22.211 Lecture 7

Homogeneous Resonance Absorption

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Outline

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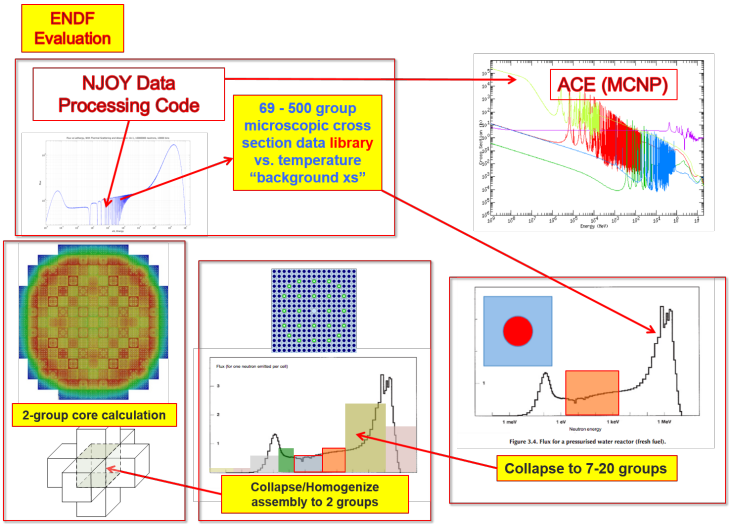
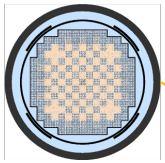
Objectives

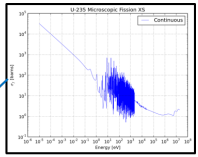
- Plazcek transients
- Slowing down with localized absorbers
- Generating resonance integral tables

Big Picture

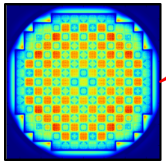

ENDF Evaluation
NJOY Data Processing Code
69 - 500 group microscopic cross section data library vs. temperature "background xs"
ACE (MCNP)
2-group core calculation
Collapse/Homogenize assembly to 2 groups
Collapse to 7-20 groups

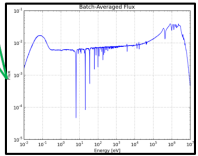
Multigroup Cross-section





$$\sigma_{n,x,k,g} = \frac{\int_{E_g}^{E_{g-1}} \int_{\mathbf{r}\in V_k} \sigma_{n,x}(\mathbf{r},E)\Phi(\mathbf{r},E)\mathrm{d}\mathbf{r}\mathrm{d}E}{\int_{E_g}^{E_{g-1}} \int_{\mathbf{r}\in V_k} \Phi(\mathbf{r},E)\mathrm{d}\mathbf{r}\mathrm{d}E}$$





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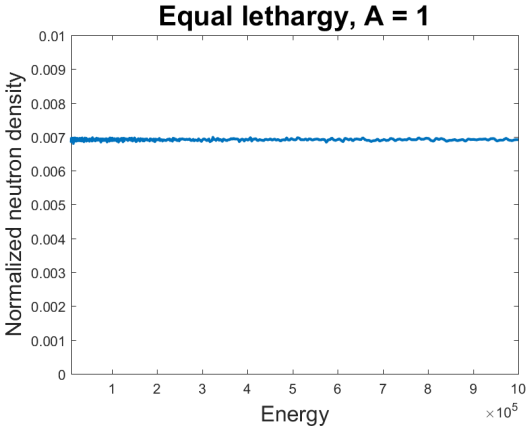
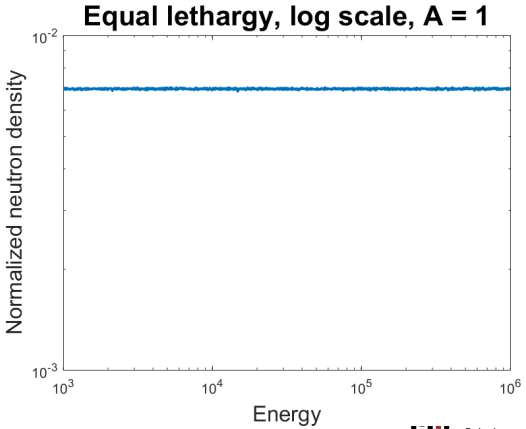
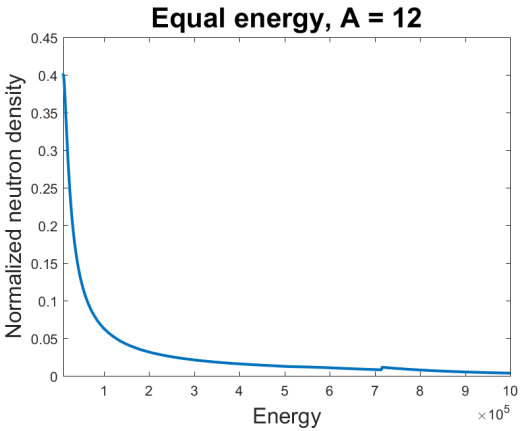
1/E flux - Lethargy Spacing - H1

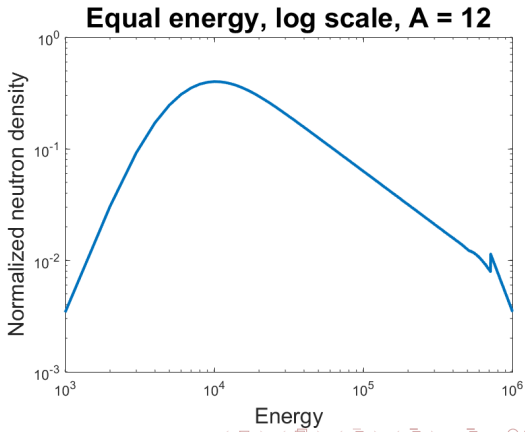




1/E flux - Energy Spacing - C12

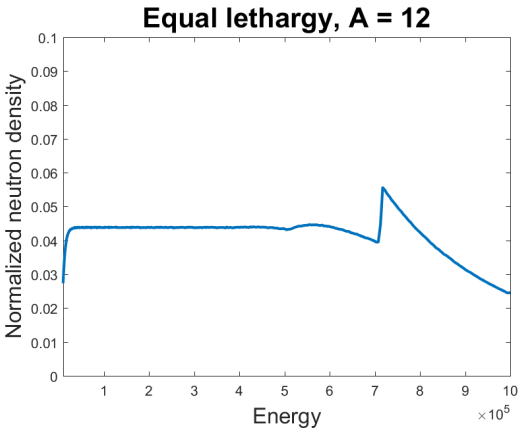
Truncated at 30 scattering events

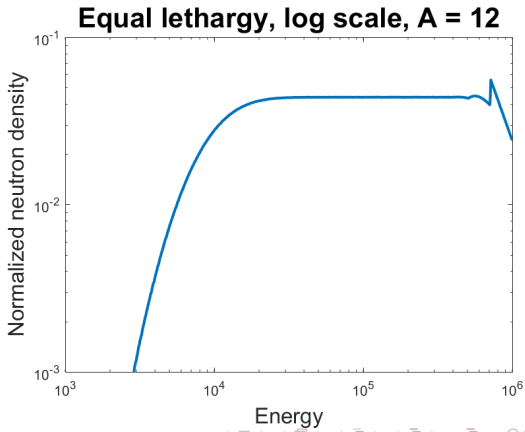




1/E flux - Lethargy Spacing - C12

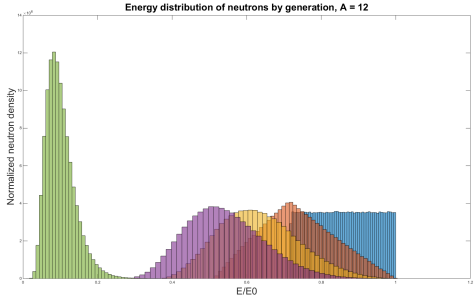
Truncated at 30 scattering events





Plazcek transient - Explanation

Physical explanation (from Reuss - Neutron Physics): "All neutrons have their first collision at the lethargy of origin; at ϵ^- there are neutrons that have undergone 1,2,3, or more collisions; at ϵ^+ it is possible to find neutrons that have undergone 2,3 or more collisions, but no first collision neutrons that were not able to exceed ϵ at the first collision, which explains the discontinuity in the flux." The only exception is H-1 which immediately goes to the asymptotic behavior.



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Homogeneous Slowing Down

In the resonance region we can neglect the direct contributions from fission. We will also assume that the source is exclusively made up of elastic down scattering.

$$\begin{aligned}(\Sigma_{t1}(u)+\Sigma_{t0}(u))\phi(u) &= \int_{u-\epsilon_0}^{u} \frac{e^{-(u-u')}}{1-\alpha_0}\Sigma_{s0}(u')\phi(u')du' \\ &+ \int_{u-\epsilon_1}^{u} \frac{e^{-(u-u')}}{1-\alpha_1}\Sigma_{s1}(u')\phi(u')du' \\ &= R_0\left[\phi(u)\right] + R_1\left[\phi(u)\right]\end{aligned}$$

0 is the resonant nuclide and 1 is the moderator

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After Factorization

We factorize the flux into two components

$$\phi(u) = \psi(u)\varphi(u)$$

The flux outside of the resonance (where there is no significant absorption, $\psi(u)$, can be approximated by (for H-1)

$$\psi(u) = \frac{R_1\left[\phi(u)\right]}{\Sigma_{t1}}$$

We can thus cancel $\psi(u)$ everywhere to get

$$(\Sigma_{t1} + \Sigma_{t0}(u))\varphi(u) = R_0\left[\varphi(u)\right] + \Sigma_{t1}$$

We then divide by N_0 to obtain (with $\sigma_d = \Sigma_{t1}/N_0$)

$$(\sigma_d + \sigma_{t0}(u))\varphi(u) = r_0\left[\varphi(u)\right] + \sigma_d$$

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Narrow Resonance Approximation

If resonance width is narrow compared to the maximum lethargy gain or average lethargy gain (with respect to the resonant nuclide 0), every scattering event will miss the resonance. The scattering cross section in the scattering kernel can thus be assumed equal to the potential scattering σ_{p0}

$$\begin{aligned} r_{0,NR}\left[\varphi(u)\right] &= \int_{u-\epsilon_0}^{u} \frac{e^{-(u-u')}}{1-\alpha_0}\sigma_{s0}(u')\varphi(u')du' \\ &= \sigma_{p0}\int_{u-\epsilon_0}^{u} \frac{e^{-(u-u')}}{1-\alpha_0}\varphi(u')du' = \sigma_{p0} \end{aligned}$$

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Narrow Resonance Model

If we replace in homogeneous slowing down equation

$$(\sigma_d + \sigma_{t0}(u))\varphi(u) = \sigma_{p0} + \sigma_d$$

where

$$\sigma_d = \frac{\Sigma_{t1}}{N_0}$$

which yields

$$\varphi_{NR}(u) = \frac{\sigma_{p0} + \sigma_d}{\sigma_{t0}(u) + \sigma_d}$$

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Narrow Resonance Model - In Energy

In energy

$$\phi(E) = \frac{1}{E}\frac{\sigma_{p0} + \sigma_d}{\sigma_{t0}(E) + \sigma_d}$$

and the effective RI for absorption

$$\begin{aligned} RI_{eff,NR} &= \int_{E_{min}}^{E_{max}} \frac{dE}{E}\sigma_{a0}(E)\frac{\sigma_{p0} + \sigma_d}{\sigma_{t0}(E) + \sigma_d} \\ &= (\sigma_{p0} + \sigma_d)\int_{E_{min}}^{E_{max}} \frac{dE}{E}\frac{\sigma_{a0}(E)}{\sigma_{t0}(E) + \sigma_d} \end{aligned}$$

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Wide Resonance Approximation (Infinite Mass Model)

In the opposite case where the resonance can be considered wide with respect to the scattering potential of the resonant nuclide, every scattering event will keep the neutron in the resonance. This model is also called the infinite mass model. In this case the reaction rate can be assumed to be constant.

$$r_{0,WR}\left[\varphi(u)\right] = \int_{u-\epsilon_0}^{u} \frac{e^{-(u-u')}}{1-\alpha_0}\sigma_{s0}(u')\varphi(u')du'$$
$$= \sigma_{s0}(u)\varphi(u)\int_{u-\epsilon_0}^{u} \frac{e^{-(u-u')}}{1-\alpha_0}du' = \sigma_{s0}(u)\varphi(u)$$

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Wide Resonance Model

If we replace in homogeneous slowing down equation

$$(\sigma_d + \sigma_{t0}(u))\varphi(u) = \sigma_{s0}(u)\varphi(u) + \sigma_d$$

$$(\sigma_d + \sigma_{a0}(u))\varphi(u) = \sigma_d$$

where

$$\sigma_d = \frac{\Sigma_{t1}}{N_0}$$

which yields

$$\varphi_{WR}(u) = \frac{\sigma_d}{\sigma_{t0}(u) - \sigma_{s0}(u) + \sigma_d}$$

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Wide Resonance Model

In energy

$$\phi_{WR}(E) = \frac{1}{E}\frac{\sigma_d}{\sigma_{a0}(E)+\sigma_d}$$

and the effective RI for absorption

$$\begin{aligned} RI_{eff,WR} &= \int_{E_{min}}^{E_{max}} \frac{dE}{E}\sigma_{a0}(E)\frac{\sigma_d}{\sigma_{a0}(E)+\sigma_d} \\ &= \sigma_d \int_{E_{min}}^{E_{max}} \frac{dE}{E}\frac{\sigma_{a0}(E)}{\sigma_{a0}(E)+\sigma_d} \end{aligned}$$

Intermediate Resonance Model

Resonances are neither wide or narrow and can be better represented as a mixture of both models.

$$r_{0,IR}\left[\varphi(u)\right] = \lambda_g \sigma_{p0} + (1-\lambda_g)\sigma_{s0}(u)\varphi(u)$$

The λ_g parameter is group dependent and will also depend on dilution.

$$\varphi_{IR}(u) = \frac{\lambda_g \sigma_{p0} + \sigma_d}{\sigma_{t0}(u) - (1-\lambda_g)\sigma_{s0}(u) + \sigma_d}$$

$$\varphi_{IR}(u) = \frac{\lambda_g \sigma_{p0} + \sigma_d}{\sigma_{a0}(u) + \lambda_g \sigma_{s0}(u) + \sigma_d}$$

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IR flux in Energy

$$\phi(E) = \frac{1}{E} \frac{\lambda_0 \sigma_{p0} + \sigma_d}{\sigma_{a0} + \lambda_0 \sigma_{p0} + \sigma_d}$$

where

$$\sigma_d = \frac{\sum_{i=1}^{N} \lambda_i N_i \sigma_{pi}}{N_0}$$

λ measures how effective the scattering nuclide is compared to H-1 which is considered perfect ($\lambda = 1$). This parameter λ is highly dependent on the scatter (mass A) and dilution.

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IR Values

Table 11

IR parameter for the microscopic absorption cross section of ^{238}U in 172-groups XMAS energy structure

Group number	Upper energy	a= 16 Background XS (barn)			a= 40 Background XS (barn)			a= 238 Background XS (barn)		
(XMAS172)	[eV]	50	1,000	10,000	50	1,000	10,000	50	1,000	10,000
57	1.01E+03	0.995	0.997	0.998	0.990	0.993	0.994	0.897	0.949	0.956
60	6.77E+02	0.998	0.999	0.999	0.997	0.999	0.999	0.899	0.942	0.946
63	3.04E+02	0.991	0.994	0.994	0.980	0.987	0.987	0.764	0.872	0.890
64	2.04E+02	0.965	0.983	0.990	0.916	0.951	0.968	0.494	0.704	0.808
66	1.37E+02	0.970	0.988	0.990	0.919	0.968	0.975	0.357	0.715	0.793
69	6.79E+01	1.000	1.000	1.000	0.992	0.982	0.985	0.328	0.682	0.776
75	3.73E+01	1.000	1.000	1.000	1.000	1.000	1.000	0.191	0.466	0.648
80	2.26E+01	0.986	0.993	0.994	0.949	0.983	0.984	0.148	0.415	0.591
88	7.52E+00	0.872	0.977	0.988	0.654	0.938	0.969	0.043	0.198	0.348

293 K; *a* represents the mass number; Lower energy for group 88 is 6.16 eV.

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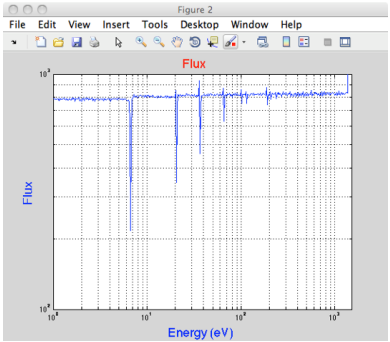
Accuracy of models - H/U238 = 100

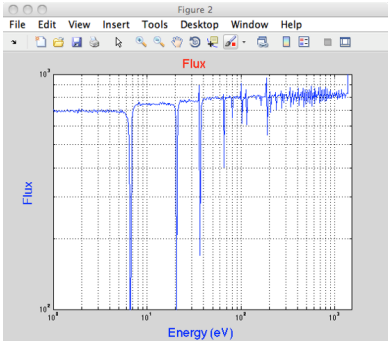
MC	NR	WR	IR(0.5)
5.13	5.57	5.57	5.57
1.69	1.70	1.70	1.70
1.46	1.45	1.45	1.45
48.3	47.6	49.0	48.3
23.1	20.5	24.8	22.4
13.9	11.3	17.9	13.6
6.7	6.2	8.0	7.7
109.6	99.8	118.4	107.1



- NR tends to underestimate absorption
- WR tends to overestimate absorption
- IR works best but is group dependent, dilution dependent and isotope dependent

Accuracy of models - H/U238 = 10

MC	NR	WR	IR(0.5)	IR(0.2)
5.11	5.57	5.57	5.57	5.57
1.69	1.70	1.70	1.70	1.70
1.45	1.45	1.45	1.45	1.45
13.6	13.8	13.9	13.9	13.8
6.6	5.6	6.7	6.5	6.1
4.6	3.3	5.2	4.5	3.9
2.4	2.1	2.8	2.4	2.4
36.0	31.0	38.8	36.1	33.5



- NR tends to underestimate absorption
- WR tends to overestimate absorption
- IR works best but is group dependent, dilution dependent and isotope dependent

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In the "real" world

- NR and WR models are never really used
- IR tables are generated for each nuclide with respect to A and dilution
- RI tables are generated from ultrafine slowing down calculations or Monte Carlo as a function of H-1 dilution with a constant cross-section
- IR model is only used to lookup that tables (i.e. calculate the dilution)
- Note that the real world is not homogeneous! Heterogeneous resonance absorption will be the topic of the next lecture

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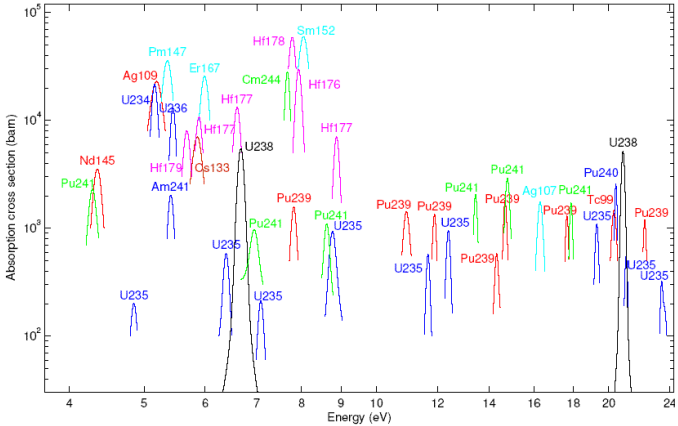
Resonance interference



Resonance Interference Factors

The idea is to pre-compute the cross-sections using a continuous-energy code and calculating the ratio of cross-sections to those computed from the self-shielding model.

$$f(\sigma_b, N_i, N_j, N_k, \ldots) = \frac{\sigma(\sigma_b, N_i, N_j, N_k, \ldots)}{\sigma(\sigma_b, N_i)}$$

$$\tilde{\sigma} = f(\sigma_b, N_i, N_j, N_k, \ldots)\sigma(\sigma_b, N_i)$$

In theory, you can compute RIFs for any combination of nuclides and dilution, but typically this is done as a function of burnup.

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RIF in PWR

Table 7. RIF's for Fresh PWR Fuel Pin

Group	238U A(a)	235U F(a)	235U A	239Pu F	239Pu A	240Pu A
43	1.013	1.001	1.001	0.991	1.007	1.025
44	1.013	1.007	1.006	1.052	1.042	1.054
45	1.012	1.000	1.000	1.016	1.006	1.021
46	1.015	1.010	1.008	1.004	1.017	1.074
47	1.015	1.013	1.015	1.019	1.021	1.022
48	1.018	1.044	1.031	0.835	0.778	0.837
49	1.005	1.000	1.000	1.013	1.016	1.004
50	1.017	1.017	1.020	1.026	1.023	0.974
51	1.034	1.067	1.088	1.084	1.029	0.642
52	0.991	0.999	0.998	1.014	1.016	0.991
53	1.031	1.005	0.939	1.267	1.276	0.605
54	1.003	1.002	1.001	1.006	1.008	1.002
55	1.010	1.002	1.002	1.044	1.040	1.001
56	1.003	1.030	1.003	1.051	1.044	0.993
57	1.004	0.970	0.919	1.394	1.406	0.921
58	0.961	0.864	0.776	0.997	0.992	1.030
59	0.998	0.999	0.998	1.000	1.000	1.004
60	1.000	1.000	1.000	1.000	1.000	0.999
61	1.000	1.000	1.000	1.000	1.000	1.001
62	1.000	1.000	1.000	1.000	1.000	0.999

Table 8. RIF's for PWR Fuel Pin at 26GWD/T Burnup

Group	238U A(a)	235U F(a)	235U A	239Pu F	239Pu A	240Pu A
43	1.007	1.001	1.001	0.992	1.007	1.024
44	1.008	1.007	1.006	1.051	1.043	1.055
45	1.007	1.000	1.000	1.017	1.007	1.025
46	1.010	1.011	1.008	1.004	1.016	1.072
47	1.012	1.011	1.013	1.020	1.019	0.992
48	1.003	1.042	1.029	0.846	0.790	0.851
49	1.002	1.004	1.004	1.005	1.006	1.002
50	1.009	1.012	1.015	1.016	1.016	0.967
51	1.013	1.060	1.080	1.086	1.038	0.646
52	0.997	1.000	0.998	1.007	1.008	0.995
53	1.022	1.021	0.955	1.241	1.249	0.622
54	1.004	1.006	1.005	1.004	1.005	1.002
55	1.005	1.020	1.019	1.019	1.018	0.999
56	1.002	1.004	1.004	1.021	1.018	0.998
57	1.023	0.980	0.930	1.395	1.408	0.923
58	0.985	0.843	0.744	0.997	0.992	1.028
59	0.999	0.998	0.997	1.000	1.000	1.002
60	1.000	1.000	1.000	1.000	1.000	1.000
61	1.000	1.000	1.000	1.000	1.000	1.000
62	1.000	1.000	1.000	1.000	1.000	1.000

Groups 51, 53 and 57 include the three main U238 resonances.

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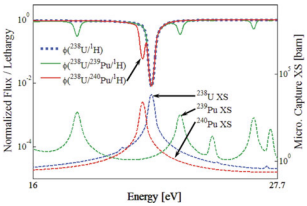
RIF as a function of background



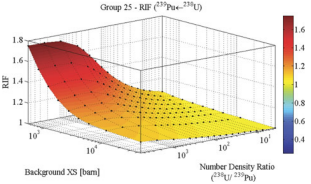


Figure 4. Capture reaction RIF of ^{239}Pu interfered by ^{238}U (group 25: 15.968 to 27.7 eV).

- For a large ratio of U-238 to Pu-239, treating the absorption in U238 as a constant across the group over-amplified the loss of neutrons in U-238 which makes us underestimate the Pu-239 microscopic cross-section significantly, since many resonances exist just above the U-238 resonance.
- In the case of a large ratio of U-238 to Pu-240, there is a direct overlap of resonances, so we are mispredicting the drop in flux, which leads to an overestimation of the Pu-240 microscopic cross-section.

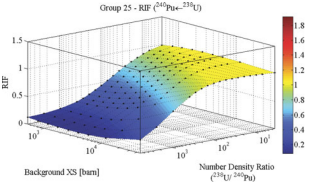


Figure 5. Capture reaction RIF of ^{240}Pu interfered by ^{238}U (group 25: 15.968 to 27.7 eV).

RIF results (PWR assembly)

Red shaded boxes are pins with Gd. NC stands for no correction, BI for background iteration and RIFL for resonance integration factor library.

IT								
-0.20 -0.19 -0.22	-0.14 -0.14 -0.16							
-0.06 -0.06 -0.08	-0.01 0.00 -0.01	-0.54 -0.46 -0.09						
GT	-0.05 -0.04 -0.06	0.01 0.00 0.01	GT					
0.06 0.06 0.06	0.02 0.03 0.02	0.05 0.05 0.05	0.03 0.04 0.04	-0.55 -0.42 -0.06				
-0.49 -0.37 -0.01	0.06 0.06 0.06	0.02 0.02 0.01	-0.05 -0.05 -0.06	0.01 0.01 0.02	GT			
GT	0.00 0.00 -0.01	-0.03 -0.03 -0.04	GT	0.03 0.02 0.02	0.04 0.04 0.04	-0.43 -0.31 0.06		
0.02 0.02 -0.01	0.05 0.05 0.04	0.06 0.06 0.06	-0.46 -0.37 -0.01	0.06 0.06 0.07	0.03 0.02 0.02	0.08 0.07 0.07	0.05 0.03 0.02	
0.04 0.03 0.00	0.04 0.03 0.02	-0.03 -0.03 -0.04	0.05 0.04 0.06	0.01 0.00 0.00	0.02 0.01 0.00	0.06 0.04 0.03	0.09 0.07 0.06	0.05 0.03 0.01

Pin Power Error (%)
NC
BI
RIFL

Figure 13. Difference of pin power distribution (octant symmetry).

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Process

- Generate IR parameter tables (comparing nuclide of mass A to hydrogen)
- Generate RI tables for each nuclide, each group, each reaction as a function of hydrogen slowing down
- For each nuclide and each group, calculate the dilution cross-section and look up RI in table

$$\sigma_d = \frac{\sum_{i=1}^{N} \lambda_i N_i \sigma_{pi}}{N_0}$$

- The RIF factor is applied to the RI to correct for interference effects
- Most fission product will be near infinite dilution
- For certain nuclides with a near constant concentration, RI tables will be generated in presence of that nuclide (e.g. U-238 in a LWR)

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Outline

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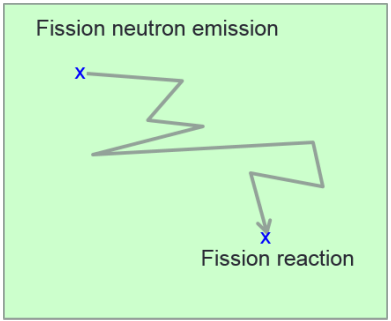
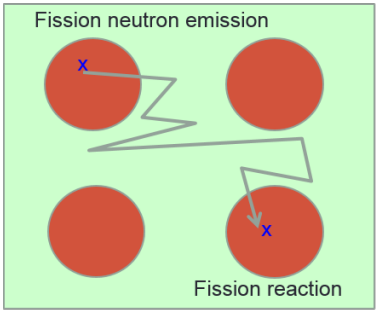
Homogeneous vs Heterogeneous

But most reactors are heterogeneous, how can we use the homogeneous tables for an heterogeneous system?



Homogeneous Reactor



Heterogeneous Reactor

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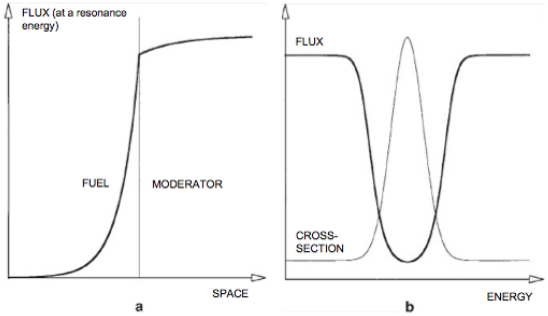
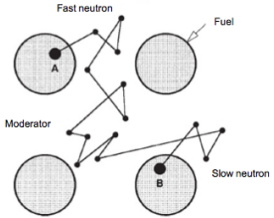
Flux shape in fuel pin





Heterogeneous Slowing Down

Goal: Identify a problem that is equivalent to the homogeneous equation so that we can solve cross-sections in the homogeneous system as a function of dilution and establish an equivalence to the heterogeneous system.
2 region problem, balance of neutrons

$$V_f R_f \left[\phi_f(u)\right] P_{f\to f} + V_m R_m \left[\phi_m(u)\right] P_{m\to f} = V_f \Sigma_{tf} \phi_f(u)$$

$$V_f R_f \left[\phi_f(u)\right] P_{f\to m} + V_m R_m \left[\phi_m(u)\right] P_{m\to m} = V_m \Sigma_{tm} \phi_m(u)$$

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Collision probabilities

Probability of going from an isotropic source at point A and making a first collision at B

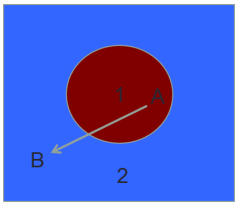
$$\frac{e^{-\tau}}{4\pi R^2}\Sigma_{tm} \quad where \;\; \tau = \int_0^R \Sigma_t(s)ds$$

Probability of going to any point in region 2

$$\int \frac{e^{-\tau}}{4\pi R^2}\Sigma_{tm} dV_m$$

If A is arbitrary and we want the average probability per unit volume in region 1, we get

$$P_{f\to m} = \frac{\Sigma_{tm}}{V_f}\int\int \frac{e^{-\tau}}{4\pi R^2} dV_m dV_f$$



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Reciprocity relation

Likewise, we can define

$$P_{m\to f} = \frac{\Sigma_{tf}}{V_m}\int\int \frac{e^{-\tau}}{4\pi R^2} dV_f\, dV_m$$

The integrals are symmetric, thus

$$P_{f\to m}\frac{V_f}{\Sigma_{tm}} = P_{m\to f}\frac{V_m}{\Sigma_{tf}}$$

which we can express

$$P_{f\to m} V_f \Sigma_{tf} = P_{m\to f} V_m \Sigma_{tm}$$

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Readings

- Reuss, chapter 8
- Nuclear Handbook, chapter 9, section 3

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