# THE POWER OF MEMORY

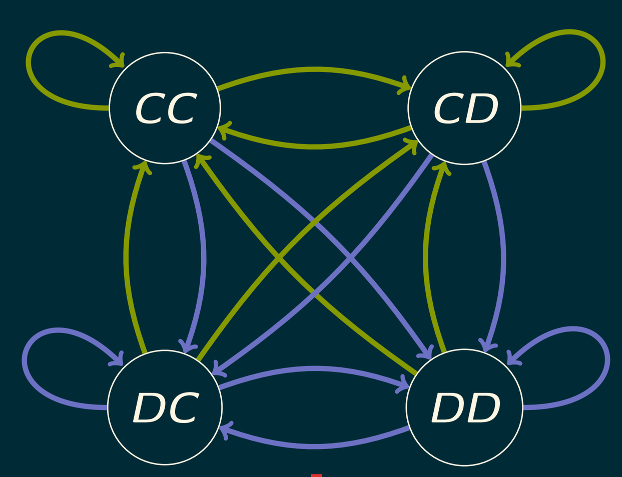
Is memory size edvantageous in interactions (social, biological, ...) ?

| | C | D |
|---|---|---|
| C | 3, 3 | 0, 5 |
| D | 5, 0 | 1, 1 |

| | 1 | 2 | 3 | $\cdots$ | $n-1$ | $n$ |
|---|---|---|---|---|---|---|
| player $p$ | $C$ | $C$ | $C$ | $\cdots$ | $D$ | $\cdots$ |
| player $q$ | $C$ | $C$ | $D$ | $\cdots$ | $D$ | $\cdots$ |

memory-1

memory size-$n-1$




$$\begin{bmatrix} p_1q_1 & p_1(-q_1+1) & q_1(-p_1+1) & (-p_1+1)(-q_1+1) \\ p_2q_3 & p_2(-q_3+1) & q_3(-p_2+1) & (-p_2+1)(-q_3+1) \\ p_3q_2 & p_3(-q_2+1) & q_2(-p_3+1) & (-p_3+1)(-q_2+1) \\ p_4q_4 & p_4(-q_4+1) & q_4(-p_4+1) & (-p_4+1)(-q_4+1) \end{bmatrix}$$

W. H. Press and F. J. Dyson. **Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent** PNAS 2012.

$$p^* \rightarrow \text{ manipulates } \rightarrow q$$

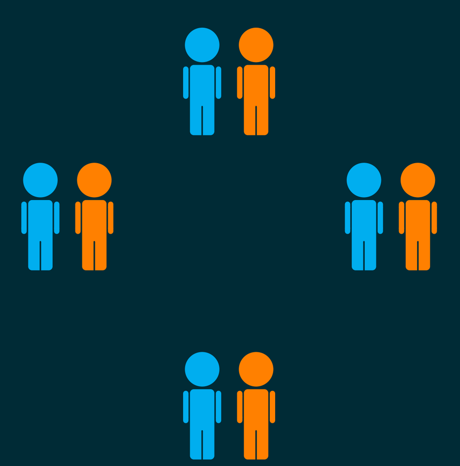
This work considers an optimisation approach to identify:

$$p^* \rightarrow \text{ best response } \rightarrow q$$

$$\max_q : \frac{\frac{1}{2}\ pQp^T + c^Tp + a}{\frac{1}{2}\ p\bar{Q}p^T + \bar{c}^Tp + \bar{a}}$$

## PURELY RANDOM STRATEGIES $p = (p, p, p, p)$

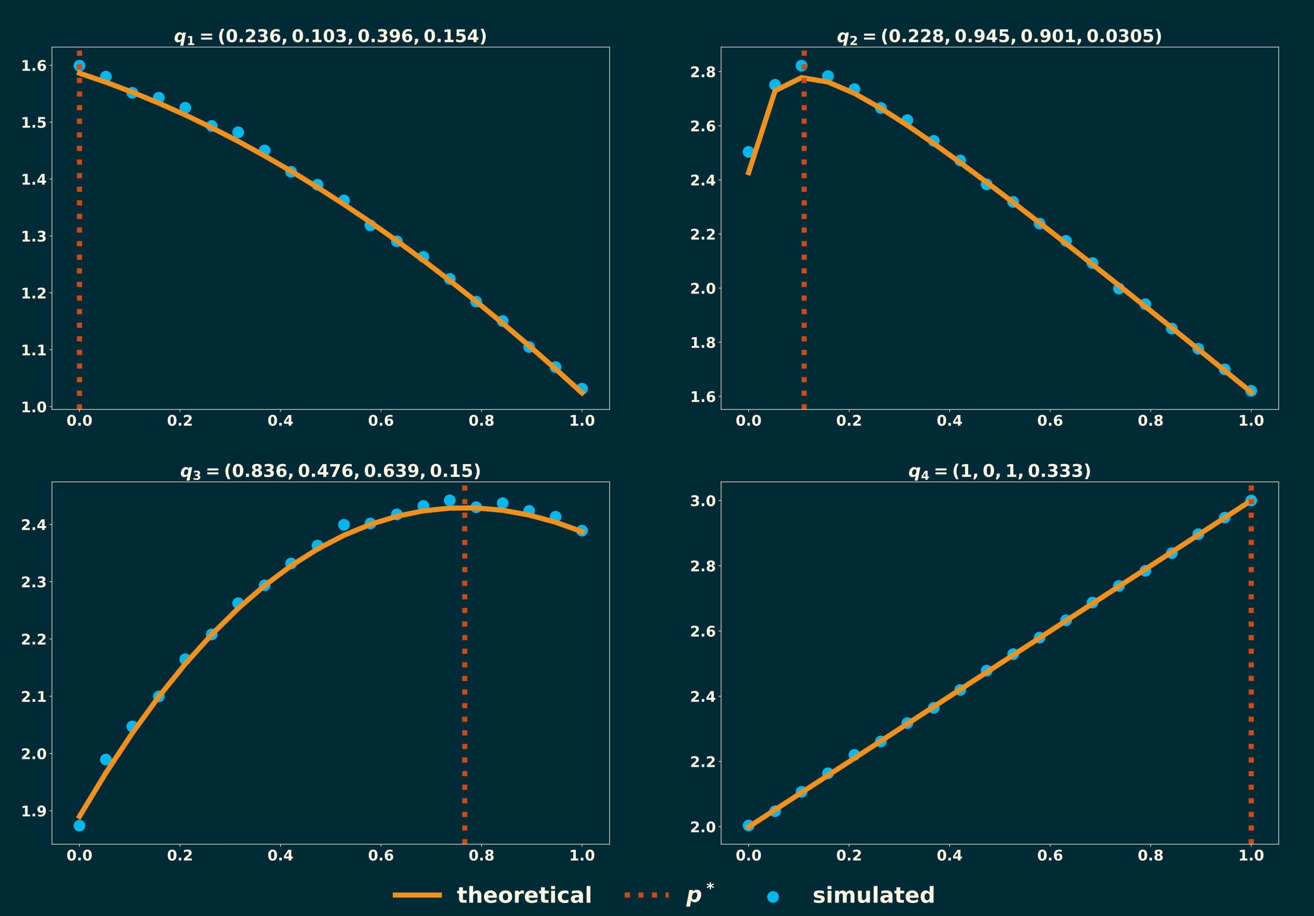
### AGAINST A SINGLE OPPONENT



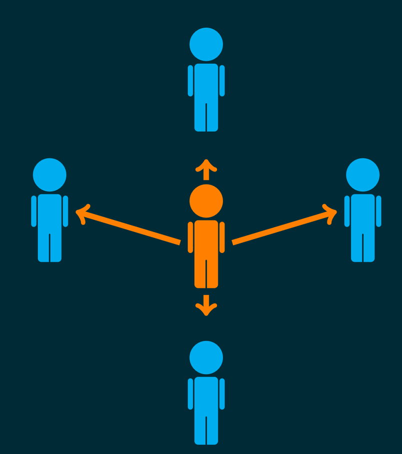
$$p^* = \text{argmax}(u_q(p)),\ p \in S_q,$$

where the set $S_q$ is defined as:

$$S_q = \left\{0, p_{\pm}, 1 \,\middle|\, \begin{array}{l} 0 < p_{\pm} < 1, \\ p_{\pm} \neq \frac{-d_0}{d_1} \end{array} \right\}$$



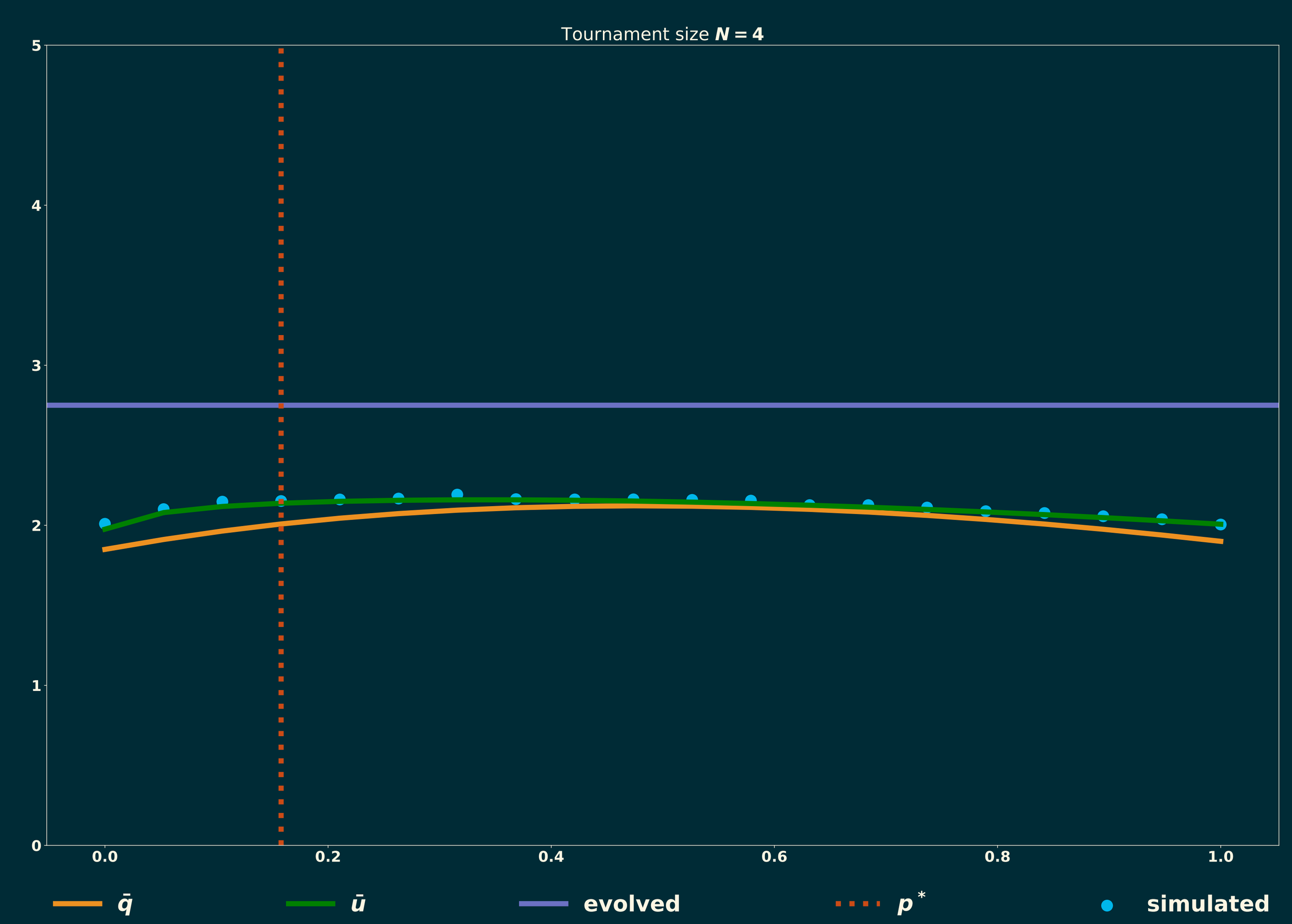

### AGAINST MULTIPLE OPPONENTS



$$p^* = \text{argmax}(\sum_{i=1}^{N} u_q{}^{(i)}(p)),\ p \in S_{q(i)},$$

where the set $S_{q(i)}$ is defined as:

$$S_{q(i)} = \bigcup_{\substack{i=1 \\ \lambda_i \neq \frac{do_i}{d1_i}}}^{2N} \lambda_i \cup \{0, 1\}$$

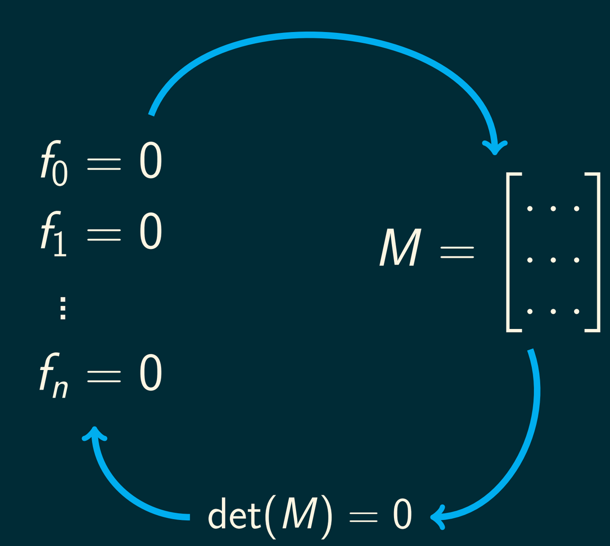



## RESULTS

1. The utility of a given player $p$ against a given opponent $q$ can be written in a compact way.
2. The optimal purely random player.
3. Complex strategies outperform optimal purely random.

## FUTURE WORK

### RESULTANT THEORY





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**In case you missed me:** `nikoleta-v3.github.io/blog/2017/08/23/power-of-memoty.html`

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