Master Equation for a chemical reaction

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1 Exercise 4: Master equation for a chemical reaction

We attach below the code used to compute the derivatives of the generating function (called **H** in what follows).
An array of values for α, β has been chosen in a range which seems compatible with the hypotheses of small fluxes. It has been noted that values of one parameter of the order 1E-4 or 1E-5 do not differ sensibly from the same computation in the 1E-3 order of magnitude. Moreover, the quantity we have defined as *fluctuation* does not seem to be a function of the ratio of the parameters, but instead a function of each of them individually. This makes our problem bi-dimensional.

The definition of the fluctuation is:

$$\delta(\alpha, \beta) = \frac{\sqrt{\sigma_n^2}}{\langle n \rangle} \tag{1}$$

The smallest fluctuation is found to be $\delta \sim 0.44$ and a realization for this value is, for example, $\beta = 9.82$, $\alpha = 0.03$.

```
# -*- coding: utf-8 -*-
"""
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"""

from pylab import *
from scipy import *
from scipy import special
from scipy import misc



def H(z, alpha, beta):
    l = len(alpha);
    p1 = ((z+1)/2)**((1-2*beta)/2)
```

```
    p2 = special.iv(2*beta-1,sqrt(8*ones(l)*alpha*(z+1)))
    p3 = special.iv(2*beta-1,sqrt(16*ones(l)*alpha))
    return p1*p2/p3

N = 1000
alpha = logspace(-3,1,N)
beta = logspace(-3,1,N)

mean_val = zeros((N,N))
variance = zeros((N,N))
for i in range(N):
    mean_val[i][:] = misc.derivative(H, 1, dx=1e-10, n=1, args=(alpha,beta[i]))
    variance[i][:] = sqrt(misc.derivative(H, 1, dx=1e-6, n=2, args=(alpha,beta[
    + mean_val[i][:] - mean_val[i][:]**2)

print(mean_val)


fluct = divide(variance, mean_val)

print('nonzero', nonzero(fluct[where( fluct<0.1 )]))
print(fluct[where( fluct<0.1 )])
```