

# Optimisation of short memory strategies in the Iterated Prisoners Dilemma

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Supervised by:

Dr. Vincent KNIGHT

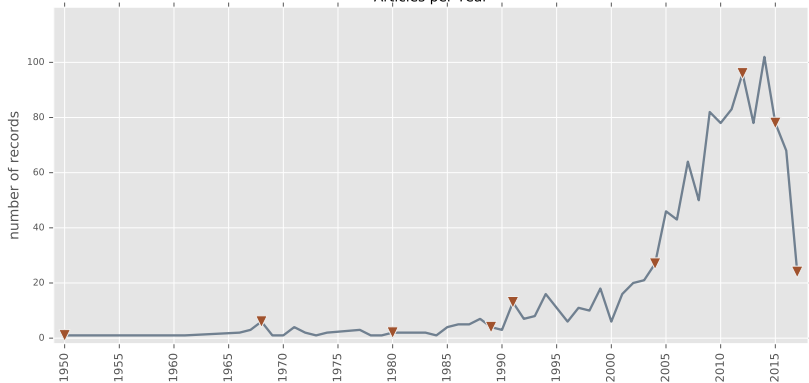
Dr. Jonathan GILLARD

$$\begin{bmatrix} (3, 3) & (0, 5) \\ (5, 0) & (1, 1) \end{bmatrix}$$

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$$(R, P, S, T) = (3, 1, 0, 5)$$

Articles per Year

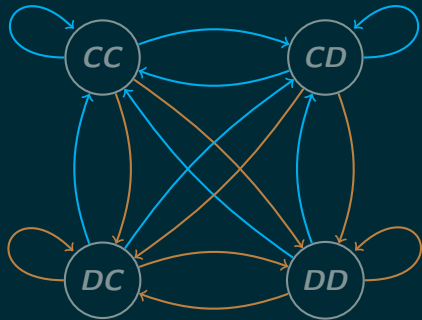




$$p = (p_1, p_2, p_3, p_4) \in \mathbb{R}_{[0,1]}^4$$

- ▶ *Christopher Lee, Marc Harper, and Dashiell Fryer.* The art of war: Beyond memory-one strategies in population games. 2015.

How good are memory one strategies ?







$$\max_p u_q(p) \text{ such that } p \in \mathbb{R}_{[0,1]}^4$$

## Lemma

$$u_q(p) = \frac{\frac{1}{2}pQp^T + c^T p + a}{\frac{1}{2}p\bar{Q}p^T + \bar{c}^T p + \bar{a}}$$

- ▶  $Q, \bar{Q} \in \mathbb{R}^{4 \times 4}$
- ▶  $c, \bar{c} \in \mathbb{R}^{4 \times 1}$
- ▶  $a, \bar{a} \in \mathbb{R}$

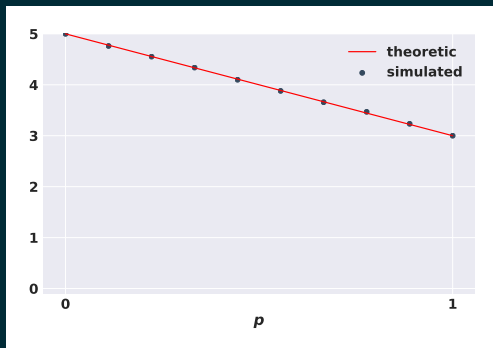
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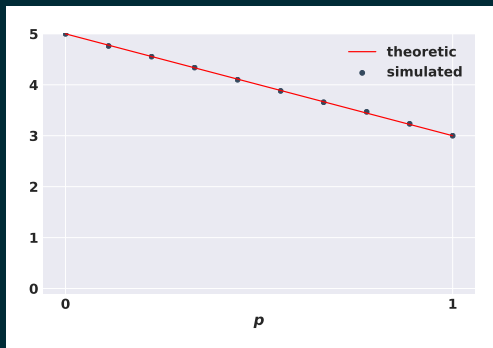
$$\text{subject to } p_1 = p_2 = p_3 = p_4 = p$$



$$q = \left(1, 1, 0, \frac{2}{3}\right)$$



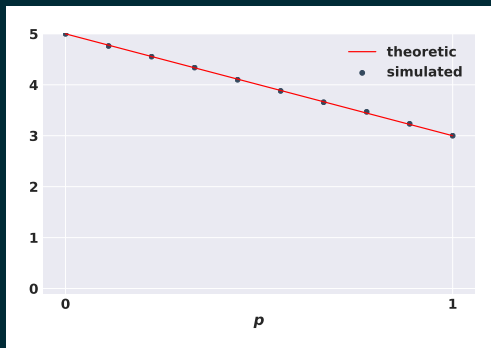
$$q = \left(1, 1, 0, \frac{2}{3}\right)$$



$$u_q(p) = \frac{-\frac{4p^2}{3} + \frac{14p}{3} - \frac{10}{3}}{\frac{2p}{3} - \frac{2}{3}}$$

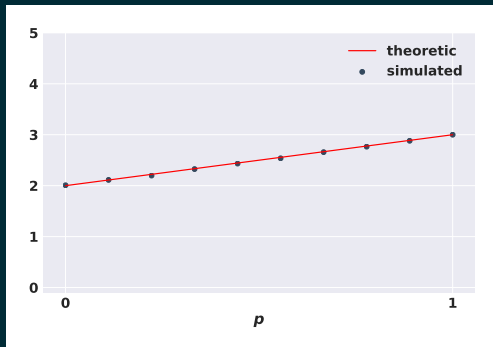


$$q = \left(1, 1, 0, \frac{2}{3}\right)$$

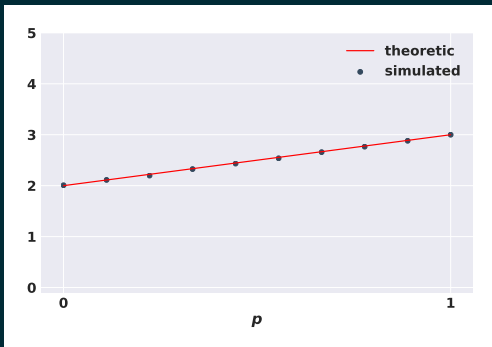


$$u_q(p) = \frac{-\frac{4p^2}{3} + \frac{14p}{3} - \frac{10}{3}}{\frac{2p}{3} - \frac{2}{3}} = -2p + 5$$

$$q = \left(1, 0, 1, \frac{1}{3}\right)$$

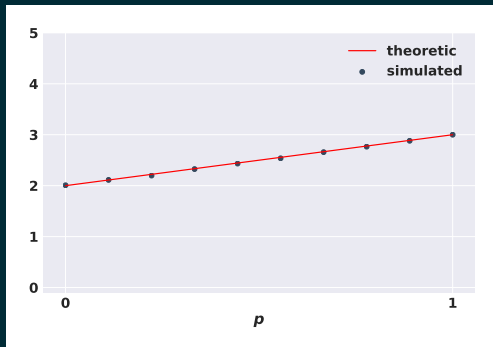


$$q = \left(1, 0, 1, \frac{1}{3}\right)$$



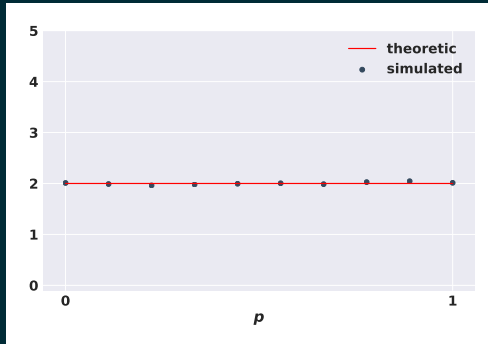
$$u_q(p) = \frac{\frac{p^2}{3} + \frac{8p}{3} - \frac{10}{3}}{\frac{p}{3} - \frac{4}{3}}$$

$$q = \left(1, 0, 1, \frac{1}{3}\right)$$

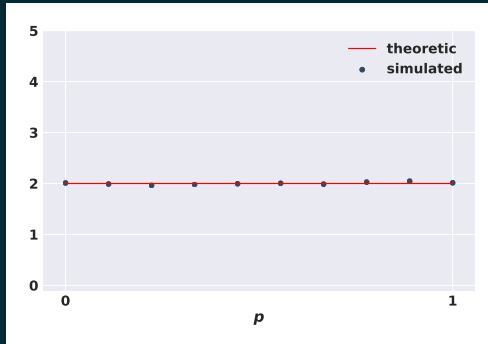


$$u_q(p) = \frac{\frac{p^2}{3} + \frac{8p}{3} - \frac{10}{3}}{\frac{p}{3} - \frac{4}{3}} = p + 2$$

$$q = \left( \frac{2}{3}, 0, \frac{2}{3}, \frac{1}{3} \right)$$

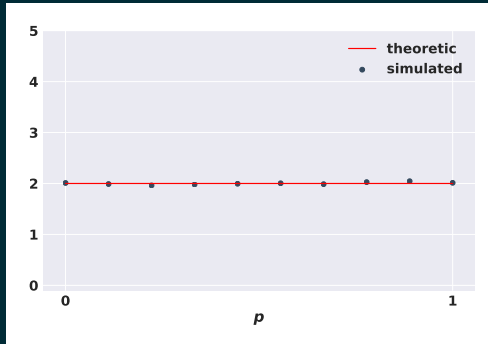


$$q = \left( \frac{2}{3}, 0, \frac{2}{3}, \frac{1}{3} \right)$$



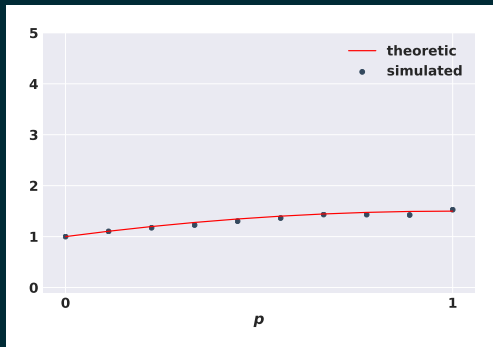
$$u_q(p) = \frac{\frac{2p}{3} - \frac{8}{3}}{\frac{p}{3} - \frac{4}{3}}$$

$$q = \left( \frac{2}{3}, 0, \frac{2}{3}, \frac{1}{3} \right)$$



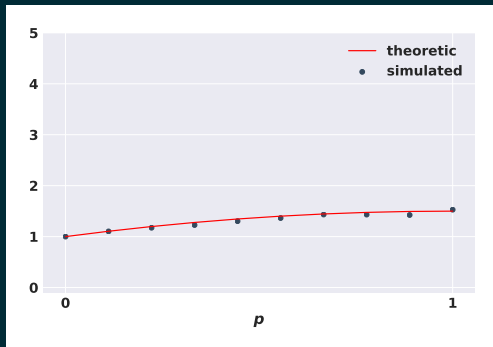
$$u_q(p) = \frac{\frac{2p}{3} - \frac{8}{3}}{\frac{p}{3} - \frac{4}{3}} = 2$$

$$q = \left( \frac{2}{3}, \frac{1}{3}, \frac{1}{3}, 0 \right)$$



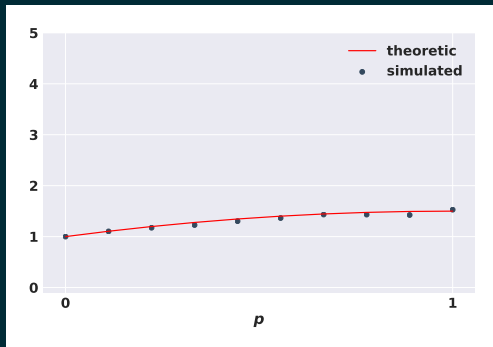


$$q = \left( \frac{2}{3}, \frac{1}{3}, \frac{1}{3}, 0 \right)$$



$$u_q(p) = \frac{\frac{p^2}{3} - \frac{2p}{3} - \frac{2}{3}}{-\frac{2}{3}}$$

$$q = \left( \frac{2}{3}, \frac{1}{3}, \frac{1}{3}, 0 \right)$$



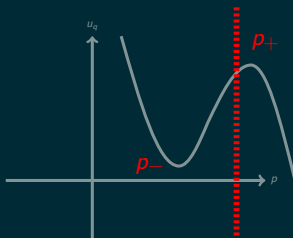
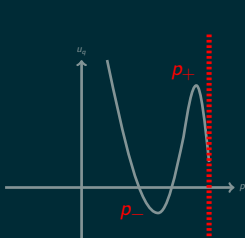
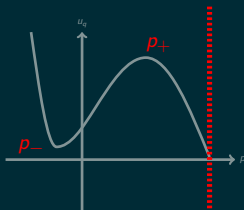
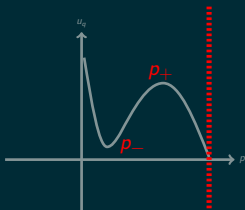
$$u_q(p) = \frac{\frac{p^2}{3} - \frac{2p}{3} - \frac{2}{3}}{-\frac{2}{3}} = -\frac{p^2}{2} + p + 1$$





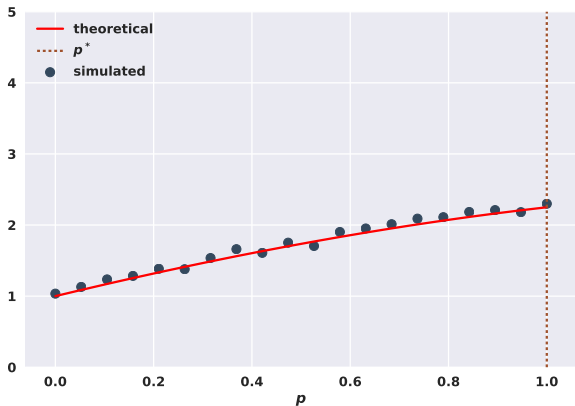


$$\frac{du}{dp} = \frac{m_2 p^2 + m_1 p + m_0}{(d_1 p + d_0)^2}$$



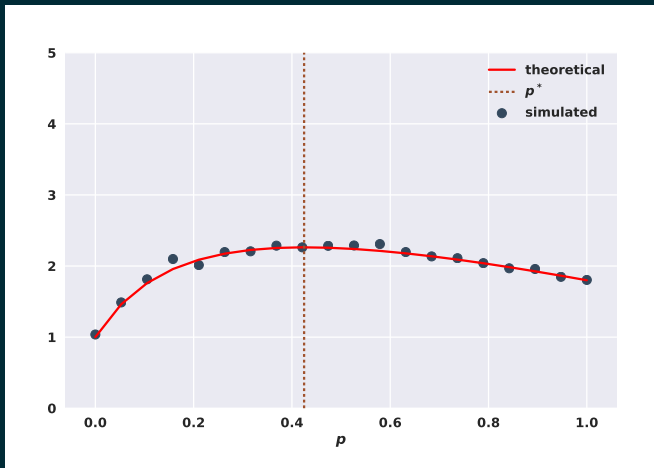


$$q = \left( \frac{7}{8}, \frac{7}{16}, \frac{3}{8}, 0 \right)$$





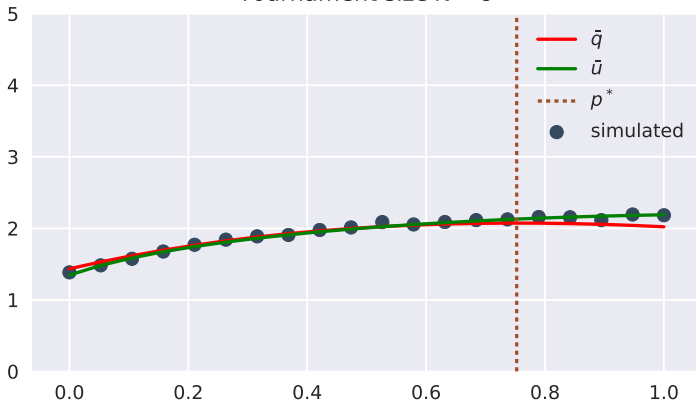
$$q = \left( \frac{1}{3}, \frac{2}{3}, 1, 0 \right)$$







Tournament size  $N = 9$





$$p^* = \operatorname{argmax}_{S_{q_1, \dots, q_n}} u(p)$$

where,  $|S_{q_1, \dots, q_n}| \leq 2N + 2$

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<https://github.com/Nikoleta-v3>