THE POWER OF MEMORY

In interactions both social and biological is memory size advantageous?

- Both players are better of choosing Cooperation (3)
- there is always a temptation for a player to Defect (5).

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} (3,3) & (5,0) \\ (0,5) & (1,1) \end{pmatrix} \end{array} \quad (1)$$

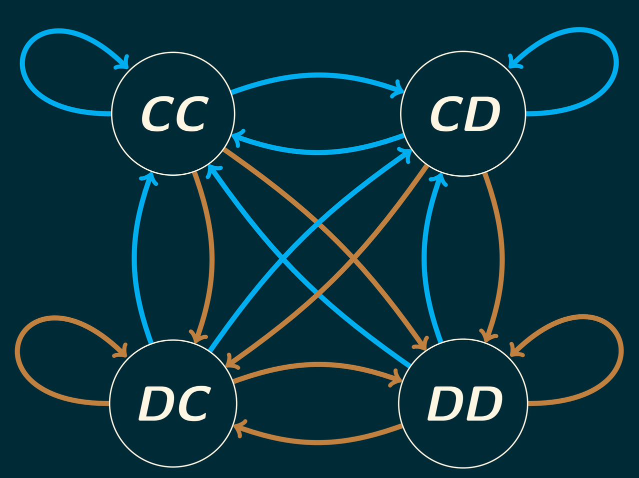
memory-1

$$\begin{array}{lcccccc} & 1 & 2 & 3 & \cdots & n-1 & n \\ \text{player } p & C & C & C & \cdots & D & \cdots \\ \text{player } q & C & C & D & \cdots & D & \cdots \end{array}$$

memory size-$n-1$

1. OPTIMAL MEMORY ONE STRATEGY IN A MATCH

Depending on the simultaneous moves of the two players, there are four possibles 'states':



A memory one strategy is denoted by the probabilities of cooperating after each of these states $p = (p_1, p_2, p_3, p_4) \in \mathbb{R}^4_{[0,1]}$ [3]. A match between two memory one players p and q can be modelled as a Markov chain.

$$M = \begin{bmatrix} p_1q_1 & p_1(-q_1+1) & q_1(-p_1+1) & (-p_1+1)(-q_1+1) \\ p_2q_3 & p_2(-q_3+1) & q_3(-p_2+1) & (-p_2+1)(-q_3+1) \\ p_3q_2 & p_3(-q_2+1) & q_2(-p_3+1) & (-p_3+1)(-q_2+1) \\ p_4q_4 & p_4(-q_4+1) & q_4(-p_4+1) & (-p_4+1)(-q_4+1) \end{bmatrix}$$

Thus the utility of player p against an opponent q is given by:

$$u_q(p) = v \times S_p$$

where v denotes the stationary vector of M and S_p the payoffs of player p given by equation (1).

Against a single opponent:

$$\max_q : u_q(p) = \frac{\frac{1}{2}pQp^T + c^Tp + a}{\frac{1}{2}p\bar{Q}p^T + \bar{c}^Tp + \bar{a}}$$
$$st: \ p \in \mathbb{R}^4_{[0,1]}$$

where $Q, \bar{Q}$ are matrices of 4×4, and $c, \bar{c}$ are 4×1 vectors defined with the transition probabilities of the opponent's transition probabilities q_1, q_2, q_3, q_4.

3. OPTIMAL MEMORY ONE IN A TOURNAMENT

In order to find the optimal memory on player against a set of opponents we need to explore the numeration of the differentiation of:
This will be explored using Resultant Theory. The resultant will equal zero if and only if the polynomials of a multivariate system have at least one common root.

- Dixon's resultant;
- Maycalay resultant.

2. WHAT IS THE OPTIMAL PURE RANDOM STRATEGY?

A memory one strategy where the transition probabilities of each state are the same is called a **purely random strategy**.

Against a single opponent:

$$\max_q : u_q(p) = \frac{n_2p^2 + n_1p + n_0}{d_1p + d_0}$$

$$st: \ p_1 = p_2 = p_3 = p_4 = p$$
$$p \in \mathbb{R}_{[0,1]}$$

The optimal behaviour of a **purely random** player (p,p,p,p) against a memory one opponent q is given by:

$$p^* = \text{argmax}(u_q(p)), \ p \in S_q,$$

where the set S_q is defined as,

$$S_q = \left\{0, p_\pm, 1 \,\middle|\, \begin{array}{l} 0 < p_\pm < 1, \\ p_\pm \neq \frac{-d_0}{d_1} \end{array}\right\}$$

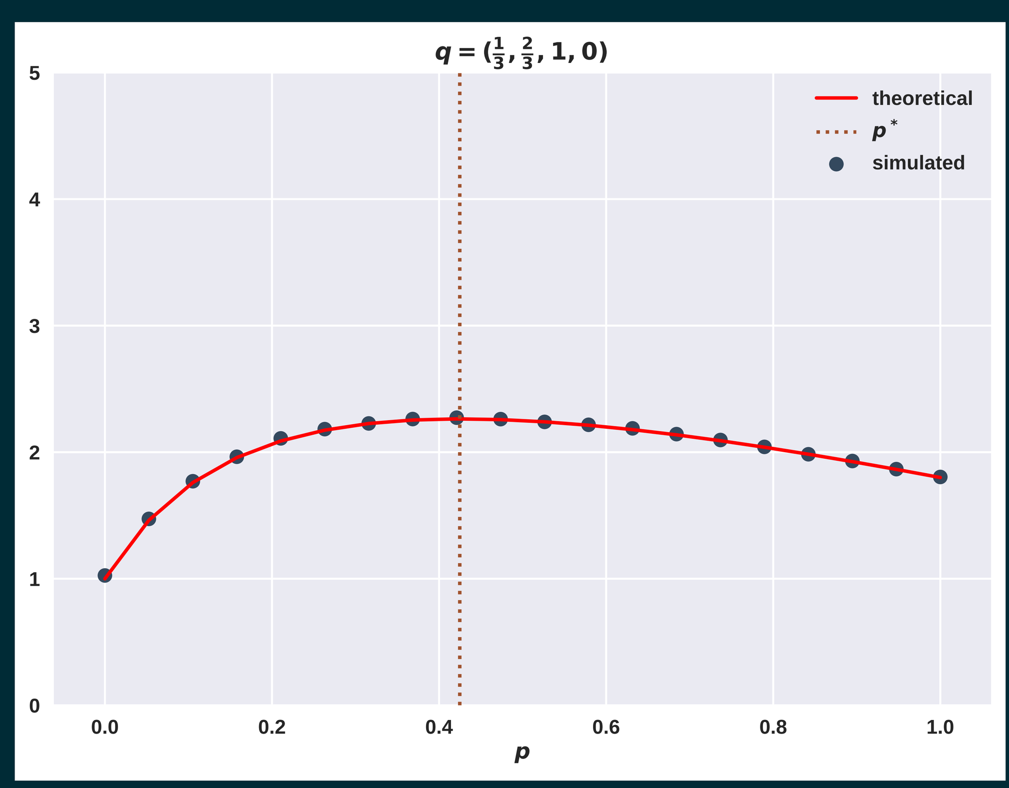


Figure: The theoretical results were generated with [2, 4], the simulated ones with [1].

Against multiple opponents:

$$\max_q : \frac{\sum_{i=1}^N u_q^{(i)}(p)}{N}$$
$$st: \ p_1 = p_2 = p_3 = p_4 = p$$
$$p \in \mathbb{R}_{[0,1]}$$

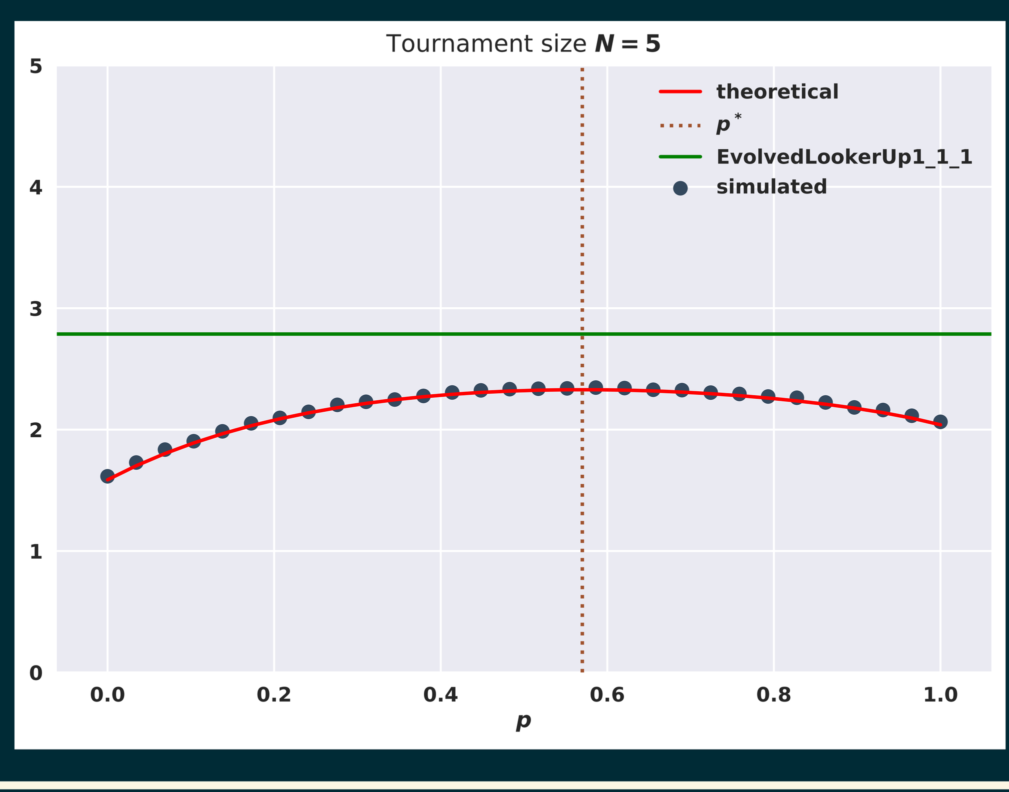
The optimal behaviour of a **purely random** player (p,p,p,p) in an $N-$memory one player tournament, $\{q_{(1)}, q_{(2)} \dots, q_{(N)}\}$ is given by:

$$p^* = \text{argmax}(\sum_{i=1}^N u_q^{(i)}(p)), \ p \in S_{q(i)},$$

where the set $S_{q(i)}$ is defined as:

$$S_{q(i)} = \bigcup_{\substack{i=1 \\ \lambda_i \neq \frac{d_{0_i}}{d_{1_i}}}}^{2N} \lambda_i \cup \{0, 1\}$$

Note the size of candidate solutions is $1 \leq |S_{q(i)}| \leq 2N + 2$.



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