

# 22.211 Lecture 5

## MGXS

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# Outline

1 Objectives

2 Resonance integral

3 MGXS

4 SPH Factors

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1 Objectives

2 Resonance integral

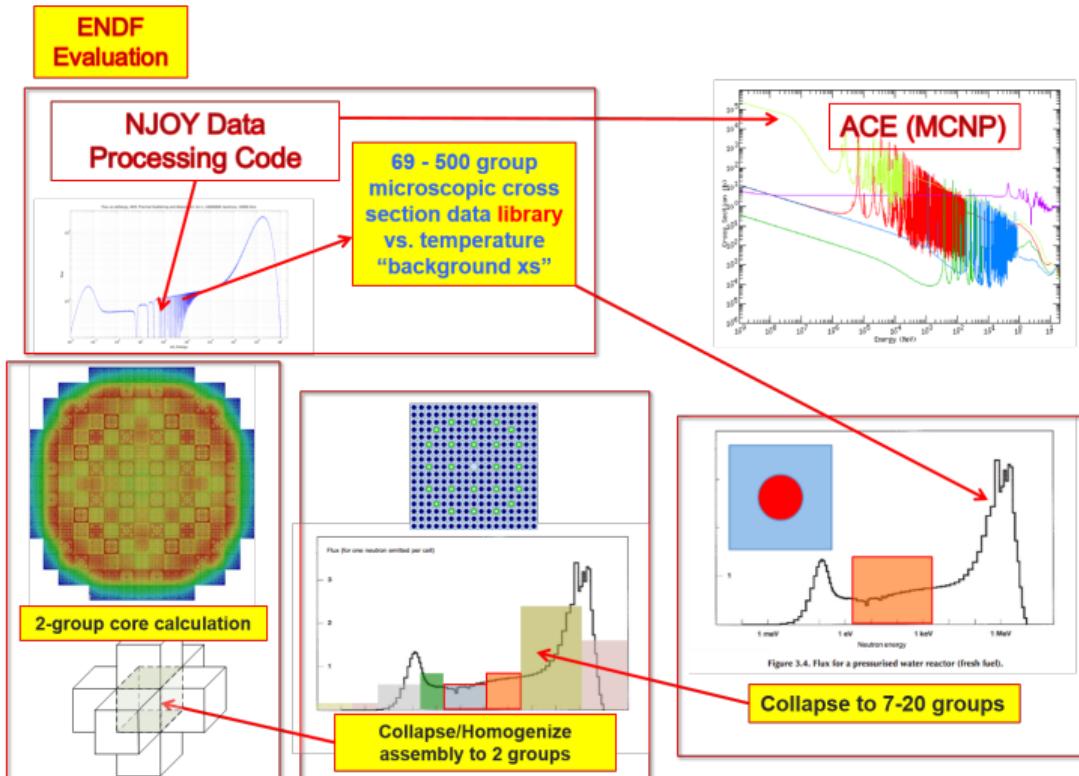
3 MGXS

4 SPH Factors

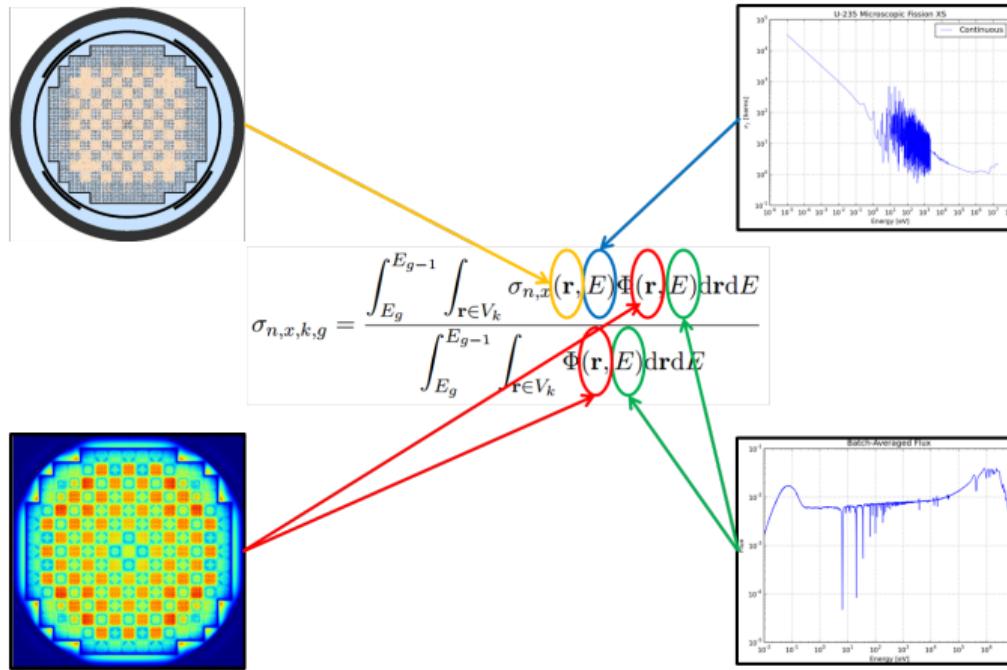
# Objectives

- Understanding why we need MGXS
- Understanding the need for equivalence factors
- Introduction to Resonance integral

# Big Picture



# Multigroup Cross-section



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# Resonance Integral - Infinite Dilution

$$RI = - \int_{u_1}^{u_2} \sigma(u) du$$

Recalling that

$$u = \ln(E_0/E) = \ln(E_0) - \ln(E)$$

which implies that

$$du = -\frac{1}{E} dE$$

$$RI = \int_{E_1}^{E_2} \sigma(E) \frac{1}{E} dE$$

- RI defined directly from cross section data
- No flux calculation is required,  $1/E$  flux is assumed
- RI depends on flux normalization; implicitly normalized that flux equals 1 at  $E=1$
- For isolated resonances, RI is independent of energy bounds
- RI is useful to compare nuclear libraries or cross-section models

# Use of RI - Comparing libraries

Comparison between ENDF VII and ENDF VI libraries for U-238 capture

**Table 9. Capture infinite-dilution resonance integrals; present values and ENDF/B-VI values**

Energy range (eV)	Present values (barn)	ENDF/B-VI (barn)	Difference
0.01–0.1	5.85	5.93	1.2%
0.1–1.0	2.47	2.51	1.5%
1.0–6.0	1.74	1.77	1.6%
6.0–10.0	127.56	128.94	1.1%
10.0–25.0	66.35	67.22	1.3%
25.0–50.0	41.84	42.13	0.7%
50.0–100.0	12.52	12.57	0.4%
0.5–10000	271.33	274.01	1.0%

# Cross sections and effective RI

$$\sigma_g = \frac{\int_{E_g}^{E_{g-1}} \sigma(E) \phi(E) dE}{\int_{E_g}^{E_{g-1}} \phi(E) dE}$$

Define

$$RI_{eff} = \sigma_g \times \int_{E_g}^{E_{g-1}} \phi(E) dE$$

- Multigroup XS depend on data and spectrum
- Multigroup XS do not depend on flux normalization
- XS depend on energy bounds
- Knowing energy bounds and group cross section, we can calculate an effective RI

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# Multigroup Approximation

Define the group flux

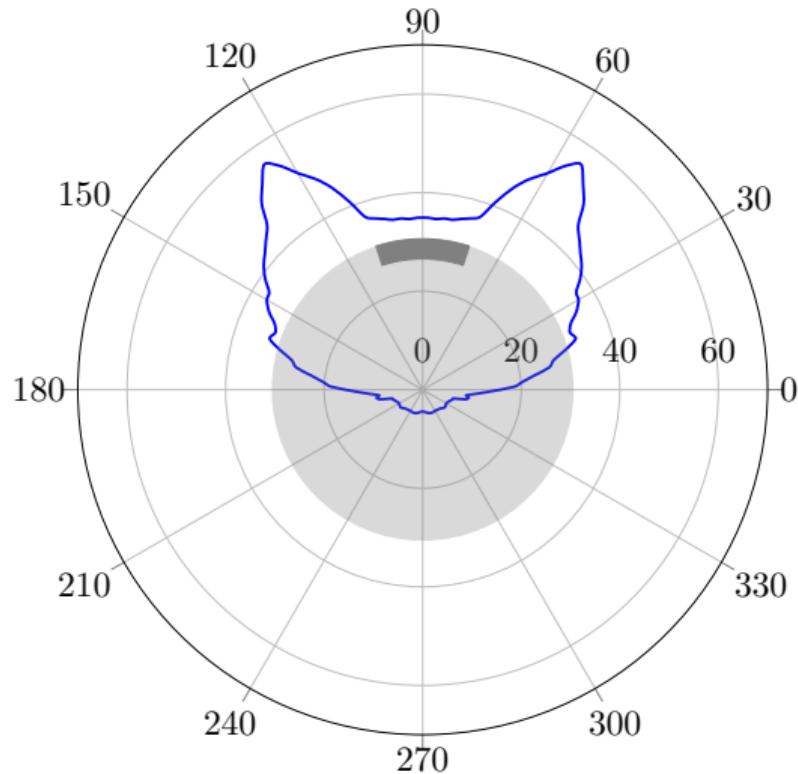
$$\psi_g(\mathbf{r}, \Omega) = \int_{E_g}^{E_{g-1}} \psi(\mathbf{r}, \Omega, E) dE$$

And group constant that preserve reaction rates

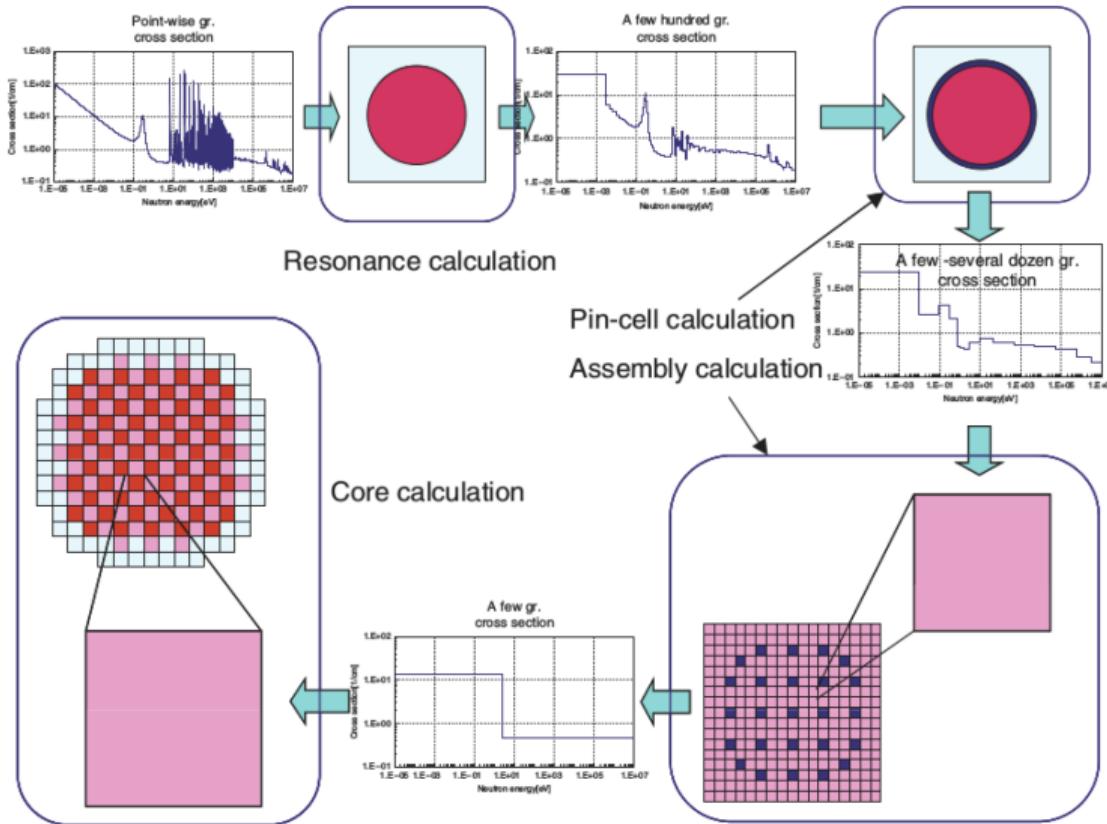
$$\Sigma_{tg}(\mathbf{r}) = \frac{\int_{E_g}^{E_{g-1}} \Sigma_t(\mathbf{r}, E) \phi(\mathbf{r}, E) dE}{\int_{E_g}^{E_{g-1}} \phi(\mathbf{r}, E) dE}$$

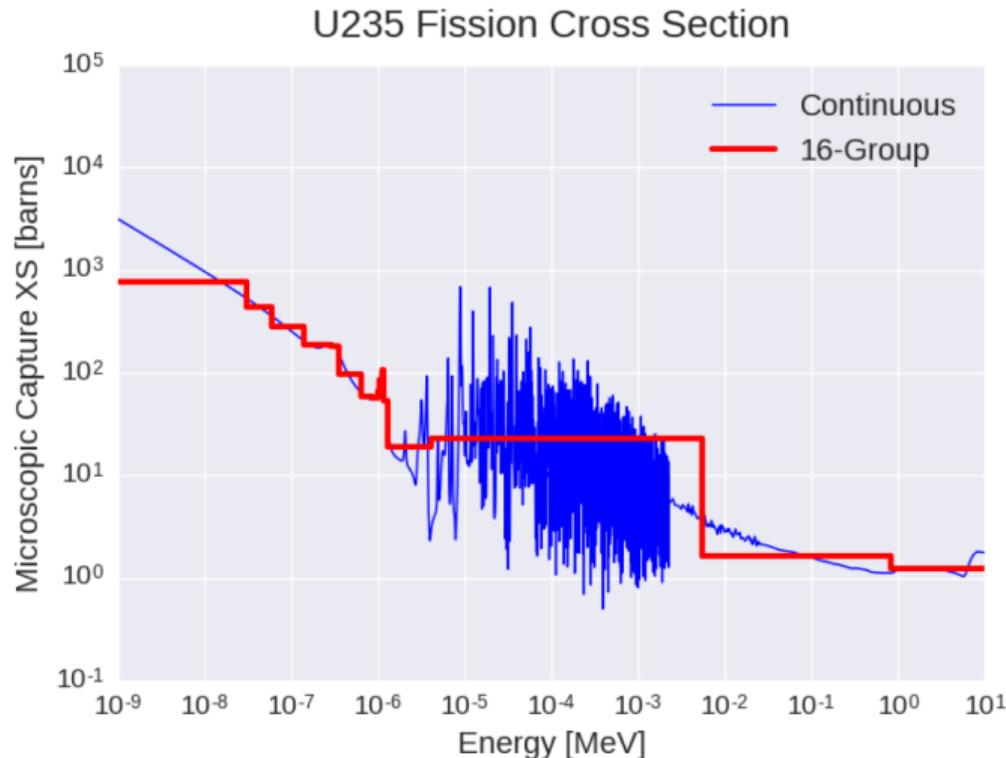
We usually use  $\phi$  instead of  $\psi$  to avoid having directional dependent group constants.

Condensation Errors (U-238 6.67eV resonance group)



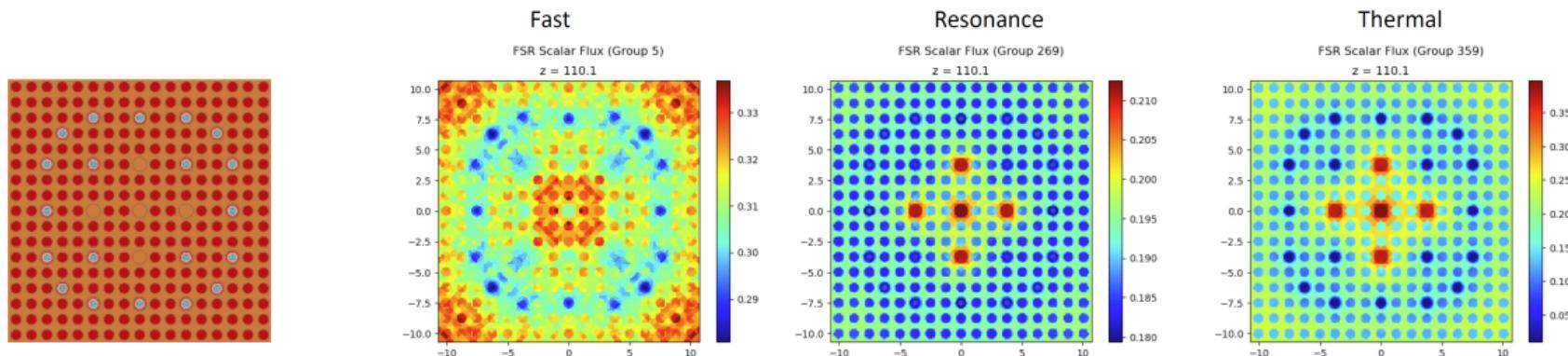
# Big Picture of MGXS





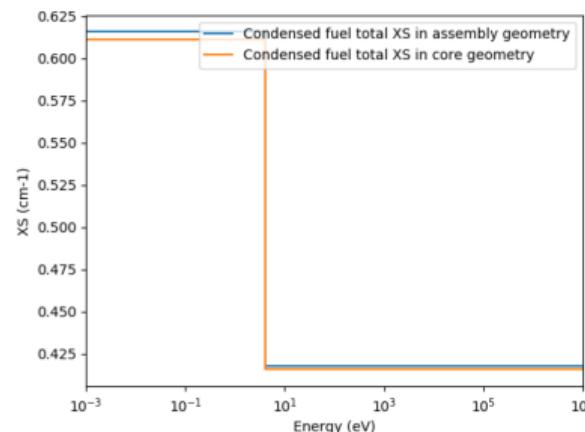
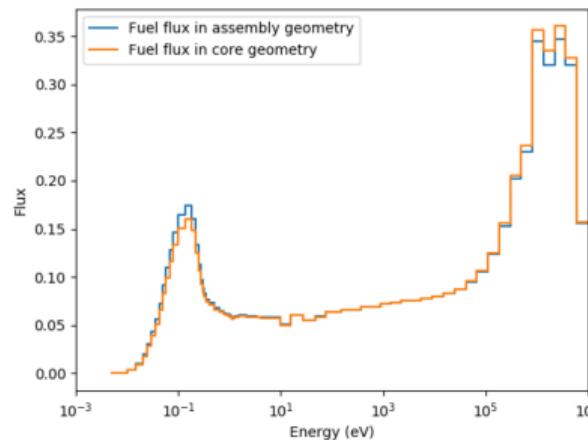
# Sources of Errors - Spatial Dependencies

The flux shape can change dramatically in space in assemblies with burnable poisons. Individual pins cannot all be considered similar based on their proximity to the absorber.



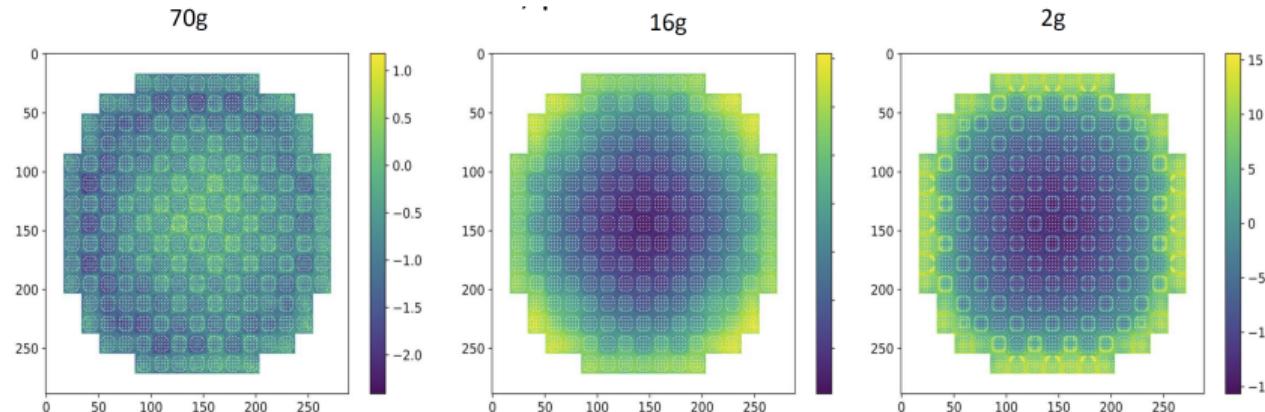
## Sources of Errors - Spectrum Differences

Using fluxes from an assembly differs sufficiently from the flux in the core.



## Sources of Errors - Energy Condensation

Condensing in energy cannot preserve both reaction rates and leakage simultaneously in heterogeneous systems.



# Sources of Errors - Spatial homogenization

- Spatial regions are homogenized to simplify problem
- Homogenized data makes the problem better suited for diffusion theory (less angular effects, less abrupt material changes).
- Often group condensation is performed at the same time.
- Equivalence factors are essential to preserve the accurate assembly transport calculation in many groups when using a few group diffusion calculation.

$$\Sigma_g = \frac{\int_V \phi_g(r) \Sigma_g(r) dV}{\int_V \phi_g(r) dV}$$

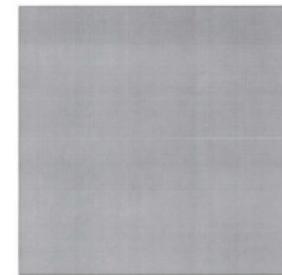
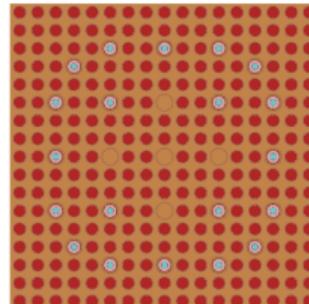


Figure 17.7. Complete homogenisation of the assembly.

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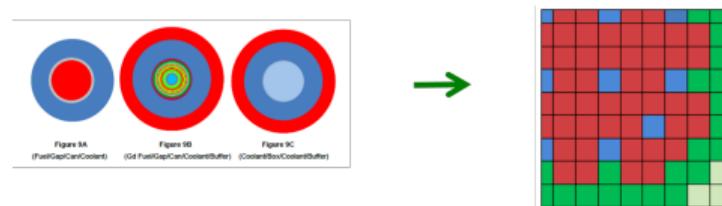
2 Resonance integral

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4 SPH Factors

## SPH Factors

Spatial Homogenization and Energy Condensation cannot preserve both reaction rates and leakage rates. SPH Factors are a simple iterative way of forcing preservation.



The idea is that you solve the pin cell using high fidelity transport theory, you then condense in energy and spatially homogenize. You then solve the homogeneous problem using diffusion theory. The scalar fluxes between the transport solution and diffusion solution will not match, so you will adjust the  $x_s$  by the flux ratio until the diffusion solution matches the transport one.

# Condensation to 1 group

We can condense the cross section with the exact scalar fluxes

$$\bar{\Sigma} = \frac{\Sigma_1\phi_1 + \Sigma_2\phi_2 + \Sigma_3\phi_3}{\phi_1 + \phi_2 + \phi_3}$$

If we solve the 1 group problem, the scalar flux in the cell will be different than the sum of the scalar fluxes. We thus define a correction factor:

$$\bar{\Sigma}^* = \mu \bar{\Sigma}$$

with

$$\mu = \frac{\bar{\phi}}{\phi_1 + \phi_2 + \phi_3}$$

and we iterate

## Example Problem

Following 3 group cross sections

$$\Sigma_{t1} = 1 \quad \Sigma_{t,2} = 2 \quad \Sigma_{t,3} = 5$$

In a 1 cm purely absorbing slab with vacuum BC, with a unit source of left surface in each group with direction  $\mu = 1$  will have the following solution:

$$\psi(x) = \psi_{in} e^{-\Sigma_t x}$$

where  $\psi_{in} = 1$  and from this we can get the cell integrated scalar flux

$$\phi = \int_0^1 dx \int d\Omega \quad \psi_{in} e^{-\Sigma_t x} = (1 - e^{-\Sigma_t})$$

# Solution of 3 group problem

Scalar flux in each group

$$\phi_1 = 0.632 \quad \phi_2 = 0.865 \quad \phi_3 = 0.993$$

which gives a total scalar flux and reaction rate

$$\phi_{total} = 2.497 \quad \sum_{g=1}^3 \Sigma_g \phi_g = 7.33$$

# 1 group problem - Iteration 1

Homogenized to 1 group

$$\Sigma_{hom} = 2.943$$

Using an incoming source of 3 (sum of the group sources)

$$\phi = 2.842 \quad \Sigma_{hom}\phi = 8.363$$

We can calculate the SPH factor

$$SPH_1 = \frac{2.497}{2.842} = 0.876$$

# 1 group problem - Iteration 2

Updating the homogenized 1 group cross section

$$\Sigma_{hom}^* = 2.943 \times 0.876 = 2.579$$

Using an incoming source of 3 (sum of the group sources)

$$\phi = 2.772 \quad \Sigma_{hom}^* \phi = 7.149$$

We can calculate the SPH factor

$$SPH_2 = \frac{2.497}{2.772} = 0.898$$

# 1 group problem - Iteration 3

Updating the homogenized 1 group cross section

$$\Sigma_{hom}^* = 2.943 \times 0.898 = 2.643$$

Using an incoming source of 3 (sum of the group sources)

$$\phi = 2.787 \quad \Sigma_{hom}^* \phi = 7.365$$

We can calculate the SPH factor

$$SPH_3 = 0.921$$

# 1 group problem - Iteration 4

Updating the homogenized 1 group cross section

$$\Sigma_{hom}^* = 2.943 \times 0.894 = 2.630$$

Using an incoming source of 3 (sum of the group sources)

$$\phi = 2.784 \quad \Sigma_{hom}^* \phi = 7.329$$

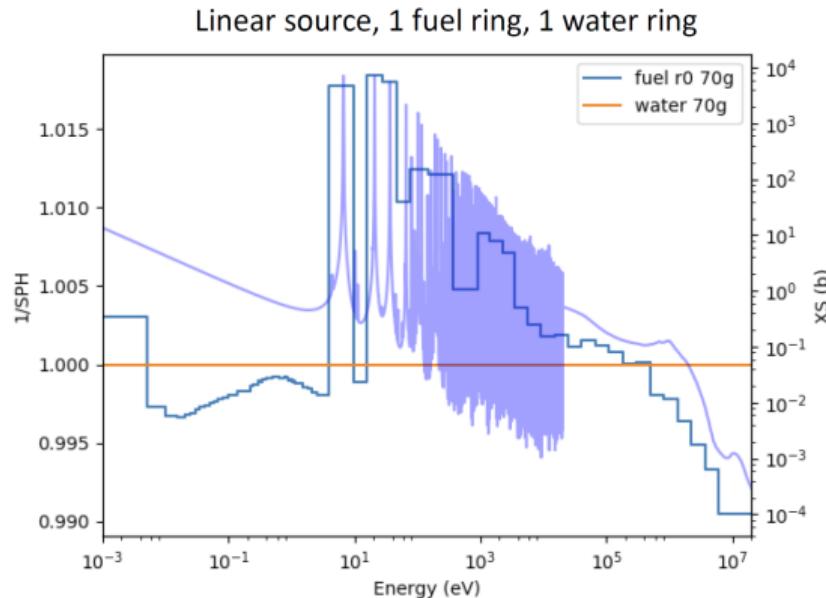
# SPH Summary

- Reaction rates are the most important quantity to be preserved
- Assume you have a high-order reference solution
- Solve the low order model and get the low-order group
- Adjust all group cross-sections of low-order model by a constant factor for each group computed from the ratio of the fluxes

$$\Sigma_g^* = \mu_g \Sigma_g \quad \mu_g = \frac{\phi_{exact}}{\phi_{low-order}}$$

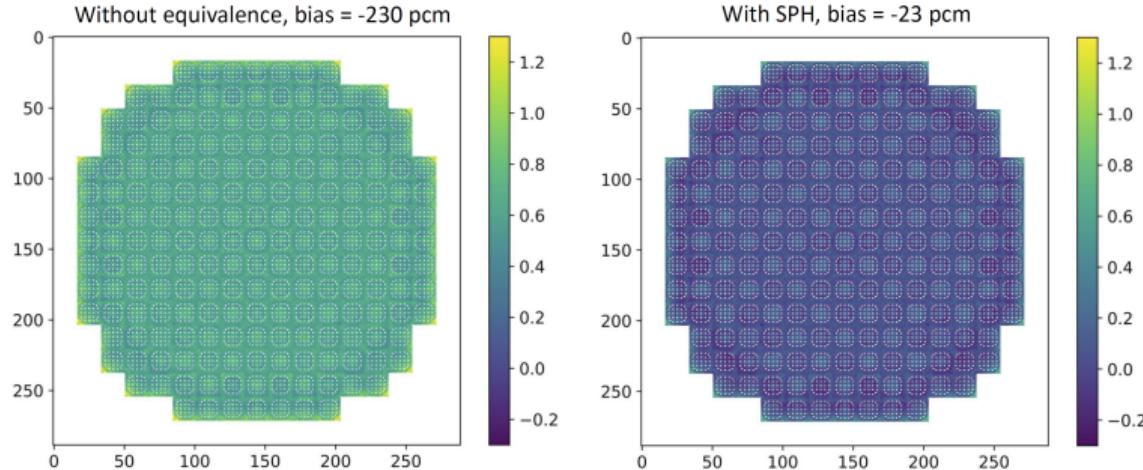
- Solve the low-order model with the adjusted cross-sections and get the low-order group flux
- Iterate until converged

# Pin cell example - Energy



SPH factors can be used in every region, but matter most in fuel. Typically, we only solve for SPH factors in the fuel regions since accounting for all regions can make the problem difficult to converge.

## At the core level - Fission to Capture Ratio



This demonstrates that SPH factors are preserving the ratio of fission rate to absorption rate throughout the core. Without SPH, the absorption rate from the U238 resonances is too large. SPH factors are typically generated in a smaller geometry (pin or assembly) and used in the larger system.

# Discussion

- Why do we need to generate MGXS?
- Can we use Monte Carlo methods to generate MGXS?
- If we use Monte Carlo, do we still need equivalence factors (like SPH)?
- What do equivalence correct?
- What are the pitfalls of equivalence factors?

# Goal

- The goal is to develop an efficient and accurate means of getting MGXS that can capture the spectral effects and spatial effects.
- There are 2 main approaches currently in the industry: equivalence theory and subgroup methods.
- They both rely on pre-computing information (resonance integrals or subgroup parameters) and finding simple effective ways to connect the environment to the pre-computed data.