

Memory size in the Prisoner's Dilemma

Nikoleta E. Glynatsi

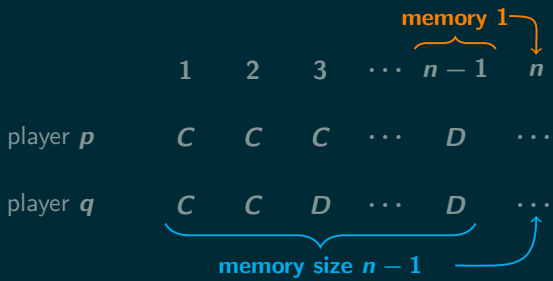


Supervised by:

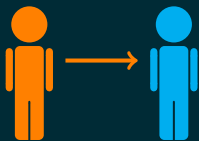
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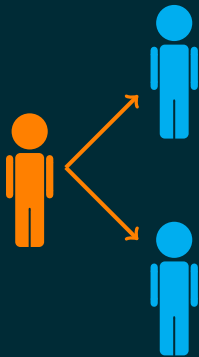
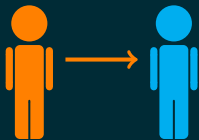
Dr. Jonathan GILLARD

	C	D
C	3, 3	0, 5
D	5, 0	1, 1



William H. Press and Freeman J. Dyson. Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent. 2012





**WHICH IS THE BEST MEMORY ONE
STRATEGY?**

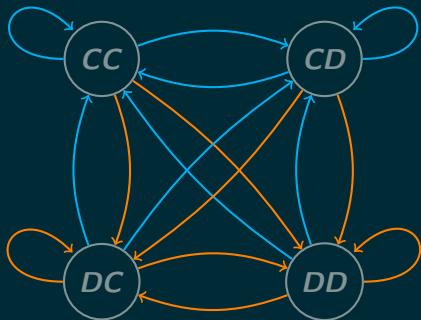
**ARE THERE LIMITATIONS TO MEMORY ONE
STRATEGIES?**

**WHICH IS THE BEST MEMORY ONE
STRATEGY?**

**ARE THERE LIMITATIONS TO MEMORY ONE
STRATEGIES?**



$$p = (p_1, p_2, p_3, p_4) \in \mathbb{R}_{[0,1]}^4$$



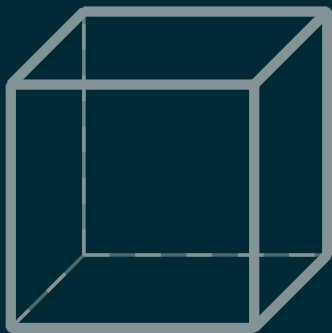
$$\begin{bmatrix} p_1 q_1 & p_1 (-q_1 + 1) & q_1 (-p_1 + 1) & (-p_1 + 1) (-q_1 + 1) \\ p_2 q_3 & p_2 (-q_3 + 1) & q_3 (-p_2 + 1) & (-p_2 + 1) (-q_3 + 1) \\ p_3 q_2 & p_3 (-q_2 + 1) & q_2 (-p_3 + 1) & (-p_3 + 1) (-q_2 + 1) \\ p_4 q_4 & p_4 (-q_4 + 1) & q_4 (-p_4 + 1) & (-p_4 + 1) (-q_4 + 1) \end{bmatrix}$$

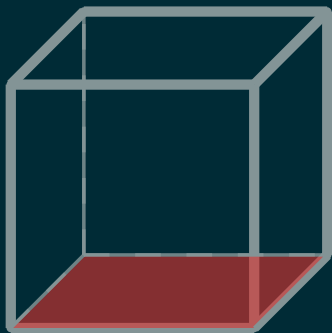
$$\max_p u_q(p) \text{ such that } p \in \mathbb{R}_{[0,1]}^4$$

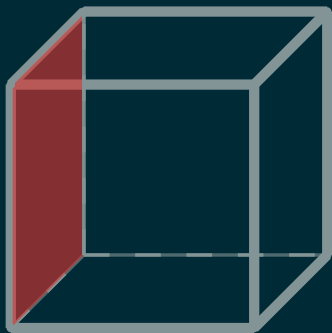
Lemma

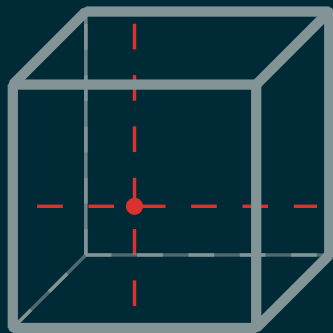
$$u_q(p) = \frac{\frac{1}{2}pQp^T + c^T p + a}{\frac{1}{2}p\bar{Q}p^T + \bar{c}^T p + \bar{a}}$$

- ▶ $Q, \bar{Q} \in \mathbb{R}^{4 \times 4}$
- ▶ $c, \bar{c} \in \mathbb{R}^{4 \times 1}$
- ▶ $a, \bar{a} \in \mathbb{R}$







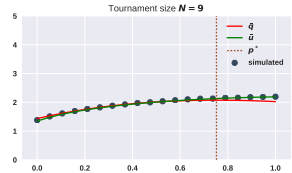
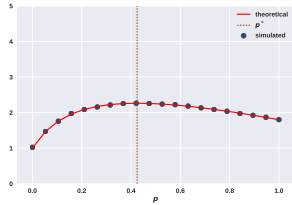


PURELY RANDOM

$$p = (p, p, p, p)$$

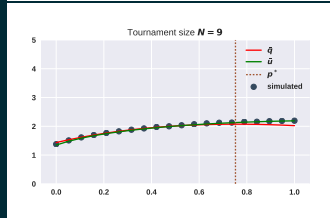
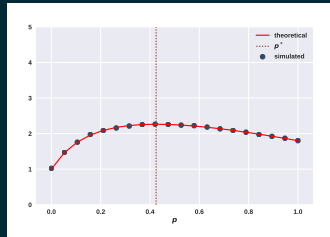
$$S_q = U_{i=1}^{2N} \lambda_i \cup \{0, 1\}$$

$$1 \leq |S_{q(i)}| \leq 2N + 2$$



$$S_q = \bigcup_{i=1}^{2N} \lambda_i \cup \{0, 1\}$$

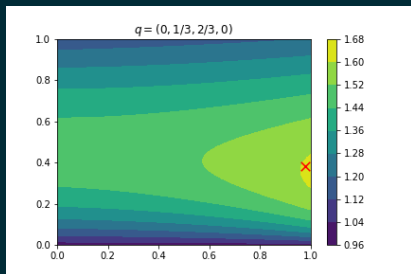
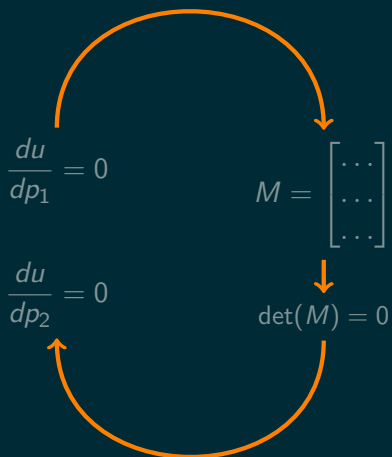
$$1 \leq |S_{q(i)}| \leq 2N + 2$$

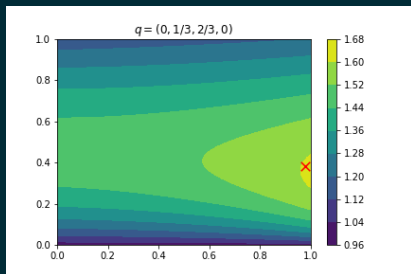
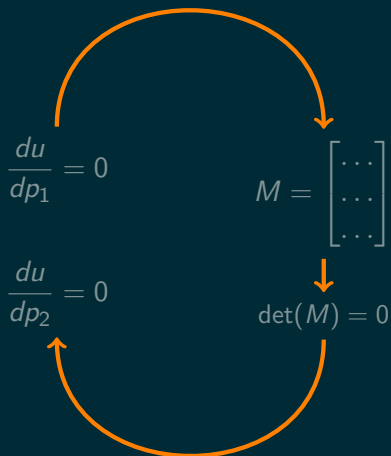


Result: optimal behaviour using eigenvalues of companion matrix

REACTIVE

$$p = (p_1, p_2, p_1, p_2)$$

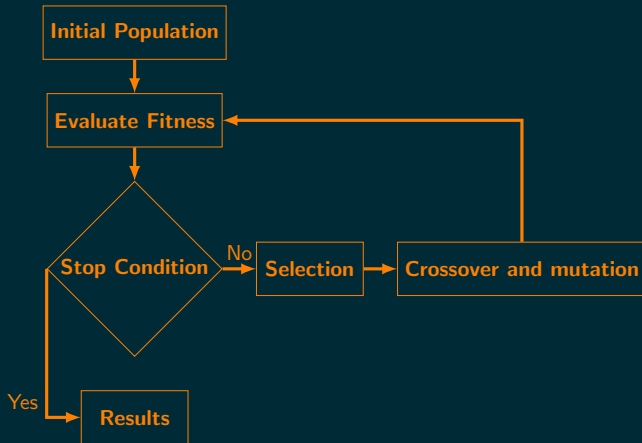




**Result: optimal behaviour using Sylvester's resultant
(Sylvester 1840)**

MEMORY ONE

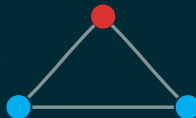
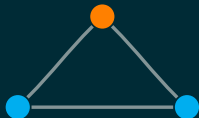
$$\mathbf{b}' = \mathbf{b}_0 + \mathbf{m} \times (\mathbf{p}^{(i)} - \mathbf{p}^{(j)})$$



	q_1	q_2	q_3	q_4	p_1	p_2	p_3	p_4	u_q	U_q
0	0.208461	0.481681	0.420538	0.859182	0.603430	0.435408	0.0	0.0	3.494901	3.467
1	0.781368	0.692829	0.969659	0.032401	0.000000	0.000000	0.0	1.0	3.266885	3.328
2	0.546571	0.964307	0.063893	0.383576	0.389439	0.491920	0.0	0.0	4.659477	4.544
3	0.930557	0.381203	0.665347	0.999155	0.145812	0.480583	0.0	0.0	3.470172	3.454
4	0.309831	0.129804	0.346928	0.770327	0.566760	0.039395	0.0	0.0	2.878247	2.886

**WHICH IS THE BEST MEMORY ONE
STRATEGY?**

**ARE THEIR LIMITATIONS TO MEMORY ONE
STRATEGIES?**



— memory one strategy

— complex strategy

	q_1	q_2	q_3	q_4	p_1	p_2	p_3	p_4	u_q	U_q
0	0.208461	0.481681	0.420538	0.859182	0.603430	0.435408	0.0	0.0	3.494901	3.467
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This repository

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Wiki

Insights

implementation of multivariate resultants. #14370

Edit

Merged

jksuom merged 19 commits into sympy:master from nikoleta-v3:multivariate-resultants on 30 Mar

Conversation 63

Commits 19

Checks 0

Files changed 6

+3,033 -0



Nikoleta-v3 commented on 2 Mar

Contributor



Brief description

Adds a new file `multivariate_resultants` that contains two classes.

These classes are implementations of the following multivariate resultants:

- Dixons
- Macaulay

They are a natural follow up from the resultants implemented withing the library:

- [subresultants_qq_zz.py](#)

Resultants are used to identify if polynomials have common roots. The multivariate version is for multivariate systems.

Other comments

Tests have been implemented and are currently passing. I added relevant literature and some notebooks with examples in the [example/notebooks/](#) directory.

References to other Issues or PRs

Reviewers

asmeurer

smichr

jksuom

normalhuman

Assignees

No one assigned

Labels

PR: sympy's turn

Projects

None yet

Milestone

No milestone

Limitations of memory size on the Iterated Prisoner's dilemma. (In preparation)

@NikoletaGlyn

<https://github.com/Nikoleta-v3>