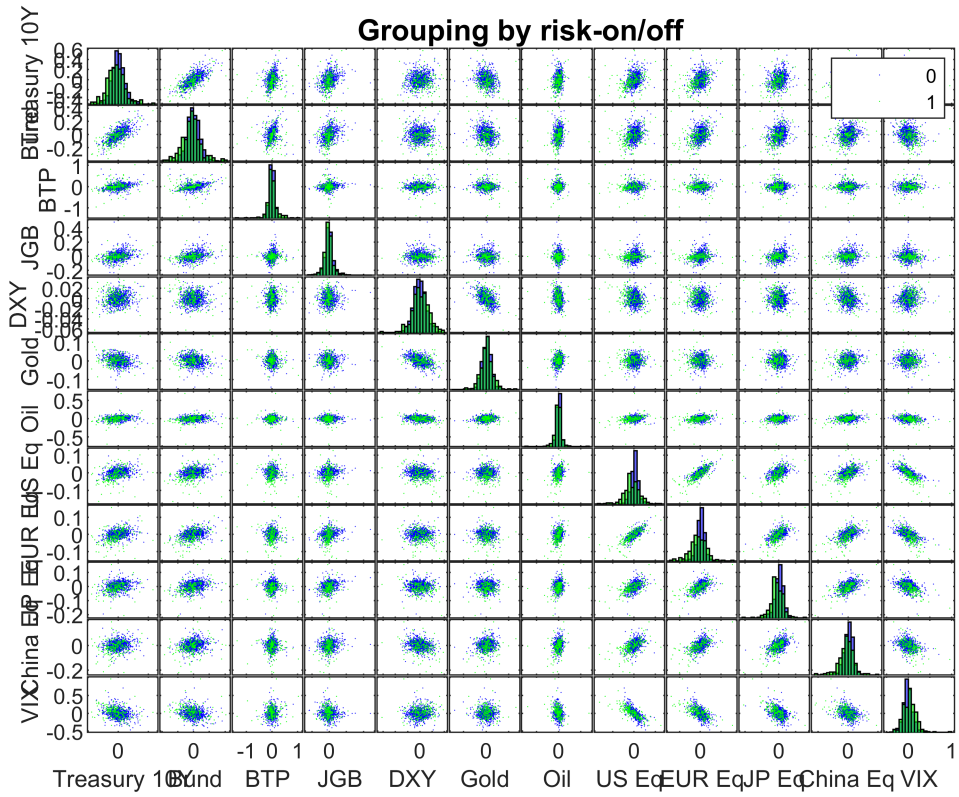
```
% % Always positive variables => log-differences (log-returns)
Indices_Currencies = [XAUBGNL BDIY CRY Cl1 DXY EMUSTRUU GBP JPY LF94TRUU LF98TRUU LG30TRUU LMBI
       LUACTRUU LUMSTRUU MXBR MXCN MXEU MXIN MXJP MXRU MXUS VIX];

% Possibly negative variables => first differences
InterestRates = [EONIA GTDEM10Y GTDEM2Y GTDEM30Y GTGBP20Y GTGBP2Y GTGBP30Y...
    GTITL10YR GTITL2YR GTITL30YR GTJPY10YR GTJPY2YR GTJPY30YR US0001M USGG3M USGG2YR GT10 USGG3

Data = [diff(log(Indices_Currencies)) ECSURPUS(2:end) diff(InterestRates) GTITL10YR(2:end) GTDE
Response = Y(2:end); % we lose one observation (the first)
```

Multivariate perspective

Matrix of scatter plots comparing our Xs by using the our Y as grouping variable (I plot just a selection of features, and they are already too many).

```
figure
Dat4Plot = [diff(GT10) diff(GTDEM10Y) diff(GTITL10YR) diff(GTJPY10YR)...
    diff(log(DXY)) diff(log(XAUBGNL)) diff(log(Cl1)) diff(log(MXUS))...
    diff(log(MXEU)) diff(log(MXJP)) diff(log(MXCN)) diff(log(VIX))];
xnames = {'Treasury 10Y', 'Bund', 'BTP', 'JGB', 'DXY', 'Gold', 'Oil', 'US Eq', 'EUR Eq', 'JP Eq
gplotmatrix(Dat4Plot,[],Response, [],[],[],[],'grpbars', xnames)
title('Grouping by risk-on/off')
```



Step1: creating training/cross-validation/test set

First we reshuffle.

Then we divide our data as follows:

- Training set = 80% of the examples, all of which are good (Y = 0)
- Cross-validation set = 10% of the examples such that Y = 0, and 50% of the anomalous examples, i.e. Y = 1;
- Test set = same as above.

```
nObs = length(Response); % #data
nObsNorm = sum(Response == 0); % #normal data points
nObsAbNorm = nObs - nObsNorm; % #abnormal data points
nObsTrain = round(0.80*nObsNorm); % #length training set
nObsCVNorm = round(0.10*nObsNorm); % #length normal portion of CV set
nObsCVabNorm = round(nObsAbNorm/2); % #length abnormal portion of CV set
nObsTest_abNorm = nObsAbNorm - nObsCVabNorm; % #length abnormal portion of Train set

% Reshuffling the sample
idxPermutation = randperm(nObs);
X = Data(idxPermutation,:);
Response = Response(idxPermutation);

% dividing normal/abnormal
Xnormal = X(Response == 0,:);
Xabnormal = X(Response == 1,:);
Yabnormal = Response(Response == 1,:);

XTrain = Xnormal(1:nObsTrain,:);
XCV = [Xnormal(nObsTrain+1:nObsTrain+1+nObsCVNorm,:); Xabnormal(1:nObsCVabNorm,:)];
XTest = [Xnormal(nObsTrain+1+nObsCVNorm+1:end,:); Xabnormal(nObsCVabNorm+1:end,:)];

yCV = zeros(length(XCV),1);
yCV(end-nObsCVabNorm+1:end) = Yabnormal(1:nObsCVabNorm);
yTest = zeros(length(XTest),1);
yTest(end-nObsTest_abNorm+1:end) = Yabnormal(nObsCVabNorm+1:end);
```

Step 2: training the model

Let's model p(X) from the training set data:

- We still assume a multivariate Gaussian distribution for our data (even if we know that it's not true);
- We train the model, i.e., we estimate the parameters, the vector of means and the covariance matrix.

```
sigma = cov(XTrain);
sigma = 0.5*(sigma+sigma'); % ensures symmetry
mu = mean(XTrain);
```

Step 3: fine tuning the hyperparameter, the threshold ϵ

On the cross-validation set:

- We compute the probability density function p(X) for each example in X;
- For different values of ϵ we check if p(X) < ϵ
- If p(X) < ϵ (anomaly), then Y = 1;
- If p(X) ≥ ϵ (normal), then Y = 0;
- We select the value of ϵ that obtains the best score in terms of *[select the most appropriate metric according to the specific problem: F1 score, Precision, Recall, True positive, False positive, False negative, True negative]*.
- Usually the different values of are in the range [min(p(X), max(p(X)].

```
p = mvnpdf(XCV,mu,sigma); % compute the probability density function for each example

[bestEpsilon, bestF1] = OptimThreshold(yCV, p) % find the optimal epsilon
```

```
bestEpsilon = 3.6540e+45
bestF1 = 0.7838
```

Step 4: testing the model

On the test set:

- We compute the probability density function p(X) for each example in X;
- Using the optimal hyperparameters ϵ, we check if p(X) < ϵ;
- If p(X) < ϵ (anomaly), then we predict Y = 1;
- If p(X) ≥ ϵ (normal), then we predict Y = 0;
- We compare our predictions with the ground truth (and we calculate usual performance metrics).

```
p = mvnpdf(XTest,mu,sigma);
predictions = p < bestEpsilon;
tp = sum((predictions == 1) & (yTest == 1))
```

```
tp = 115
```

```
fp = sum((predictions == 1) & (yTest == 0))
```

```
fp = 67
```

```
fn = sum((predictions == 0) & (yTest == 1))
```

```
fn = 3
```

```
prec = tp / (tp + fp)
```

```
prec = 0.6319
```

```
rec = tp / (tp + fn)
```

```
rec = 0.9746
```

```
F1 = 2 * prec * rec / (prec + rec)
```

```
F1 = 0.7667
```

New data arrives...

Let's assume we are at the beginning of a week on the markets: all the updated data arrives (Asian closures, European market openings, etc.) and the question for the Early warning System is:

is a risk-on or a risk-off day?

(Let's artificially define new fake data - for the sake of simplicity, we create them from existing data - sampling and averaging two points from, respectively, abnormal and normal data, and setting all the equity last week returns equal to -15% for the abnormal case.)

```
NewDataAbnormal = mean(Data(randi(length(X(Response == 1,:)),2,1), :));
NewDataAbnormal(16:22) = -0.15;
NewDataNormal = mean(Data(randi(length(X(Response == 0,:)),2,1), :));
```

Supposedly risk-off day:

```
p = mvnpdf(NewDataAbnormal,mu,sigma);
prediction = p < bestEpsilon;
if prediction
    'prediction = Abnormal'
else
    'prediction = Normal'
end
```

```
ans =
'prediction = Abnormal'
```

Supposedly risk-on day:

```
p = mvnpdf(NewDataNormal,mu,sigma);
prediction = p < bestEpsilon;
if prediction
    'prediction = Abnormal'
else
    'prediction = Normal'
end
```

```
ans =
'prediction = Normal'
```

Not too bad, overall.

Plus VIX (option implied volatility)

```
% equity indices are column 16-22 (see above)
```

```
globalEquityCV = mean(XCV(:,16:22),2); % global equity index for the CV set
globalEquityTest = mean(XTest(:,16:22),2); % global equity index for the test set
VixCV = XCV(:,23);
Vixtest = XTest(:,23);
```

Compute the probability density function for each example in the cross-validation set.

```
p = mvnpdf(XCV,mu,sigma);
```

Do the grid search over the parameter space of epsilon and pPercentile.

```
% Find the point in X with the greatest/smallest pdf value
maxEpsilon = max(p);
minEpsilon = min(p);

% Initialize accuracy metrics and hyperparameters
best_pPercentile = 0;
best_p1Percentile = 0;
bestEpsilon = 0;
bestF1 = 0;
bestPrecision = 0;
bestRecall = 0;
besttp = 0;
bestfp = 0;
bestfn = 0;

%loop for pQuantile
for pPercentile = 1:1:99
for p1Percentile = 1:1:99

    equityThreshold = prctile(globalEquityCV,pPercentile);
    VixThreshold = prctile(VixCV,p1Percentile);

    % loop for epsilon
    stepsize = (maxEpsilon - minEpsilon)/ 1000;
    for epsilon = minEpsilon:stepsize:maxEpsilon

        predictions = (p < epsilon) & (globalEquityCV<equityThreshold) & (VixCV<VixThreshold);
        tp = sum((predictions == 1) & (yCV == 1));
        fp = sum((predictions == 1) & (yCV == 0));
        fn = sum((predictions == 0) & (yCV == 1));
        prec = tp / (tp + fp);
        rec = tp / (tp + fn);
        F1 = 2 * prec * rec / (prec + rec);

        if F1 > bestF1
            bestF1 = F1;
            bestPrecision = prec;
            bestRecall = rec;
            bestEpsilon = epsilon;
            best_pPercentile = pPercentile;
            best_p1Percentile = p1Percentile;
            besttp = tp;
```

```
            bestfp = fp;
            bestfn = fn;
        end
    end
end
end

bestF1
```

bestF1 = 0.7740

```
bestPrecision
```

bestPrecision = 0.6532

```
bestRecall
```

bestRecall = 0.9496

```
bestEpsilon
```

bestEpsilon = 3.6540e+45

```
best_pPercentile
```

best_pPercentile = 99

```
best_p1Percentile
```

best_p1Percentile = 99

```
besttp
```

besttp = 113

```
bestfp
```

bestfp = 60

```
bestfn
```

bestfn = 6

```
p = mvnpdf(XTest,mu,sigma);
equityThreshold = prctile(globalEquityTest,best_pPercentile);
VixThreshold = prctile(Vixtest,best_p1Percentile);
predictions = (p < bestEpsilon) & (globalEquityTest<equityThreshold) & (Vixtest<VixThreshold);
tp = sum((predictions == 1) & (yTest == 1))
```

tp = 112

```
fp = sum((predictions == 1) & (yTest == 0))
```

fp = 66

```
fn = sum((predictions == 0) & (yTest == 1))
```

```
fn = 6
```

```
prec = tp / (tp + fp)
```

```
prec = 0.6292
```

```
rec = tp / (tp + fn)
```

```
rec = 0.9492
```

```
F1 = 2 * prec * rec / (prec + rec)
```

```
F1 = 0.7568
```

Only Vix

```
p = mvnpdf(XCV,mu,sigma);
VixCV = XCV(:,23);
Vixtest = XTest(:,23);
```

```
maxEpsilon = max(p);
minEpsilon = min(p);

% Initialize accuracy metrics and hyperparameters
best_p1Percentile = 0;
bestEpsilon = 0;
bestF1 = 0;
bestPrecision = 0;
bestRecall = 0;
besttp = 0;
bestfp = 0;
bestfn = 0;
VixThreshold=0;

%loop for pQuantile
for p1Percentile = 1:1:99

    VixThreshold = prctile(VixCV,p1Percentile);

    % loop for epsilon
    stepsize = (maxEpsilon - minEpsilon)/ 1000;
    for epsilon = minEpsilon:stepsize:maxEpsilon

        predictions = (p < epsilon) & (VixCV<VixThreshold);
        tp = sum((predictions == 1) & (yCV == 1));
        fp = sum((predictions == 1) & (yCV == 0));
        fn = sum((predictions == 0) & (yCV == 1));
        prec = tp / (tp + fp);
```

```
        rec = tp / (tp + fn);
        F1 = 2 * prec * rec / (prec + rec);

        if F1 > bestF1
            bestF1 = F1;
            bestPrecision = prec;
            bestRecall = rec;
            bestEpsilon = epsilon;
            best_p1Percentile = p1Percentile;
            besttp = tp;
            bestfp = fp;
            bestfn = fn;
        end
    end
end

bestF1
```

```
bestF1 = 0.7823
```

```
bestPrecision
```

```
bestPrecision = 0.6571
```

```
bestRecall
```

```
bestRecall = 0.9664
```

```
bestEpsilon
```

```
bestEpsilon = 3.6540e+45
```

```
best_p1Percentile
```

```
best_p1Percentile = 99
```

```
besttp
```

```
besttp = 115
```

```
bestfp
```

```
bestfp = 60
```

```
bestfn
```

```
bestfn = 4
```

```
p = mvnpdf(XTest,mu,sigma);
VixThreshold = prctile(Vixtest,best_p1Percentile);
predictions = (p < bestEpsilon) & (Vixtest<VixThreshold);
tp = sum((predictions == 1) & (yTest == 1))
```

```
tp = 114
```

```
fp = sum((predictions == 1) & (yTest == 0))
```

```
fp = 66
```

```
fn = sum((predictions == 0) & (yTest == 1))
```

```
fn = 4
```

```
prec = tp / (tp + fp)
```

```
prec = 0.6333
```

```
rec = tp / (tp + fn)
```

```
rec = 0.9661
```

```
F1 = 2 * prec * rec / (prec + rec)
```

```
F1 = 0.7651
```

BTP-BUND spread and equity index mean

```
p = mvnpdf(XCV,mu,sigma);
spreadCV = (XCV(:,end-1)-XCV(:,end))*100;
spreadtest = (XTest(:,end-1)-XTest(:,end))*100;

% Find the point in X with the greatest/smallest pdf value
maxEpsilon = max(p);
minEpsilon = min(p);

% Initialize accuracy metrics and hyperparameters
best_pPercentile = 0;
best_p1Percentile = 0;
bestEpsilon = 0;
bestF1 = 0;
bestPrecision = 0;
bestRecall = 0;
besttp = 0;
bestfp = 0;
bestfn = 0;

%loop for pQuantile
for pPercentile = 1:1:99
for p1Percentile = 1:1:99

    equityThreshold = prctile(globalEquityCV,pPercentile);
    spreadThreshold = prctile(spreadCV,p1Percentile);

    % loop for epsilon
    stepsize = (maxEpsilon - minEpsilon)/ 1000;
```

```
    for epsilon = minEpsilon:stepsize:maxEpsilon

        predictions = (p < epsilon) & (globalEquityCV<equityThreshold) & (spreadCV<spreadThresh
        tp = sum((predictions == 1) & (yCV == 1));
        fp = sum((predictions == 1) & (yCV == 0));
        fn = sum((predictions == 0) & (yCV == 1));
        prec = tp / (tp + fp);
        rec = tp / (tp + fn);
        F1 = 2 * prec * rec / (prec + rec);

        if F1 > bestF1
            bestF1 = F1;
            bestPrecision = prec;
            bestRecall = rec;
            bestEpsilon = epsilon;
            best_pPercentile = pPercentile;
            best_p1Percentile = p1Percentile;
            besttp = tp;
            bestfp = fp;
            bestfn = fn;
        end
    end
end
end

bestF1
```

```
bestF1 = 0.7671
```

```
bestPrecision
```

```
bestPrecision = 0.6474
```

```
bestRecall
```

```
bestRecall = 0.9412
```

```
bestEpsilon
```

```
bestEpsilon = 3.6540e+45
```

```
best_pPercentile
```

```
best_pPercentile = 99
```

```
best_p1Percentile
```

```
best_p1Percentile = 99
```

```
besttp
```

```
besttp = 112
```

```
bestfp
```

```
bestfp = 61
```

```
bestfn
```

```
bestfn = 7
```

```
p = mvnpdf(XTest,mu,sigma);
equityThreshold = prctile(globalEquityTest,best_pPercentile);
spreadThreshold = prctile(spreadtest,best_p1Percentile);
predictions = (p < bestEpsilon) & (globalEquityTest<equityThreshold) & (spreadtest<spreadThresh
tp = sum((predictions == 1) & (yTest == 1))
```

```
tp = 111
```

```
fp = sum((predictions == 1) & (yTest == 0))
```

```
fp = 67
```

```
fn = sum((predictions == 0) & (yTest == 1))
```

```
fn = 7
```

```
prec = tp / (tp + fp)
```

```
prec = 0.6236
```

```
rec = tp / (tp + fn)
```

```
rec = 0.9407
```

```
F1 = 2 * prec * rec / (prec + rec)
```

```
F1 = 0.7500
```

Only BTP-Bund spread

```
p = mvnpdf(XCV,mu,sigma);
spreadCV = (XCV(:,end-1)-XCV(:,end))*100;
spreadtest = (XTest(:,end-1)-XTest(:,end))*100;

% Find the point in X with the greatest/smallest pdf value
```

```
maxEpsilon = max(p);
minEpsilon = min(p);

% Initialize accuracy metrics and hyperparameters
best_pPercentile = 0;
bestEpsilon = 0;
bestF1 = 0;
bestPrecision = 0;
bestRecall = 0;
besttp = 0;
bestfp = 0;
bestfn = 0;

%loop for pQuantile
for pPercentile = 1:1:99

    spreadThreshold = prctile(spreadCV,pPercentile);


    % loop for epsilon
    stepsize = (maxEpsilon - minEpsilon)/ 1000;
    for epsilon = minEpsilon:stepsize:maxEpsilon

        predictions = (p < epsilon) & (spreadCV<spreadThreshold);
        tp = sum((predictions == 1) & (yCV == 1));
        fp = sum((predictions == 1) & (yCV == 0));
        fn = sum((predictions == 0) & (yCV == 1));
        prec = tp / (tp + fp);
        rec = tp / (tp + fn);
        F1 = 2 * prec * rec / (prec + rec);

        if F1 > bestF1
            bestF1 = F1;
            bestPrecision = prec;
            bestRecall = rec;
            bestEpsilon = epsilon;
            best_pPercentile = pPercentile;
            besttp = tp;
            bestfp = fp;
            bestfn = fn;
        end
    end
end

bestF1
```

```
bestF1 = 0.7755
```

```
bestPrecision
```

```
bestPrecision = 0.6514
```

```
bestRecall
```

```
bestRecall = 0.9580
```

```
bestEpsilon
```

```
bestEpsilon = 3.6540e+45
```

```
best_pPercentile
```

```
best_pPercentile = 99
```

```
besttp
```

```
besttp = 114
```

```
bestfp
```

```
bestfp = 61
```

```
bestfn
```

```
bestfn = 5
```

```
p = mvnpdf(XTest,mu,sigma);
spreadThreshold = prctile(spreadtest,best_pPercentile);
predictions = (p < bestEpsilon) & (spreadtest<spreadThreshold);
tp = sum((predictions == 1) & (yTest == 1))
```

```
tp = 113
```

```
fp = sum((predictions == 1) & (yTest == 0))
```

```
fp = 67
```

```
fn = sum((predictions == 0) & (yTest == 1))
```

```
fn = 5
```

```
prec = tp / (tp + fp)
```

```
prec = 0.6278
```

```
rec = tp / (tp + fn)
```

```
rec = 0.9576
```

```
F1 = 2 * prec * rec / (prec + rec)
```

```
F1 = 0.7584
```