22.211 Lecture 6

Neutron Slowing Down

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Outline

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1. Objectives
2. Constant Absorber
3. Localized absorber

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Objectives

- Slowing in the absence of absorption
- Slowing down with constant absorption
- Slowing down with localized absorption

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Slowing down density

Let's assume that we are in the resolved resonance range (below threshold reactions, below prompt fission neutrons, above up-scatter region) in a purely isotropic elastic scattering medium. Define the slowing down density, $q(E)$ as the number of neutrons per second slowing down past energy E

$$q(E) = -\int_{E}^{\infty} \Sigma_a(E')\phi(E')dE' + S_f$$

Invoking second fundamental theorem of calculus

$$\frac{d}{dE}q(E) = \Sigma_a(E)\phi(E)$$

The slowing down density decreases with increasing absorption. If $\Sigma_a = 0$, then q(E) is a constant equal to the source

Slowing down density - Relating to flux

In the resonance range (below the source $E_1 < E_0$), with no absorption, we can write the following balance equation

$$\Sigma_s(E)\phi(E) = \int_0^{E_1} \Sigma_s(E' \to E)\phi(E')dE'$$

if we further assume, elastic scattering with a single nuclide, we can write

$$\Sigma_s(E' \to E) = \Sigma_s(E')P(E' \to E) = \frac{1}{(1-\alpha)E'}\Sigma_s(E')$$

replacing in balance equation

$$\Sigma_s(E)\phi(E) = \int_E^{E/\alpha} \frac{1}{(1-\alpha)E'}\Sigma_s(E')\phi(E')dE'$$

Slowing down density

We can re-define the slowing down density as

$$q(E) = \int_E^{E/\alpha} \left(\int_{\alpha E'}^{E} \frac{1}{(1-\alpha)E'} \Sigma_s(E')\phi(E')dE'' \right) dE'$$

Assume that the solution of the integral is of the form

$$\Sigma_s(E)\phi(E) = \frac{C}{E}$$

and integrate twice

$$q = \left[1 + \frac{\alpha}{(1-\alpha)} ln\alpha \right] C = \xi C$$

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Relating to flux

Replacing $C = \Sigma_s(E)\phi(E)E$

$$q = \xi\Sigma_s(E)\phi(E)E \Rightarrow \phi(E) = \frac{q}{\xi\Sigma_s(E)E}$$

We know that q is a constant in the absence of absorption, and if scattering is constant

$$\phi(E) \propto \frac{1}{E}$$

or in lethargy space

$$\phi(u) = \frac{q}{\xi\Sigma_s(u)}$$

Slowing Down in H-1 with the source

The flux at the source will be a delta function

$$\Sigma_s(E)\phi(E) = \int_0^{E_0^-} \Sigma_s(E' \rightarrow E)\phi(E')dE' + S_f\delta(E - E_0)$$

Asymptotic flux solution is the same form as previously with a jump at E_0 that is equal to

$$\phi(E) = \frac{S_f}{E\Sigma_s} + \frac{S_f\delta(E_0 - E_0)}{\Sigma_s}$$

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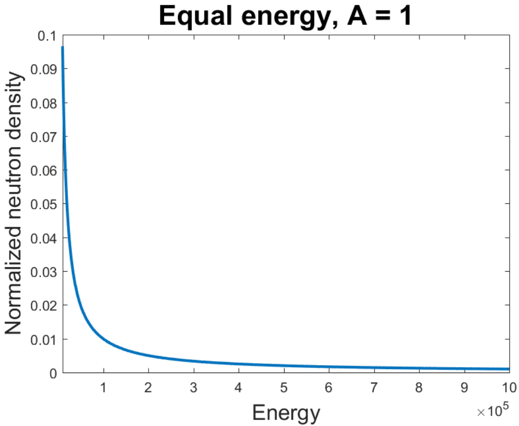
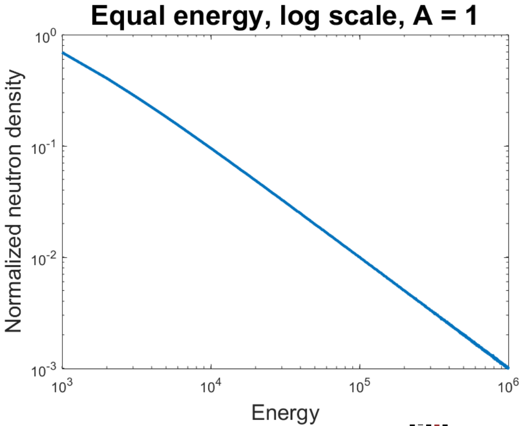
1/E flux - Energy Spacing - H1





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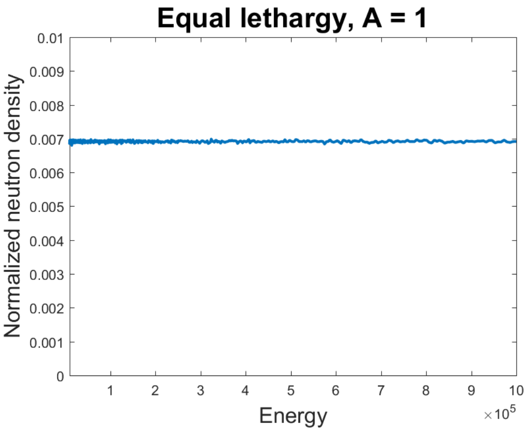
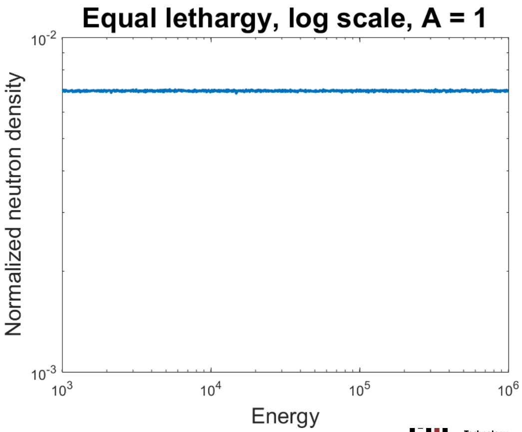
1/E flux - Lethargy Spacing - H1





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Outline

Slowing Down in H-1 with the source and constant absorption

The flux at the source will be a delta function $(b\Sigma_s = \Sigma_t = \Sigma_s + \Sigma_a)$

$$b\Sigma_s(E)\phi(E) = \int_0^{E_0^-} \Sigma_s(E' \rightarrow E)\phi(E')dE' + S_f\delta(E - E_0)$$

Asymptotic flux solution becomes

$$\phi(E) = \frac{S_f}{b^2 E^{1/b}\Sigma_s}\frac{E_0^{1/b}}{E_0} + \frac{S_f\delta(E_0 - E_0)}{b\Sigma_s}$$

Resonance escape probability

Using the previous solution, we can estimate the number of neutrons born fast that will reach thermal energies

$$p = \frac{S_f - \int_{E_t}^{E_0} \Sigma_a \phi(E) dE}{S_f}$$

$$p = 1 - \frac{\Sigma_a}{b^2 \Sigma_s} \frac{E_0^{1/b}}{E_0} \left(\frac{b E_0^{(b-1)/b}}{b-1} - \frac{b E_{th}^{(b-1)/b}}{b-1} \right) - \frac{\Sigma_a}{b \Sigma_s}$$

If $b = 1.01$ (from 1MeV to 1 eV), p $=$ 0.864. If $b = 1.02$ (from 1MeV to 1 eV), p $=$ 0.748.

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Homogeneous Slowing Down

In the resonance region we can neglect the direct contributions from fission. We will also assume that the source is exclusively made up of elastic down scattering.

$$
\begin{aligned}
(\Sigma_{t1}(u) + \Sigma_{t0}(u))\phi(u) &= \int_{u-\epsilon_0}^{u} \frac{e^{-(u-u')}}{1-\alpha_0} \Sigma_{s0}(u')\phi(u')du' \\
&+ \int_{u-\epsilon_1}^{u} \frac{e^{-(u-u')}}{1-\alpha_1} \Sigma_{s1}(u')\phi(u')du' \\
&= R_0\left[\phi(u)\right] + R_1\left[\phi(u)\right]
\end{aligned}
$$

0 is the resonant nuclide and 1 is the moderator

After Factorization

We factorize the flux into two components

$$\phi(u) = \psi(u)\varphi(u)$$

The flux outside of the resonance (where there is no significant absorption, $\psi(u)$, can be approximated by (for H-1)

$$\psi(u) = \frac{R_1\left[\phi(u)\right]}{\Sigma_{t1}}$$

We can thus cancel $\psi(u)$ everywhere to get

$$(\Sigma_{t1} + \Sigma_{t0}(u))\varphi(u) = R_0\left[\varphi(u)\right] + \Sigma_{t1}$$

We then divide by N_0 to obtain (with $\sigma_d = \Sigma_{t1}/N_0$)

$$(\sigma_d + \sigma_{t0}(u))\varphi(u) = r_0\left[\varphi(u)\right] + \sigma_d$$

Narrow Resonance Approximation

If resonance width is narrow compared to the maximum lethargy gain or average lethargy gain (with respect to the resonant nuclide 0), every scattering event will miss the resonance. The scattering cross section in the scattering kernel can thus be assumed equal to the potential scattering σ_{p0}

$$
\begin{aligned}
r_{0,NR}\left[\varphi(u)\right] &= \int_{u-\epsilon_0}^{u} \frac{e^{-(u-u')}}{1-\alpha_0}\sigma_{s0}(u')\varphi(u')du' \\
&= \sigma_{p0}\int_{u-\epsilon_0}^{u} \frac{e^{-(u-u')}}{1-\alpha_0}\varphi(u')du' = \sigma_{p0}
\end{aligned}
$$

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Narrow Resonance Model

If we replace in homogeneous slowing down equation

$$(\sigma_d + \sigma_{t0}(u))\varphi(u) = \sigma_{p0} + \sigma_d$$

where

$$\sigma_d = \frac{\Sigma_{t1}}{N_0}$$

which yields

$$\varphi_{NR}(u) = \frac{\sigma_{p0} + \sigma_d}{\sigma_{t0}(u) + \sigma_d}$$

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Narrow Resonance Model - In Energy

In energy

$$\phi(E) = \frac{1}{E}\frac{\sigma_{p0} + \sigma_d}{\sigma_{t0}(E) + \sigma_d}$$

and the effective RI for absorption

$$RI_{eff,NR} = \int_{E_{min}}^{E_{max}} \frac{dE}{E}\sigma_{a0}(E)\frac{\sigma_{p0} + \sigma_d}{\sigma_{t0}(E) + \sigma_d}$$
$$= (\sigma_{p0} + \sigma_d)\int_{E_{min}}^{E_{max}} \frac{dE}{E}\frac{\sigma_{a0}(E)}{\sigma_{t0}(E) + \sigma_d}$$

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Wide Resonance Approximation (Infinite Mass Model)

In the opposite case where the resonance can be considered wide with respect to the scattering potential of the resonant nuclide, every scattering event will keep the neutron in the resonance. This model is also called the infinite mass model. In this case the reaction rate can be assumed to be constant.

$$
\begin{aligned}
r_{0,WR}\left[\varphi(u)\right] &= \int_{u-\epsilon_0}^{u} \frac{e^{-(u-u')}}{1-\alpha_0}\sigma_{s0}(u')\varphi(u')du' \\
&= \sigma_{s0}(u)\varphi(u)\int_{u-\epsilon_0}^{u} \frac{e^{-(u-u')}}{1-\alpha_0}du' = \sigma_{s0}(u)\varphi(u)
\end{aligned}
$$

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Wide Resonance Model

If we replace in homogeneous slowing down equation

$$(\sigma_d + \sigma_{t0}(u))\varphi(u) = \sigma_{s0}(u)\varphi(u) + \sigma_d$$

$$(\sigma_d + \sigma_{a0}(u))\varphi(u) = \sigma_d$$

where

$$\sigma_d = \frac{\Sigma_{t1}}{N_0}$$

which yields

$$\varphi_{WR}(u) = \frac{\sigma_d}{\sigma_{t0}(u) - \sigma_{s0}(u) + \sigma_d}$$

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Wide Resonance Model

In energy

$$\phi_{WR}(E) = \frac{1}{E}\frac{\sigma_d}{\sigma_{a0}(E) + \sigma_d}$$

and the effective RI for absorption

$$RI_{eff,WR} = \int_{E_{min}}^{E_{max}} \frac{dE}{E}\sigma_{a0}(E)\frac{\sigma_d}{\sigma_{a0}(E) + \sigma_d}$$

$$= \sigma_d \int_{E_{min}}^{E_{max}} \frac{dE}{E}\frac{\sigma_{a0}(E)}{\sigma_{a0}(E) + \sigma_d}$$

Resonance Escape Probability

Define the resonance escape probability as the fraction of neutrons that escape absorption in the resonance range, which we can write as (assuming no absorption in moderator)

$$p(E) = \frac{q(E)}{S} = \frac{S - \int_E^\infty N_0 \sigma_{a0}(E)\phi(E)dE}{S}$$

From the definition of slowing down density

$$\frac{d}{dE}q(E) = \Sigma_a(E)\phi(E)$$

Resonance Escape Probability

We've also shown that in the absence of absorption

$$q(E) = \Sigma_s(E)\phi(E)E\xi$$

If we now add absorption, assuming that scattering is dominant and that absorption only happens over a narrow range, we can write

$$q(E) = \left(\Sigma_s(E) + \Sigma_a(E)\right)\phi(E)E\xi$$

We can now combine the two definitions of $q(E)$ to get

$$\frac{dq(E)}{q(E)} = d\left(\ln q(E)\right) = \frac{\Sigma_a(E)}{\xi\left(\Sigma_s(E) + \Sigma_a(E)\right)}\frac{dE}{E}$$

Resonance Escape Probability

If we now integrate from energy E_{th} to ∞, and assume that energy E_{th} is below the resonance range, we get

$$p = \frac{q(E)}{S} = e^{-\int_{E_{th}}^{\infty} \frac{\Sigma_a(E)}{\xi(\Sigma_s(E)+\Sigma_a(E))} \frac{dE}{E}}$$

which we can relate to our definitions of effective resonance integrals (assuming no absorption in the moderator and constant scattering), to get

$$p = e^{-\frac{1}{\xi}\int_{E_{th}}^{\infty} \frac{N_0\sigma_{a0}(E)}{\left(N_0\sigma_{p0}+N_1\sigma_{p1}+N_0\sigma_{a0}(E)\right)} \frac{dE}{E}} = e^{-\frac{1}{\xi}\int_{E_{th}}^{\infty} \frac{\sigma_{a0}(E)}{\left(\sigma_{p0}+\sigma_{d}+\sigma_{a0}(E)\right)} \frac{dE}{E}} = e^{-\frac{RI_{eff}}{\xi\sigma_d}}$$

Final Thoughts

- If absorption was distributed evenly across the energy range, very few neutrons would survive thermalization.
- Localized absorption in resonances allows neutrons to avoid absorption.
- RI_{eff} or σ_g can easily be computed in an infinite homogeneous system as a function of dilution.
- The more dilute the resonant nuclide is, the less likely the neutrons will get absorbed and the more likely they will thermalize.

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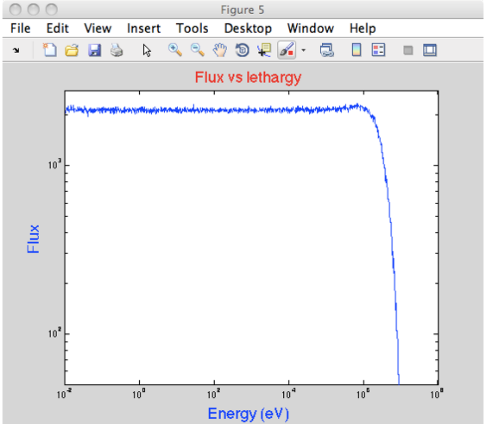
Flux Spectrum

Fission spectrum with constant cross-section



PWR Flux Spectrum

Real PWR spectrum shows a well-defined fission emission spectrum

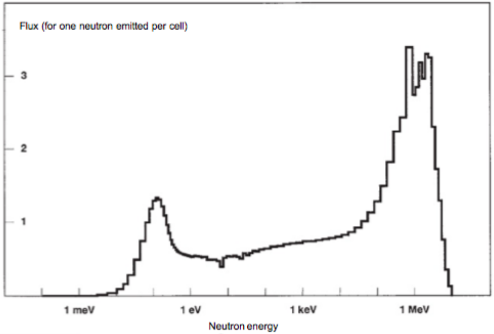


Figure 3.4. Flux for a pressurised water reactor (fresh fuel).

What is our model missing?

Hydrogen-1 cross section is not completely flat