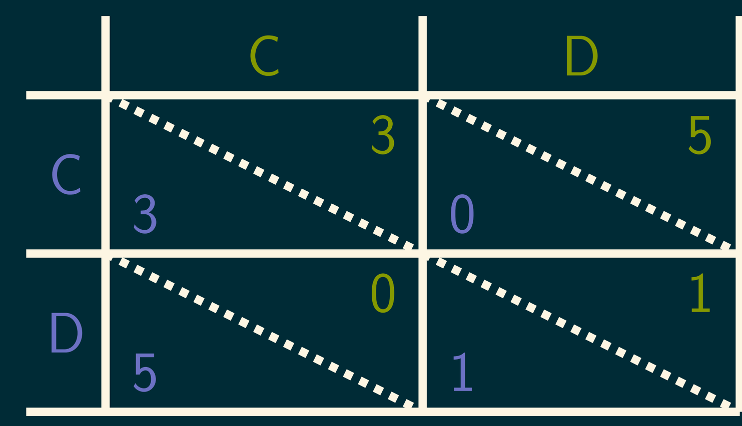
THE POWER OF MEMORY

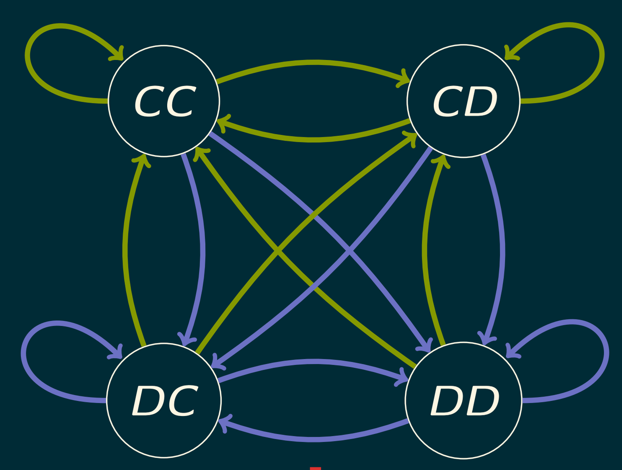
Is memory size advantageous in interactions?



	1	2	3	$\cdots$	$n-1$	n
player p	C	C	C	$\cdots$	D	$\cdots$
player q	C	C	D	$\cdots$	D	$\cdots$

memory-1

memory size-$n-1$



$$\begin{bmatrix} p_1q_1 & p_1(-q_1+1) & q_1(-p_1+1) & (-p_1+1)(-q_1+1) \\ p_2q_3 & p_2(-q_3+1) & q_3(-p_2+1) & (-p_2+1)(-q_3+1) \\ p_3q_2 & p_3(-q_2+1) & q_2(-p_3+1) & (-p_3+1)(-q_2+1) \\ p_4q_4 & p_4(-q_4+1) & q_4(-p_4+1) & (-p_4+1)(-q_4+1) \end{bmatrix}$$

W. H. Press and F. J. Dyson. **Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent** PNAS 2012.

$p^* \rightarrow$ manipulates $\rightarrow q$

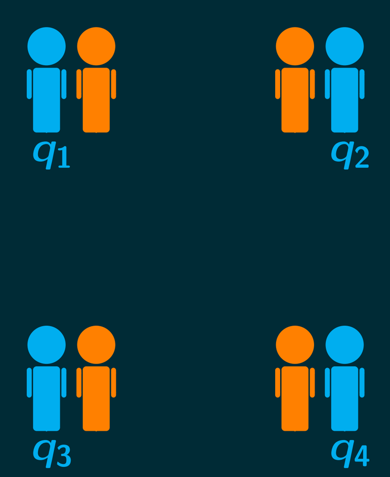
This work considers an optimisation approach to identify:

$p^* \rightarrow$ best response $\rightarrow q$

$$u_q(p) = \frac{\frac{1}{2}\, pQp^T + c^T p + a}{\frac{1}{2}\, p\bar{Q}p^T + \bar{c}^T p + \bar{a}}, \quad \text{where} \quad p \in \mathbb{R}^4_{[0,1]}$$

PURELY RANDOM STRATEGIES $p = (p, p, p, p)$

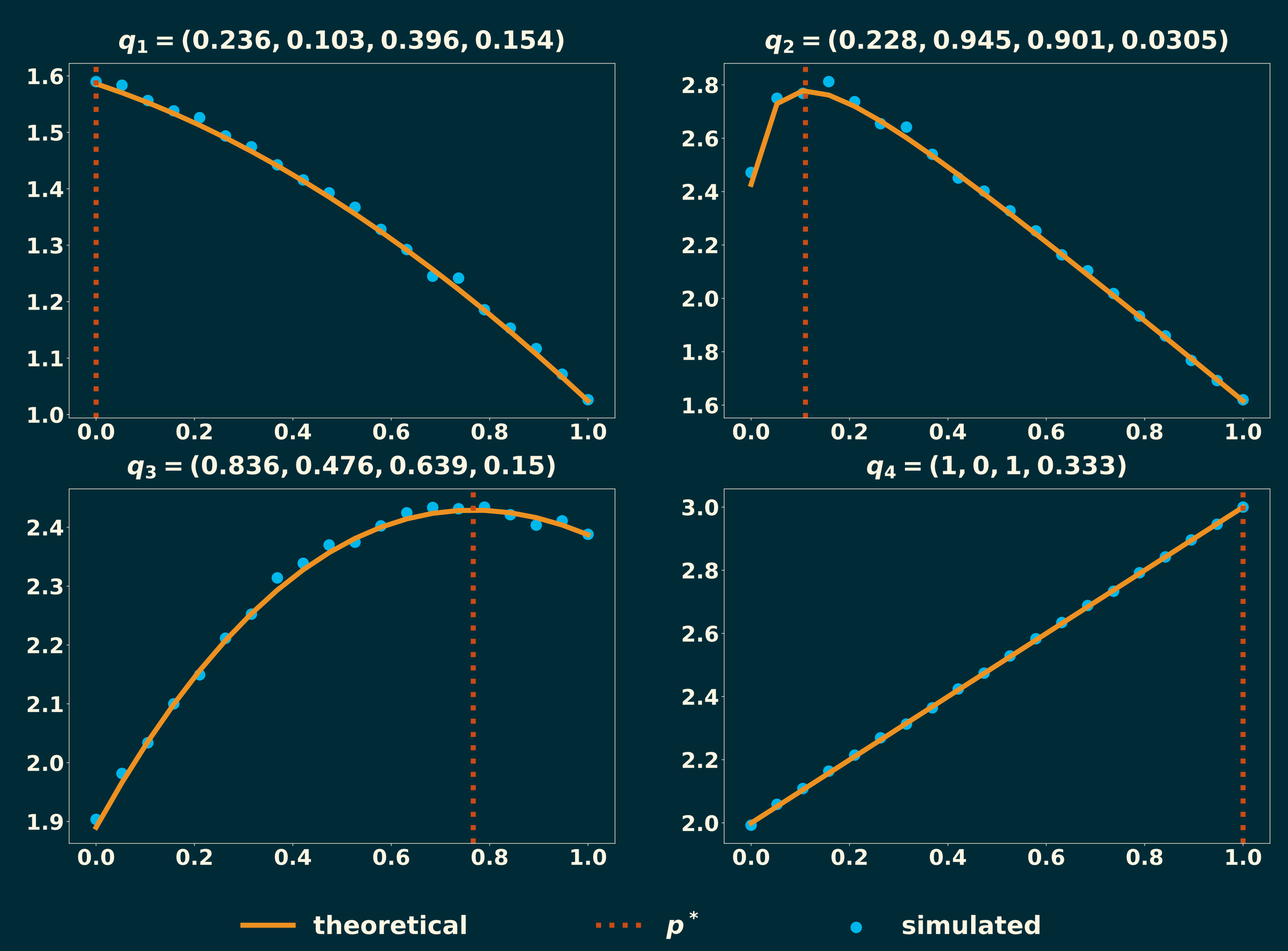
AGAINST A SINGLE OPPONENT



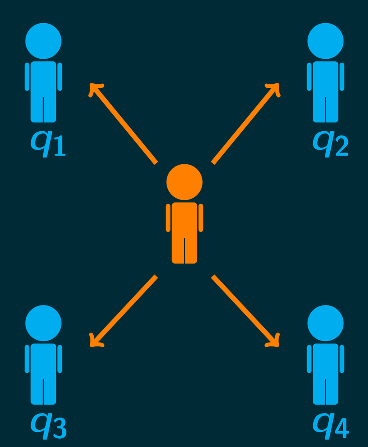
$$p^* = \text{argmax}(u_q(p)), \ p \in S_q,$$

where:

$$S_q = \left\{ 0, p_{\pm}, 1 \,\middle|\, \begin{matrix} 0 < p_{\pm} < 1, \\ p_{\pm} \neq \frac{-d_0}{d_1} \end{matrix} \right\}$$



AGAINST MULTIPLE OPPONENTS



$$p^* = \text{argmax}\left(\sum_{i=1}^{N} {u_q}^{(i)}(p)\right), \ p \in S_{q(i)},$$

where:

$$S_{q(i)} = \bigcup_{\substack{i=1 \\ \lambda_i \neq \frac{d_{0i}}{d_{1i}}}}^{2N} \lambda_i \cup \{0, 1\}$$

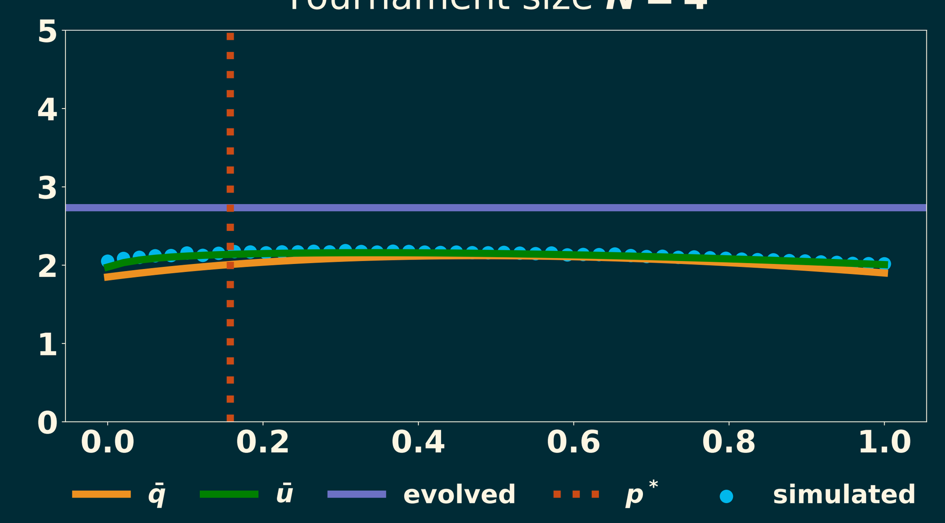
$$u_q(p)$$

$$\downarrow$$

$$\begin{bmatrix} 0 & 0 & \dots & 0 & -a_0 \\ 1 & 0 & \dots & 0 & -a_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & -a_{2N} \end{bmatrix}$$

$$\downarrow$$

Tournament size $\boldsymbol{N = 4}$



FURTHER WORK

$p = (p_1, p_2, p_3, p_4) \rightarrow$ RESULTANT THEORY

$$f_0 = 0$$
$$f_1 = 0$$
$$\vdots$$
$$f_n = 0$$

$$M = \begin{bmatrix} \dots \\ \dots \\ \dots \end{bmatrix}$$

$$\det(M) = 0$$

SUMMARY

1. The utility of a given player p against a given opponent q can be written in a compact way.
2. Obtaining the optimal random behaviour p^* reduces to a search over a small finite set.
3. Optimising against the mean utility cannot be captured by optimising against the mean opponent.
4. Memory is important in interactions with multiple agents.

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In case you missed me: `https://nikoleta-v3.github.io/blog/2018/01/05/power-of-memory.html`

Nikoleta-v3