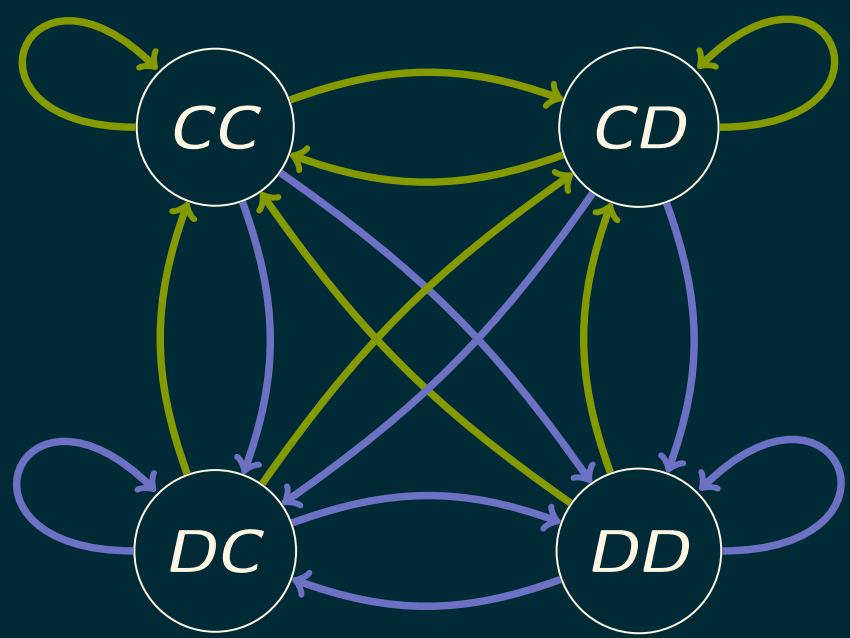


THE POWER OF MEMORY

In interactions both social and biological is memory size advantageous?



$$\begin{bmatrix} p_1 q_1 & p_1 (-q_1 + 1) & q_1 (-p_1 + 1) & (-p_1 + 1) (-q_1 + 1) \\ p_2 q_3 & p_2 (-q_3 + 1) & q_3 (-p_2 + 1) & (-p_2 + 1) (-q_3 + 1) \\ p_3 q_2 & p_3 (-q_2 + 1) & q_2 (-p_3 + 1) & (-p_3 + 1) (-q_2 + 1) \\ p_4 q_4 & p_4 (-q_4 + 1) & q_4 (-p_4 + 1) & (-p_4 + 1) (-q_4 + 1) \end{bmatrix}$$

	C	D
C	3	0
D	5	1

	1	2	3	...	$\overbrace{n-1}^{\text{memory-1}}$	$\overbrace{n}^{\text{memory-1}}$
player p	C	C	C	...	D	...
player q	C	C	D	...	D	...

W. H. Press and F. J. Dyson. **Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent** PNAS 2012.

$$p^* \rightarrow \text{manipulates} \rightarrow q$$

This work considers an optimisation approach to identify:

$$p^* \rightarrow \text{best response} \rightarrow q$$

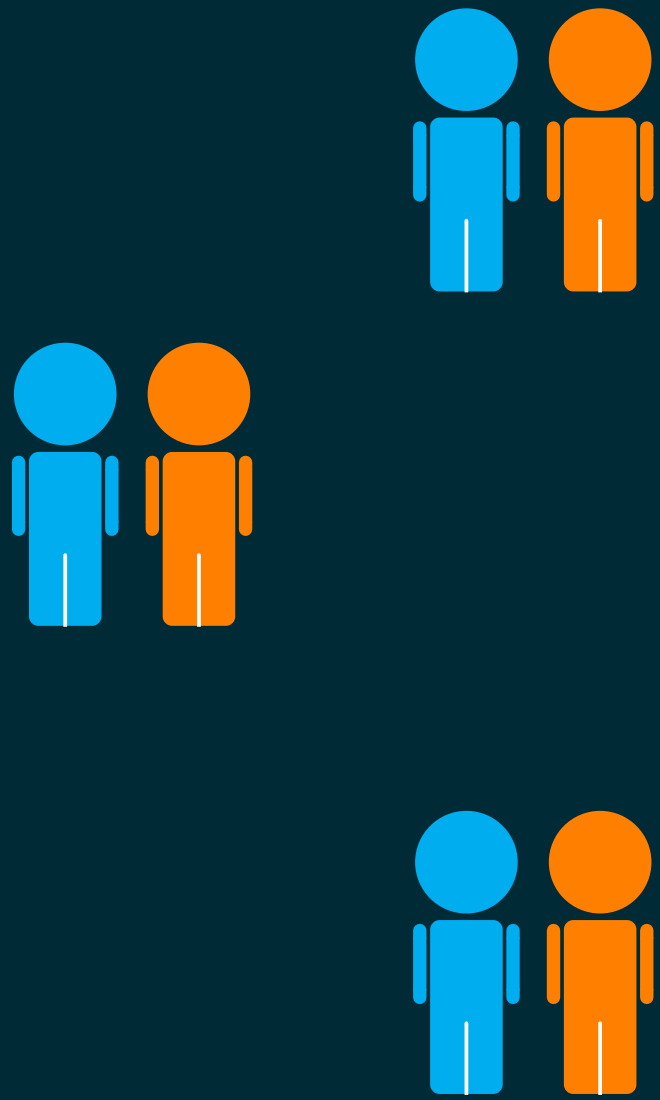
PURELY RANDOM STRATEGIES $p = (p, p, p, p)$

AGAINST A SINGLE OPPONENT

$$p^* = \operatorname{argmax}(u_q(p)), \quad p \in S_q,$$

where the set S_q is defined as,

$$S_q = \left\{ 0, p_{\pm}, 1 \mid \begin{array}{l} 0 < p_{\pm} < 1, \\ p_{\pm} \neq \frac{-d_0}{d_1} \end{array} \right\}$$



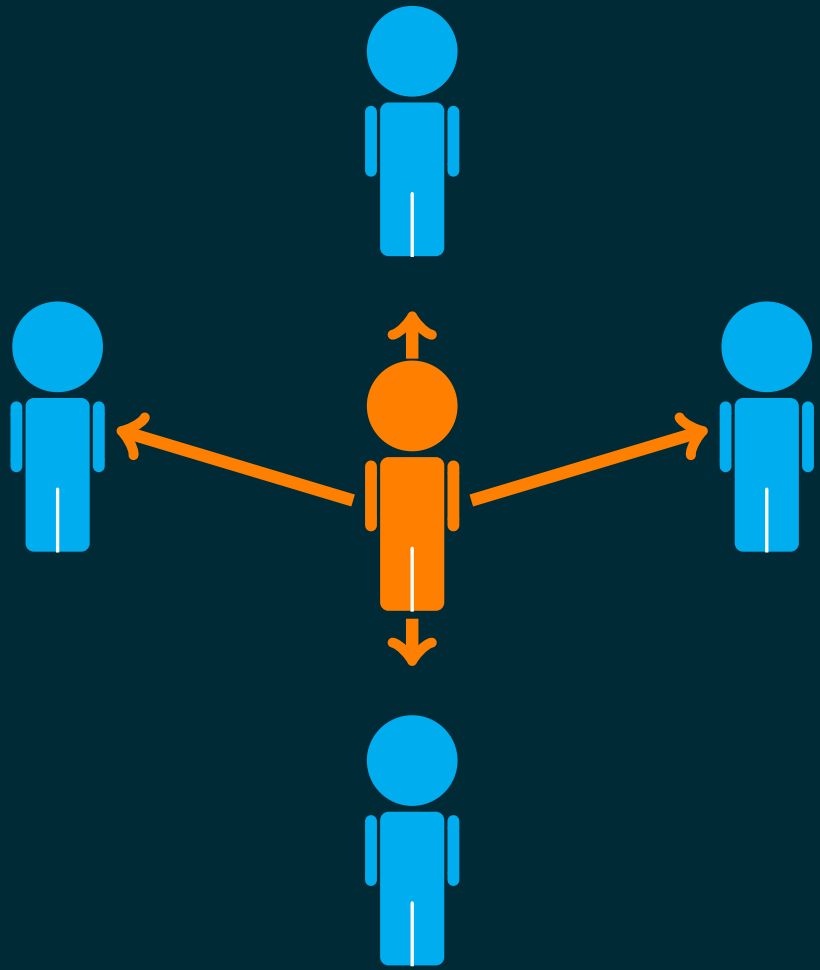
AGAINST MULTIPLE OPPONENTS

$$p^* = \operatorname{argmax}\left(\sum_{i=1}^N u_q^{(i)}(p)\right), \quad p \in S_{q(i)},$$

where the set $S_{q(i)}$ is defined as:

$$S_{q(i)} = \bigcup_{\substack{i=1 \\ \lambda_i \neq \frac{d_0}{d_1}}}^{2N} \lambda_i \cup \{0, 1\}$$

Note the size of candidate solutions is $1 \leq |S_{q(i)}| \leq 2N + 2$.



RESULTS

1. The utility of a given player p against a given opponent q is a ratio of quadratic forms.
2. Optimization procedures, often reducing the complex optimisation problem to a search over a small finite set are found.
3. Complex strategies outperform optimal purely random.

FUTURE WORK
RESULTANT THEORY

$$\begin{array}{l} f_0 = 0 \\ f_1 = 0 \\ \vdots \\ f_n = 0 \end{array} \quad M = \begin{bmatrix} \dots \\ \dots \\ \dots \end{bmatrix} \quad \det(M) = 0$$

