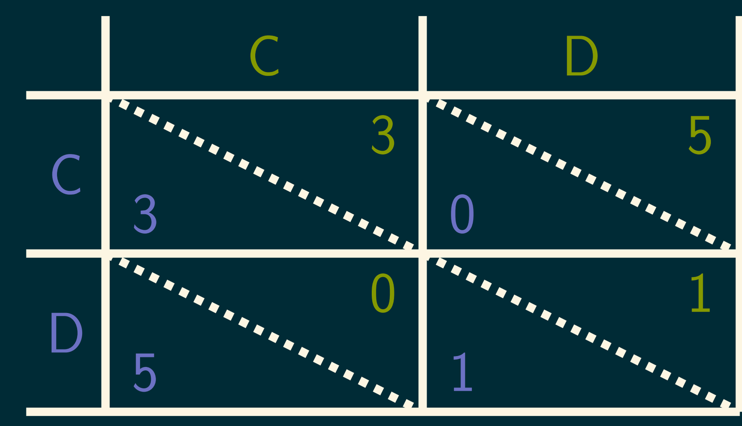
CARDIFF UNIVERSITY
PRIFYSGOL CAERDYDD

THE POWER OF MEMORY

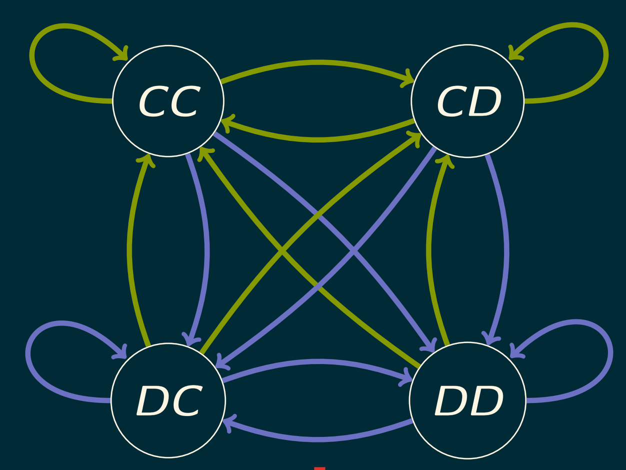
Is memory size advantageous in interactions (social, biological, etc) ?



	1	2	3	$\cdots$	$n-1$	n
player p	C	C	C	$\cdots$	D	$\cdots$
player q	C	C	D	$\cdots$	D	$\cdots$

memory-1

memory size-$n-1$



$$\begin{bmatrix} p_1q_1 & p_1(-q_1+1) & q_1(-p_1+1) & (-p_1+1)(-q_1+1) \\ p_2q_3 & p_2(-q_3+1) & q_3(-p_2+1) & (-p_2+1)(-q_3+1) \\ p_3q_2 & p_3(-q_2+1) & q_2(-p_3+1) & (-p_3+1)(-q_2+1) \\ p_4q_4 & p_4(-q_4+1) & q_4(-p_4+1) & (-p_4+1)(-q_4+1) \end{bmatrix}$$

W. H. Press and F. J. Dyson. **Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent** PNAS 2012. The **zero determinant** strategies.

$p^* \to$ manipulates $\to q$

This work considers an optimisation approach to identify:

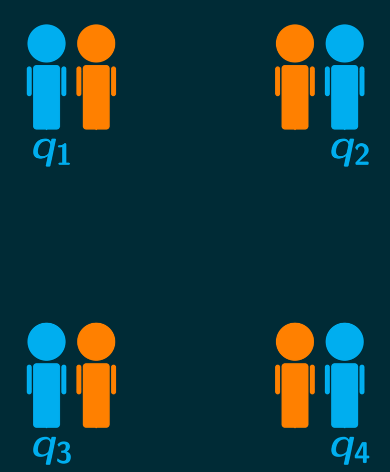
$p^* \to$ best response $\to q$

$$u_q(p) = \frac{\frac{1}{2}\, pQp^T + c^Tp + a}{\frac{1}{2}\, p\bar{Q}p^T + \bar{c}^Tp + \bar{a}},$$

where $p \in \mathbb{R}^4_{[0,1]}$

PURELY RANDOM STRATEGIES $p = (p, p, p, p)$

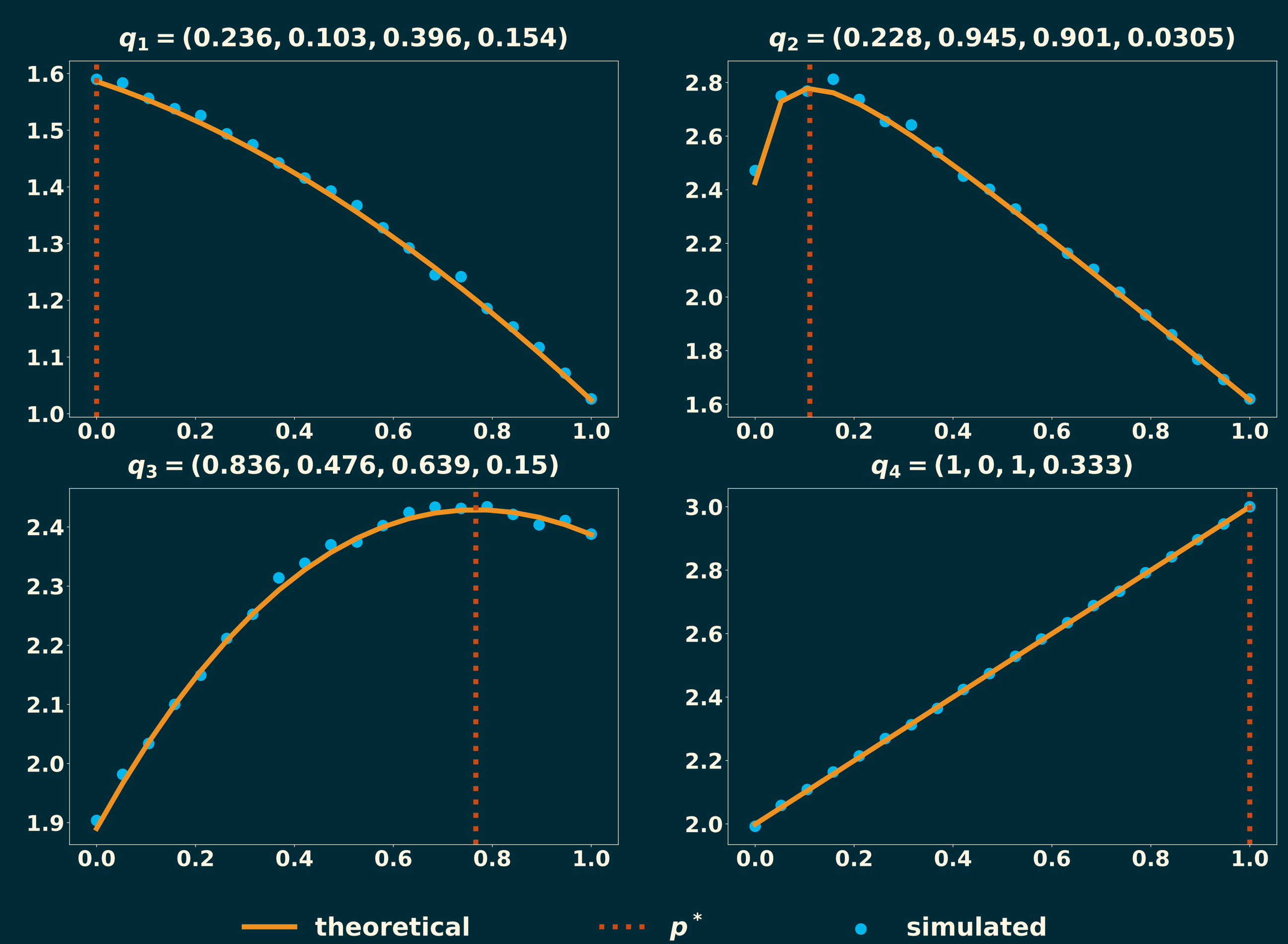
AGAINST A SINGLE OPPONENT



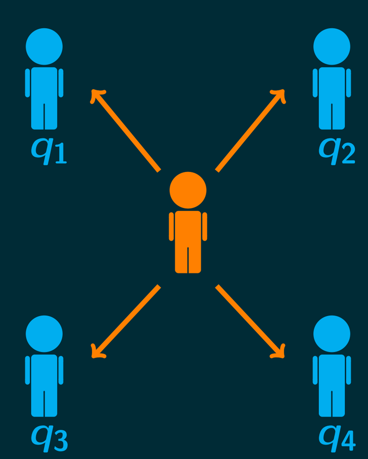
$$p^* = \text{argmax}(u_q(p)),\ p \in S_q,$$

where the set S_q is defined as:

$$S_q = \left\{ 0, p_{\pm}, 1 \,\middle|\, \begin{array}{l} 0 < p_{\pm} < 1, \\ p_{\pm} \neq \frac{-d_0}{d_1} \end{array} \right\}$$



AGAINST MULTIPLE OPPONENTS



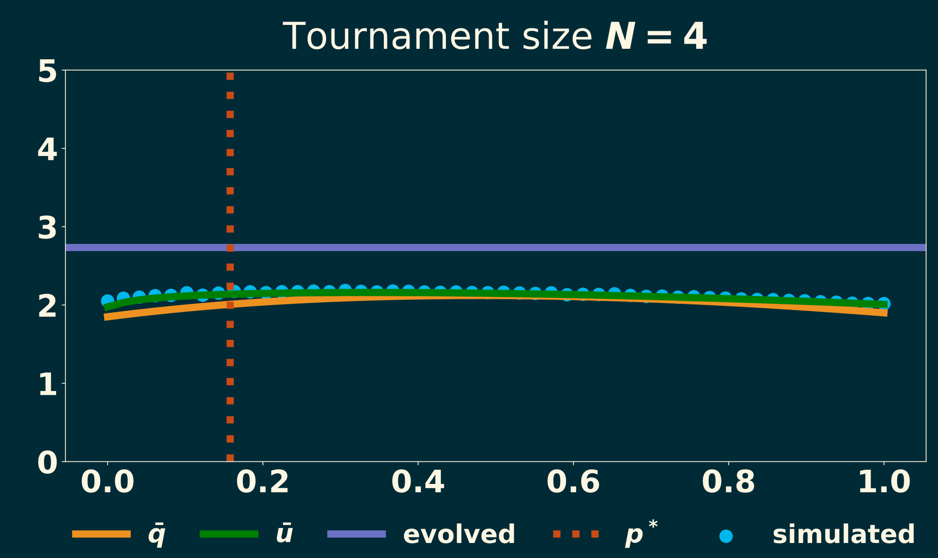
$$p^* = \text{argmax}\Big(\sum_{i=1}^{N} {u_q}^{(i)}(p)\Big),\ p \in S_{q(i)},$$

where the set $S_{q(i)}$ is defined as:

$$S_{q(i)} = \bigcup_{\substack{i=1 \\ \lambda_i \neq \frac{do_i}{d1_i}}}^{2N} \lambda_i \cup \{0, 1\}$$

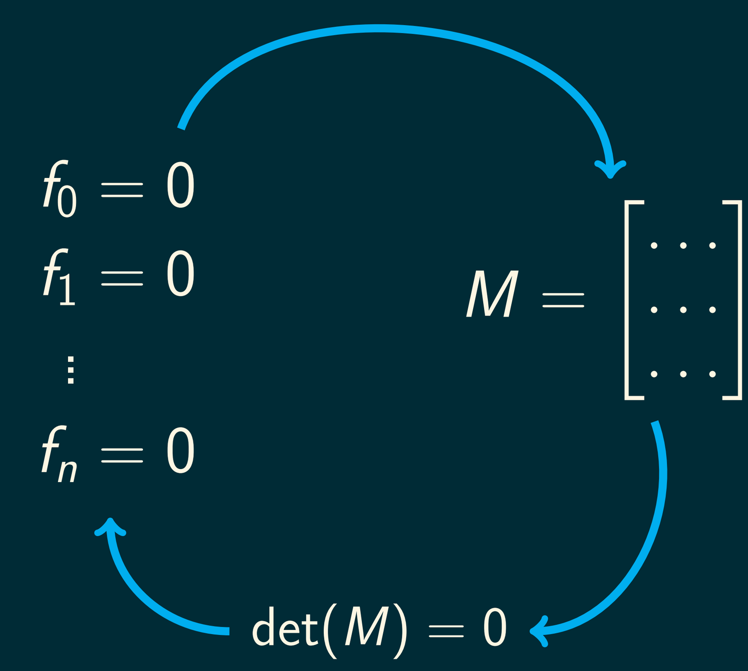
$u_q(p)$

$$\begin{bmatrix} 0 & 0 & \dots & 0 & -a_0 \\ 1 & 0 & \dots & 0 & -a_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & -a_{2N} \end{bmatrix}$$



FURTHER WORK

$p = (p_1, p_2, p_3, p_4) \to$ RESULTANT THEORY



SUMMARY

1. The utility of a given player p against a given opponent q can be written in a compact way.
2. Obtaining the optimal random behaviour p^* reduces to a search over a small finite set.
3. Optimising against the mean utility can not be captured by optimising against the mean opponent.

NikoletaGlyn

In case you missed me: `https://nikoleta-v3.github.io/blog/2018/01/05/power-of-memory.html`

Nikoleta-v3