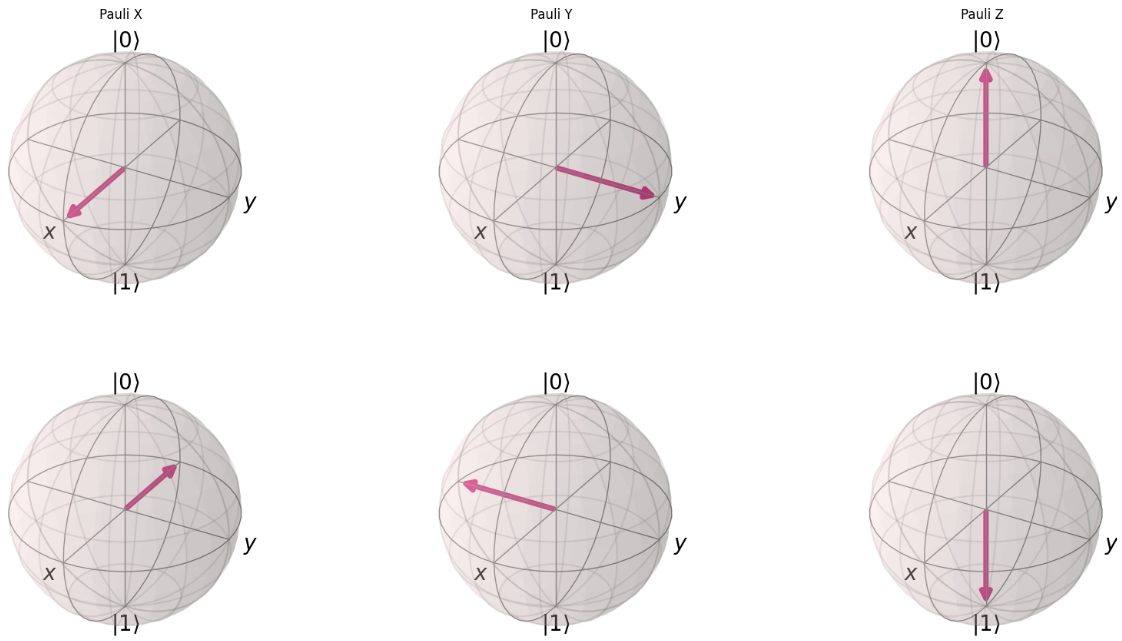
Exercise 4.1

The eigenvectors are as follows:
Pauli Z: $|0\rangle$, $|1\rangle$
Pauli X: $|0\rangle + |1\rangle$, $|0\rangle - |1\rangle$
Pauli Y: $|0\rangle + i\,|1\rangle$, $|0\rangle - i\,|1\rangle$
Bloch sphere representations:



Exercise 4.2

$$\exp(iAx) = \sum_n (iAx)^n = \sum_n (-1)^n x^{2n} I + \sum_n (-1)^n i x^n A = \cos(x) I + i \sin x A$$

Exercise 4.3

Up to a global phase:
$$T = \begin{bmatrix} e^{-i\pi/8} & 0 \\ 0 & e^{i\pi/8} \end{bmatrix} = \begin{bmatrix} e^{-i\frac{\pi}{4}/2} & 0 \\ 0 & e^{i\frac{\pi}{4}/2} \end{bmatrix} = R_z(\pi/4)$$

Exercise 4.4

First consider $R_z R_x R_z$:
$$R_z R_x R_z = \begin{bmatrix} \cos\frac{\theta}{2} e^{-i\theta} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} e^{i\theta} \end{bmatrix}$$
For $\theta = \frac{\pi}{2}$:
$$R_z R_x R_z = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{-i\frac{\pi}{2}} & e^{-i\frac{\pi}{2}} \\ e^{-i\frac{\pi}{2}} & e^{i\frac{\pi}{2}} \end{bmatrix}$$
Hence, by multiplying by $e^{i\frac{\pi}{2}}$ we get,

$e^{i\frac{\pi}{2}} R_z R_x R_z = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = H$

Exercise 4.5

We have $n_x^2 + n_y^2 + n_z^2 = 1$
$\hat{n} \cdot \vec{\sigma} = \begin{bmatrix} n_z & n_x - in_y \\ n_x + in_y & n_z \end{bmatrix}$
Therefore,
$(\hat{n} \cdot \vec{\sigma})^2 = \begin{bmatrix} n_x^2 + n_y^2 + n_z^2 & 0 \\ 0 & n_x^2 + n_y^2 + n_z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$
Consider, $R_n(\theta) R_n(-\theta)$
$I = R_n(\theta) R_n(-\theta) = (\cos(\frac{\theta}{2}) I - \sin(\frac{\theta}{2}) \hat{n} \cdot \vec{\sigma})(\cos(\frac{\theta}{2}) I + \sin(\frac{\theta}{2}) \hat{n} \cdot \vec{\sigma}) = \cos^2(\frac{\theta}{2}) I + \sin^2(\frac{\theta}{2}) (\hat{n} \cdot \vec{\sigma})^2 = (\cos^2(\frac{\theta}{2}) + \sin^2(\frac{\theta}{2})) I = I$

Exercise 4.6