Memory size in the Prisoner's Dilemma

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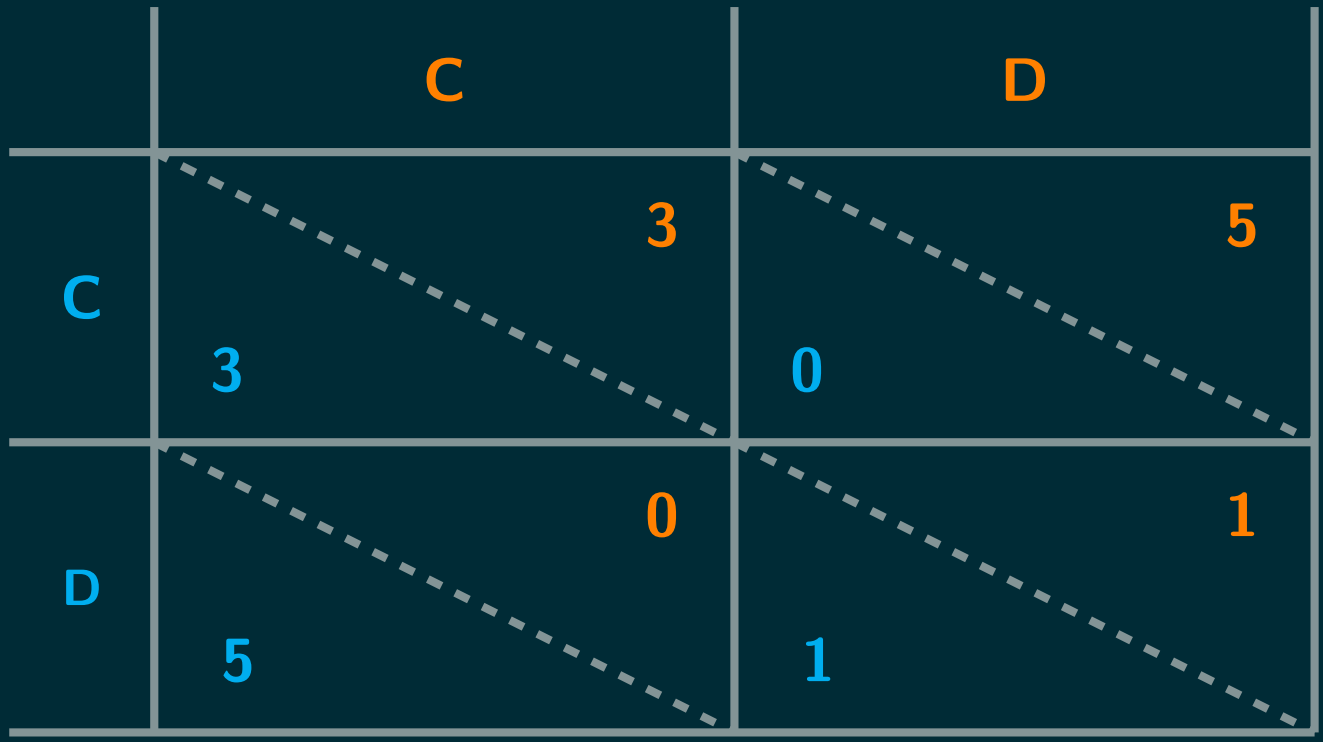
Supervised by:

Dr. Vincent Knight Dr. Jonathan Gillard


C
D
C
3
3
5
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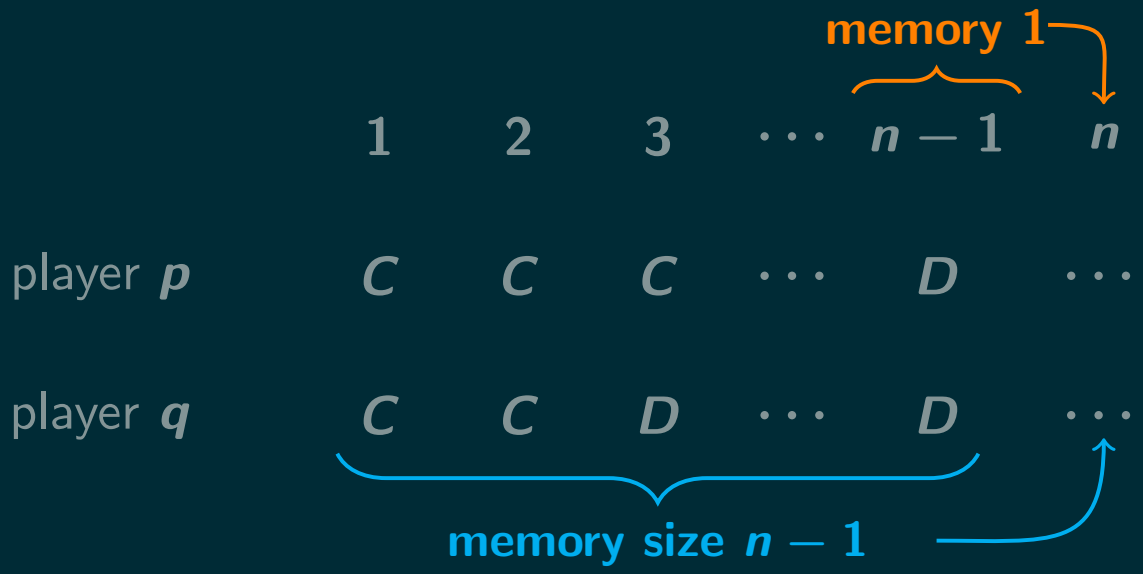
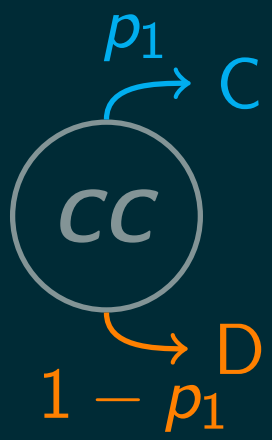
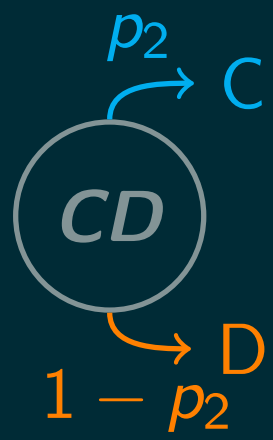
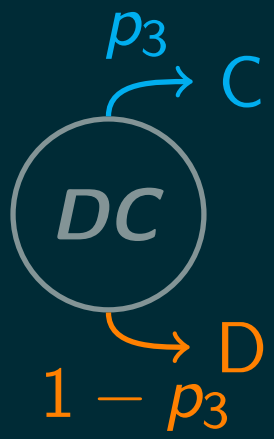
1

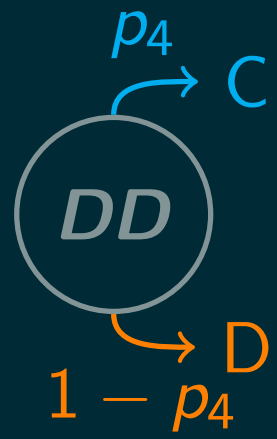

memory 1
1 2 3 ··· n − 1 n
player p C C C ··· D ···
player q C C D ··· D ···
memory size n − 1

William H. Press and Freeman J. Dyson. Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent. 2012.

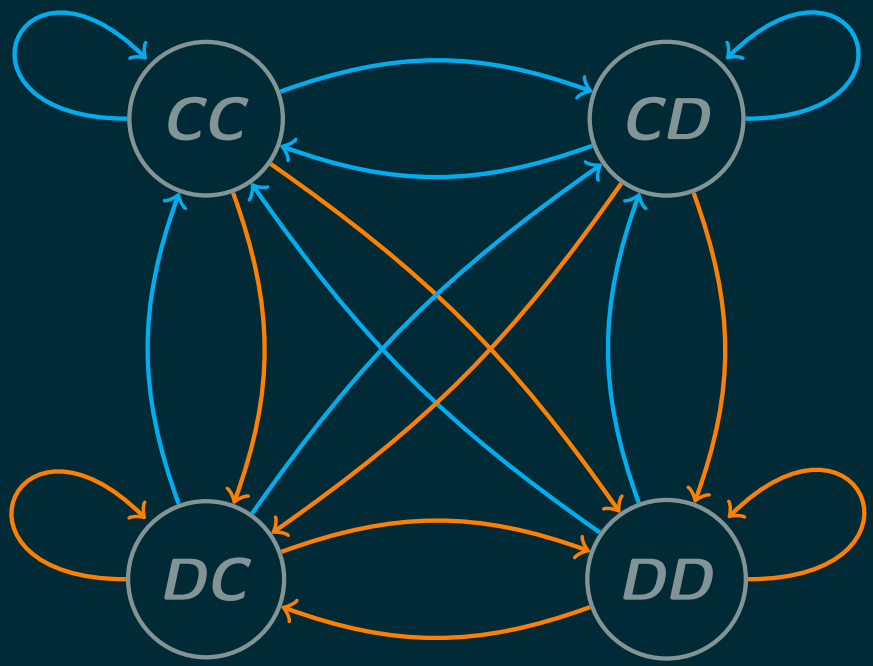








$$p = (p_1, p_2, p_3, p_4) \in \mathbb{R}^4_{[0,1]}$$



$$\begin{bmatrix} p_1 q_1 & p_1 (-q_1 + 1) & q_1 (-p_1 + 1) & (-p_1 + 1)(-q_1 + 1) \\ p_2 q_3 & p_2 (-q_3 + 1) & q_3 (-p_2 + 1) & (-p_2 + 1)(-q_3 + 1) \\ p_3 q_2 & p_3 (-q_2 + 1) & q_2 (-p_3 + 1) & (-p_3 + 1)(-q_2 + 1) \\ p_4 q_4 & p_4 (-q_4 + 1) & q_4 (-p_4 + 1) & (-p_4 + 1)(-q_4 + 1) \end{bmatrix}$$

WHICH IS THE BEST MEMORY ONE STRATEGY?

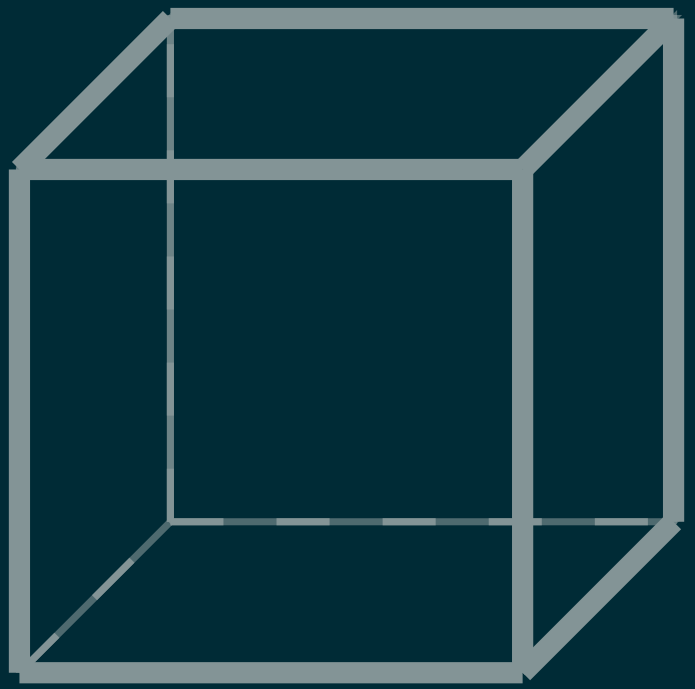
ARE THEY LIMITATIONS TO MEMORY ONE STRATEGIES?

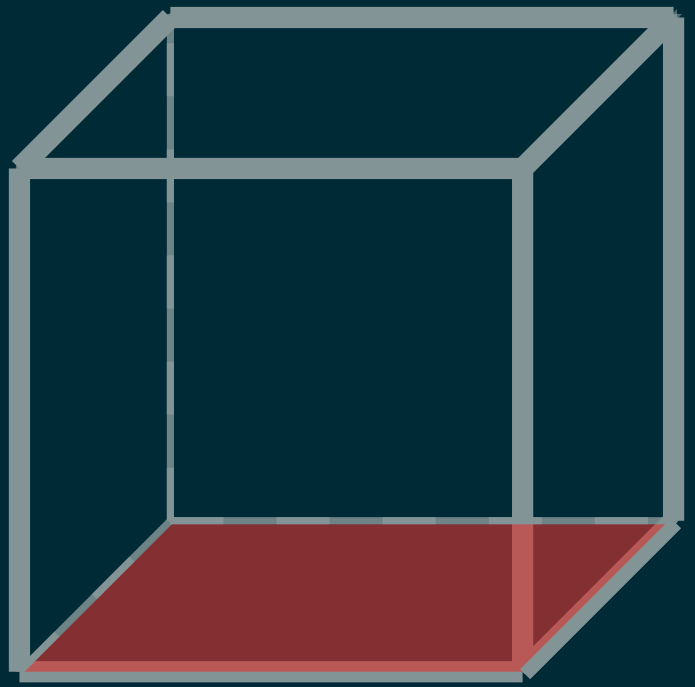
$$\max_p u_q(p) \text{ such that } p \in \mathbb{R}^4_{[0,1]}$$

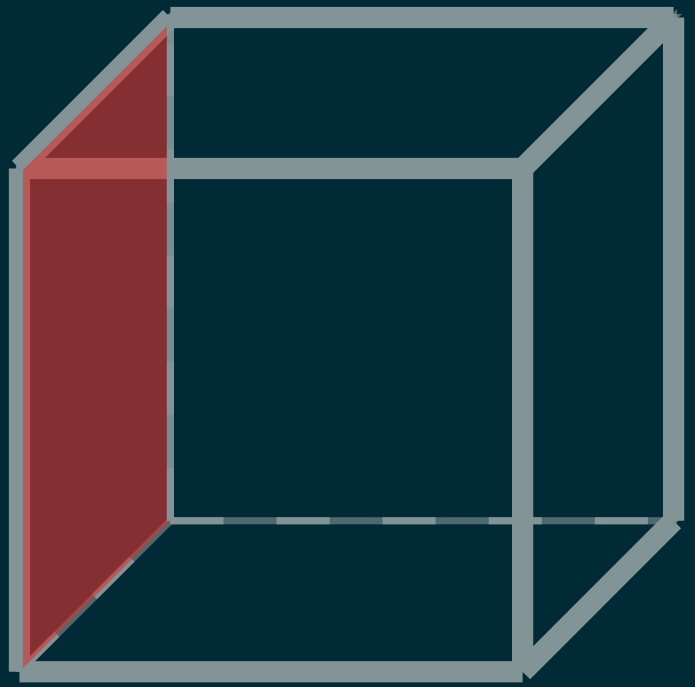
Lemma

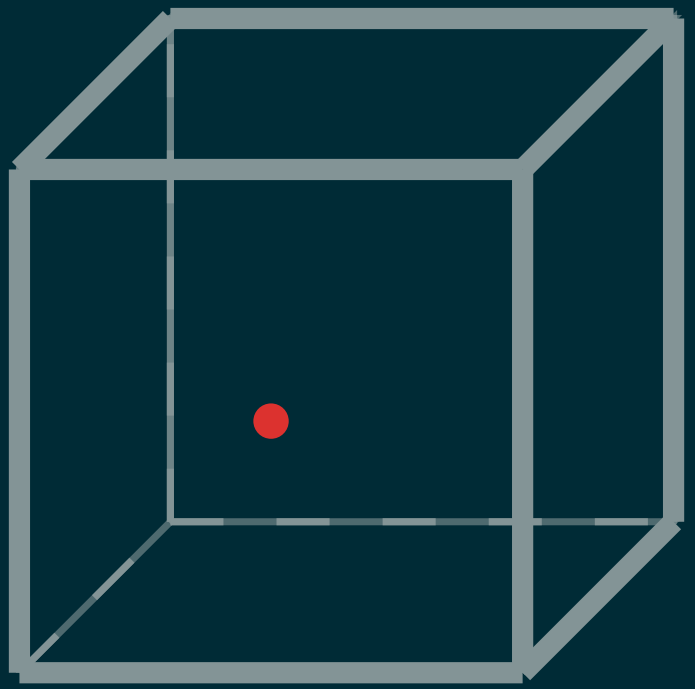
$$u_q(p) = \frac{\frac{1}{2} p Q p^T + c^T p + a}{\frac{1}{2} p \bar{Q} p^T + \bar{c}^T p + \bar{a}}$$

- $Q, \bar{Q} \in \mathbb{R}^{4\times4}$
- $c, \bar{c} \in \mathbb{R}^{4\times1}$
- $a, \bar{a} \in \mathbb{R}$









PURELY RANDOM

$$p = (p, p, p, p)$$

REACTIVE

$$p = (p_1, p_2, p_1, p_2)$$

DIFFERENTIAL EVOLUTION

memory one strategy

comple strategy

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