

Exercise 10.1

Let $|\psi\rangle = a|0\rangle + b|1\rangle$ and the initial state be $|\psi_0\rangle = a|000\rangle + b|100\rangle$.

Applying a CNOT to the first two qubits we get,

$$|\psi_1\rangle = a|000\rangle + b|110\rangle$$

Applying a CNOT to the first and last qubits we get,

$$|\psi_2\rangle = a|000\rangle + b|111\rangle$$

Exercise 10.2

$$P_{\pm} = \frac{1}{2}(|0\rangle \pm |1\rangle)(\langle 0| \pm \langle 1|) = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1| \pm |1\rangle\langle 0| \pm |0\rangle\langle 1|) = \frac{1}{2}(I \pm X)$$

Therefore,

$$\mathcal{E}(\rho) = (1-2p)\rho + 2pP_+\rho P_+ + 2pP_-\rho P_- = (1-2p)\rho + \frac{1}{2}p(I+X)\rho(I+X) + \frac{1}{2}p(I-X)\rho(I-X) = (1-2p)\rho + p\rho + pX\rho X = (1-p)\rho + pX\rho X$$

Exercise 10.3

$$\begin{aligned} Z_2 Z_3 Z_1 Z_2 &= [I \otimes (|00\rangle\langle 00| + |11\rangle\langle 11|) - I \otimes (|01\rangle\langle 01| + |10\rangle\langle 10|)][(|00\rangle\langle 00| + |11\rangle\langle 11|) \otimes \\ &I - (|01\rangle\langle 01| + |10\rangle\langle 10|) \otimes I] = \underbrace{|000\rangle\langle 000| + |111\rangle\langle 111|}_{P_0} - \underbrace{(|100\rangle\langle 100| + |011\rangle\langle 011|)}_{P_1} \\ &+ \underbrace{|010\rangle\langle 010| + |101\rangle\langle 101|}_{P_2} - \underbrace{(|001\rangle\langle 001| + |110\rangle\langle 110|)}_{P_3} \end{aligned}$$

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