

Exercise 5

Context:

On the boosting sequence classifier $\varphi = ((\alpha_1, \Psi_1), \dots, (\alpha_n, \Psi_n))$ of Exercise 4 we can extract a probability function for an unknown sample $(0, \hat{x})$ as follows

$$p_{\hat{x}} = \frac{\sum_{1 \leq i \leq n, \Psi_i(vt_i, \hat{x}_i)=1} \alpha_i}{\sum_{1 \leq i \leq n} \alpha_i}.$$

Assignment:

Build a multi-class conformal sequence classifier based on boosting binary classifier. Given (X, Y) , with $Y \in \{0, \dots, C-1\}$ as a training set, for the training phase we have the following steps for each class $0 \leq cl < C$

1. let Y_t^{cl} be the $\{1, -1\}$ array with $Y_t^{cl}[i] = \begin{cases} 1 & Y[i] = cl \\ -1 & \text{otherwise} \end{cases}$ for each $0 \leq i < |Y|$
2. split (X, Y) into (X_t, Y_t^{cl}) and (X_c, Y_c^{cl}) making sure that both 1 and -1 belong to both Y_t and Y_c ;
3. train a boosting sequence classifier φ_{cl} on (X_t, Y_t^{cl}) ;
4. calibrate φ_{cl} on (X_c, Y_c^{cl}) obtaining a conformal binary classifier Φ_{cl} .

For the predict phase given an unknown sample \hat{x} and $0 \leq \epsilon \leq 1$ returns the prediction region:

$$\{cl : \Phi_{cl}((0, \hat{x}), \epsilon) = \{1\}\} \cup \{\overline{cl} : \Phi_{cl}((0, \hat{x}), \epsilon) = \{-1\}\} \cup \{cl? : \Phi_{cl}((0, \hat{x}), \epsilon) = \{1, -1\}\}$$