Anonymizing Graphs: k-Degree Anonymity

UNIVERSITÀ DEGLI STUDI DI GENOVA | Dibris

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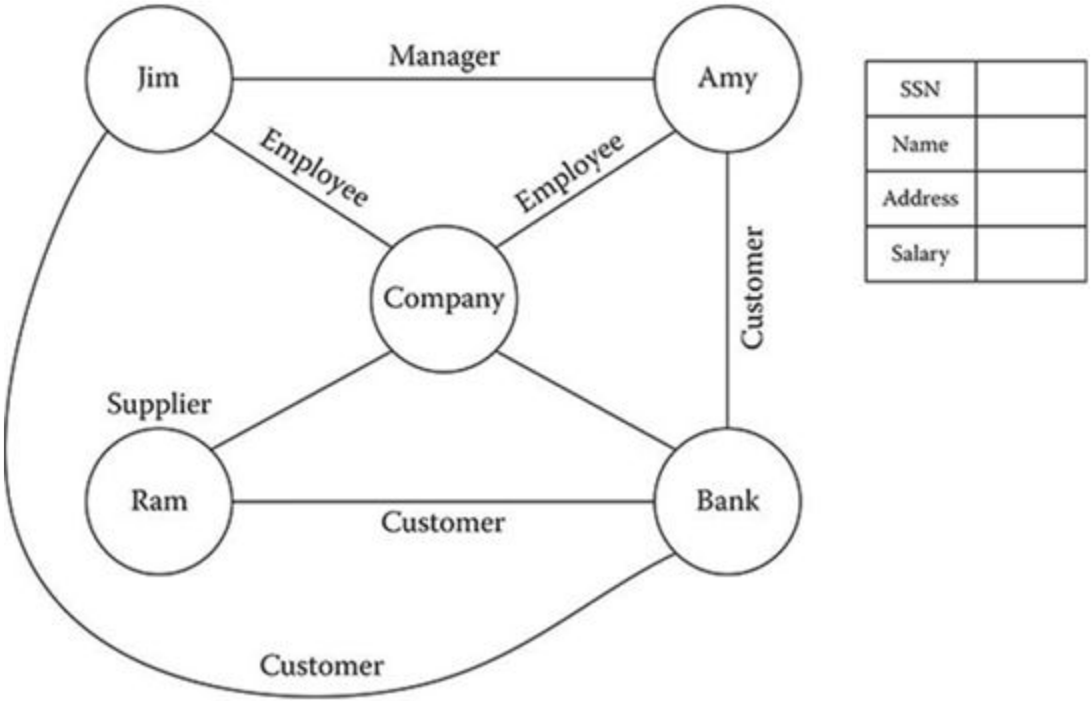
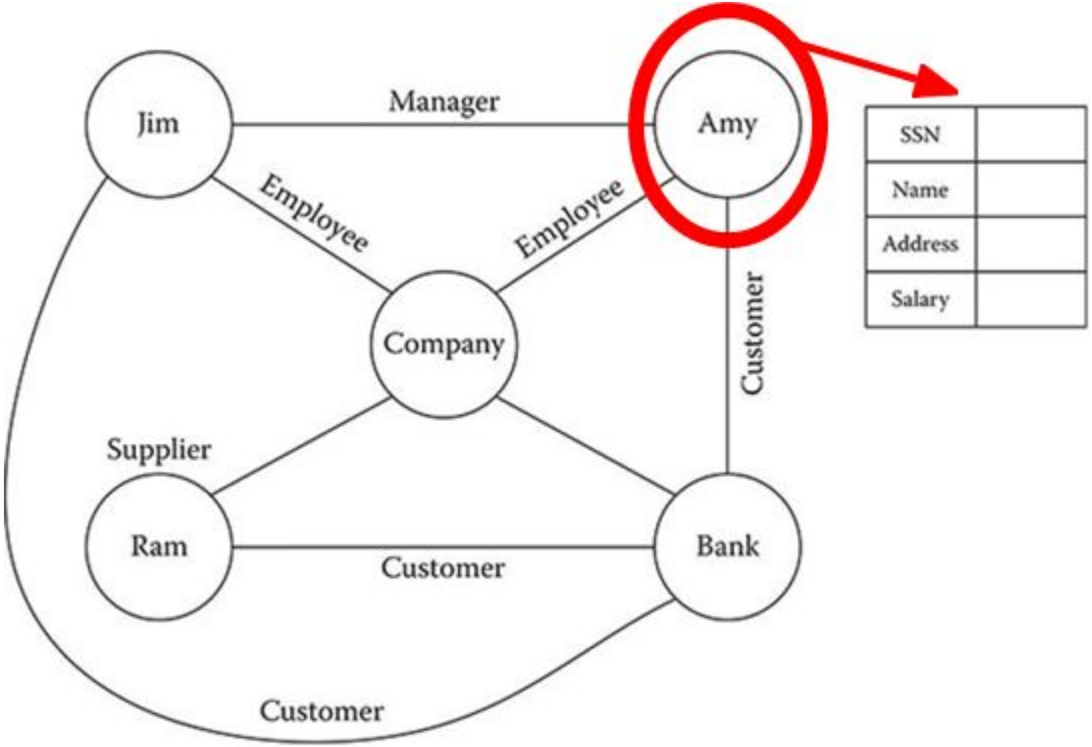
Graphs


Jim
Manager
Amy
Employee
Employee
Company
Customer
Supplier
Ram
Bank
Customer
Customer
SSN
Name
Address
Salary

Graphs



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Graphs vs. Multidimensional Data

- In multidimensional data each record or tuple can be transformed independent of each other.

- In graph data any change in the nodes or edges affects the characteristics of the graph and also the utility of the anonymized graph.
    - Vertex properties, Vertex Labels, Link relationship
    - Graph metrics

How Protect?

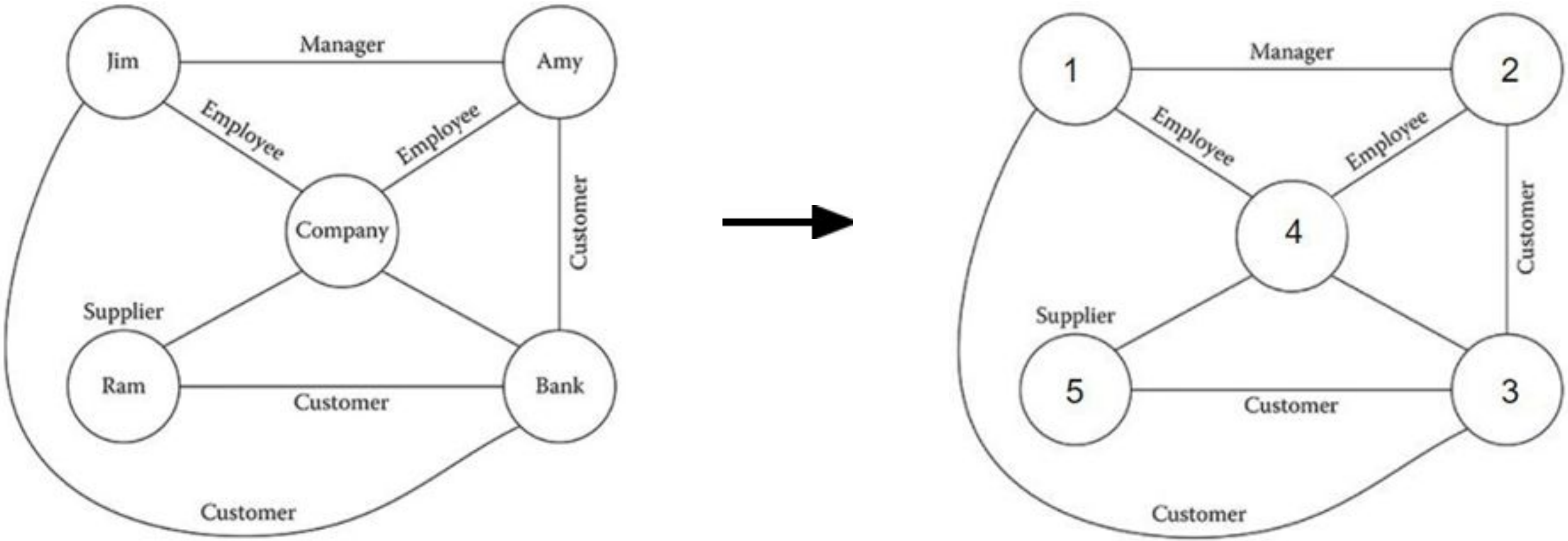
- Identity Protection → entities identification (EI in multidimensional data)
- Content Protection → entities information (QI, SD in multidimensional data)
- Link Protection → Relationship between entities

How to Anonymize?

- Naive anonymization → effective when adversary has no background knowledge
- Random Perturbation or graph modification → utility loss
- Clustering → grouping similar objects (i.e., vertices, edges, vertices-edges)

Identity Protection: Naive Anonymization

- Identiers (EI) are replaced by random values → high utility, low privacy. Weak against external knowledge.



Identity Protection: Graph Modification

- Idea: the degree of a node is informative (i.e., social networks) → it could allow to reveal identities

- **k-degree Anonymity** for each node v there exists other k - 1 nodes with the same degree as v.

k-Degree: Problem

- **Problem Formulation**
  - **G(V,E):** *simple graph*
  - $\mathbf{d_g}$: *denotes the degree sequences of G* ($|d_g| == n == |V|$)
  - $\mathbf{d_g(i)}$: *is the degree of the i-th node of G*

- **Assumption**
  - Entries in d are ordered in decreasing order of the degrees $d_g(1) >= d_g(2) >= \ldots >= d_g(n)$
  - d[i; j] (for i < j) denotes the subsequence of dg that contains elements i ; i + 1; ... ;j - 1; j

- **Definition**
  - A vector of integers v is k-anonymous, if every distinct value in v appears at least k times.
    - For example, vector v = [5, 5, 3, 3, 2, 2, 2] is 2-anonymous
  - A graph G(V; E) is k-degree anonymous if the degree sequence of G, dg ,is k-anonymous.

Example of k-degree graph

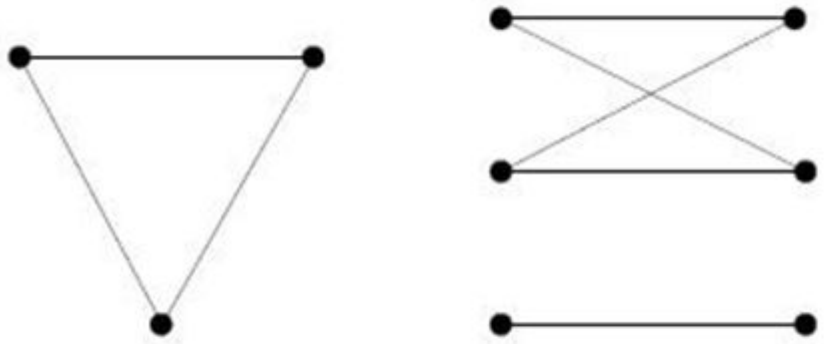


Figure 1: Examples of a 3-degree anonymous graph (left) and a 2-degree anonymous graph (right).

Graph Anonymization Problem

1. First, starting from $\mathbf{d}$, we construct a new degree sequence $\widehat{\mathbf{d}}$ that is k-anonymous and such that the *degree-anonymization* cost

$$\text{DA}(\widehat{\mathbf{d}}, \mathbf{d}) = L_1(\widehat{\mathbf{d}} - \mathbf{d}),$$

   is minimized.

2. Given the new degree sequence $\widehat{\mathbf{d}}$, we then construct a graph $\widehat{G}(V, \widehat{E})$ such that $\mathbf{d}_{\widehat{G}} = \widehat{\mathbf{d}}$ and $\widehat{E} \cap E = E$ (or $\widehat{E} \cap E \approx E$ in the relaxed version).

$$L_1\left(\widehat{\mathbf{d}} - \mathbf{d}\right) = \sum_i \left|\widehat{\mathbf{d}}(i) - \mathbf{d}(i)\right|,$$

Degree Anonymization

- **Problem**
  - Given d, the degree sequence of graph G(V;E), and an integer k construct a k-anonymous sequence d^ such that L1(d^ - d) is minimized.

- **Solutions**
  - Dynamic Programming Algorithm
  - Greedy Algorithm

Dynamic Programming Algorithm

for $i < 2k$,

$$\mathrm{DA}\,(\mathbf{d}[1,i]) \quad = \quad I\,(\mathbf{d}[1,i])\,. \tag{2}$$

For $i \geq 2k$,

$$\mathrm{DA}\,(\mathbf{d}[1,i]) = \min \tag{3}$$
$$\left\{ \min_{k \leq t \leq i-k} \left\{ \mathrm{DA}\,(\mathbf{d}[1,t]) + I\,(\mathbf{d}[t+1,i]) \right\}, I\,(\mathbf{d}[1,i]) \right\}.$$

$$I\,(\mathbf{d}[i,j]) = \sum_{\ell=i}^{j} (\mathbf{d}(i) - \mathbf{d}(\ell))\,.$$

Dynamic Programming Algorithm (1)

for $i < 2k$,

$$\textsc{Da}\left(\mathbf{d}[1,i]\right) \;=\; I\left(\mathbf{d}[1,i]\right). \qquad (2)$$

- Equation (2). When i < 2k, it is impossible to construct two different anonymized groups each of size k. As a result, the optimal degree consists of a single group in which all nodes are assigned the same degree equal to d(1)

Dynamic Programming Algorithm (2)

For $i \geq 2k$,

$$\text{DA}\left(\mathbf{d}[1,i]\right) = \min \left\{ \min_{k \leq t \leq i-k} \left\{ \text{DA}\left(\mathbf{d}[1,t]\right) + I\left(\mathbf{d}[t+1,i]\right) \right\}, I\left(\mathbf{d}[1,i]\right) \right\}. \quad (3)$$

- Equation (3) handles the case where i >= 2k. In this case, the degree-anonymization cost for the subsequence d[1; i] consists of optimal degree-anonymization cost of the sub-sequence d[1; t], plus the anonymization cost incurred by putting all nodes t + 1;...; i in the same group (provided that this group is of size k or larger).

Greedy Algorithm (1)

1. The Greedy algorithm first forms a group consisting of the first k highest-degree nodes and assigns to all of them degree d(1).

2. Then it checks whether it should merge the (k+1)-th node into the previously formed group or start a new group at position (k + 1)

Greedy Algorithm (2)

$$C_{\text{merge}} = (\mathbf{d}(1) - \mathbf{d}(k+1)) + I\left(\mathbf{d}[k+2, 2k+1]\right),$$

and

$$C_{\text{new}} = I\left(\mathbf{d}[k+1, 2k]\right).$$

1. If $C_{merge} > C_{new}$, a new group starts with the (k + 1)-th node and the algorithm proceeds recursively for the sequence d[k + 1; n].

2. Else the (k + 1)-th node is merged to the previous group and the (k + 2)-th node is considered for merging or as a starting point of a new group.

Construct Graph Algorithm

Algorithm 1 The `ConstructGraph` algorithm.

Input: A degree sequence $\mathbf{d}$ of length n.
Output: A graph $G(V, E)$ with nodes having degree sequence $\mathbf{d}$ or "No" if the input sequence is not realizable.

$V \leftarrow \{1, \ldots, n\}$, $E \leftarrow \emptyset$
if $\sum_i \mathbf{d}(i)$ is odd **then**
  Halt and return "No"
while 1 **do**
  if there exists $\mathbf{d}(i)$ such that $\mathbf{d}(i) < 0$ **then**
    Halt and return "No"
  if the sequence $\mathbf{d}$ are all zeros **then**
    Halt and return $G(V, E)$
  Pick a random node v with $\mathbf{d}(v) > 0$
  Set $\mathbf{d}(v) = 0$
  $V_{\mathbf{d}(v)} \leftarrow$ the $\mathbf{d}(v)$-highest entries in $\mathbf{d}$ (other than v)
  for each node $w \in V_{\mathbf{d}(v)}$ **do**
    $E \leftarrow E \cup (v, w)$
    $\mathbf{d}(w) \leftarrow \mathbf{d}(w) - 1$

Complete the algorithm

Download the code from Aulaweb and complete it

```python
def dp_graph_anonymization():
    # complete this function
    do_stuff()
```

```python
def greedy_rec_algorithm():
    # complete this function
    do_stuff()
```