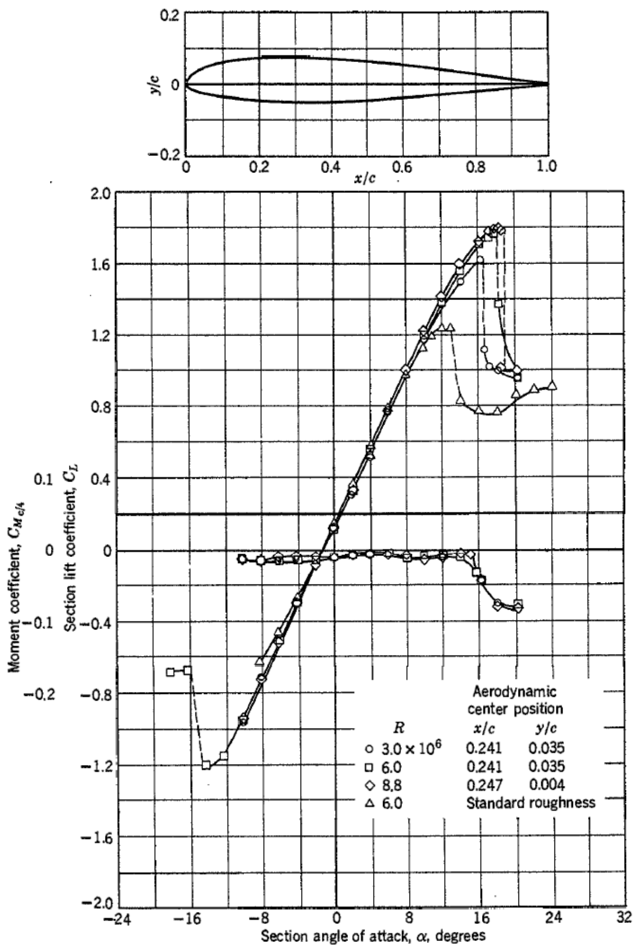
Aerodynamic Center (AC): the point of the section around which the steady aerodynamic moment coefficient is constant with respect to changes of angle of attack

$$\frac{\partial C_{m_{ac}}}{\partial \alpha} = 0$$

if α is small i.e., far from the stall region.



Aerodynamics (incompressible 2D)



Lift	L	qSC_L	N
Wing surface	S		m^2
Dynamic pressure	q	$\frac{1}{2}\rho U^2$	Pa
Density	ρ		$\mathrm{kg/m^3}$
Asymptotic Flight Speed	U		m/s
Lift Coefficient	C_L		

$$C_L = \frac{L}{qS} = f(\alpha, Ma, Re, \ldots) \approx \frac{\partial C_L}{\partial \alpha}(Ma, Re)\alpha$$

$$C_L = C_{L/\alpha}\alpha = a\alpha$$

α is the angle of attack referred to the zero-lift line of the airfoil.

Aerodynamics (3D and compressible)

2D INCOMPRESSIBILE

$$C_{L\alpha}^{2D} = a_0 \approx 2\pi$$

THIN AIRFOIL THEORY (2D)

3D INCOMPRESSIBILE

$$C_{L\alpha}^{3D} = \frac{a_0}{1 + \frac{a_0}{\pi\lambda}(1+\tau)}$$

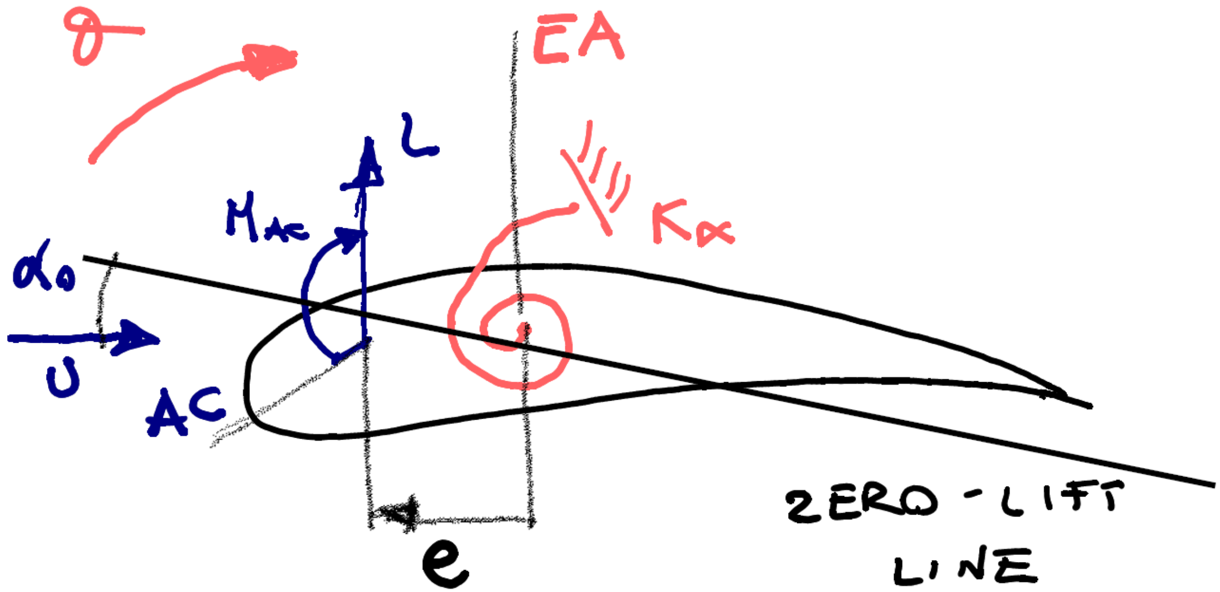
$\lambda = b^2/S$ Aspect ratio
τ non-elliptic lift distribution coefficient

COMPRESSIBLE FLOW

$$C_{L\alpha}^{C} \approx \frac{a_0}{\sqrt{1-M^2}}$$

PRANDTL-GLAUERT CORRECTION

Typical section



θ **<u>ELASTIC TWIST</u>**

$$\alpha = \alpha_0 + \theta$$

Typical Section: Elastic twist

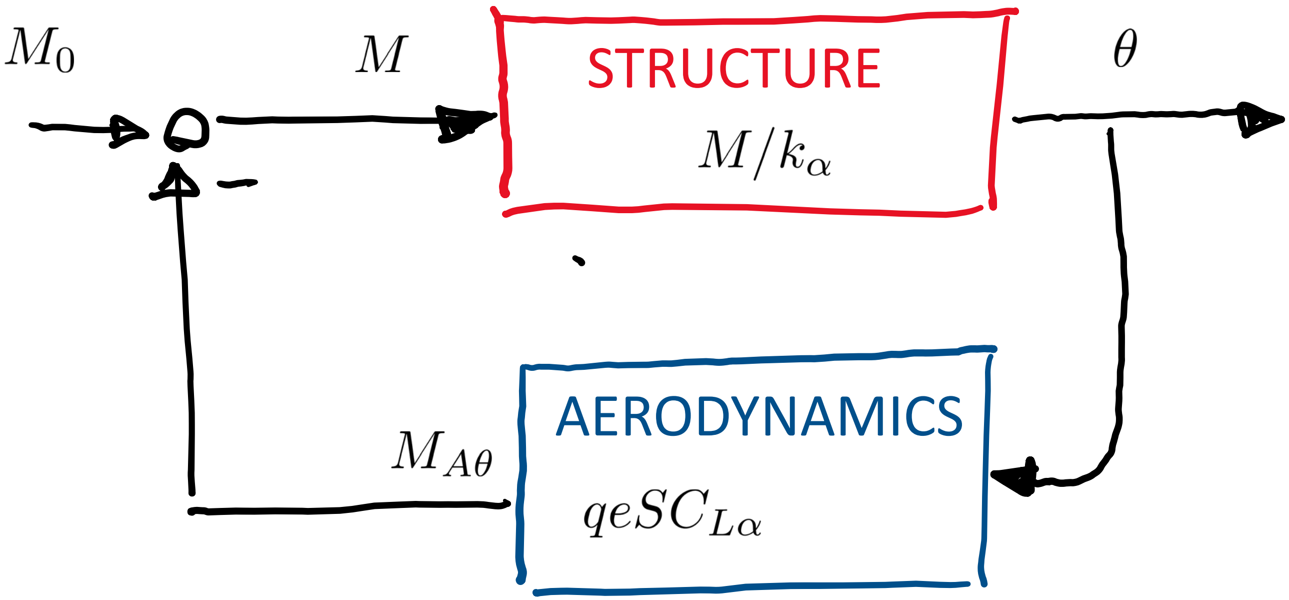
Find the equilibrium position of the elastic twist

$\underline{\theta} =$

Typical Section: Feedback structure

$$= qS(C_{L0}e + cC_{m_{CA}})$$

Initial moment applied on the rigid structure



AERODYNAMIC STIFFNESS

$$qk_A = qeSC_{L\alpha}$$

EFFECTIVE STIFFNESS

$$\tilde{k} = k_\alpha - qk_A$$

Torsional Divergence

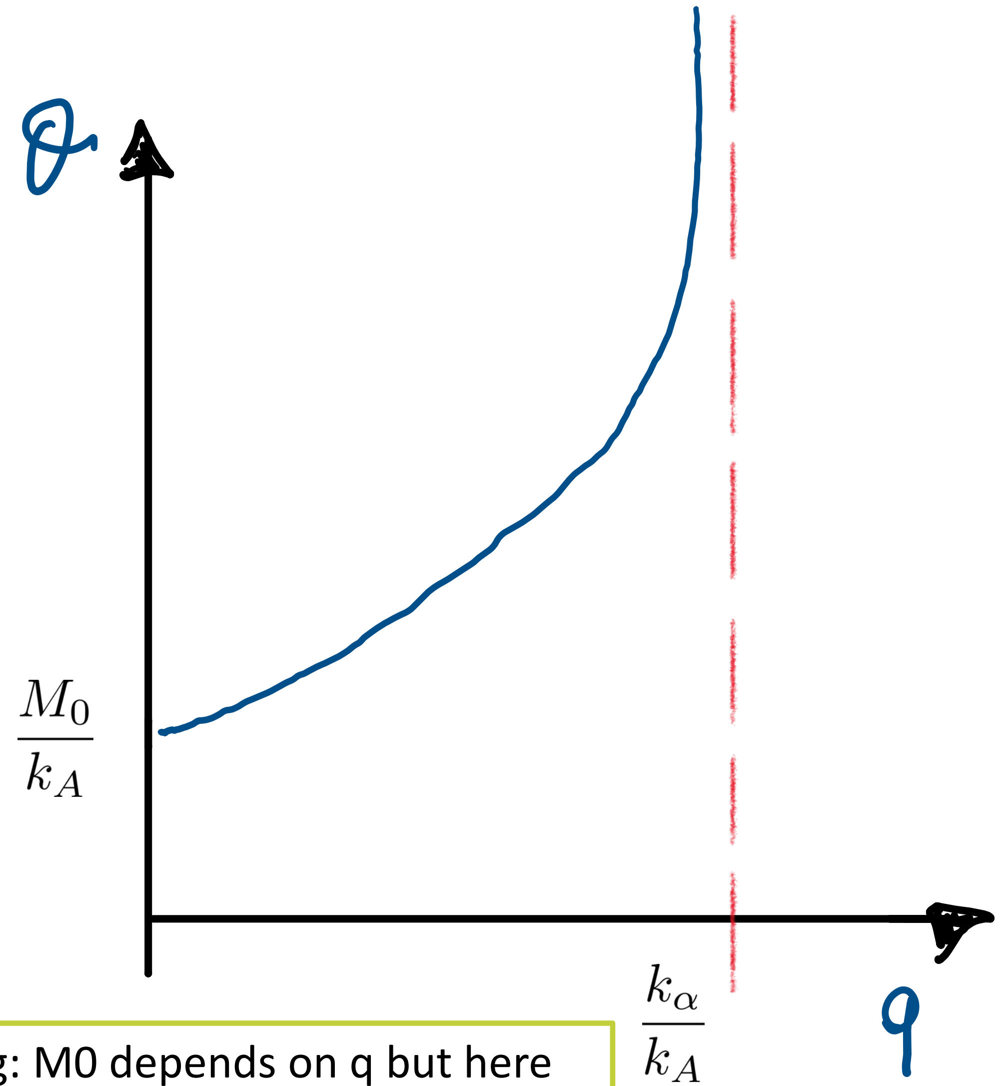
$$\theta = \frac{M_0}{k_\alpha - qk_A}$$

$$\theta = \frac{M_0}{k_\alpha}\left(\frac{1}{1 - q\frac{k_A}{k_\alpha}}\right)$$

if $q\frac{k_A}{k_\alpha} \rightarrow 1$ then $\theta \rightarrow \infty$

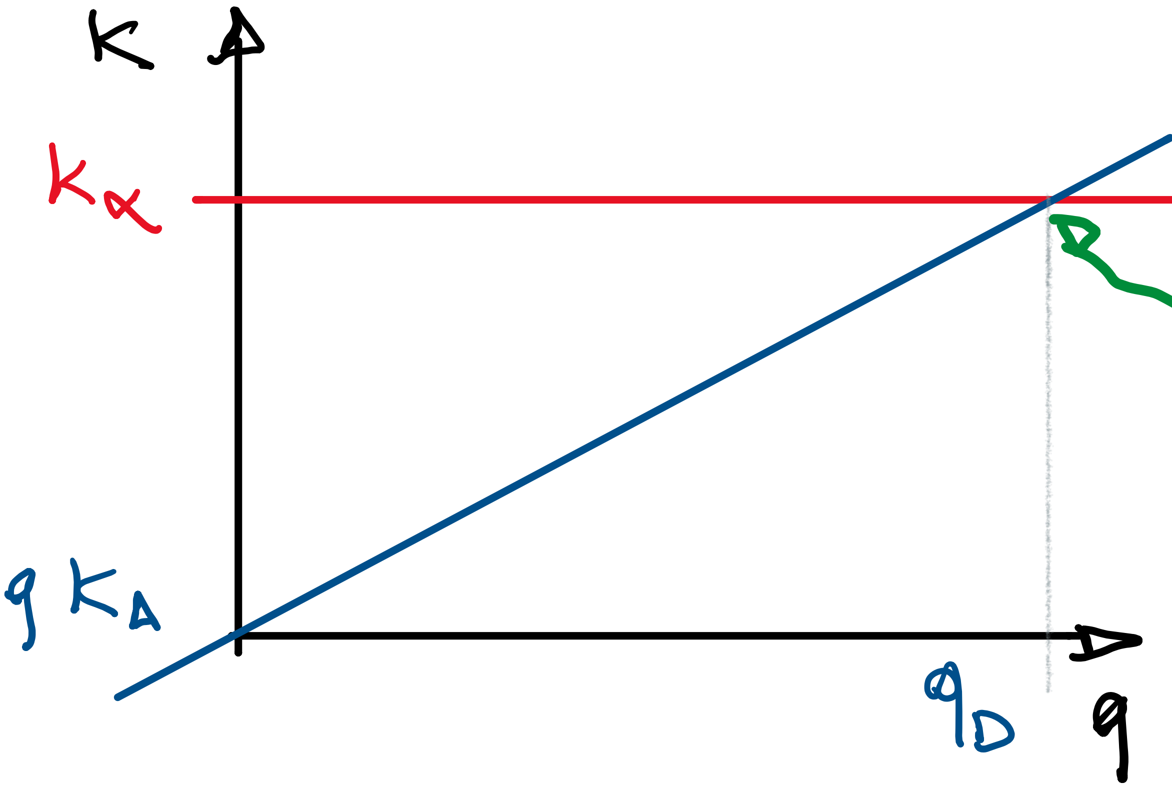
When the denominator tend to zero the torsion angle tent to become infinitely large and so airfoil "diverges"

$$q_D = \frac{k_\alpha}{k_A}$$



Warning: M0 depends on q but here is considered as a constant moment applied on the wing

Divergence dynamic pressure



Two stiffness equal and opposite are applied

This is equivalent to NO STIFFNESS

$$q_D = \frac{k_\alpha}{k_A} = \frac{k_\alpha}{eSC_{L\alpha}} \qquad q_D = \frac{1}{2}\rho U_D^2 \qquad \Rightarrow \qquad U_D = \sqrt{\frac{2k_\alpha}{\rho eSC_{L\alpha}}}$$

What happens if e < 0 ?

Aeroelastic performance index

Consider an initial moment M_0 applied to the structure

The deformation obtained by the elastic system will be

$$\theta_0 = \frac{M_0}{k_\alpha}$$

The deformation obtained by the AEROELASTIC system will be

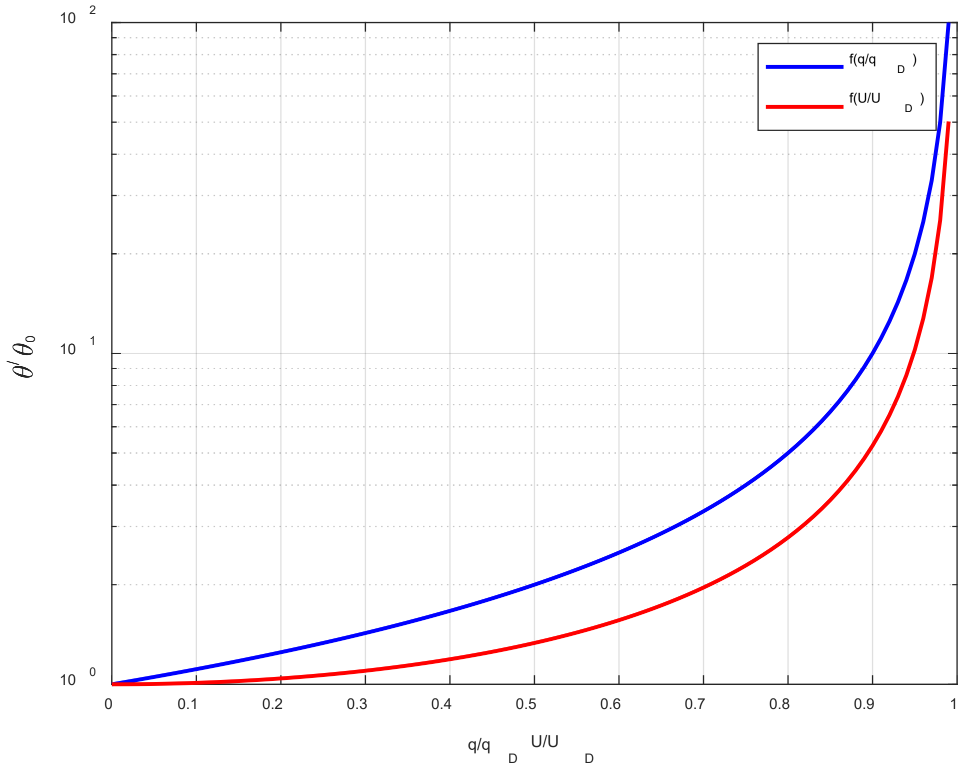
$$\frac{\theta}{\theta_0} = \frac{1}{1 - \frac{q}{q_D}}$$

The more q is close to q_D the higher will be the incidence of aeroelastic effects

Homework

Plot a diagram of the function

$$\frac{\theta}{\theta_0} = f\left(\frac{q}{q_D}\right)$$



The divergence dynamic pressure is an index of the relevance of aeroelastic effects

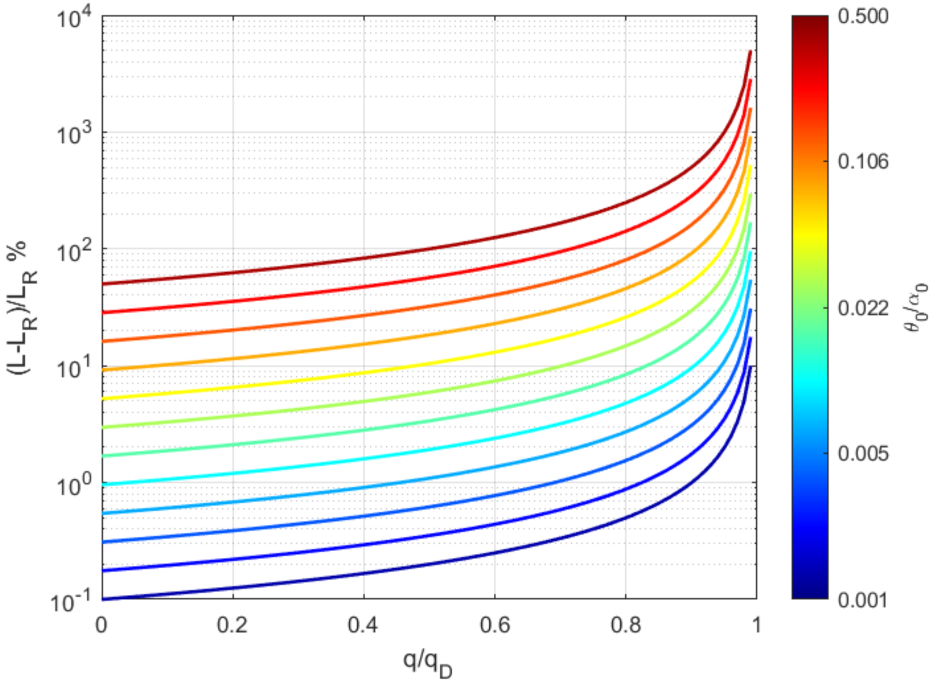
Comparison of aeroelastic and rigid Lift

$$
\begin{aligned}
L_R &= qSC_{L0} = qSC_{L\alpha}\alpha_0 \\
L &= qSC_{L\alpha}\left(\alpha_0 + \theta\right) \\
M_0 &= qSc\left(C_{m_{CA}} + \frac{e}{c}C_{L0}\right) = eqScC_{M0} \\
\theta &= \theta_0 \frac{1}{1 - \frac{q}{q_D}}
\end{aligned}
$$

$$
\frac{L - L_R}{L_R} = \frac{(\alpha_0 + \theta) - \alpha_0}{\alpha_0} = \frac{\theta}{\alpha_0} = \frac{\theta_0}{\alpha_0}\left(\frac{1}{1 - \frac{q}{q_D}}\right)
$$

Homework

Plot a diagram of the function $$\frac{L - L_R}{L_R} = \frac{\theta_0}{\alpha_0} \frac{1}{1 - \frac{q}{q_D}}$$



Homework

What will happen to CL vs α curve?

Torsional divergence as stability problem

Static stability: taken a system that is in equilibrium, the equilibrium is said statically stable if given a slight perturbation to the equilibrium there is a tendency to return to the equilibrium.

Consider the aeroelastic moment applied as a nonlinear function of

$$M_{AE} = f(\theta, q)$$

$$\begin{aligned} M_{AE} &= f(\theta_T, q) = 0 \\ \theta &= \theta_T + \Delta\theta \end{aligned}$$

Linearize the system

$$M_{AE} \approx f(\theta_T, q) + \frac{\partial f}{\theta} \Delta\theta$$

Stable if

$$\frac{\partial f}{\partial \theta} < 0$$

Torsional divergence as static stability problem

$$M_{AE} = (qSeC_{L0} + qScC_{m_{CA}} + qSeC_{L\alpha}\theta) - k_\alpha \theta$$

$$M_{AE} = f(\theta_T, q) = 0$$
$$-qSeC_{L0} - qScC_{m_{CA}} + (qSeC_{L\alpha} - k_\alpha)\theta_T = 0$$

The perturbation of this nonlinear system is

$$\frac{\partial f}{\partial \theta}\Delta\theta = \boxed{(qSeC_{L\alpha} - k_\alpha)}\ \Delta\theta$$

if < 0 Stable

If > 0 Unstable

The critical condition that transforms the system from stable to unstable is

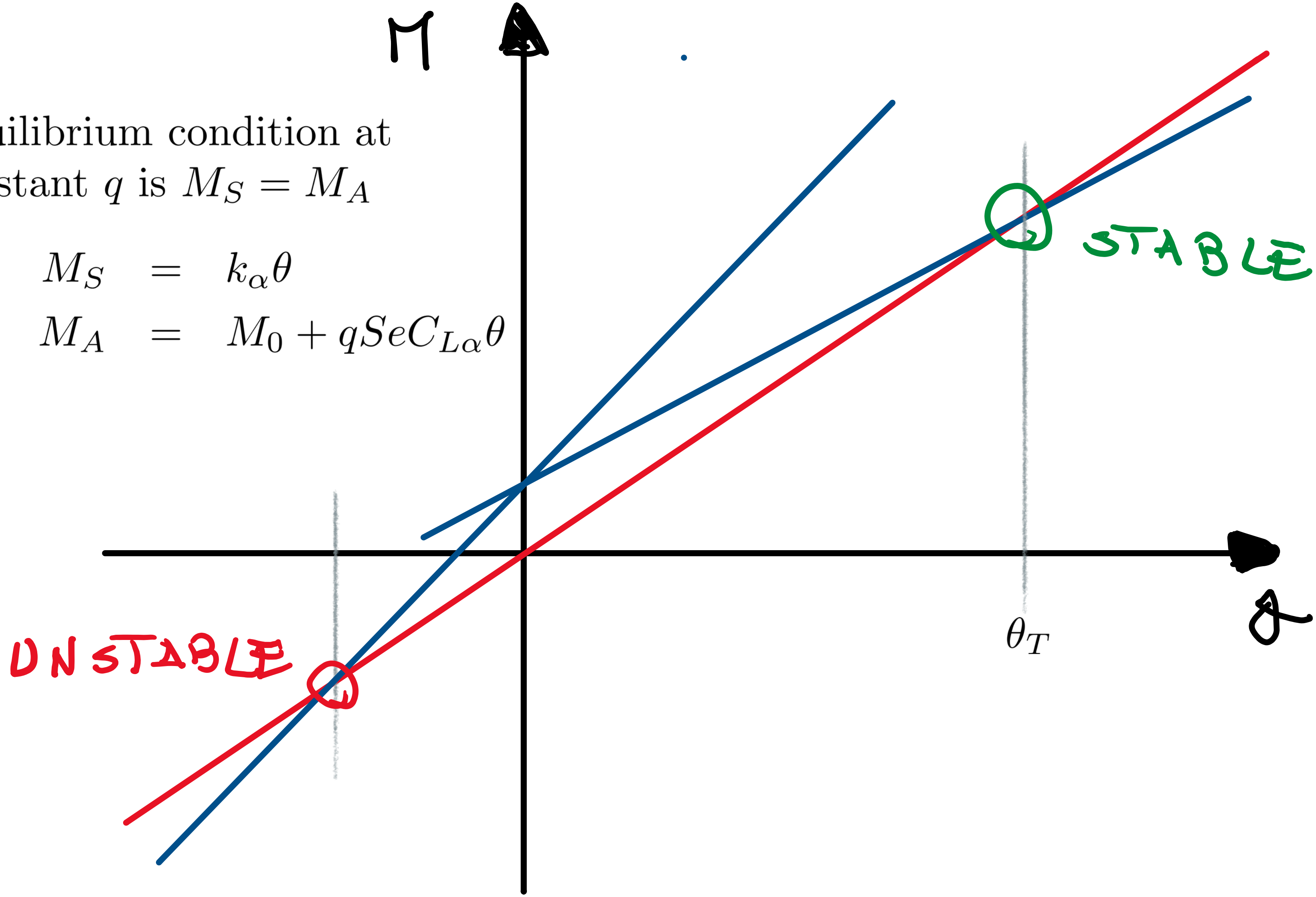
$$k_\alpha - qeSC_{L\alpha} = 0$$

$$\boxed{q_D = \frac{k_\alpha}{eSC_{L\alpha}} = \frac{k_\alpha}{k_A}}$$

Torsional divergence as static stability problem

Equilibrium condition at constant q is $M_S = M_A$

$$
\begin{aligned}
M_S &= k_\alpha \theta \\
M_A &= M_0 + qSeC_{L\alpha}\theta
\end{aligned}
$$



Torsional Divergence as Eigenvalue problem

Since θ_T is an equilibrium condition, we expect at that condition that

$$(k_\alpha - qSeC_{L\alpha})\Delta\theta = 0$$

This is an homogeneous equation that has a trivial solution that is $\Delta\theta = 0$. However, when

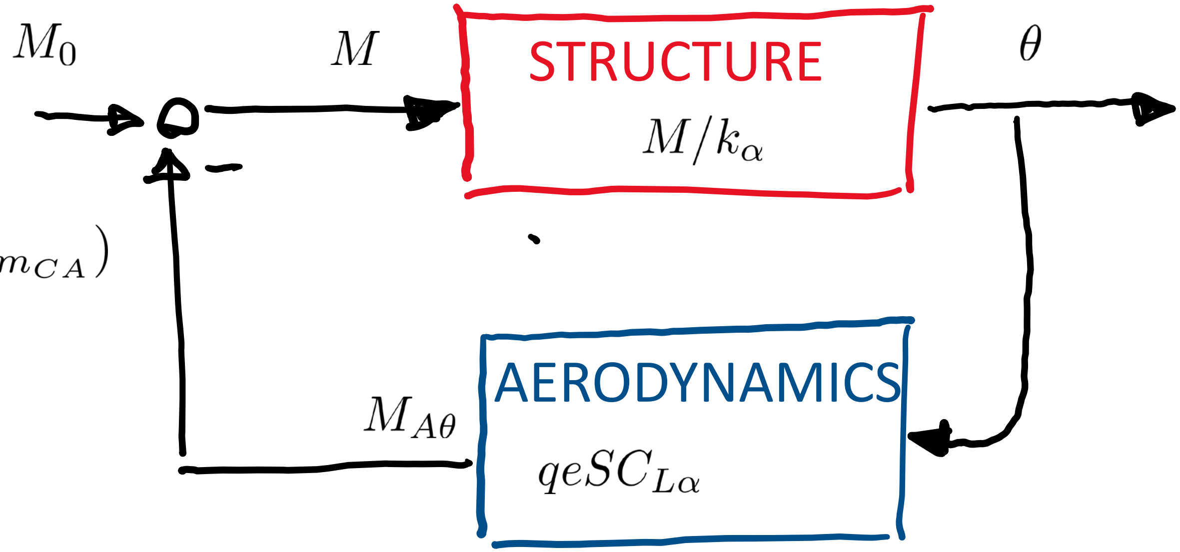
$$(k_\alpha - qSeC_{L\alpha}) = 0$$

any $\Delta\theta \neq 0$ will be valid.

This type of problem should recall you an EIGENVALUE problem, where q are the eigenvalues of the system that lead to a nontrivial solution of the homogeneous problem

Iterative solution to identify static aeroelastic solution

Procedure based on the feedback scheme:



$$M = M_0 = qS(eC_{L0} + cC_{m_{CA}})$$
$$\theta_0 = 0$$
$$\theta_1 = M/k_\alpha$$
$$i = 1$$
$$\text{While } |\theta_{i-1} - \theta_i| > \text{Toll.}$$
$$\quad i{+}{+}$$
$$\quad M_{A\theta} = qk_A\theta_{i-1}$$
$$\quad M = M_0 + M_{A\theta}$$
$$\quad \theta_i = M/k_\alpha$$
$$\text{EndWhile}$$

Toll. Tolerance below which the error is considered numerically to be zero.

Analysis of the iterative solution algorithm

Compute the solution of the linear difference equation

$$k_\alpha \theta_{i+1} - qSeC_{L\alpha}\theta_i = M$$

Check if the algorithm converges to the correct solution

Effects of Mach number

The lift curve slope tends to grow with Mach number, at least for thin airfoils before reaching the critical Mach number.

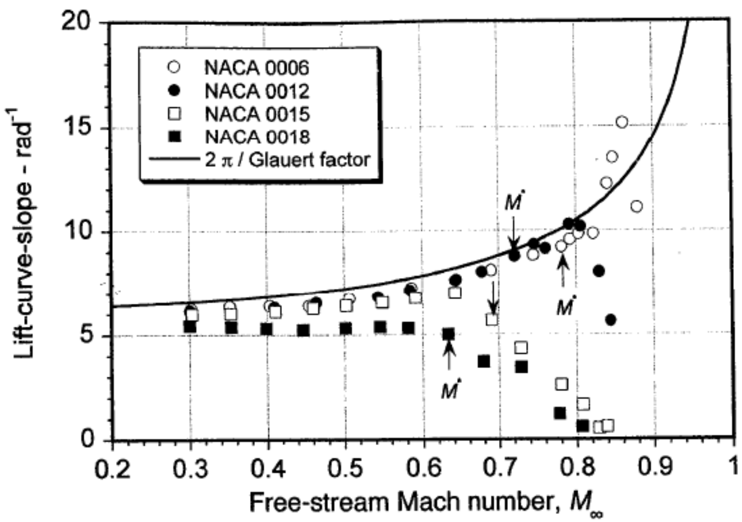
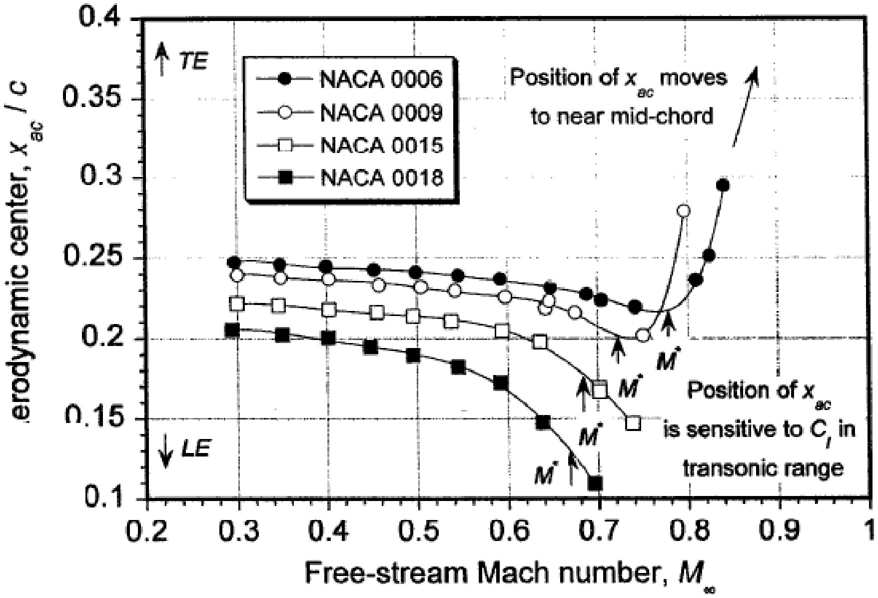


Figure 7.19 Comparison of measured lift-curve-slope for the NACA 00-series with predictions made by using Glauert factor. Data source: Riegels (1961).



The Aerodynamic Center tends to move forward initially while the Mach number increases.

Effect of Mach number

Try to guess what will happen to divergence speed if you consider the effect of Mach number

Calling c the speed of sound

$$q_D = \frac{1}{2}\rho U_D^2 = \frac{1}{2}\rho c^2 M_D^2 \qquad c^2 = \gamma R T = \gamma \frac{p}{\rho}$$

$$\begin{aligned} q_D &= \frac{1}{2}\gamma p_\infty M_D^2 = A M_D^2 \\ A &= \frac{1}{2}\gamma p_\infty \end{aligned} \qquad C_{L\alpha} = \frac{C_{L\alpha}^{M=0}}{\sqrt{1-M^2}}$$

The condition for divergence is

$$k_\alpha - q_D S e C_{L\alpha} = 0$$

Effect of Mach number

$$k_\alpha = q_D S e \frac{C_{L\alpha}^{M=0}}{\sqrt{1 - M_D^2}}$$

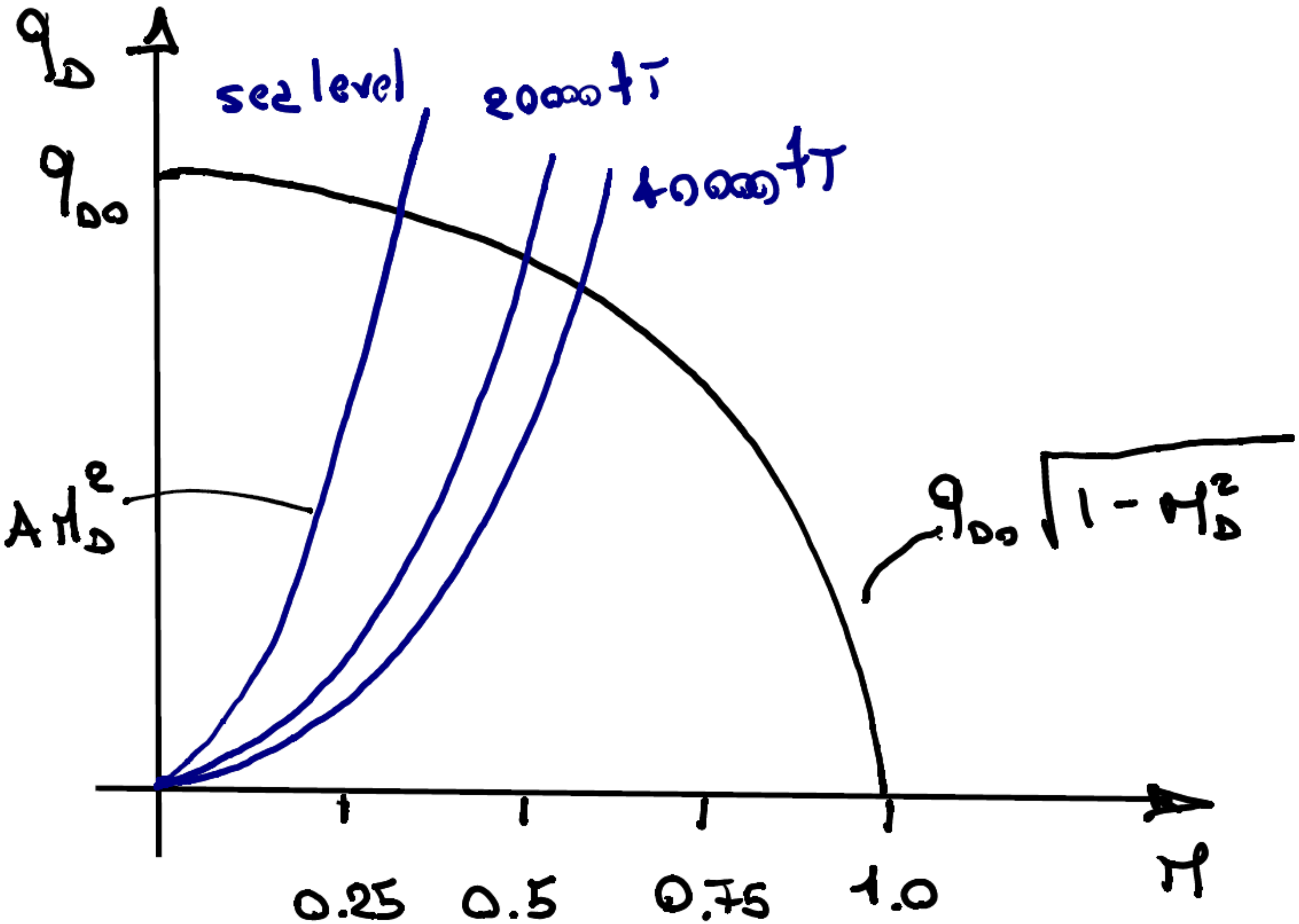
$$q_D = \frac{k_\alpha}{S e C_{L\alpha}^{M=0}} \sqrt{1 - M_D^2} = q_D^{M=0} \sqrt{1 - M_D^2}$$

$$\begin{aligned} q_D^2 &= q_D^{M=0}(1 - M_D^2) \\ A^2 M_D^4 &= q_D^{M=0}(1 - M_D^2) \end{aligned}$$

$$M_D^4 + \left(\frac{q_D^{M=0}}{A} \right)^2 M_D^2 - \left(\frac{q_D^{M=0}}{A} \right)^2 = 0$$

The constant A is function of the altitude. So, for every altitude it is possible to identify several divergence Mach number, but only one is physically meaningful.

Effect of Mach Number



Appendix: Shear Center

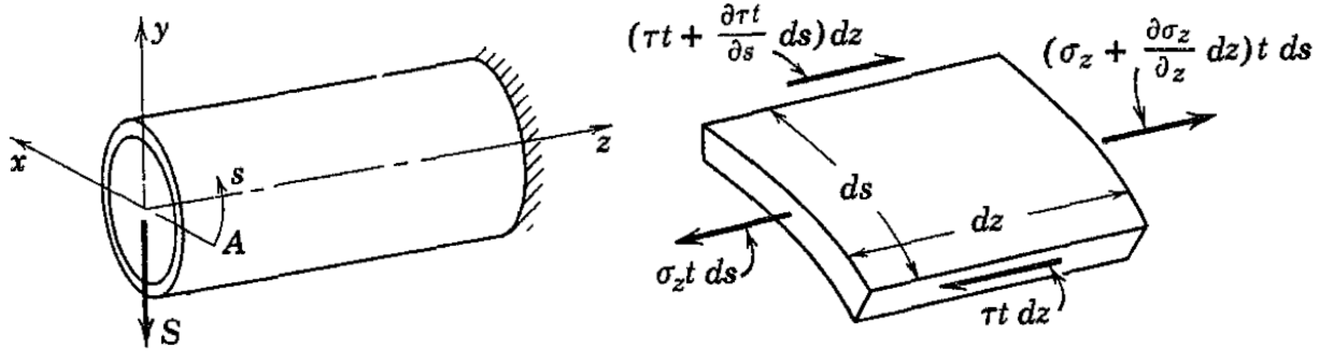


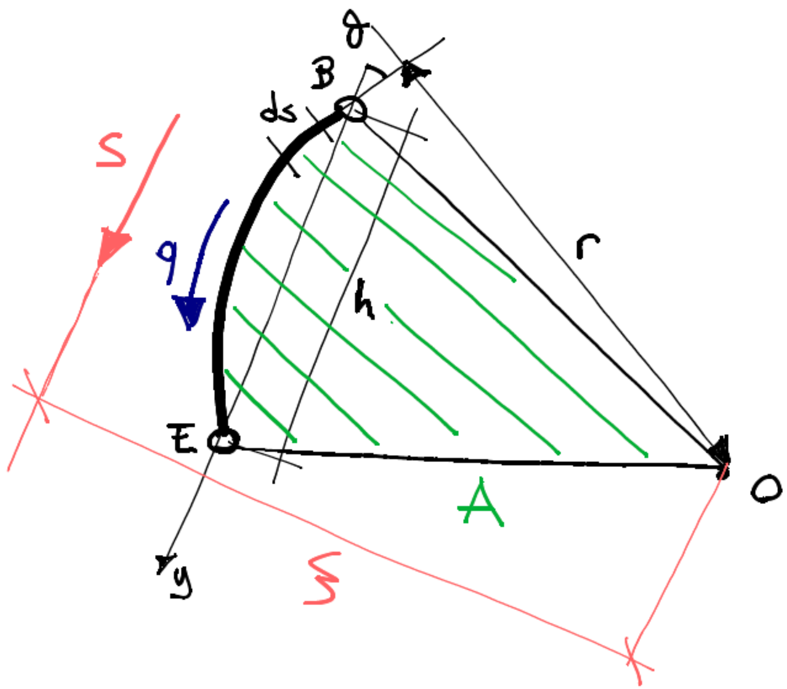
Fig. 1.1. Equilibrium of forces acting on an element of a thin-walled section.

SHEAR FLOW $q = \tau t$ N/m

Equilibrium in the axial direction

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau}{\partial s} = 0$$

Appendix: Shear Center



How to compute the shear center position

Consider a curved structural panel along which a constant shear flow q is acting

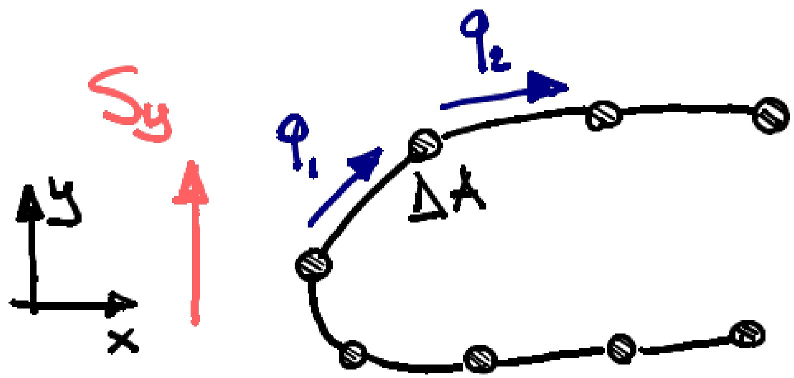
- The total shear goes in the direction parallel to the line BE
- The application point (SC) can be found using equilibrium

$$
\begin{aligned}
S_y &= \int q \cos\theta \mathrm{d}s = \int q \mathrm{d}y = qh \\
M_0 &= \int q r \mathrm{d}s = q \int r \mathrm{d}s = 2qA
\end{aligned}
$$

$$\xi = \frac{M_0}{S} = \frac{2A}{h}$$

Appendix: Semi-monocoques approximation

If you have a beam with multiple stringers, the equation that consider the balance of the shear flows at each stringer must be considered



$$\Delta q = q_2 - q_1 = -\frac{S_y}{I} y \Delta A$$

with I the moment of inertia about the x axis.