Superconductivity and the Meissner Effect

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1 Introduction

Ever since its discovery in the first decades of the 20^{th} century, superconductivity has attracted more and more physicists that right from the beginning saw many possible applications of this phenomenon. This shared interest of the scientific community is testified by the number of Nobel prizes won by people who made interesting findings related to superconductors. This field of research is still pretty active and many of the world-leading scientists are currently working on finding a room temperature and easy to implement superconductor. This can be considered the "Holy Grail" of superconductivity and would be worth at least another Nobel-prize. To better understand why superconductivity is so important we will give in this paper an overview of what this phenomenon is and what are its peculiarities.

DISCLAIMER: Even though this is a quantum mechanics phenomenon we will try to describe it using classical electrodynamics as good as possible.

2 Perfect conductors

One of the defining properties of superconductors is that their electrical resistance abruptly drops to zero below a critical temperature. For a generic object, resistance is the measure of its opposition to the flow of electric current. In imperfect conductors, resistance is caused by the collisions of free electrons with atoms as they flow through the material, which causes energy to be lost (this is Joule's heating law). Consequently, higher temperatures induce more collisions between the electrons that are flowing and the atoms and so a higher electric resistance. Conversely, there is less resistance if there is a lower temperature. As side note, we should discern the difference between Superconductors and Perfect Conductor: indeed, a perfect conductor is a material which have zero electric resistance regardless of any conditions, and it is just a theoretical assumption. Superconductors are materials that obtain the particular property of having zero electric resistance only when they are below their critical temperature, and which completely expel their interior magnetic field at these temperatures, which is the Meissner effect.

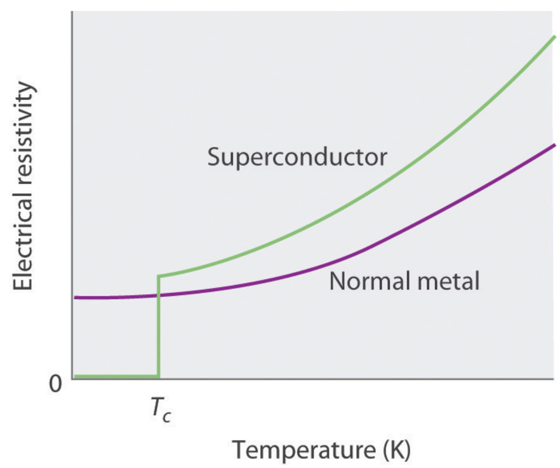

Figure 1: Relationship between resistivity and temperature in normal metals and superconductors. T_c is the critical temperature

In order to explain the cause of zero resistance in superconductors, one must rely on quantum mechanics. BCS (Bardeen-Cooper-Schrieffer) theory is the first (and, so far, most valid) theory which aims to explain superconductivity at a microscopic level. However, it does not manage to encompass all superconducting materials, which are thus split into two categories: Conventional superconductors, which conform to BCS theory, and unconventional superconductors, which do not. The latter group includes most high-temperature superconductors (superconductors with a critical temperature greater than $77K$).
Before entering into the BCS theory, we want to mention also the Ginzburg-Landau Theory. The aim of this theory is to give a phenomenological explanation of type I superconductors. Later this theory will be replaced by the one formulated by Bardeen, Cooper and Schrieffer (BCS), which is the one we are going to explain. The main results of Ginzburg-Landau theory are: penetration depth (λ) and the coherence length (ξ), which we will talk about more later, since they are essential for the classification of superconductors in type 1, 2 and 1.5.

2.1 BCS Theory

The BCS theory places an attractive bond between electrons caused by their interactions with the atomic lattice at the basis of superconductivity. This attraction is favored when the electrons have opposite spin and moments. The bonded state formed in this way is called a Cooper pair. The BCS theory wants to explain the superconductivity phenomena as an ordered and coherent set of

Cooper pairs, let's see it more in details.

Imagine that a superconducting material can be viewed microscopically as a lattice of positively charged nuclei (or ions) with a "fluid" of free electrons surrounding them, and that there is a current flowing in the material. As the electrons flow at low temperatures, they will attract the positive nuclei which are close to them by Coulomb's law (this motion of ions is known as a phonon). Let us focus on an individual electron momentarily: as the ions approach the electron, they generate an area of overall positive charge density. This collection of ions attracts a second electron. By the time this electron will reach the area of positive charge, the original one will already have moved on and will be restarting the same process with other ions. The electrons will thus maintain a large distance between each other, which renders the repulsive Coulomb force between them negligible. This attractive force between the two electrons allows them to form something called a Cooper pair, which are essential to the explanation of superconductors. The same process will take place for the other electrons. Note that the formation of Cooper pairs can only occur at very low temperatures. This is because the binding energy of a Cooper pair is of the order of $10^{-3}eV$ (electronvolt), which can be easily overcome even at low temperatures.

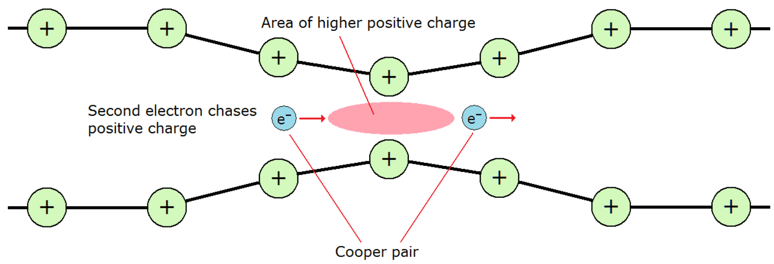

Figure 2: Formation of a Cooper pair

Electrons have a spin of $\pm\frac{1}{2}$ which means that they are fermions. By the Pauli Exclusion Principle, they cannot exist in the same quantum state. However, Cooper Pairs behave like composite bosons, since their spins sum to 0 or ± 1. Therefore, unlike electrons, Cooper pairs can exist in the same quantum state since they do not obey the Pauli Exclusion Principle. Since isolated systems always seek to exist at the lowest energy level (by the second law of thermodynamics), Cooper pairs will condense into the same ground quantum state (the lowest energy state of a system) and form a Bose-Einstein Condensate. Clearly, the bosonic nature of Cooper pairs is essential to allow them to simultaneously occupy the same energy level. In this condensed state, something called an energy gap (Δ) arises in the energy spectrum of the condensate. Essentially, this means that there is a band of energy levels that the collection of Cooper pairs cannot attain. Consequently, this energy gap imposes that a minimum amount of energy is required to excite the condensate.

The size of the energy gap is given by the difference between the bosonic ground state and the lowest possible fermionic state, which can only be obtained if the electrons are unpaired (and regain their fermionic characteristics). Therefore, the energy gap is essentially the energy needed to break a Cooper pair. It is found to vary with the temperature of the system and be dependent on the critical temperature of the material. At a temperature of 0K, the relation between Δ and the critical temperature is:

$$\Delta = 1.764 k_B T_c$$

As the temperature of the system, T, approaches T_c, the relationship becomes:

$$\Delta = 3.06 k_B T_c \sqrt{1 - \frac{T}{T_c}}$$

Once the critical temperature is surpassed, the energy gap vanishes, the Cooper pairs break and the superconductive properties are consequently lost. However, at lower temperatures, the electrons' collisions with oscillating atoms will not deliver enough energy to excite the condensate, which results in an undisturbed flow of Cooper pairs (this flow of Cooper pairs is akin to that of a superfluid, which is a fluid with zero viscosity). This is essentially what leads to a resistance-free current. The theory predicts the Meissner effect, i.e. the ejection of the magnetic field of superconductors.

One of the peculiarities of superconductors is that materials which would usually be considered the best "normal" conductors, such as copper, do not exhibit superconductivity, while metals that are "bad" conductors do (This is noticeable in Figure 1). This can be explained through the BCS theory: good conductors have very little resistance to begin with, since the interactions between electrons and the lattice is very weak. However, these interactions are the source of the formation of Cooper pairs and thus the superconductive state! In the best conductors they are so weak that they do not allow the formation of Cooper pairs. In these materials, the resistance drops with temperature but does not reach zero!

Clearly, BCS theory is heavily reliant on low temperatures to explain phenomena relating to Cooper pairs. For this reason, it is unable to accurately explain the mechanisms of superconductors with high critical temperatures. Unfortunately, there is still no valid theory which can do this.

3 Meissner Effect

We can observe that when a superconductor is cooled down to a temperature below a certain temperature T_c and it is immersed in a small enough magnetic field it expels the latter. This is due to small currents that flow on the surface of the superconductor and these currents generate an equal magnetic field but opposite in sign to the one applied, and for this reason the resulting magnetic field in the material is zero. We can say that superconductors behave like perfect diamagnets. Physically speaking superconductors have magnetic susceptibility $\chi_m = -1$. Thus the magnetic field, that can be written as $\vec{B} = \mu_0(1+\chi_m)\vec{H}$ is equal to 0, coherently with what we know about superconductors.

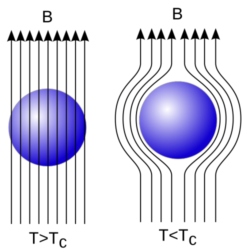

Figure 3: Meissner effect

To better understand this phenomenon let's first analyze what happens if we place a magnet above a superconductor cooled at temperature T_c. In this case the gravity will tend to move the magnet down in the direction of the superconductor and more field lines will concatenate the surface of the superconductor, hence a variation of the flux will arise. As a consequence an electromotive force will be generated, due to Faraday's law (i.e. $\varepsilon = -\frac{d\Phi(\vec{B})}{dt}$), that will put in motion the electrons generating an induced current. Given that the electrical resistance is zero this induced current will continue to flow ideally forever (experiments have been actually carried out to verify this and scientists were able to let the same current flow in a superconductor for 23 years without any dissipation of energy). This will generate a magnetic field opposed to the one that induced the current and for this reason the magnet will stay suspended. This really coincides with the intuition of the repulsion force that we experience by putting two equal poles close to one another.

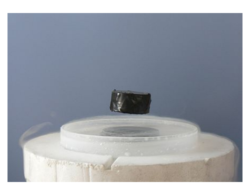

Figure 4: Magnetic levitation

Now we have to pay some attention because even though superconductors behave in certain conditions like perfect diamagnets as stated before, they're **not** the same thing! To better understand this think about the following situation: you start with a certain material that admits superconductivity and insert it in a uniform magnetic field. When you lower the temperature below the critical temperature T_c the material will become a superconductor and the magnetic field will be expelled. If you think about it, though, you don't have any change in the flux of the magnetic field and therefore you shouldn't have any kind of induced emf and indeed you don't have such an effect if the material is just a perfect diamagnet.

More precisely these surface currents are able to flow up only to a certain depth and this distance to the surface of the material, where also the magnetic field can penetrate, is called London penetration depth λ. To prove that the magnetic field can penetrate the surface at most at depth we should use a combination of Maxwell's equation and quantum mechanics, but with some clever and naive manipulations of just the Maxwell equations we can still explain this phenomenon. That is what Fritz and Heinz London did back in 1935, when they came out with the so called "London Equations".

To write the first one we start from the equations of motion of an electron inside the superconductor. Given the fact that there's no resistance, this can be modelled as a particle moving freely into an electric field.

$$m\frac{d\vec{v}}{dt} = -e\vec{E}$$

We can now rewrite the equation using the fact that $\vec{J} = -en_s\vec{v}$ (where e is the charge of the electron, m is the mass of the electron and n_e is the density of superconducting electrons per unit volume) we get:

$$\frac{d\vec{J}}{dt} = \frac{n_s e^2}{m}\vec{E}$$

This is called first London equation. If we now take the curl on both sides we obtain:

$$\frac{d}{dt}[\vec{\nabla} \times \vec{J}] = \frac{n_s e^2}{m}[\vec{\nabla} \times \vec{E}]$$

Rewriting the curl of $\vec{E}$ using the third Maxwell equation (i.e. $\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$) and integrating on both sides we eventually obtain:

$$\vec{\nabla} \times \vec{J} = -\frac{n_s e^2}{m}\vec{B}$$

And this is the second London equation. The sloppiness of this result is in the fact that we can't assume a uniform electric field $\vec{E}$ in the first place.

To see where the Meissner effect comes into play we need a further step applying the fourth Maxwell equation (i.e. $\vec{\nabla} \times \vec{B} = -\mu_0\vec{J}$), to rewrite $\vec{J}$:

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = -\frac{\mu_0 n_s e^2}{m}\vec{B}$$

$$\vec{\nabla}(\cancel{\vec{\nabla}\cdot\vec{B}}) - \nabla^2\vec{B} = -\frac{\mu_0 n_s e^2}{m}\vec{B}$$

You can cancel the first term on the LHS since the divergence of $\vec{B}$ is always zero by the second Maxwell equation. Now we introduce a new variable λ (this is exactly the London penetration depth mentioned before) defined as:

$$\lambda = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

The cleaned equation that comes out is:

$$\nabla^2 B = \frac{1}{\lambda^2}B$$

and solving the differential equation we conclude that:

$$\vec{B}(x) = \vec{B}_1 e^{\frac{x}{\lambda}} + \vec{B}_2 e^{-\frac{x}{\lambda}}$$

That implies that the magnetic field vanishes at a distance λ from the surface as mentioned above. To have an idea of the order of magnitude of this length, that in the end depends on the material, you can think that it ranges more or less from $50nm$ to $500nm$

To have a more intuitive idea of why the current in a superconductor is confined almost all to the surface let's see what we can derive from Maxwell's equations using the assumption that the conductivity of the material is infinite and B inside the medium is zero (that we just stated is almost true). The infinite conductivity σ implies $\vec{E} = 0$ since $\vec{J} = \sigma\vec{E} \Rightarrow \vec{E} = \frac{\vec{J}}{\sigma} \Rightarrow \vec{E} = 0$

Using Ampere-Maxwell ($\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\ \frac{\partial\vec{E}}{\partial t}$) and the fact that $\vec{E} = 0$ and $\vec{B} = 0$ we deduce that $\vec{J} = 0$ and hence the current can only be localized on the surface.

4 Types of Superconductors

Superconductors vary a lot between themselves. That's the reason why we classify and divide them in three categories: Type I, Type II and Type 1.5. Let's analyze and discuss each of the types of superconductors we mentioned.

4.1 Type I

The most common examples of superconductors of type I are metals.

They present a very low critical temperature T_c (between 0 K and 10 K): when cooled down to these temperatures, they start to be superconductors!

Subsequently, in this condition they expel completely the magnetic field, due to induced currents on the surface.

They retain this state of superconductivity until even a relatively low magnetic field is applied on them, after which they act as normal conductors: this critical value is called H_c.

Superconductor of types I obey perfectly the Meissner Effect : the magnetic field cannot penetrate them! Connecting us with the previous part of the article, superconductors of this type are described by the BSC theory.

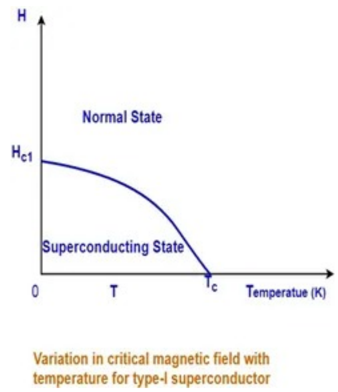

Figure 5: Phase Diagram of Type 1 superconductors

4.2 Type II

Superconductors of type 2 present a slightly more complicated and different behaviour.

They have a very different critical temperature T_c(up to 250 K), usually higher than the one for Type I materials. Superconductors of this type presents 2 different critical values, leading to a quite different behaviour and phenomena. The phase transition proceeds in this way: first, when cooled down, they act as type I superconductors, but if now an external magnetic field is applied above the critical value H_{c1}, they don't lose immediately their properties, but instead they do let the magnetic field penetrate them. We can consider this as an intermediate step.

In this stage of the process, analyzing what happens at a microscopic level, in specific points where the magnetic field has penetrated, we can see the formation of the so called magnetic field vortex. In this stage, the bulk of the material is still superconducting, but at these vortices the material acts as a normal conductor. Basically, in those specific points the magnetic field goes through them in a certain small amount called magnetic flux quanta, and each one of those points will be surrounded by a swirling supercurrent, which invalidate the properties of the superconductors in those points.

The vortex densities increase proportionally with the increase of the field strength. When this fields reach the higher critical level H_{c2}, superconductivity is lost and the material goes back to a normal conducting state.

It is quite obvious from what explained above that those superconductors do not obey completely the Meissner effect.

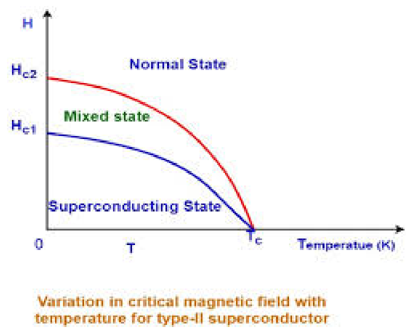

Figure 6: Phase Diagram of Type 2 superconductors

4.3 Type 1.5 Superconductors

Recently, the idea of the existence of type 1.5 superconductors has been carried on, besides is not widely recognized across the scientific community.

This theory was introduced by Egor Babaev, and it states that superconductors of type 1.5 presents two or more coherence lengths at least one of which is longer than the magnetic field London penetration length λ. The magnetic field London penetration is a quantity that measures the depth of penetration of a magnetic field inside a superconductor. Indeed, it decreases exponentially as it moves away from the surface of the material.

In superconductivity, the coherence length, is a quantity that expresses the attenuation of the perturbations of the material. It is usually denoted with ξ. To be more precise, the change in density of the electrons needs a minimum length in which that given change can happen to not destroy the superconducting state.

Theoretically, this material would let be possible to allow the existence simultaneously of properties of both type I and II superconductors. This theory goes in contrast with the properties of the previous two type of superconductors, in which there is only one coherence length. For type I superconductors $\xi > \lambda$, while for type II superconductors, $\xi < \lambda$.

Those are the main results from the Ginzburg-Landau theory.

5 Exploitation of superconductivity today

The advances of modern science promise to deepen the theoretical and practical understanding of superconductivity and superconductors, with potential benefits for humankind. State-of-the-art research is currently trying to achieve superconductivity at room temperature (or higher than 77 Kelvin), since cooling down materials to low temperatures around proves itself to be a costly activity. However, there is a vast range of areas where superconductivity plays quite a role. In particle accelerators, superconductive phenomena serve for the creation of intense magnetic fields, under whose influence collisions of small particles is studied. At the same time MRI, also known as magnetic resonance imaging, relies, as the name suggests, on stable magnetic fields generated by superconductors. MRI is revolutionary since no humanly harmful radiation is generated. Most exciting though, is the application of superconductivity in the field of transportation in the form of magnetic levitation. In particular, the celebrated maglev bullet-trains reach high velocities, mediated by drastic acceleration since friction is fantastically reduced.

High temperature superconductivity on the other hand, which is not explained by the Bardeen-Cooper-Schrieffer theory (since quantum physics phenomena play a bigger part), has been successful on the small scale with the SQUID for example, a device that is capable of detecting the most sensible magnetic fields on earth. This last extremely complex device is based on something called Josephson junction and it is worth a paper itself, but given its complexity as quantum device we don't have the tools to properly describe it. Among its applications we can find the above mentioned MRI and fMRI machines, some efficient radio frequency antennas and quantum computers. In this last application superconductors (Josephson junctions, actually) are used to create a superposition of currents and this superposition is the leading principle of Q-bits and quantum information theory.

We look forward to see developments in this field as they can be a milestone in the simulation and comprehension of complex systems and more in general of the world around us.