THE POWER OF MEMORY

In interactions both social and biological is memory size advantageous?

	C	D
C	3 / 3	3 / 0 ...

(payoff matrix C,C = 3,3 ; C,D = 3,0 ; D,C = 5,0 ; D,D = 5,1)

memory-1

	1	2	3	$\cdots$	$n-1$	n
player p	C	C	C	$\cdots$	D	$\cdots$
player q	C	C	D	$\cdots$	D	$\cdots$

memory size-$n-1$
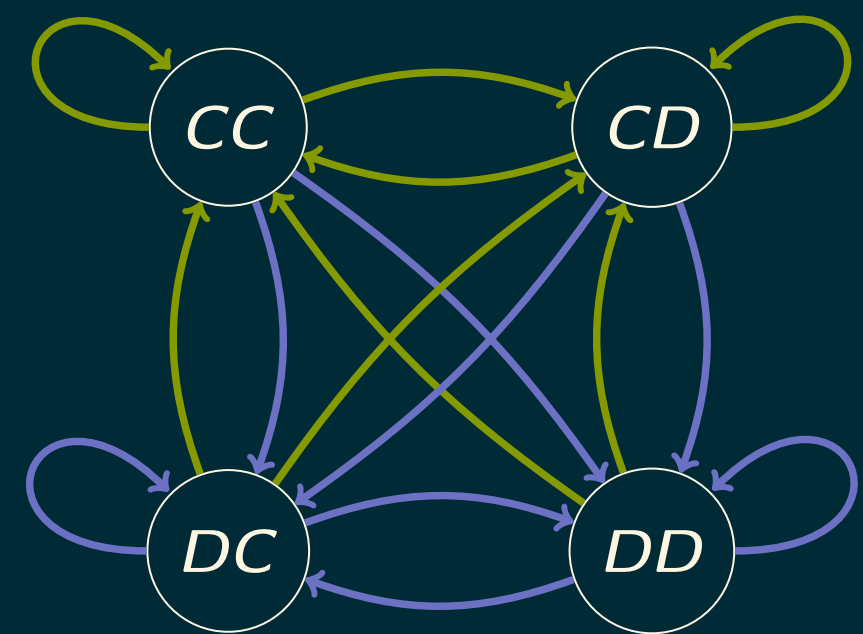
$$\begin{bmatrix} p_1 q_1 & p_1(-q_1+1) & q_1(-p_1+1) & (-p_1+1)(-q_1+1) \\ p_2 q_3 & p_2(-q_3+1) & q_3(-p_2+1) & (-p_2+1)(-q_3+1) \\ p_3 q_2 & p_3(-q_2+1) & q_2(-p_3+1) & (-p_3+1)(-q_2+1) \\ p_4 q_4 & p_4(-q_4+1) & q_4(-p_4+1) & (-p_4+1)(-q_4+1) \end{bmatrix}$$
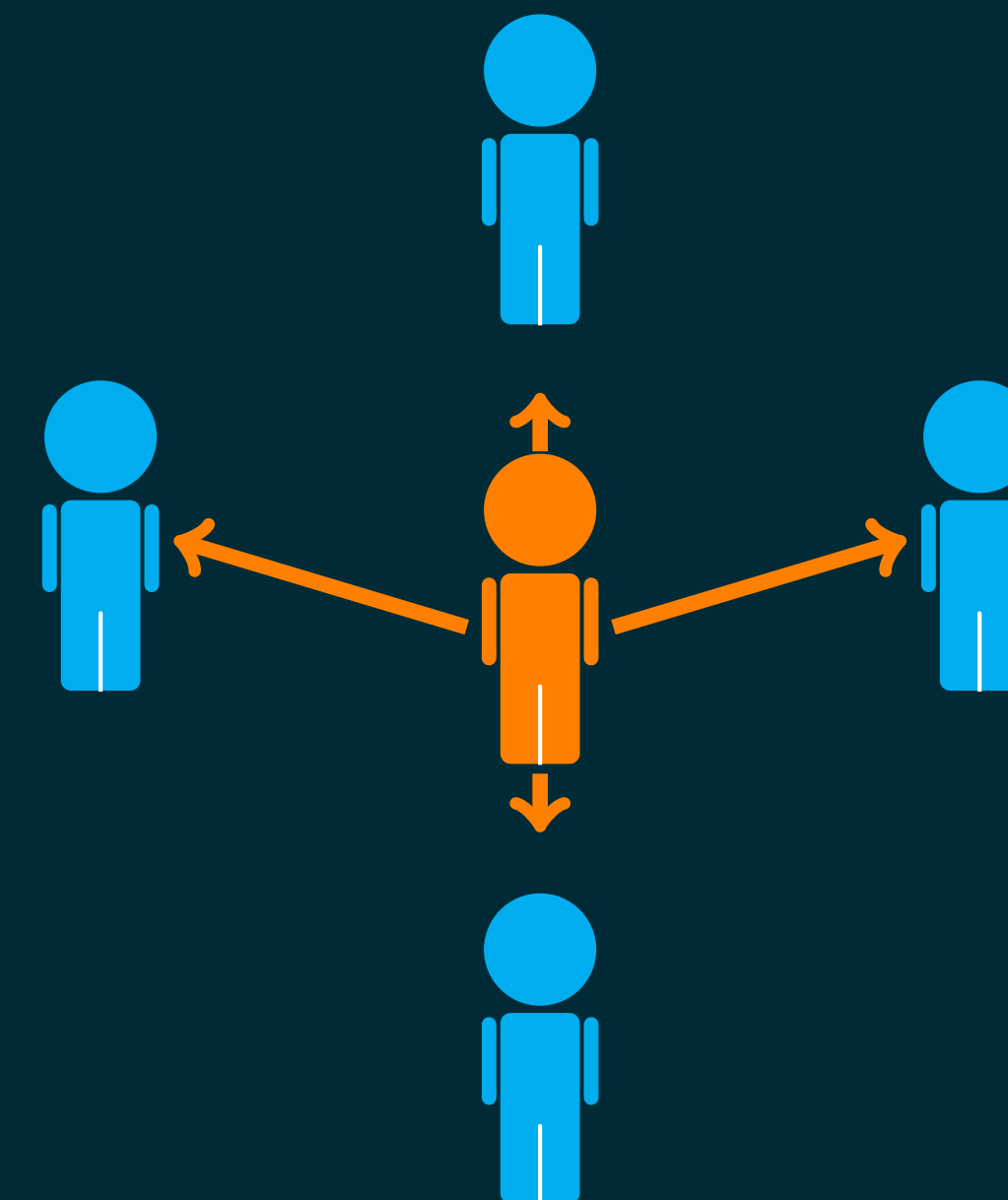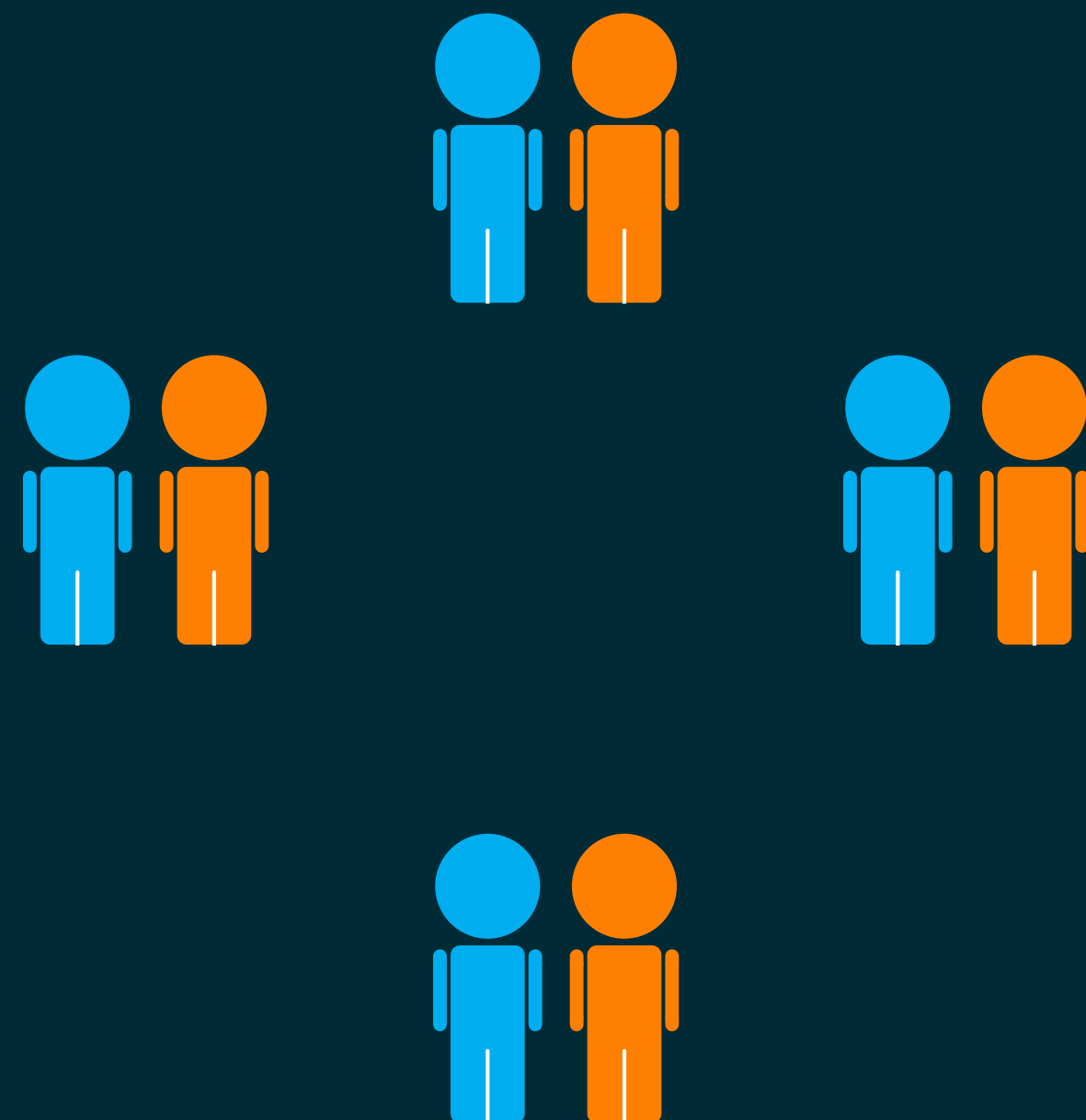
W. H. Press and F. J. Dyson. **Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent** PNAS 2012.

$$p^* \to \text{ manipulates } \to q$$

This work considers an optimisation approach to identify:

$$p^* \to \text{ best response } \to q$$

PURELY RANDOM STRATEGIES $p = (p, p, p, p)$

$$p^* = \text{argmax}(u_q(p)), \ p \in S_q,$$

where the set S_q is defined as,

$$S_q = \left\{ 0, p_{\pm}, 1 \ \middle| \ \begin{array}{l} 0 < p_{\pm} < 1, \\ p_{\pm} \neq \frac{-d_0}{d_1} \end{array} \right\}$$

$$p^* = \text{argmax}\left(\sum_{i=1}^{N} u_q^{(i)}(p)\right), \ p \in S_{q(i)},$$

where the set $S_{q(i)}$ is defined as:

$$S_{q(i)} = \bigcup_{\substack{i=1 \\ \lambda_i \neq \frac{d0_i}{d1_i}}}^{2N} \lambda_i \cup \{0, 1\}$$

RESULTS

1. The utility of a given player p against a given opponent q is a ratio of quadratic forms.
2. Optimization procedures, often reducing the complex optimisation problem to a search over a small finite set are found.
3. Complex strategies outperform optimal purely random.

NUMERICAL EXPERIMENTS

$q = (0.228, 0.945, 0.901, 0.0305)$ — theoretical, p^*, simulated

$q = (0.236, 0.103, 0.396, 0.154)$ — theoretical, p^*, simulated

$q = (0.836, 0.476, 0.639, 0.15)$ — theoretical, p^*, simulated

$q = (1, 0, 1, 0.333)$ — theoretical, p^*, simulated

Tournament size $N = 4$ — $\bar{q}$, $\bar{u}$, evolved, p^*, simulated