POLITECNICO
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055738 – STRUCTURAL DYNAMICS AND AEROELASTICITY

04 Static Aeroelasticity: swept wings

Giuseppe Quaranta
Dipartimento di Scienze e Tecnologie Aerospaziali

Material

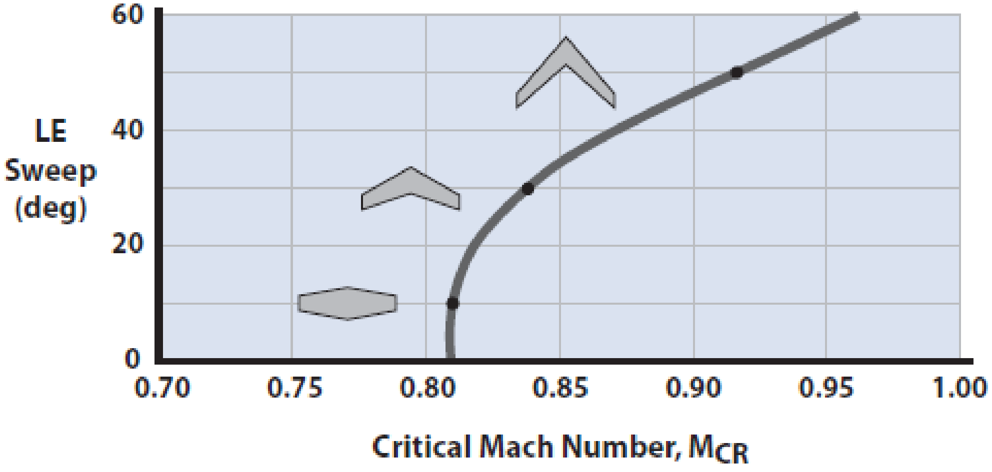
BAH Section 8.4 Swept wing

Masarati DCFA Section 8.1.6

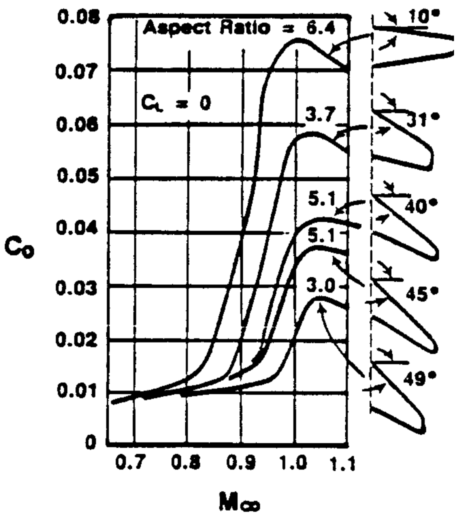
Dowell Section 2.6

Cooper & Wright Section 7.4

Reasons to use sweep angle

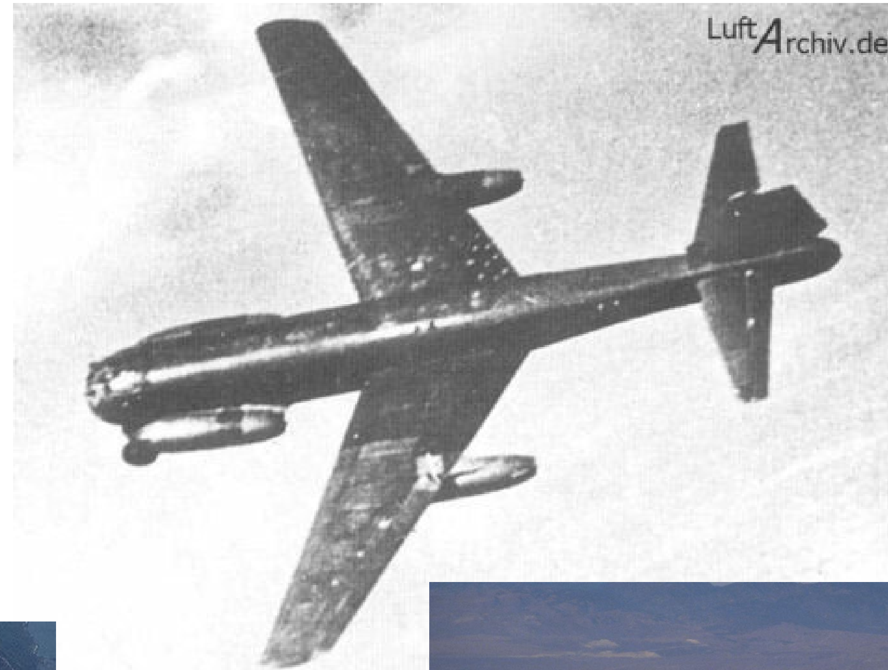


to delay transonic drag rise (compressibility).

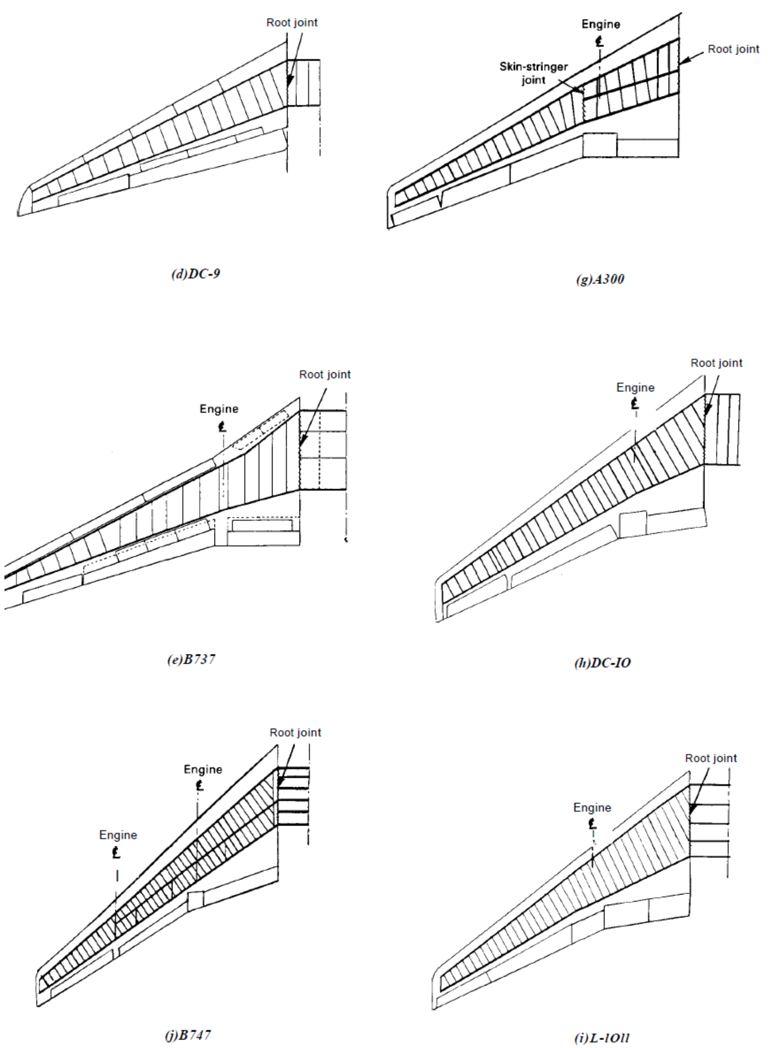


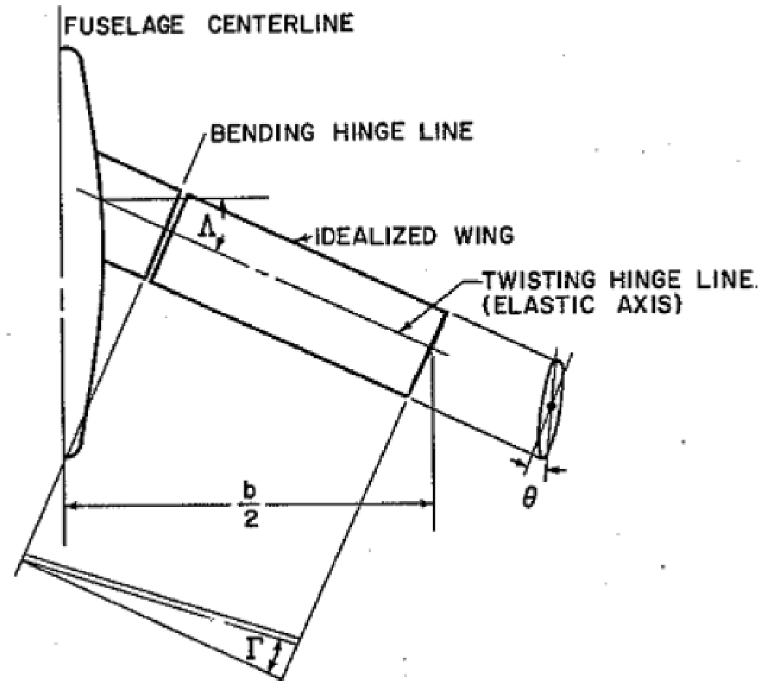
To improve longitudinal stability by changing the distance between the AC and the CG of the wing

Swept back and swept forward wings



Structure of the swept wing

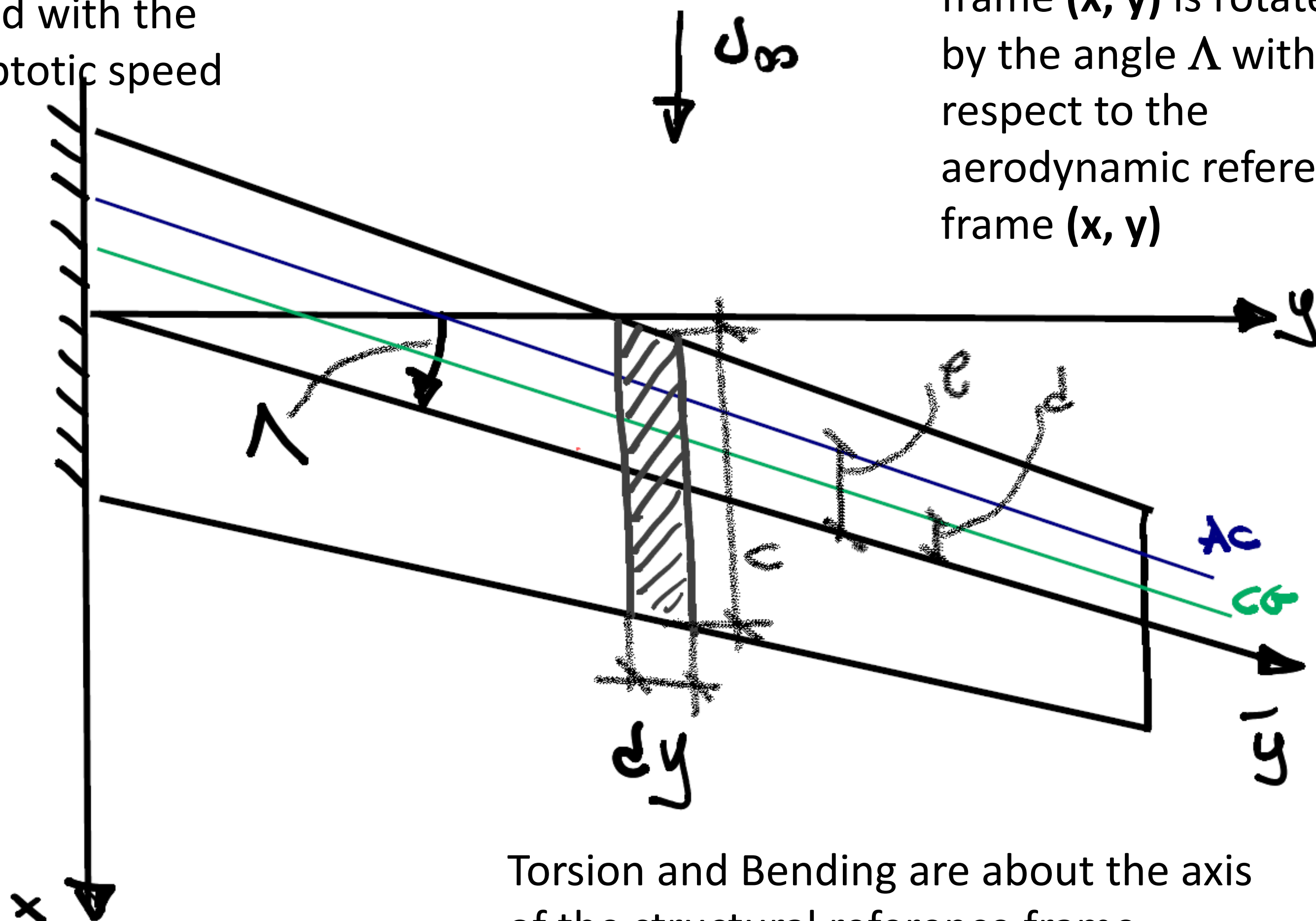




Streamwise segment aligned with asymptotic speed

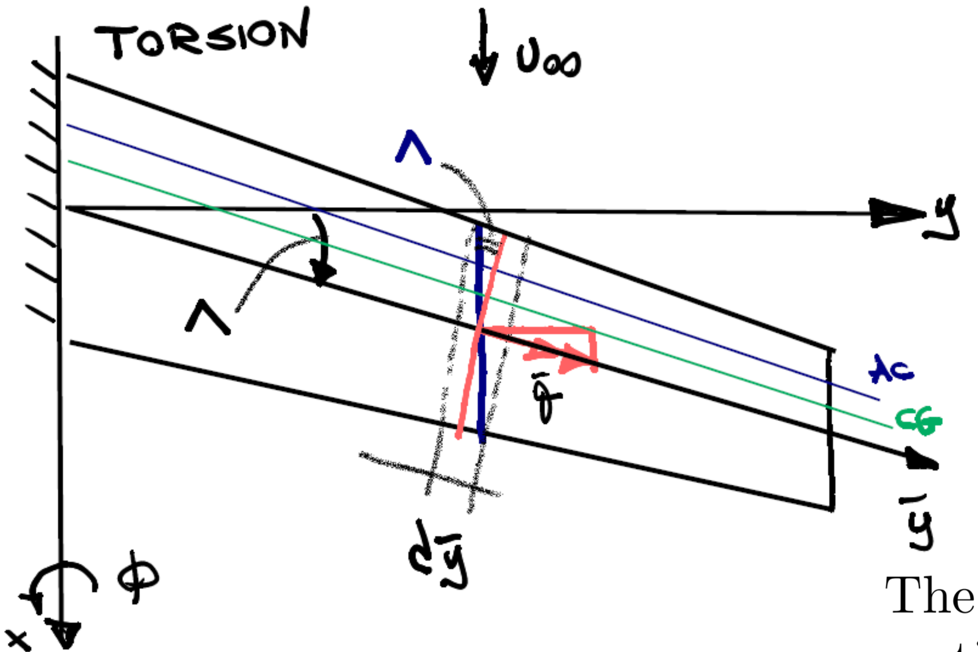
Aerodynamic forces are developed by airfoils aligned with the asymptotic speed

The structural reference frame **($\bar{x}$, $\bar{y}$)** is rotated by the angle Λ with respect to the aerodynamic reference frame **(x, y)**



Torsion and Bending are about the axis of the structural reference frame

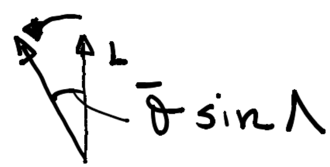
Swept wing: torsion



$$
\begin{aligned}
\Delta\alpha &= \bar{\theta}\cos\Lambda \\
\phi &= \bar{\theta}\sin\Lambda
\end{aligned}
$$

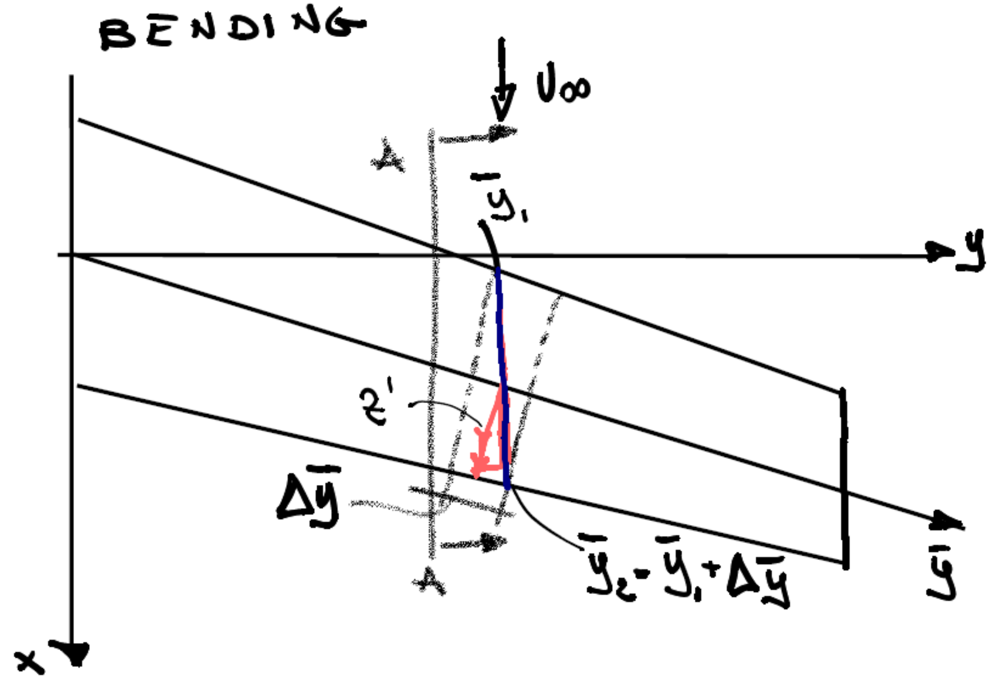
The angle ϕ is causing a rotation of the sectional Lift vector. However the angle $\bar{\theta}\sin\Lambda$ is small,
so this effect can be neglected.

Only a portion of the torsion of the structure results in a change of angle of attack.
The rest is change of direction of Lift



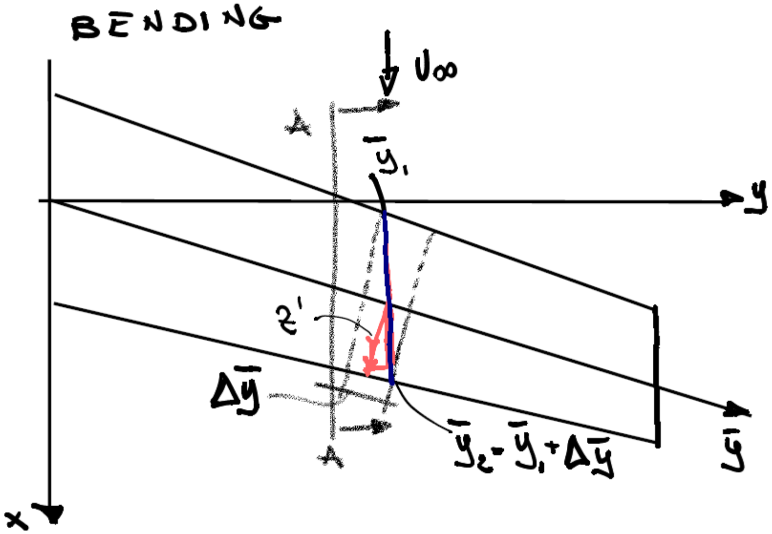
Swept Wing: Bending

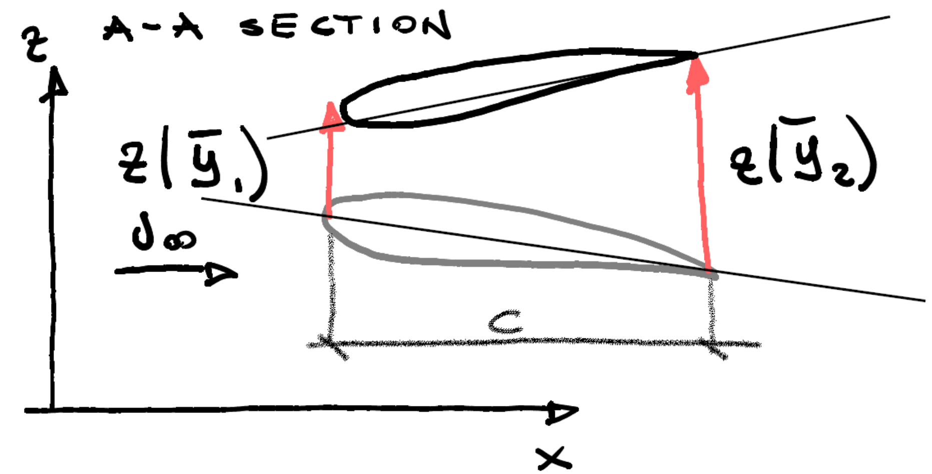
When a swept back wing bends up the angle of attack of streamwise sections will reduce.



$$\Delta\bar{y} = c\sin\Lambda$$

Swept Wing: bending





$c \sin\Lambda = \Delta\bar{y}$

$$\Delta\alpha_B = \frac{z(\bar{y}_1) - z(\bar{y}_2)}{c} = -\frac{1}{c}\frac{\mathrm{d}z}{\mathrm{d}\bar{y}}\Delta\bar{y}$$

$$\Delta\alpha_B = -z' \sin\Lambda$$

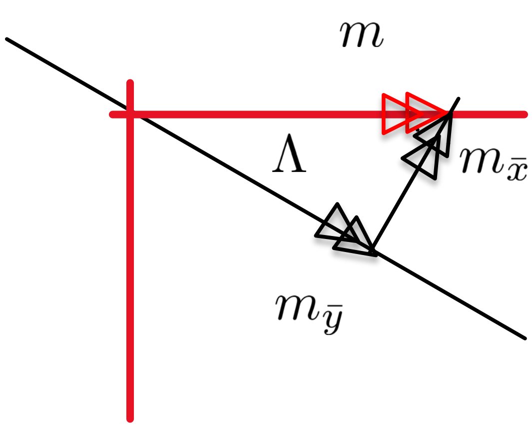
Swept wing: total change of angle of attack

$$\Delta\alpha = \Delta\alpha_T + \Delta\alpha_B = \bar{\theta}\cos\Lambda - z'\sin\Lambda$$

The change of the streamwise angle of attack resulting from the elastic deformation is made up of a component coming from the structure twist and o component of slope coming from the wing bending

Swept wing: forces and moments acting along the elastic axis

To solve the bending and torsional problems it is necessary to compute the shear and torsional load along the structure



$$
\begin{aligned}
L(y) &= qcC_L - mNg \\
m(y) &= qecC_L + qc^2 C_{m_{AC}} - mNgd
\end{aligned}
$$

The total lift is

$$
L_T = \int L(y)\mathrm{d}y = \int L(\bar{y})\mathrm{d}\bar{y}
$$

If we consider that $\mathrm{d}y = \mathrm{d}\bar{y}\cos\Lambda$, then

$$
\begin{aligned}
L(\bar{y}) &= (qcC_L - mNg)\cos\Lambda \\
m(\bar{y}) &= (qecC_L + qc^2 C_{m_{AC}} - mNgd)\cos\Lambda
\end{aligned}
$$

The aerodynamic moment could be decomposed in a torsional component along the beam axis $m_{\bar{y}}$ and a bending component about the $\bar{x}$ axis $m_{\bar{x}}$

$$
\begin{aligned}
m_{\bar{y}} &= m(\bar{y})\cos\Lambda \\
m_{\bar{x}} &= m(\bar{y})\sin\Lambda
\end{aligned}
$$

Swept wing: forces and moments acting along the elastic axis

The wing bending is caused by the shear load $L(\bar{y})$ and the moment $m_{\bar{x}}$. This second effect is often neglected.

$$L(\bar{y}) = qcC_L = qcC_{L\alpha}\alpha = qcC_{L\alpha}\left(\alpha_0 + \bar{\theta}(\bar{y})\cos\Lambda - z'(\bar{y})\sin\Lambda\right)\cos\Lambda$$

The torsion is caused by the moment $m_{\bar{y}}$

$$\begin{aligned}
m_t &= (qecC_L + qc^2C_{m_{AC}} - mNgd)\cos^2\Lambda \\
m_t &= (qecC_{L\alpha}\alpha_0 + qc^2C_{m_{AC}} - mNgd)\cos^2\Lambda + qecC_{L\alpha}\Delta\alpha\cos^2\Lambda \\
m_t &= (qecC_L\alpha_0 + qc^2C_{m_{AC}} - mNgd)\cos^2\Lambda + \\
&\quad + qecC_{L\alpha}\left(\bar{\theta}(\bar{y})\cos\Lambda - z'(\bar{y})\sin\Lambda\right)\cos^2\Lambda
\end{aligned}$$

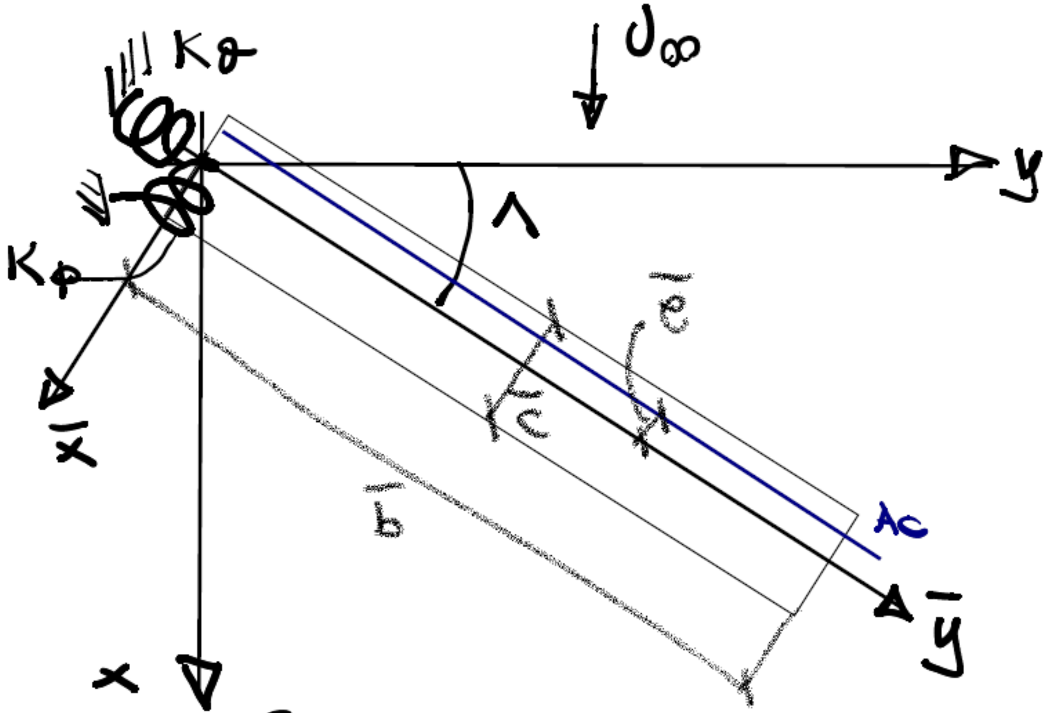
$$\begin{aligned}
m_b &= (qecC_L\alpha_0 + qc^2C_{m_{AC}} - mNgd)\cos\Lambda\sin\Lambda + \\
&\quad + qecC_{L\alpha}\left(\bar{\theta}(\bar{y})\cos\Lambda - z'(\bar{y})\sin\Lambda\right)\cos\Lambda\sin\Lambda
\end{aligned}$$

Simple problem (Typical section)

Rigid model with

✓ spring at root to represent TORSIONAL STIFFNESS and

✓ another spring to represent BENDING STIFFNESS



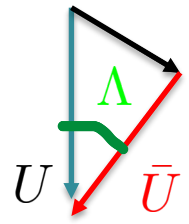
Write the equilibrium equation using PVW

$$
\begin{aligned}
\delta W_i &= \delta\bar{\theta} k_\theta \bar{\theta} + \delta\bar{\varphi} k_\varphi \bar{\varphi} \\
\delta W_e &= \int_0^{\bar{b}} \delta\bar{\theta}^T \bar{e} L(\bar{y}) \mathrm{d}\bar{y} + \int_0^{\bar{b}} \delta\bar{\varphi}^T \bar{y} L(\bar{y}) \mathrm{d}\bar{y}
\end{aligned}
$$

$$z(\bar{y}) = \bar{\varphi}\bar{y}, \quad z'(\bar{y}) = \bar{\varphi}$$

To understand the equivalence that exist between the quantities defined using the difference reference systems it is possible to say that

$$
\begin{aligned}
\mathrm{d}y &= \cos\Lambda \mathrm{d}\bar{y} \\
c &= \frac{\bar{c}}{\cos\Lambda} \longrightarrow c\mathrm{d}y = \bar{c}\mathrm{d}\bar{y} \\
\bar{q} &= \frac{1}{2}\rho\bar{U}^2 = \frac{1}{2}\rho U^2 \cos^2\Lambda = q\cos^2\Lambda
\end{aligned}
$$



$$
\begin{aligned}
&L\mathrm{d}y = \bar{L}\mathrm{d}\bar{y} \\
&qcC_{L\alpha}\alpha\mathrm{d}y = \bar{q}\bar{c}\bar{C}_{L\alpha}\bar{\alpha}\mathrm{d}\bar{y} \\
&C_{L\alpha}\alpha = \cos^2\Lambda\bar{C}_{L\alpha}\bar{\alpha}
\end{aligned}
\longrightarrow
\begin{aligned}
\bar{C}_{L\alpha} &= \frac{C_{L\alpha}}{\cos\Lambda} \\
\bar{\alpha} &= \frac{\alpha}{\cos\Lambda}
\end{aligned}
$$

Write the equilibrium equation using PVW

$$L(\bar{y}) \quad = \quad \bar{q}\bar{c}\bar{C}_{L\alpha}\left(\frac{\alpha_0}{\cos\Lambda} + \bar{\theta} - \bar{\varphi}\tan\Lambda\right)$$

Given the arbitrariness of $\delta\bar{\theta}$ and $\delta\bar{\phi}$

$$k_\theta\bar{\theta} \quad = \quad \bar{q}\bar{c}\bar{e}\bar{b}\bar{C}_{L\alpha}\left(\frac{\alpha_0}{\cos\Lambda} + \bar{\theta} - \bar{\varphi}\tan\Lambda\right)$$

$$k_\varphi\bar{\varphi} \quad = \quad \bar{q}\bar{c}\frac{\bar{b}^2}{2}\bar{C}_{L\alpha}\left(\frac{\alpha_0}{\cos\Lambda} + \bar{\theta} - \bar{\varphi}\tan\Lambda\right)$$

$$\left(\begin{bmatrix} k_\theta & 0 \\ 0 & k_\varphi \end{bmatrix} - \bar{q}\bar{c}\bar{b}\bar{C}_{L\alpha}\begin{bmatrix} \bar{e} & -\tan\Lambda\bar{e} \\ \frac{\bar{b}}{2} & -\tan\Lambda\frac{\bar{b}}{2} \end{bmatrix}\right)\begin{Bmatrix} \bar{\theta} \\ \bar{\varphi} \end{Bmatrix} = \frac{\bar{q}\bar{c}\bar{b}\bar{C}_{L\alpha}}{\cos\Lambda}\begin{Bmatrix} \bar{e} \\ \frac{\bar{b}}{2} \end{Bmatrix}\alpha_0$$

$$\left(\mathbf{K}_s - \bar{Q}\mathbf{K}_A\right)\begin{Bmatrix} \bar{\theta} \\ \bar{\varphi} \end{Bmatrix} = \frac{\bar{Q}}{\cos\Lambda}\begin{Bmatrix} \bar{e} \\ \frac{\bar{b}}{2} \end{Bmatrix}\alpha_0$$

with $\bar{Q} = \bar{q}\bar{c}\bar{b}\bar{C}_{L\alpha} = \bar{q}S\bar{C}_{L\alpha}$

$$\det(\mathbf{K}_A) = 0$$

The aerodynamic stifffness is singular!

Compute the divergence speed

$$\det\left(\mathbf{K}_s - \bar{Q}\mathbf{K}_A\right) = 0$$

That is equivalent to

$$\left(k_\theta - \bar{Q}\bar{e}\right)\left(k_\varphi + \bar{Q}\frac{\bar{b}}{2}\tan\Lambda\right) + \bar{Q}^2\bar{e}\frac{\bar{b}}{2}\tan\Lambda = 0$$

$$k_\theta k_\varphi + \bar{Q}\left(k_\theta\frac{\bar{b}}{2}\tan\Lambda - k_\varphi\bar{e}\right) = 0$$

There is only one divergence speed, even if the system is 2-dofs.

$$\bar{Q}_D = \frac{k_\theta k_\varphi}{\bar{e}k_\varphi - k_\theta\frac{\bar{b}}{2}\tan\Lambda}$$

$$\boxed{q_D = \frac{1}{\cos^2\Lambda}\frac{k_\theta}{S\bar{e}\bar{C}_{L\alpha}}\frac{1}{1 - \frac{k_\theta}{k_\varphi}\frac{\bar{b}}{\bar{e}}\frac{\tan\Lambda}{2}}}$$

$$\bar{Q} = \bar{q}\bar{c}\bar{b}\bar{C}_{L\alpha} = qS\cos^2\Lambda\bar{C}_{L\alpha}$$

Divergence speed

1. if $\Lambda > 0$, i.e. backward sweep angle, and $\bar{e} > 0$

$$\frac{k_\theta}{k_\varphi}\frac{\bar{b}}{\bar{e}}\frac{\tan\Lambda}{2} > 0$$

In this case it is possible to identify a critical sweep angle $\Lambda_{\text{CRIT}} > 0$ above which no divergence exists, because $q_D < 0$.

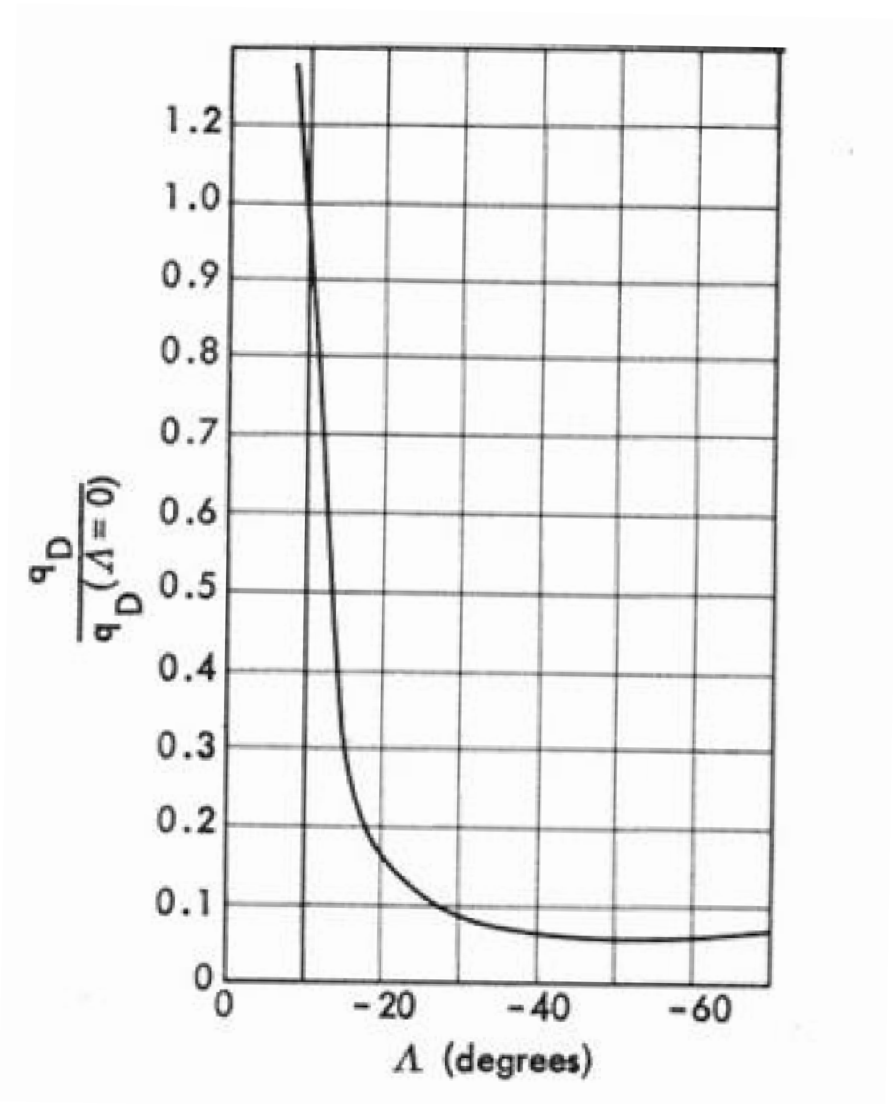
$$\frac{\bar{e}}{\bar{b}} = \frac{1}{60} \begin{cases} \frac{k_\varphi}{k_\theta} = 10 & \Lambda_{\text{CRIT}} = 18^\circ \\ \frac{k_\varphi}{k_\theta} = 3 & \Lambda_{\text{CRIT}} = 5.7^\circ \end{cases}$$

$$\frac{k_\theta}{k_\varphi}\frac{\bar{b}}{\bar{e}}\frac{\tan\Lambda_{\text{CRIT}}}{2} = 1, \quad \Lambda_{\text{CRIT}} = \tan^{-1}\left(2\frac{k_\varphi}{k_\theta}\frac{\bar{e}}{\bar{b}}\right)$$

2. $\Lambda < 0$, i.e. forward sweep angle, and $\bar{e} > 0$. The higher is $|\Lambda|$ the lower is q_D

N.B. q_D grows until it reaches ∞ at Λ_{CRIT} and then starts becoming negative with a modulus that reduces the higher is Λ.

Divergence speed of an elastic swept wing



Fast drop of divergence speed for negative sweep angles. Below -30° divergence speed is close to zero

Compute the Lift effectiveness

Ratio of Lift developed by the elastic model with respect to Lift developed by the rigid model

$$L_R = \bar{q} S \bar{C}_{L\alpha} \frac{\alpha_0}{\cos\Lambda} = \frac{\bar{Q}\alpha_0}{\cos\Lambda}$$

$$\bar{Q}_D = \frac{k_\theta k_\varphi}{\bar{e}k_\varphi - k_\theta \frac{\bar{b}}{2}\tan\Lambda}$$

$$\left(\mathbf{K}_s - \bar{Q}\mathbf{K}_A\right)\begin{Bmatrix}\bar{\theta}\\ \bar{\varphi}\end{Bmatrix} = \frac{\bar{Q}}{\cos\Lambda}\begin{Bmatrix}\bar{e}\\ \frac{\bar{b}}{2}\end{Bmatrix}\alpha_0$$

$$\bar{\theta} = \frac{\bar{Q}\bar{e}\alpha_0}{\cos\Lambda}\frac{k_\varphi}{k_\theta k_\varphi + \bar{Q}\left(k_\theta \frac{\bar{b}}{2}\tan\Lambda - k_\varphi \bar{e}\right)} = \frac{\bar{Q}\bar{e}\alpha_0}{\cos\Lambda}\frac{k_\varphi}{k_\theta k_\varphi\left(1 - \frac{\bar{Q}}{\bar{Q}_D}\right)}$$

$$\bar{\varphi} = \frac{\bar{Q}\bar{b}\alpha_0}{2\cos\Lambda}\frac{k_\theta}{k_\theta k_\varphi + \bar{Q}\left(k_\theta \frac{\bar{b}}{2}\tan\Lambda - k_\varphi \bar{e}\right)} = \frac{\bar{Q}\bar{b}\alpha_0}{2\cos\Lambda}\frac{k_\theta}{k_\theta k_\varphi\left(1 - \frac{\bar{Q}}{\bar{Q}_D}\right)}$$

Compute the Lift effectiveness

$$L_R = \bar{q} S \bar{C}_{L\alpha} \frac{\alpha_0}{\cos\Lambda} = \frac{\bar{Q}\alpha_0}{\cos\Lambda}$$

$$L = \bar{q} S \bar{C}_{L\alpha} \left(\frac{\alpha_0}{\cos\Lambda} + \bar{\theta} - \bar{\varphi} \tan\Lambda \right)$$

$$L = \frac{\bar{Q}\alpha_0}{\cos\Lambda} \left(1 + \frac{\bar{Q}\bar{e}}{k_\theta \left(1 - \frac{\bar{Q}}{\bar{Q}_D}\right)} - \frac{\bar{Q}\frac{\bar{b}}{2}}{k_\varphi \left(1 - \frac{\bar{Q}}{\bar{Q}_D}\right)} \tan\Lambda \right)$$

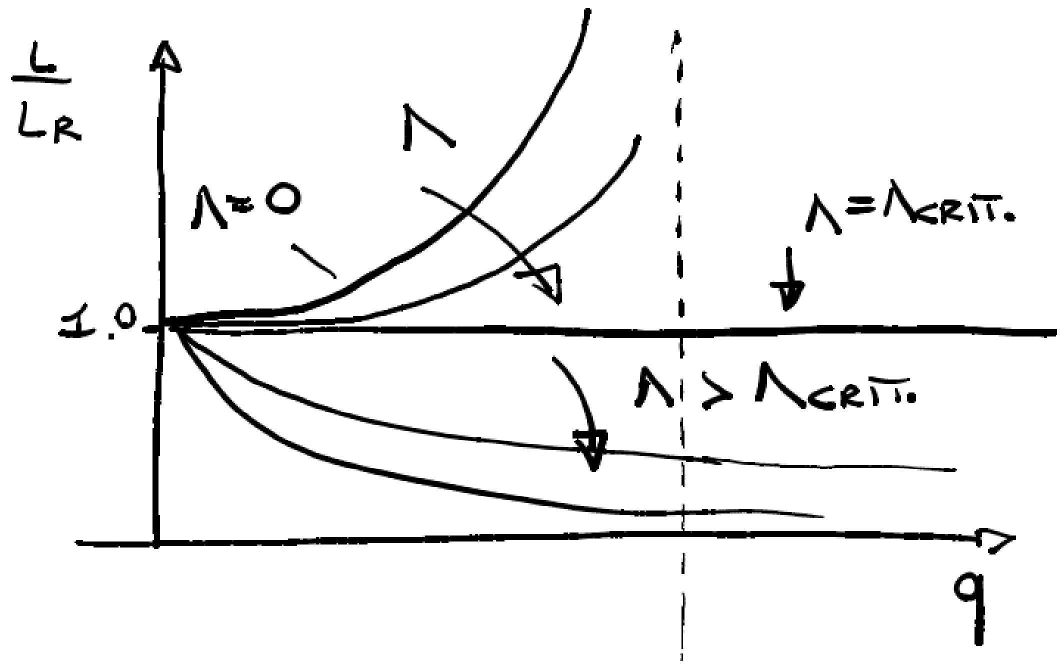
$$L = L_R \frac{1}{k_\theta k_\varphi \left(1 - \frac{\bar{Q}}{\bar{Q}_D}\right)} \left(k_\theta k_\varphi \left(1 - \frac{\bar{Q}}{\bar{Q}_D}\right) + k_\varphi \bar{Q}\bar{e} - k_\theta \bar{Q} \frac{\bar{b}}{2} \tan\Lambda \right)$$

$$L = L_R \frac{1}{k_\theta k_\varphi \left(1 - \frac{\bar{Q}}{\bar{Q}_D}\right)} \left(k_\theta k_\varphi \left(1 - \frac{\bar{Q}}{\bar{Q}_D}\right) + k_\theta k_\varphi \frac{\bar{Q}}{\bar{Q}_D} \right)$$

$$L = L_R \frac{1}{\left(1 - \frac{\bar{Q}}{\bar{Q}_D}\right)} \left(1 - \frac{\bar{Q}}{\bar{Q}_D} + \frac{\bar{Q}}{\bar{Q}_D} \right)$$

$$\frac{L}{L_R} = \frac{1}{1 - \frac{\bar{Q}}{\bar{Q}_D}} = \frac{1}{1 - \frac{\bar{q}}{\bar{q}_D}}$$

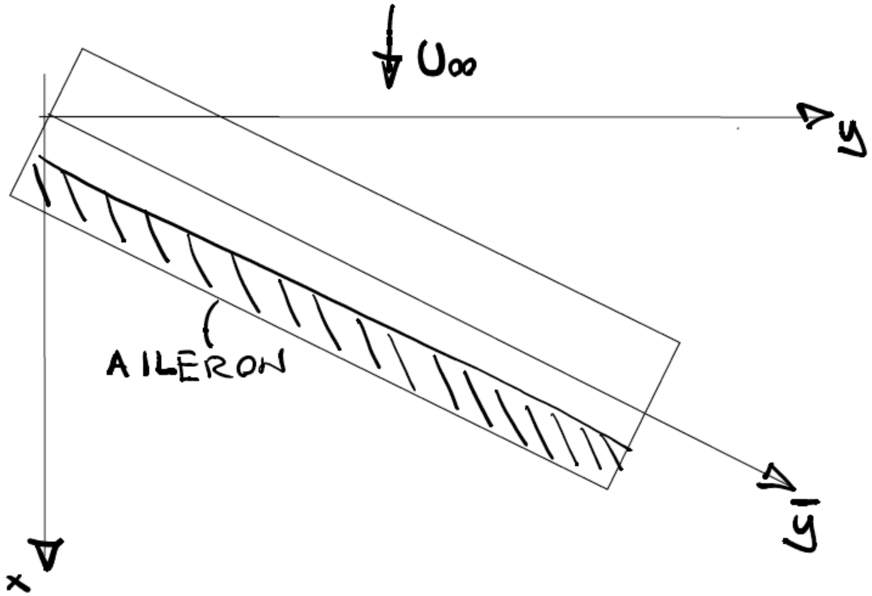
Lift Effectiveness



$$\begin{cases} \Lambda < \Lambda_{\text{CRIT}},\ q_D > 0 \quad L/L_R > 1 \\ \Lambda = \Lambda_{\text{CRIT}},\ q_D = \infty \quad L/L_R = 1 \\ \Lambda > \Lambda_{\text{CRIT}},\ q_D < 0 \quad L/L_R < 1 \end{cases}$$

For large positive sweep angles, the divergence disappears but there is a reduction of the lift effectiveness

Control Reversal



Consider a rigid aileron on the rigid wing connected with two springs at the root.

- ✓ Compute torsion and bending due to a unit rotation of the aileron
- ✓ Compute the lift generated including the effects of wing deformation

$$L = \bar{q}S\bar{C}_{L\alpha}\left(\bar{\theta} - \bar{\varphi}\tan\Lambda\right) + \bar{q}S\bar{C}_{L\beta}\beta = \bar{Q}\left(\bar{\theta} - \bar{\varphi}\tan\Lambda\right) + \bar{Q}\frac{\bar{C}_{L\beta}}{\bar{C}_{L\alpha}}\beta$$

$$M_{AC} = \bar{q}S\bar{c}\bar{C}_{m\beta}\beta = \bar{Q}\bar{c}\frac{\bar{C}_{m\beta}}{\bar{C}_{L\alpha}}\beta$$

$$m_\theta = L\bar{e} + M_{AC}$$

$$m_\varphi = L\frac{\bar{b}}{2}$$

$$\left(\mathbf{K}_s - \bar{Q}\mathbf{K}_A\right)\begin{Bmatrix}\bar{\theta}\\ \bar{\varphi}\end{Bmatrix} = \bar{Q}\bar{e}\frac{\bar{C}_{L\beta}}{\bar{C}_{L\alpha}}\begin{Bmatrix}1 + \frac{\bar{c}}{\bar{e}}\frac{\bar{C}_{m\beta}}{\bar{C}_{L\beta}}\\ \frac{\bar{b}}{2\bar{e}}\end{Bmatrix}\beta$$

Control Reversal

$$\left(\mathbf{K}_s - \bar{Q}\mathbf{K}_A\right)\begin{Bmatrix}\bar{\theta}\\ \bar{\varphi}\end{Bmatrix} = \bar{Q}\bar{e}\frac{\bar{C}_{L\beta}}{\bar{C}_{L\alpha}}\begin{Bmatrix}1+\frac{\bar{c}}{\bar{e}}\frac{\bar{C}_{m\beta}}{\bar{C}_{L\beta}}\\ \frac{\bar{b}}{2\bar{e}}\end{Bmatrix}\beta$$

$$L_R = \bar{Q}\frac{\bar{C}_{L\beta}}{\bar{C}_{L\alpha}}\beta$$

$$\bar{\theta} = \bar{Q}\bar{e}\frac{\bar{C}_{L\beta}}{\bar{C}_{L\alpha}}\left(1+\frac{\bar{c}}{\bar{e}}\frac{\bar{C}_{m\beta}}{\bar{C}_{L\beta}}\right)\frac{k_\varphi}{k_\theta k_\varphi\left(1-\frac{\bar{Q}}{\bar{Q}_D}\right)}\beta$$

$$\bar{\varphi} = \bar{Q}\frac{\bar{b}\bar{C}_{L\beta}}{2\bar{C}_{L\alpha}}\frac{k_\theta}{k_\theta k_\varphi\left(1-\frac{\bar{Q}}{\bar{Q}_D}\right)}\beta$$

$$L = \bar{Q}\left(\bar{\theta}-\bar{\varphi}\tan\Lambda\right)+\bar{Q}\frac{\bar{C}_{L\beta}}{\bar{C}_{L\alpha}}\beta$$

$$L = L_R\left(1+\frac{\bar{Q}\bar{e}\left(1+\frac{\bar{c}}{\bar{e}}\frac{\bar{C}_{m\beta}}{\bar{C}_{L\beta}}\right)}{k_\theta\left(1-\frac{\bar{Q}}{\bar{Q}_D}\right)}-\frac{\bar{Q}\frac{\bar{b}}{2}}{k_\varphi\left(1-\frac{\bar{Q}}{\bar{Q}_D}\right)}\tan\Lambda\right)$$

$$\frac{L}{L_R} = \frac{1}{1-\frac{q}{q_D}}+\frac{1}{1-\frac{q}{q_D}}\frac{\bar{Q}\bar{c}\frac{\bar{C}_{m\beta}}{\bar{C}_{L\beta}}}{k_\theta} = \frac{1+\frac{\bar{Q}\bar{c}}{k_\theta}\frac{\bar{C}_{m\beta}}{\bar{C}_{L\beta}}}{1-\frac{q}{q_D}}$$

Control Reversal

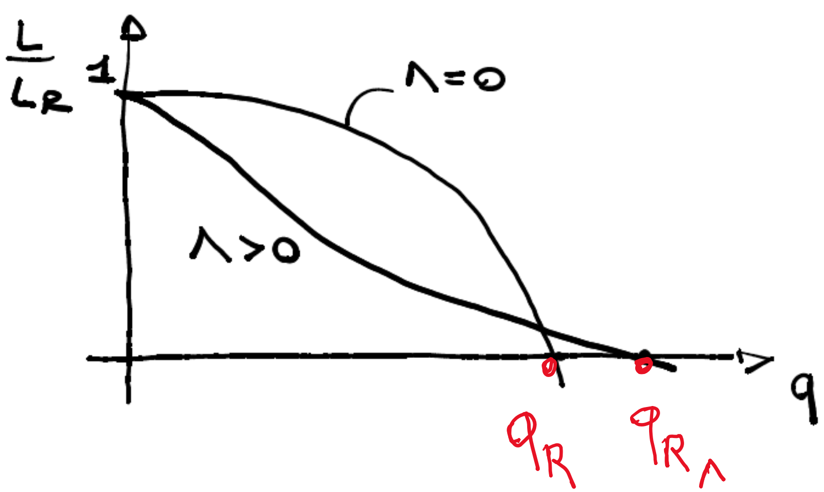
$$L_R = qS\bar{C}_{L\beta}\cos^2\Lambda\beta$$

$$\frac{L}{L_R} = \frac{1 + q\frac{S\bar{c}\bar{C}_{L\alpha}}{k_\theta}\frac{\bar{C}_{m\beta}}{\bar{C}_{L\beta}}\cos^2\Lambda}{1 - \frac{q}{q_D}}$$

$$q_{R\Lambda} = -\frac{k_\theta}{S\bar{c}\bar{C}_{L\alpha}}\frac{\bar{C}_{L\beta}}{\bar{C}_{m\beta}}\frac{1}{\cos^2\Lambda}$$

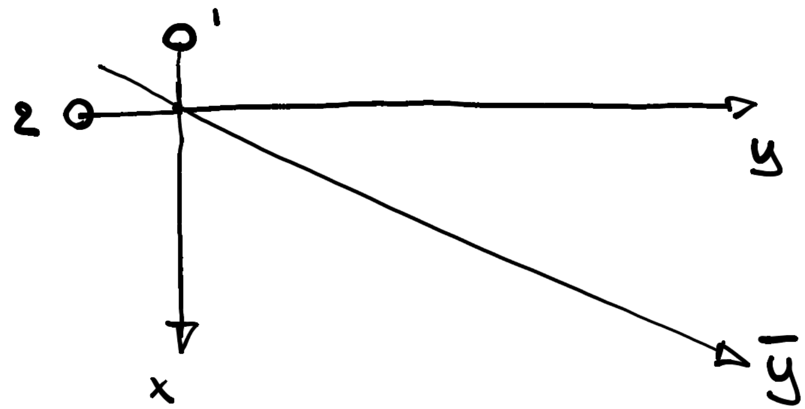
Increasing Λ, especially above Λ_{CRIT}, $q_{R\Lambda}$ is increased but the divergence dynamic pressure $q_D < 0$ becomes more negative decreasing significantly the lift effectiveness.

Reversal speed may grow, but due to elastic deformations (also bending causes a reduction of lift) the control effectiveness is rapidly reduced.



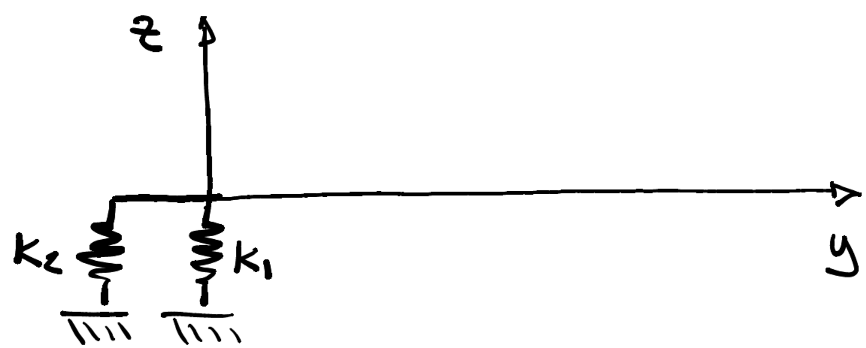
Aeroelastic Tailoring

Is it possible to obtain a torsional structural moment when the wing bends and vice-versa?



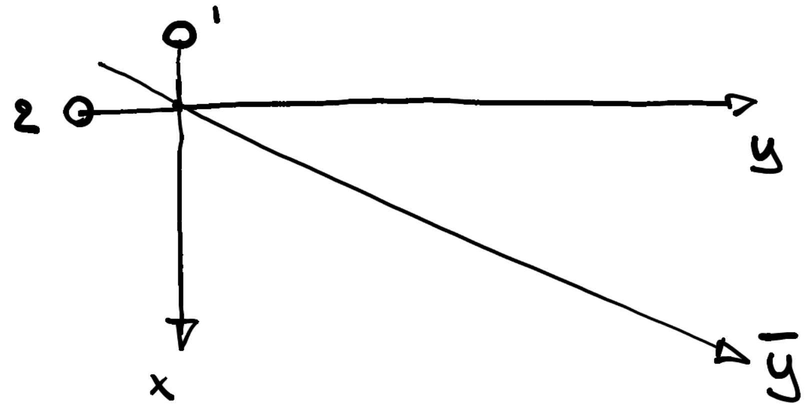
Consider the case

$$\mathbf{K}_s = \begin{bmatrix} k_\theta & k \\ k & k_\varphi \end{bmatrix}$$



Aeroelastic Tailoring

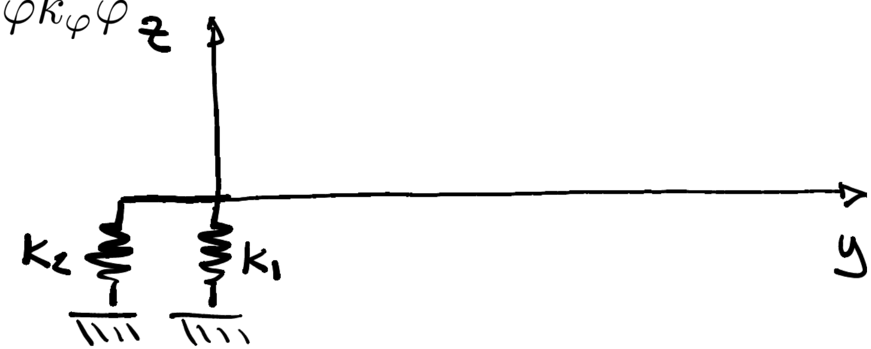
$$\begin{Bmatrix} \bar{\theta} \\ \bar{\varphi} \end{Bmatrix} = \begin{bmatrix} \cos\Lambda & \sin\Lambda \\ -\sin\Lambda & \cos\Lambda \end{bmatrix} \begin{Bmatrix} \theta \\ \varphi \end{Bmatrix}$$



$$\begin{Bmatrix} \bar{\theta} \\ \bar{\varphi} \end{Bmatrix} = \mathbf{R} \begin{Bmatrix} \theta \\ \varphi \end{Bmatrix}$$

$$\begin{aligned} z_1 &= \theta L_1 \\ z_2 &= \varphi L_2 \end{aligned}$$

$$\begin{aligned} \delta W_i &= \delta z_1 k_1 z_1 + \delta z_2 k_2 z_2 \\ \delta W_i &= \delta\theta L_1^2 k_1 \theta + \delta\varphi L_2^2 k_2 \varphi = \delta\theta k_\theta \theta + \delta\varphi k_\varphi \varphi \end{aligned}$$



$$\delta W_i = \delta \begin{Bmatrix} \bar{\theta} \\ \bar{\varphi} \end{Bmatrix}^T \mathbf{R}^T \begin{bmatrix} L_1^2 k_1 & 0 \\ 0 & L_2^2 k_2 \end{bmatrix} \mathbf{R} \begin{Bmatrix} \bar{\theta} \\ \bar{\varphi} \end{Bmatrix}$$

$$\delta W_i = \delta \begin{Bmatrix} \bar{\theta} \\ \bar{\varphi} \end{Bmatrix}^T \begin{bmatrix} k_\theta \cos^2\Lambda + k_\varphi \sin^2\Lambda & (k_\theta - k_\varphi)\sin\Lambda\cos\Lambda \\ (k_\theta - k_\varphi)\sin\Lambda\cos\Lambda & k_\theta \sin^2\Lambda + k_\varphi \cos^2\Lambda \end{bmatrix} \begin{Bmatrix} \bar{\theta} \\ \bar{\varphi} \end{Bmatrix}$$

Aeroelastic Tailoring

$$\det\left(\mathbf{K}_s - \bar{Q}_D \mathbf{K}_A\right) = 0$$

That is equivalent to

$$\left(k_\theta - \bar{Q}_D \bar{e}\right)\left(k_\varphi - \bar{Q}_D \frac{\bar{b}}{2}\tan\Lambda\right) - \left(k - \bar{Q}_D \tan\Lambda \bar{e}\right)\left(k - \bar{Q}_D \frac{\bar{b}}{2}\right) = 0$$

$$k_\theta k_\varphi - k^2 - \bar{Q}_D\left(k_\theta \frac{\bar{b}}{2}\tan\Lambda + k_\varphi \bar{e} - k\left(\bar{e}\tan\Lambda - \frac{\bar{b}}{2}\right)\right) = 0$$

$$\bar{Q}_D = \frac{k_\theta k_\varphi - k^2}{k_\theta \frac{\bar{b}}{2}\tan\Lambda + k_\varphi \bar{e} - k\left(\bar{e}\tan\Lambda - \frac{\bar{b}}{2}\right)}$$

The crtitical sweep angle is obtained when $Q_D = \infty$

$$\tan\Lambda_{\mathrm{CRIT}}\left(\frac{\bar{b}}{2}k_\theta - k\bar{e}\right) = k_\varphi \bar{e} + k\frac{\bar{b}}{2}$$

For a negative swept wing (i.e. forward swept), it is possible to obtain a negative Critical sweep angle if k < 0 and large enough in modulus. It means to implement a wash-out effect on the forward swept wing

$$\Lambda_{\mathrm{CRIT}} = \tan^{-1}\left(\frac{k_\varphi \bar{e} + k\frac{\bar{b}}{2}}{\frac{\bar{b}}{2}k_\theta - k\bar{e}}\right)$$

Aeroelastic Tailoring

More generally using composite material with appropriate direction for the deposition of the fibers, it is possible to obtain the level of coupling required

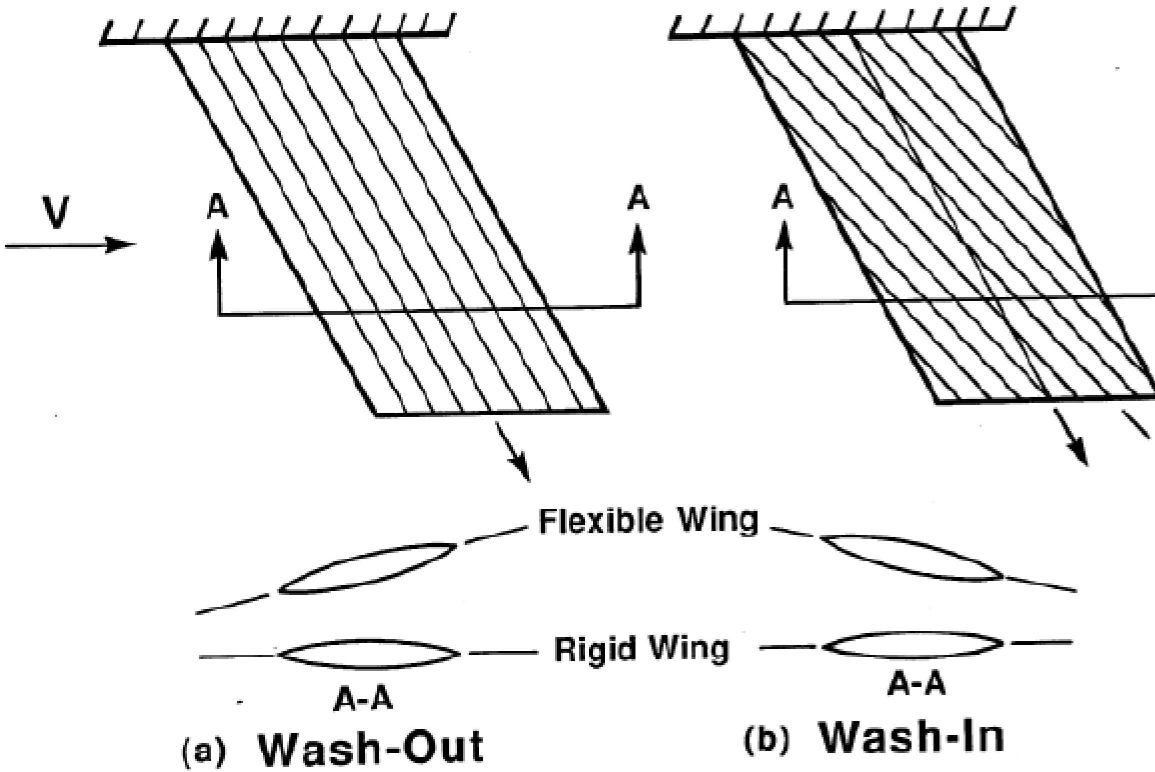


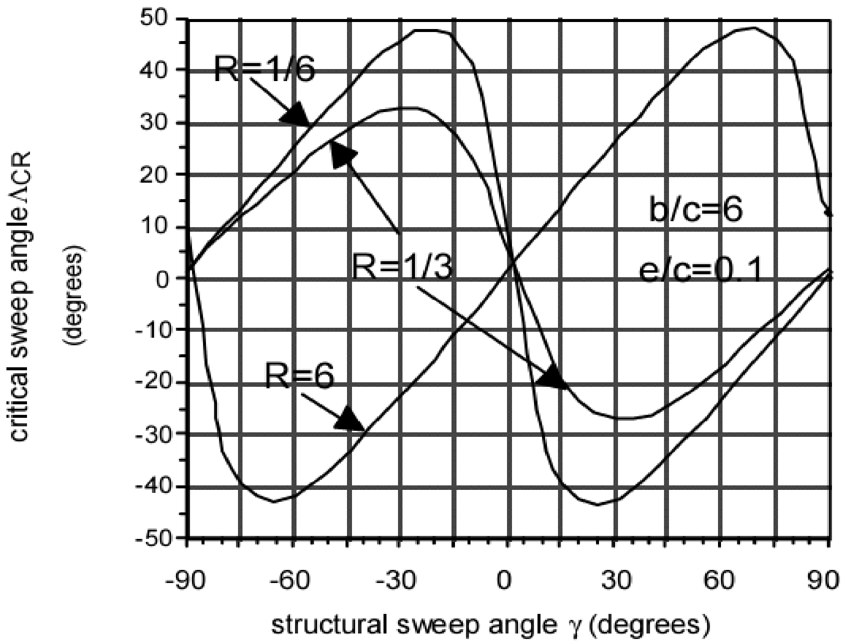
Figure 3.7.1 – Laminate re-orientation for shape control

Aeroelastic Tailoring

γ angle between the fibers and the beam axis

$$
\begin{aligned}
k &= (k_\varphi - k_\theta)\sin\gamma\cos\gamma \\
R &= \frac{k_\theta}{k_\varphi}
\end{aligned}
$$

Changes of the critical sweep angle by changing the direction of the fibers



PVW and Ritz-Galerkin approach

$$
\begin{aligned}
\delta W_i &= \int \delta\bar{\theta}'^T GJ\bar{\theta}' \mathrm{d}\bar{y} + \int \delta z''^T EJz'' \mathrm{d}\bar{y} \\
\delta W_e &= \int \delta z_{AC}^T L(\bar{y}) \mathrm{d}\bar{y} + \int \delta\bar{\theta}^T M_{AC} \mathrm{d}\bar{y}
\end{aligned}
$$

Then use the Ritz-Galerkin approch epproximating

$$
\begin{aligned}
\bar{\theta} &= \mathbf{N}_\theta \mathbf{q}_\theta \\
z &= \mathbf{N}_z \mathbf{q}_z
\end{aligned}
$$

$$z_{AC} = z + e\bar{\theta}$$

Summarizing

Positive sweep:

- ✓ Increases divergence speed (that often disappears)
- ✓ Decreases lift effectiveness
- ✓ Decreases controllability

The opposite is obtained by negative sweep

Aeroelastic tailoring can be used to change this behavior