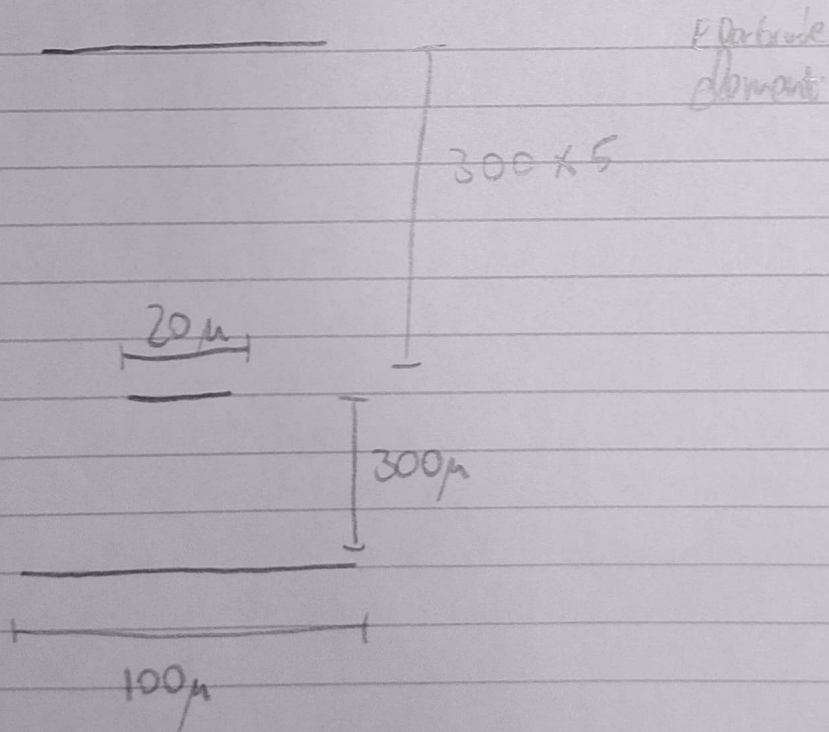
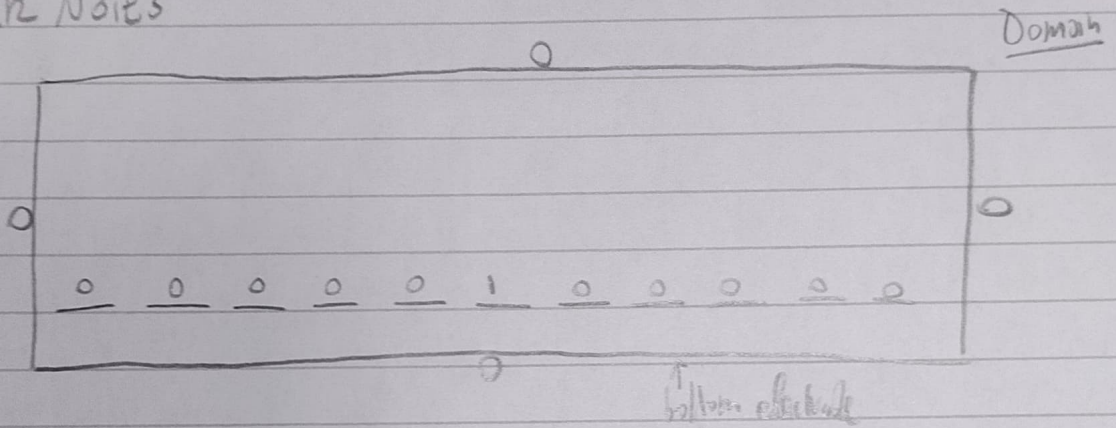


18/01/2022 - Weighting field  
Documentation

PI

# WEIGHTING FIELD

## PAPER NOTES



LAPLACES EQ FOR THE W.F.

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0$$

$$\frac{f_{i,j+1} - 2f_{i,j} + f_{i,j-1}}{S_x^2} + \frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{S_y^2}$$

$$= p_{i,j}$$

Using a cube mesh ~~many~~  $p=0$  since we are considering free space only

$$f_{i,j+1} + f_{i,j-1} - 4f_{i,j} + f_{i+1,j} + f_{i-1,j} = 0$$

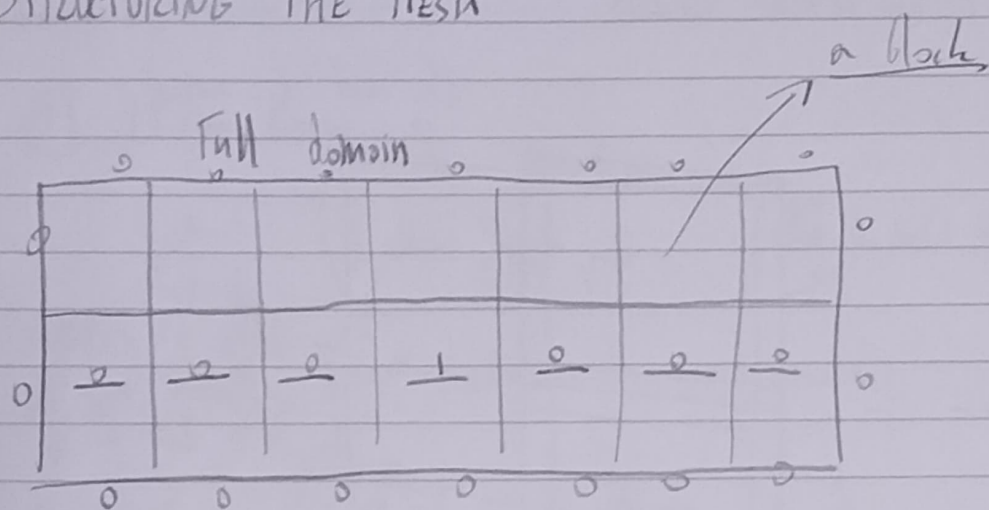
In terms of potentials:

$$V_S + V_W - 4V_P + V_E + V_N = 0$$



IP3

## STRUCTURING THE MESH



The mesh is divided into two type of blocks

### BLOCK NAMING

Naming convention for the blocks and for the transition cells

B-left-t	L-4-t	L-3-t	L-2-t	L-1-t	central	right-1	right-2	right-3	right-4	B-right-t
B-left	L-4	L-3	L-2	L-1	central	right-1	right-2	right-3	right-4	B-right

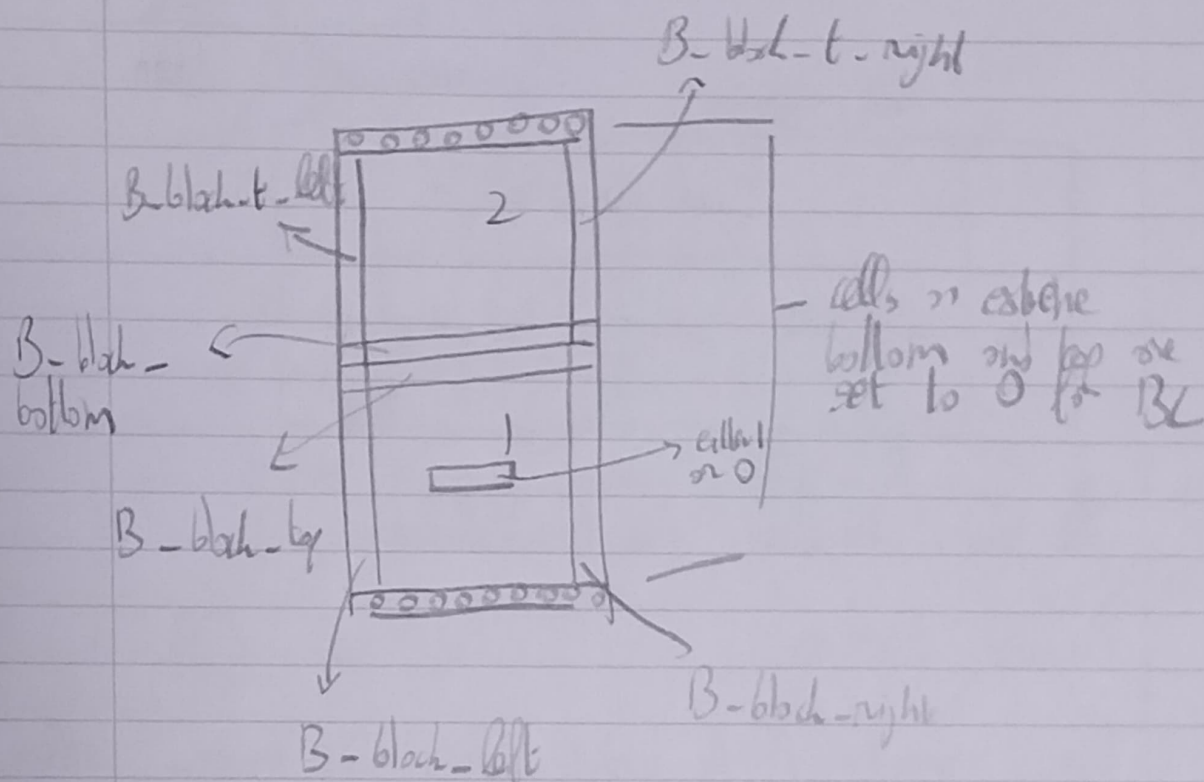
The transition <sup>cells</sup> are noted as

B - block name - boundary location  
i.e. B - L-1-t - left

(The mesh is divided in blocks) (Block <sup>type -</sup> structure mesh)

IP4

# TRANSITION CELLS



Note that  $B_{-}block_{-}l$  are  $N_y - 1$

likewise

$B_{-}block_{-}t_{-}l$  are  $N_y - 1$

the open  
rule is fixed

to 0

$B_{-}block_{-}top$   $N_x$   
 $B_{-}block_{-}bottom$

P5

## Flow Diagram

### 1 Define dimensions

$N-x-1$   
 $N-y-1$  } bottom blocks

$N-x-2$   
 $N-y-2$  } top blocks

define electrode location  
for bottom blocks

$$\frac{N-y}{3} * N-x + \frac{2}{5} N-x \quad [S]$$

$$+ \frac{3}{5} N-x \quad [E]$$

### 2 Use fns

block-with-electrode  
block-without-electrode

to define the  
matrix of coefficients  
(coeff. 1 eq. is included  
eq.)  
and matrix of positions

All bottom blocks will have block-with electrode  
(same eq.)

All top blocks

→ same position

### 3 Define initial potential pos.

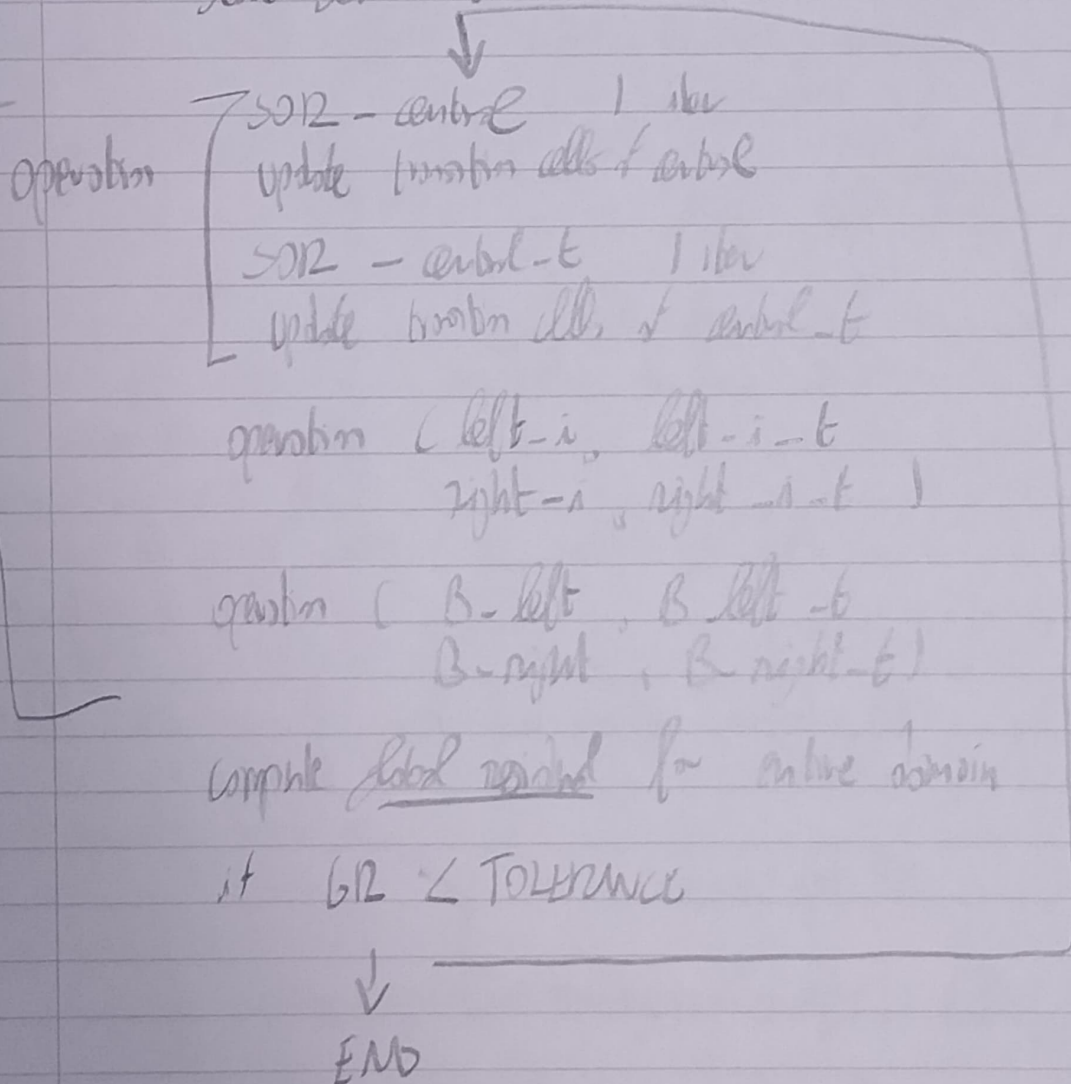
all 0 except for initial electrode set to 1  
this is also BC.

4) Define all transition cells (initially all to 0)

Notice that the transition cells for B-right (-t) (right)  
left (-t) (left)

are set to 0 and left to 0 since they are a BC

5) define source for each block  
start S012 iteration



\* At every operation step we must define the source since the transition cells are constantly updated.



P7

## SOLVING THE SYSTEM OF DISCRETISED EQUATIONS

System:

$$A \vec{V} = \vec{b}$$

matrix of coefficients  $\nearrow$  potential vector  $\nwarrow$  source vector

G.S. SOR:

$$X_i^{(n+1)} = (1-w) X_i^{(n)} + \frac{w}{2\alpha_i} \left( - \sum_{j < i} 2\alpha_j X_j^{(n+1)} - \sum_{j > i} 2\alpha_j X_j^{(n)} + b_i \right)$$

updated values  $\nearrow$

For the specific case of the weighting field:

$$V_p^{(n+1)} = (1-w) V_p^{(n)} + \frac{w}{(4)} \left( -V_s^{(n+1)} - V_w^{(n+1)} - V_e^{(n)} - V_n^{(n)} + b_i \right)$$

$b_i$  : source at the boundary cells only

P3

32	33	32	35	36			
24	25	26	27	28	28	30	31
16	17	18	19	20	21	22	23
8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7

2nd 50 51

$$2N_x \longrightarrow 3N_x - 1$$

$$N_x \longrightarrow 2N_x - 1$$

$$0 \longrightarrow N_x - 1$$

Cell positions:

S W P E N

$P - N_x$     $P - 1$     $\begin{matrix} \text{center} \\ \text{cell} \end{matrix}$     $P + 1$     $P + N_x$

mapping to  
adjacent vertices  
via

$$V_E = V[E] \text{ and so on.}$$



P. 3.

CREATING THE SOURCES FOR A BLOCK

two functions block-with-electrode-source  
block-without-electrode-source

create the source terms

GLOBAL RESIDUAL

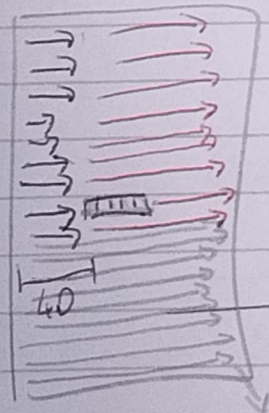
$$A \vec{V} = \vec{b}$$

$$\sum_i (A_{ij} V_j - b_i)^2 = R_{-tot}$$

How far away we are from satisfying the system of linear eq. with our iterative solution

NOTE ON THE DIFFUSION OF THE BC

0th iteration



Block with electrode

In order for the BC to diffuse to the left you require 40 iterations at least

updates nothing so far

sweep of the domain

The solution will diffuse much more quickly to the right than the left

In order to reach Left -2 at least 240 iterations are needed.

Poisson Eq

$$\frac{f_{i,j+1} - 2f_{i,j} + f_{i,j-1}}{\Delta x^2} + \frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{\Delta y^2}$$

$$= -\frac{\rho_{i,j}}{\epsilon}$$

$$\nabla^2 V = -\frac{\rho}{\epsilon}$$

since

$$\vec{D} \cdot \vec{E} = \frac{\rho}{\epsilon}$$

$$\vec{E} = -\nabla V$$

$$\nabla^2 V = -\frac{N_e}{\epsilon} \quad \rightarrow \text{dopant concentration}$$

$$\nabla^2 V = -\frac{N_e}{\epsilon_2 \epsilon_0}$$

$$\nabla^2 \left( \frac{V \epsilon_0}{e} \right) = -\frac{N}{\epsilon_2}$$

 $N_0$ : reference concentration

$$\nabla^2 \left( \frac{V \epsilon_0}{N_0 e} \right) = -\frac{N}{\epsilon_2 N_0}$$

Source

solution

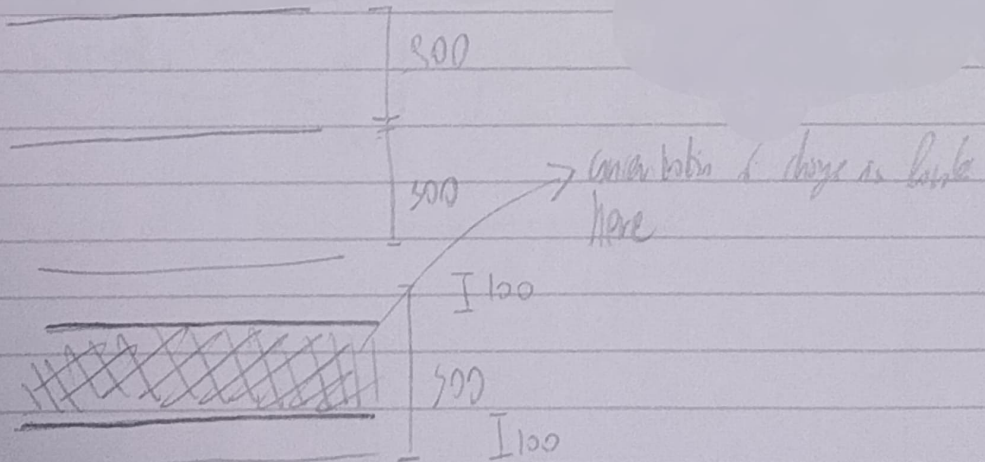
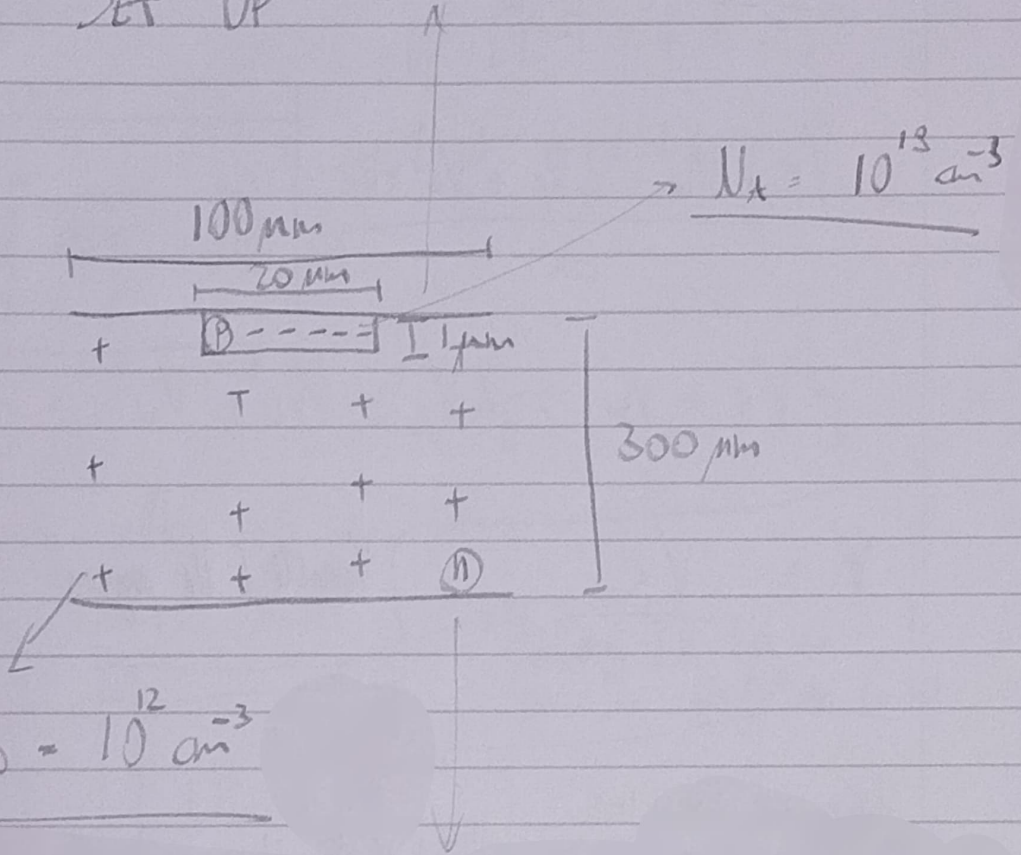
$N_0$  is dopant concentration of bulk Si,  $n$  Si



IP2

$$N_0 = 10^{12} \text{ cm}^{-3}$$

Set Up



Assume a thickness of 0.5  $\mu\text{m}$

→ In order to get surface charge density all concentrations have to be multiplied by this value



P3

DISCRETISED EQ.

$$\frac{V_S + V_W - 4V_P + V_E + V_N}{\Delta x^2} = - \frac{Ne}{\epsilon_r \epsilon_0}$$

$$\frac{V_S + V_W - 4V_P + V_E + V_N}{\Delta x^2} \frac{\epsilon_0}{en_0} = - \frac{N}{N_0 \epsilon_r}$$

$$V'_S + V'_W - 4V'_P + V'_E + V'_N = - \frac{N}{N_0 \epsilon_r} \quad \ominus$$

$$V' = \frac{V \epsilon_0}{N_0 e \Delta x^2} \quad \text{Variable of the program}$$

Hence we can use

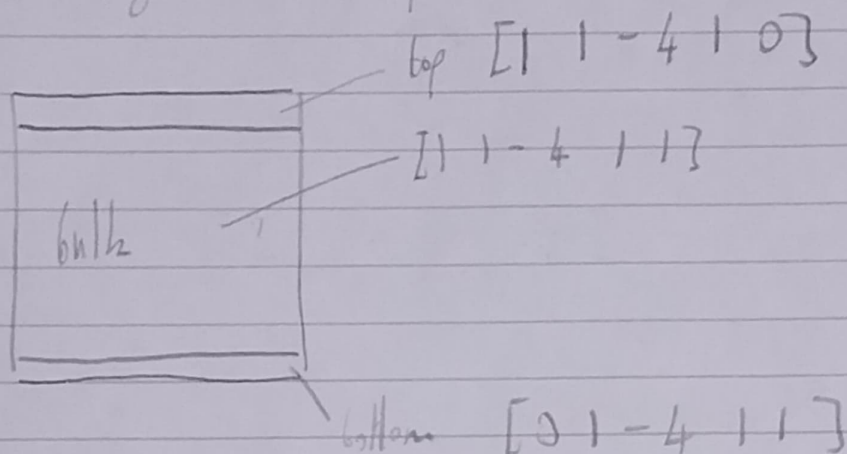
$$\begin{aligned} \frac{N_A}{N_0} &= 10^7 \\ \frac{N_D}{N_0} &= 1 \end{aligned}$$

the p impurity is almost fully depleted, so we can just say that all of it is  $\ominus$  charge

P4

# COEFFICIENTS IN THE DISCRETISED EQUATIONS

Square/rectangular bulk periodic B.C.



Neighbouring cells

