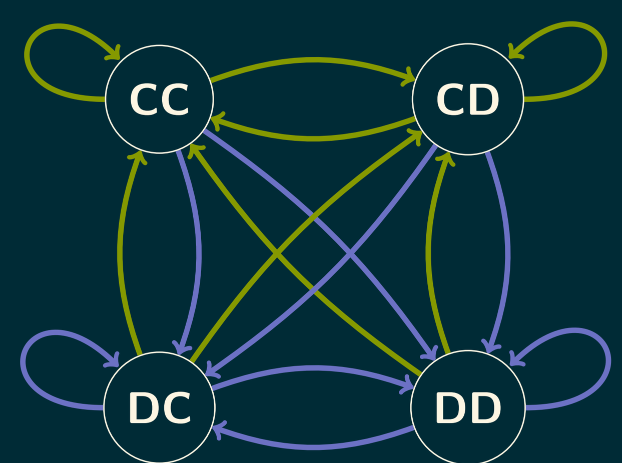
THE POWER OF MEMORY

Is memory size advantageous in interactions (social, biological, ...) ?

	C	D
C	3, 3	0, 5
D	5, 0	1, 1

	1	2	3	$\cdots$	$n-1$ (memory-1)	n
player **p**	C	C	C	$\cdots$	D	$\cdots$
player **q**	C	C	D	$\cdots$	D	$\cdots$

memory size-$n-1$



$$\begin{bmatrix} p_1q_1 & p_1(-q_1+1) & q_1(-p_1+1) & (-p_1+1)(-q_1+1) \\ p_2q_3 & p_2(-q_3+1) & q_3(-p_2+1) & (-p_2+1)(-q_3+1) \\ p_3q_2 & p_3(-q_2+1) & q_2(-p_3+1) & (-p_3+1)(-q_2+1) \\ p_4q_4 & p_4(-q_4+1) & q_4(-p_4+1) & (-p_4+1)(-q_4+1) \end{bmatrix}$$

W. H. Press and F. J. Dyson. **Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent** PNAS 2012.

$p^* \rightarrow$ manipulates $\rightarrow q$

This work considers an optimisation approach to identify:

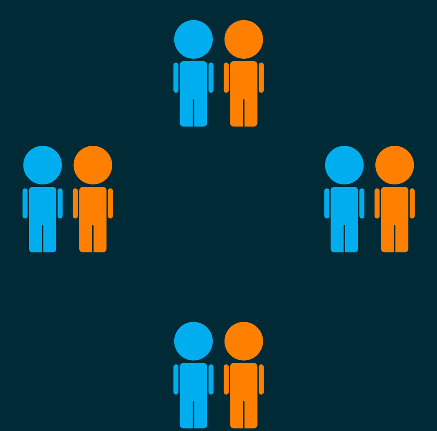
$p^* \rightarrow$ best response $\rightarrow q$

$$1.\quad \max_{q} : \frac{\frac{1}{2}\, \mathbf{p}Q\mathbf{p}^T + \mathbf{c}^T\mathbf{p} + \mathbf{a}}{\frac{1}{2}\, \mathbf{p}\bar{Q}\mathbf{p}^T + \bar{\mathbf{c}}^T\mathbf{p} + \bar{\mathbf{a}}}$$

$$\mathbf{st:}\quad \mathbf{p} \in \mathbb{R}^4_{[0,1]}$$

PURELY RANDOM STRATEGIES $p = (p, p, p, p)$

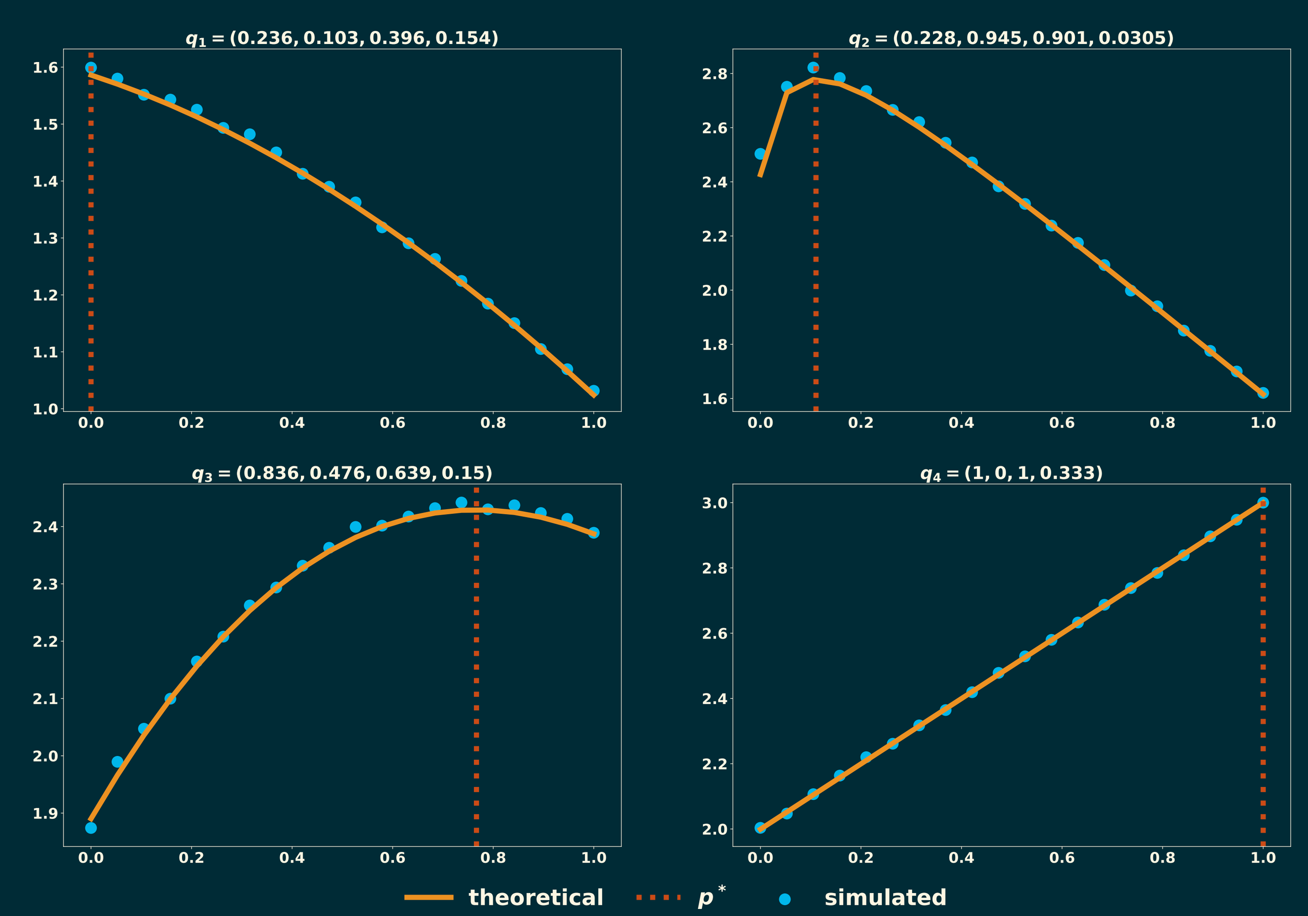
2. AGAINST A SINGLE OPPONENT



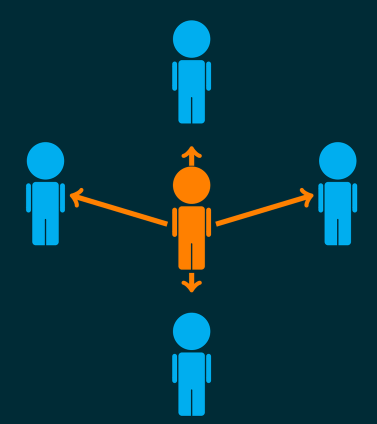
$$p^* = \text{argmax}(u_q(p)),\ p \in S_q,$$

where the set S_q is defined as:

$$S_q = \left\{0, p_\pm, 1 \;\middle|\; \begin{array}{l} 0 < p_\pm < 1, \\ p_\pm \neq \frac{-d_0}{d_1} \end{array}\right\}$$



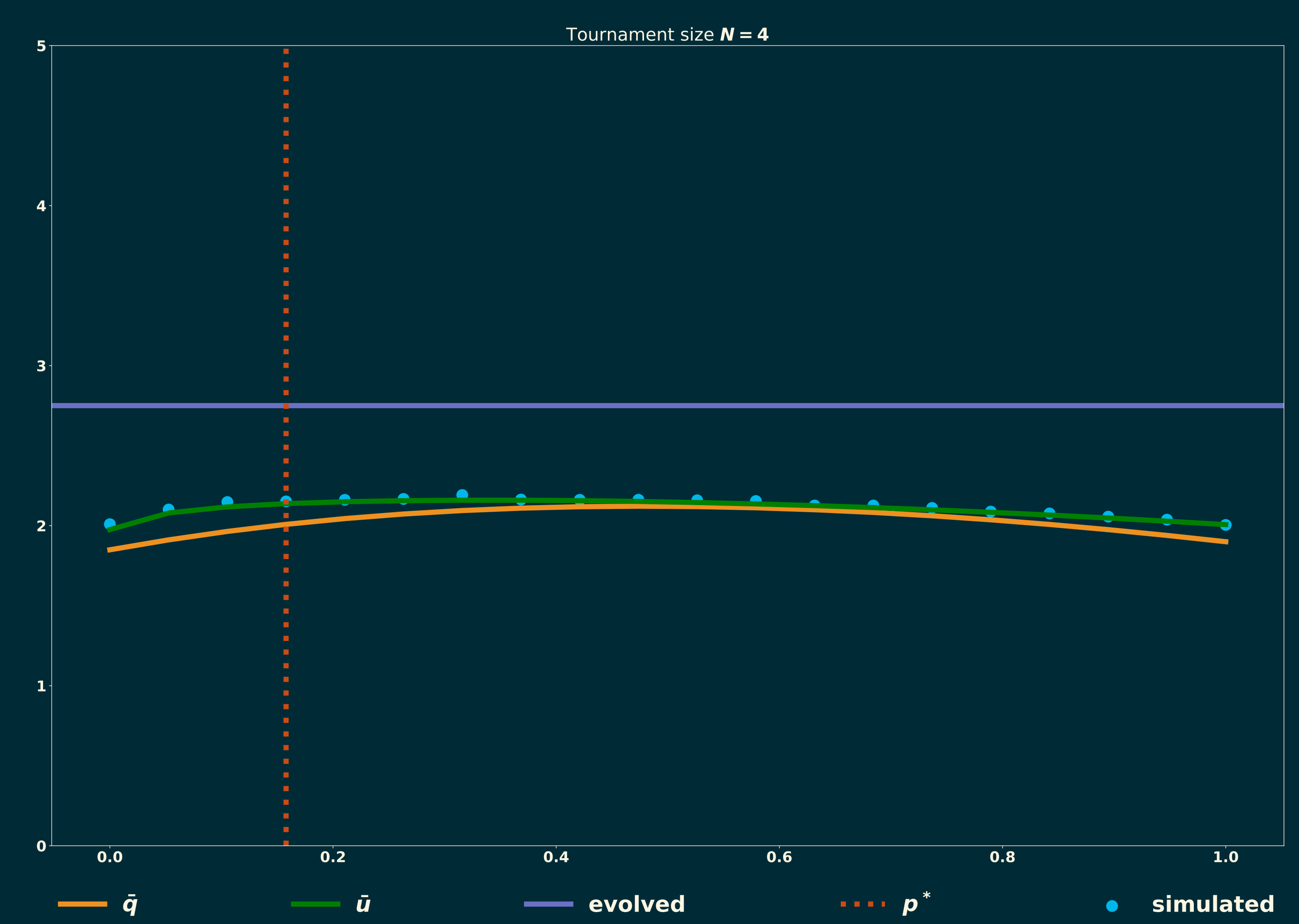
3. AGAINST MULTIPLE OPPONENTS



$$p^* = \text{argmax}\Big(\sum_{i=1}^{N} u_q^{(i)}(p)\Big),\ p \in S_{q(i)},$$

where the set $S_{q(i)}$ is defined as:

$$S_{q(i)} = \bigcup_{\substack{i=1 \\ \lambda_i \neq \frac{-d_{0i}}{d_{1i}}}}^{2N} \lambda_i \cup \{0, 1\}$$



RESULTS

1. The utility of a given player p against a given opponent q is written in a compact way.
2. Defining the optimal random behaviour p^* is reduced to a search over a small finite set.
3. Optimising against the mean utility can not be captured by optimising against the mean opponent.

FUTURE WORK

RESULTANT THEORY

$$f_0 = 0,\quad f_1 = 0,\quad \vdots,\quad f_n = 0 \;\longrightarrow\; M = \begin{bmatrix} \cdots \\ \cdots \\ \cdots \end{bmatrix} \;\longrightarrow\; \det(M) = 0$$

In case you missed me: `nikoleta-v3.github.io/blog/2018/01/05/power-of-memory.html`

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