

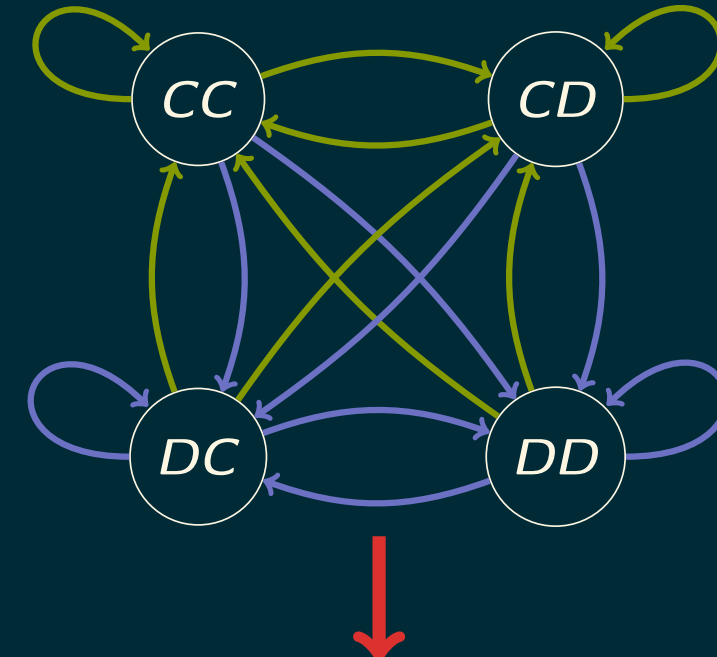
# THE POWER OF MEMORY

Is memory size advantageous in interactions (social, biological, ... ) ?

	C	D
C	3	0
D	5	1

	1	2	3	...	$n-1$	$n$
player $p$	C	C	C	...	D	...
player $q$	C	C	D	...	D	...

memory size  $n-1$



$$\begin{bmatrix} p_1 q_1 & p_1(-q_1+1) & q_1(-p_1+1) & (-p_1+1)(-q_1+1) \\ p_2 q_3 & p_2(-q_3+1) & q_3(-p_2+1) & (-p_2+1)(-q_3+1) \\ p_3 q_2 & p_3(-q_2+1) & q_2(-p_3+1) & (-p_3+1)(-q_2+1) \\ p_4 q_4 & p_4(-q_4+1) & q_4(-p_4+1) & (-p_4+1)(-q_4+1) \end{bmatrix}$$

W. H. Press and F. J. Dyson. **Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent** PNAS 2012.

$$p^* \rightarrow \text{manipulates} \rightarrow q$$

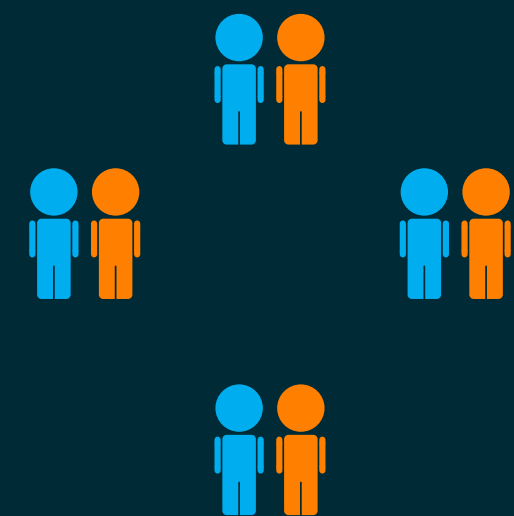
This work considers an optimisation approach to identify:

$$p^* \rightarrow \text{best response} \rightarrow q$$

$$\max_q : \frac{\frac{1}{2} p Q p^T + c^T p + a}{\frac{1}{2} p \bar{Q} p^T + \bar{c}^T p + \bar{a}}$$

## PURELY RANDOM STRATEGIES $p = (p, p, p, p)$

AGAINST A SINGLE OPPONENT

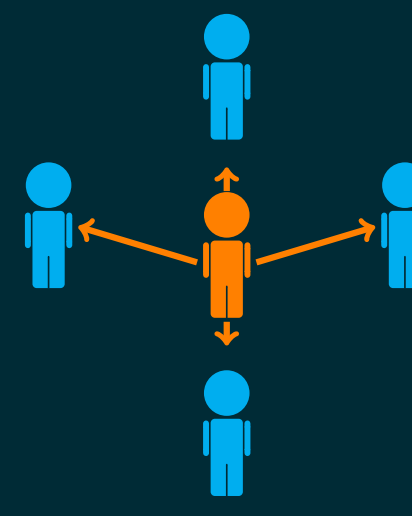


$$p^* = \operatorname{argmax}(u_q(p)), \quad p \in S_q,$$

where the set  $S_q$  is defined as:

$$S_q = \left\{ 0, p_{\pm}, 1 \mid \begin{array}{l} 0 < p_{\pm} < 1, \\ p_{\pm} \neq \frac{-d_0}{d_1} \end{array} \right\}$$

AGAINST MULTIPLE OPPONENTS



$$p^* = \operatorname{argmax} \left( \sum_{i=1}^N u_q^{(i)}(p) \right), \quad p \in S_{q(i)},$$

where the set  $S_{q(i)}$  is defined as:

$$S_{q(i)} = \frac{2N}{U} \lambda_i \cup \{0, 1\} \quad \lambda_i \neq \frac{d_0}{d_1}$$

## RESULTS

1. The utility of a given player  $p$  against a given opponent  $q$  can be written in a compact way.
2. The optimal purely random player.
3. Complex strategies outperform optimal purely random.

## FUTURE WORK

RESULTANT THEORY

