HANDIN 1

WORKSHOP 2 - S1725018 - NLA - LORENZO STIGLIANO

EXERCISE 1

i) Write an algorithm called algorithm OP that computes the so called outer product $\vec{x}\vec{y}^T \in \mathbb{R}^{n\times n}$ of vectors $\vec{x}, \vec{y} \in \mathbb{R}^n$

Algorithm OP ; outer product
Input : $\vec{x}, \vec{y} \in \mathbb{R}^n$
Output : $A = \vec{x}\vec{y}^T$

1. for i, ..., n do :
2. for j, ..., n do :
3. $A_{ij} = x_i y_j$
4. end for
5. end for
6. return A

ii) The cost of algorithm OP is $c(n,n) = \sum_{i=1}^{n} \sum_{i=1}^{n} 1^* = \sum_{i=1}^{n} n = n \cdot n = n^2 \therefore c(n) = n^2$

sum over the columns

sum over the rows

*there is a 1 because we only preform 1 computation per for loop ($x_i y_i$).

iii) $(I - \vec{u}\vec{u}^T)\vec{x}$ for $u, x \in \mathbb{R}^n$

$= (I\vec{x} - \vec{u}\vec{u}^T\vec{x})$ ← $Ix = x$ with compute cost $O(n)$

$= \vec{x} - \vec{u}\vec{u}^T\vec{x}$

$= \vec{x} - \lambda\vec{u}$

$u^T x = \lambda$ for some scalar and compute cost for dot product is $O(n)$

$\vec{x} - \lambda u = O(n)$ since we multiply each value by λ. and take away $\vec{x}$

So as a result our compute cost is:

we first re-write our expression as $I\vec{x} - \vec{u}\vec{u}^T\vec{x}$ $C(1)$
then we calculate dot product $\lambda = \vec{u}^T\vec{x}$ at cost $\underline{n}$
then we calculate $I\vec{x}$ again at cost $\underline{n}$
then we are left with $\vec{x} - \lambda\vec{u}$ where multiplying by scalar and subtracting $\vec{x}$ from $\lambda\vec{u}$ is at a cost $\underline{n}$

so cost of algorithm = $n + n + n = 3n = O(n)$ □.

EXERCISE 3

(i) Let $x^T = [4, 1, 3, -3, -1]$ $\|x\|_1$, $\|x\|_2$, $\|x\|_\infty$

$$\|x\|_1 = \sum_{i=1}^{n} |x_i| = 4+1+3+3+1 = \boxed{12}$$

$$\|x\|_2 = \left(\sum_{i=1}^{n} x_i^2\right)^{1/2} = (4^2+1^2+3^2+3^2+1^2)^{1/2} = \boxed{6}$$

$$\|x\|_\infty = \max_{1\le i\le n} |x_i| = \boxed{4}$$

(ii) $A = \begin{bmatrix} 3 & -5 & 0 \\ 2 & 3 & 2 \\ 6 & -7 & 8 \end{bmatrix}$ $\|A\|_1$ and $\|A\|_\infty$

By Theorem 1.5 $\|A\|_1 = \max_{1\le j\le n} \sum_{i=1}^{m} |a_{ij}|$, max column sum.

$$\therefore \|A\|_1 = \max(|3|+|2|+|6|, |5|+|3|+|7|, |0|+|2|+|8|) = \max(11, 15, 10) = \boxed{15}$$

$\|A\|_\infty = \max_{1\le i\le m} \sum_{j=1}^{n} |a_{ij}|$, max row sum by 1.5

$$\therefore \|A\|_\infty = \max(|3|+|-5|+|0|, |2|+|3|+|2|, |6|+|-7|+|8|)$$
$$\max(8, 7, 21) = \boxed{21}$$