# Dataset

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This thesis wants to be somehow usable by practitioners. Everything that we will show from a theoretical point of view is also implemented and used on real market data. The data used are the SPX options chain of the 23$^{\text{rd}}$ January 2023 (source: Bloomberg), with moneyness from 80% to 120% and tenors from 2 weeks to almost 10 years. The closing price of that day was 4019.81.

## Implied drift term

Something that is usually assumed in academic papers is that the forward is flat. This means that we are implicitly assuming that there is no drift term in the risk-free world dynamics. This is reasonable in a low rates environment and if we are considering an underlying with a very low yield. As you can imagine, this is not acceptable for us, especially since the high rates environment of the last year, so we will take great care into keeping the drift term in all of our calculations. The drift term is the difference between the risk-free rate and the yield, continuously compounded, of the underlying. To be completely precise it includes not only the yield, but also the borrowing cost. In order to estimate that we recall the call-put parity. We will use the following notation:

- $C_{t,K}$ is the value of a call at time $t$ with strike $K$ and tenor $T$;
- $P_{t,K}$ is the value of a put at time $t$ with strike $K$ and tenor $T$;
- $S_t$ is the spot value of the underlying;
- $r$ is the risk-free rate (continuously compounded);
- $q$ is the yield of the underlying (continuously compounded).

Then the following hold:

$$C_{t,K} - P_{t,K} = S_t e^{-q(T-t)} - K e^{-r(T-t)}$$

solving for $q$ we obtain:

$$q = \frac{1}{t-T} \log\left(\frac{C_{t,K} - P_{t,K} + K e^{-r(T-t)}}{S_t}\right)$$

now this relation still holds for $t = 0$ and we can observe the market price of spot $S_0$, the call market-price $\hat{C}_{0,K}$ and the put price $\hat{P}_{0,K}$. So the relation now is:

$$q = -\frac{1}{T}\log\left(\frac{\hat{C}_{0,K} - \hat{P}_{0,K} + Ke^{-rT}}{S_0}\right)$$

one can argue now that $q$ can depend on the strike, but this is not true. Indeed, let's suppose that this is true and consider two strikes $K_1 < K_2$ with $q_{K_1} > q_{K_2}$. Buy the following portfolio at time $t = 0$:

$$\frac{1}{S_0}(C_{0,K_1} - P_{0,K_1} + K_1 e^{-rT}) - \frac{1}{S_0}(C_{0,K_2} - P_{0,K_2} + K_2 e^{-rT})$$

due to call-put parity buying this portfolio at time zero let you receive a premium, indeed its value at time 0 is

$$\frac{(C_{0,K_1} - P_{0,K_1} + K_1 e^{-rT})}{S_0} - \frac{(C_{0,K_2} - P_{0,K_2} + K_2 e^{-rT})}{S_0} = e^{-q_{K_1}T} - e^{-q_{K_2}T} < 0$$

At maturity the value of the portfolio is 0, because $C_{T,K_1} - P_{T,K_1} + K_1 = S_T = C_{T,K_2} - P_{T,K_2} + K_2$. So this strategy is an arbitrage. The same, with obvious modifications, applies if $q_{K_1} < q_{K_2}$. So in conclusion the term $q$ depends only on the tenor. To estimate the risk-free rate we used the values of the zero coupon USD OIS Swap rates, interpolated with cubic splines for the missing tenors. Moreover, for tenors greater than 10 years, we supposed a flat rate equals to the 10-year rate. Here below there is the curve for the implied drift obtained, as we can see such curve is not zero at all.