

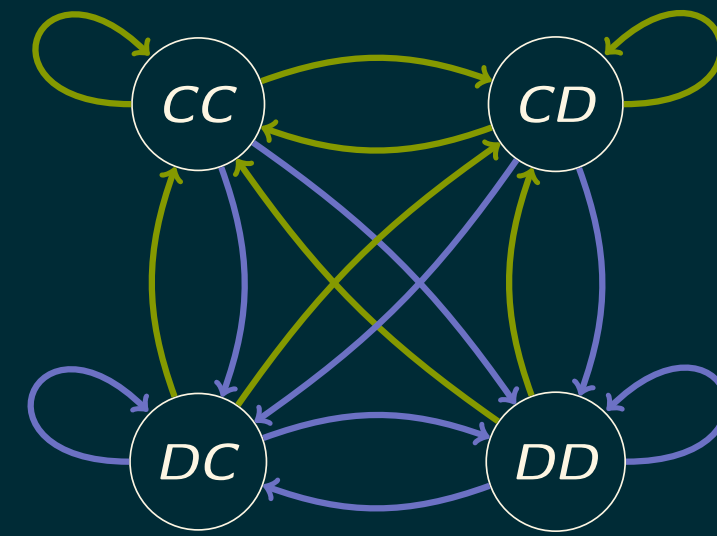
THE POWER OF MEMORY

Is memory size advantageous in interactions (social, biological, ...) ?

	C	D
C	3, 3	0, 5
D	5, 3	1, 1

	1	2	3	...	$n-1$	n
player p	C	C	C	...	D	...
player q	C	C	D	...	D	...

memory size $n-1$



$$\begin{bmatrix} p_1 q_1 & p_1(-q_1+1) & q_1(-p_1+1) & (-p_1+1)(-q_1+1) \\ p_2 q_3 & p_2(-q_3+1) & q_3(-p_2+1) & (-p_2+1)(-q_3+1) \\ p_3 q_2 & p_3(-q_2+1) & q_2(-p_3+1) & (-p_3+1)(-q_2+1) \\ p_4 q_4 & p_4(-q_4+1) & q_4(-p_4+1) & (-p_4+1)(-q_4+1) \end{bmatrix}$$

W. H. Press and F. J. Dyson. **Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent** PNAS 2012.

$p^* \rightarrow \text{manipulates} \rightarrow q$

This work considers an optimisation approach to identify:

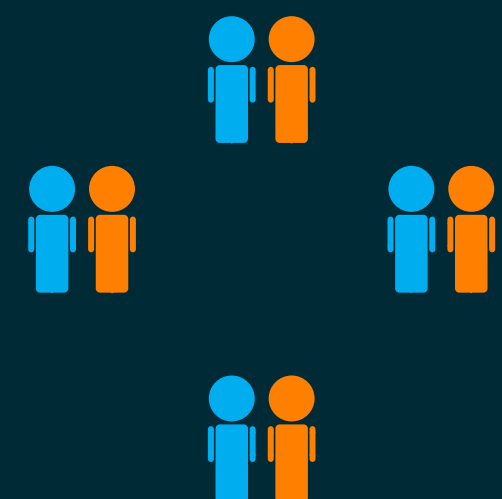
$p^* \rightarrow \text{best response} \rightarrow q$

$$1. \max_q : \frac{\frac{1}{2} p Q p^T + c^T p + a}{\frac{1}{2} p \bar{Q} p^T + \bar{c}^T p + \bar{a}}$$

$st : p \in \mathbb{R}_{[0,1]}^4$

PURELY RANDOM STRATEGIES $p = (p, p, p, p)$

AGAINST A SINGLE OPPONENT

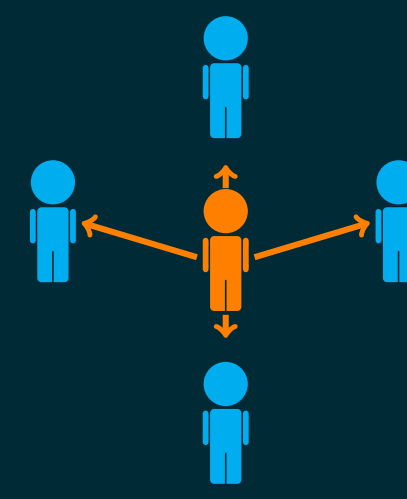


$$p^* = \operatorname{argmax}(u_q(p)), p \in S_q,$$

where the set S_q is defined as:

$$S_q = \left\{ 0, p_{\pm}, 1 \mid \begin{array}{l} 0 < p_{\pm} < 1, \\ p_{\pm} \neq \frac{-d_0}{d_1} \end{array} \right\}$$

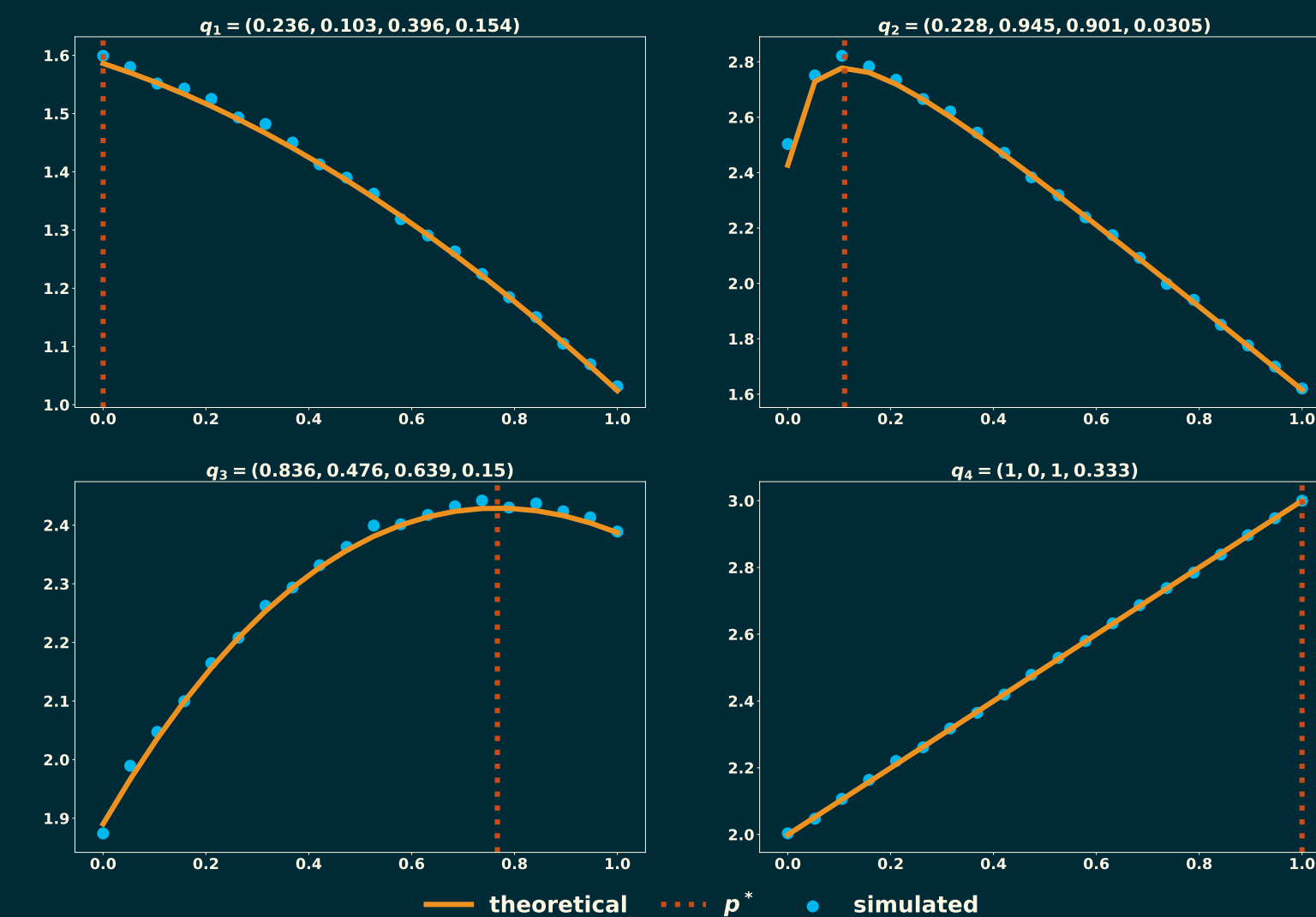
AGAINST MULTIPLE OPPONENTS



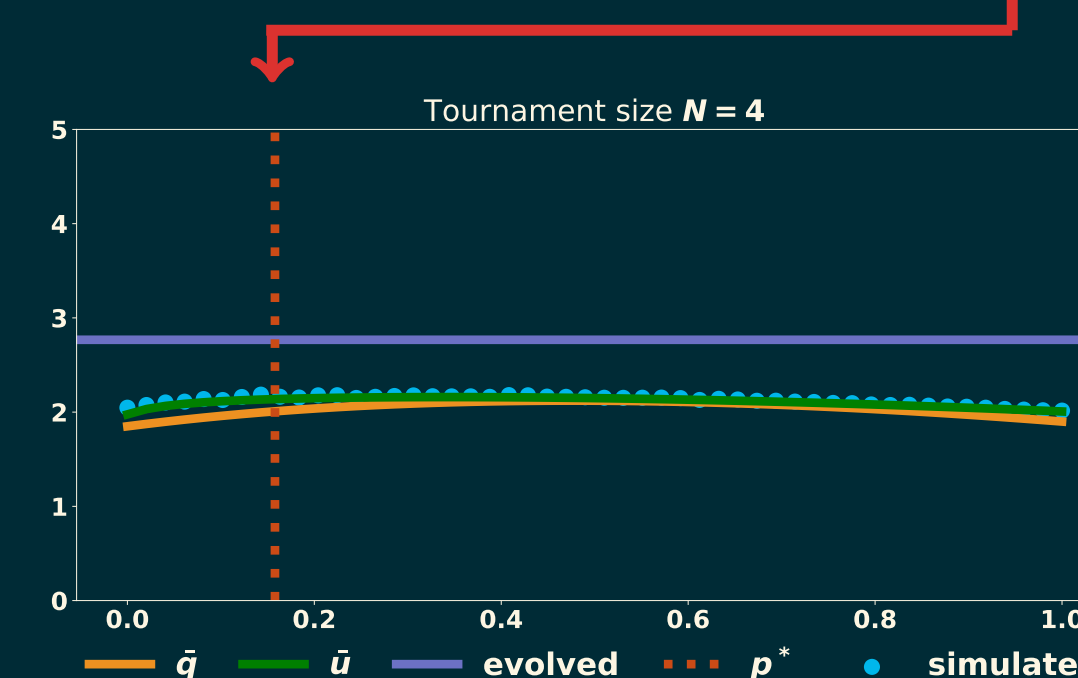
$$p^* = \operatorname{argmax} \left(\sum_{i=1}^N u_{q(i)}^{(i)}(p) \right), p \in S_{q(i)},$$

where the set $S_{q(i)}$ is defined as:

$$S_{q(i)} = \bigcup_{\substack{2N \\ i=1 \\ \lambda_i \neq \frac{d_0}{d_1}}} \lambda_i \cup \{0, 1\}$$



$$a_{2N} p^{2N} + \dots + a_1 p + a_0 \rightarrow \begin{pmatrix} 0.0 & 0.0 & \dots & 0 & -a_0 \\ 1.0 & 0.0 & \dots & 0 & -a_1 \\ 0.0 & 1.0 & \dots & 0 & -a_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0.0 & 0.0 & \dots & 1 & -a_{2N} \end{pmatrix}$$



RESULTS

1. The utility of a given player p against a given opponent q is written in a compact way.
2. Defining the optimal random behaviour p^* is reduced to a search over a small finite set.
3. Optimising against the mean utility can not be captured by optimising against the mean opponent.

FUTURE WORK RESULTANT THEORY

