Sports Tournament Scheduling problem

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1 Introduction

The problem we tackle in this project is known in the literature as the *Single Round-Robin* (SRR) tournament scheduling problem, and it is recognized to be NP-hard. In order to solve this problem, we first propose solutions for the feasibility part, secondly we focus on minimizing the so-called breaks, which consist of 2 consecutive home(HH)/away(AA) games by the same team, to guarantee fairness. Specifically, in order to do so, we will minimize the total number of breaks of all the teams, a problem well studied in the literature. To model the problem, we introduce some input parameters that we will use across all the solutions:

Symbol	**Domain**	**Meaning**
T	set	teams, $\|T\| = n$ (even)
W	set	weeks, $\|W\| = n - 1$
P	set	periods per week, $\|P\| = n/2$
$t, t' \in T$	indices	distinct teams
$w \in W$	index	week
$p \in P$	index	period

Inspired greatly by [1], we define the main decision variables used in all our modeling approaches (their actual implementation will change depending on the modeling paradigm):

Variable	**Meaning**
$opp_{w,t}$	Opponent of team t in week w
$h_{w,t}$	Whether team t plays at home in week w
$per_{w,t}$	Period assigned to team t in week w

The problem contains different hard constraints:

- Each week w pairs the n teams into $n/2$ disjoint matches.
- Each team plays once against every other team

$$\{\text{opp}_{w,t} \mid w \in W\} = T \setminus \{t\} \quad \forall t \in T. \tag{1}$$

- Each team must play a maximum of two times per period:

$$\sum_{w \in W} \big[\, t \text{ plays in period } p \,\big] \leq 2 \quad \forall (t, p) \in T \times P. \tag{2}$$

- The lower bound and upper bound of the objective function are common to all models, and the lower bound, as shown in [2] is:

$$n - 2 \tag{3}$$

- The maximum upper limit of the breaks is, as cited in [3]:

$$(n^2) - 3n + 2 = (n-1) * (n-2) \tag{4}$$

The models contain several symmetries that can be exploited to heavily shrink the search space, we chose these comparing actual improvements on the models:

- Permutation of the weeks by requiring team 1 to face successively higher-numbered opponents (strictly increasing order).
- Team1 home opener
$$h_{1,1} = 1 \tag{5}$$
helps breaking H/A complement.

We have also implemented an implied constraint, which enforces that if team i plays against team j in week w, then team j must also play against team i in the same week, this is strictly deducible from the other constraints but writing it to the solver helped in many cases fasten up the solutions.

$$\forall\, w \in W,\ \forall\, i, j \in T\ (i \neq j):\quad \mathrm{opp}(w,i) = j \implies \mathrm{opp}(w,j) = i. \tag{6}$$

2 CP model

Decision variables

The model relies on the following decision variables:

- $opp_{w,t} \in T \quad w \in W,\ t \in T$, it represents the opponent of the team t in week w
- $h_{w,t} \in \{0,1\} \quad w \in W,\ t \in T$, it represents the home/away indicator: $h_{w,t} = 1$ if team t plays at home in week w; $h_{w,t} = 0$ if team t play away in week w, used only for the optimization part.
- $per_{w,t} \in P \quad w \in W,\ t \in T$, it represents the period in which team t plays during week w.

Objective functions

As discussed in the introduction part, the objective values will focus on minimizing the number of breaks. In order to do so we introduce a binary variable

$$break_{w,t} \in \{0,1\}, \qquad w = 2, \ldots, n-1,\ t \in T, \tag{7}$$

$$break_{w,t} = \begin{cases} 1 & \text{if } h_{\,w-1,t} = h_{\,w,t}, \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

The objective function is defined as:

$$obj = min \sum_{t \in T} \sum_{w=2}^{n-1} break_{w,t}. \tag{9}$$

Constraints

The SRR problem we tackle contains several constraints that need to be respected, here we list our implementation of those who helped fastening the results:

- Forbids that a teams plays against himself

$$\forall w \in W, t \in T, \quad opp[w,t]! = t \tag{10}$$

- Channeling constraints that ensures that the opponent assignment is bijective for each week (i.e., every team is assigned a unique opponent and vice versa):

$$\forall w \in W, \quad \texttt{inverse}\left([\text{opp}[w,t] \mid t \in T], [\text{opp}[w,t] \mid t \in T]\right) \tag{11}$$

- Ensures that both opponents are scheduled in the same period:

$$\forall w \in W, \ t \in T, \quad \text{per}[w,t] = \text{per}[w, \ \text{opp}[w,t]] \tag{12}$$

- Every period must appear 2 times in the list of assignment for that week (2 teams)

$$\forall w \in W, \quad \texttt{global\_cardinality}\left([\text{per}[w,t] \mid t \in T], \ P, \ [2 \mid \ \_ \in P]\right) \tag{13}$$

- Single round robin, each team meets the other ones exactly one time during the weeks

$$\forall t \in T, \quad \texttt{alldifferent}\left([opp[w,t] \mid w \in W]\right) \tag{14}$$

- A team can play maximum 2 times per period

$$\forall\, t \in T: \quad \begin{aligned} &\texttt{global\_cardinality\_low\_up}\Big(\left[\,per[w,t] \mid w \in W\,\right], \ P, \\ &\qquad\qquad \left[\,0 \mid \_ \in P\,\right], \ \left[\,2 \mid \_ \in P\,\right]\Big) \end{aligned} \tag{15}$$

- One team play home, one play away.

$$\forall w \in W, \ t \in T \quad h[w,t] + h[w, opp[w,t]] = 1 \tag{16}$$

Symmetry breaking

We fixed 2 symmetry breaking constraint proving that improve the results:

- Fixing team 1 in home the first week, in this way we remove all possibility of permuting home and away games, as cited in [2]

$$h[1,1] = \text{true} \tag{17}$$

- Enforces not permutation of the teams over the weeks by a lexicographic order (we tried using global constraint lex_less but did not show improvement)

$$\forall w \in \{2, ..., n-1\}, \quad ([\text{opp}[w,1] > \ [\text{opp}[w-1,1]) \tag{18}$$

Validation experiments

We experimented with different solvers(Gecode, Chuffed and OR-tools) both for the feasibility and the optimization part. We divide the search, in 3 sequential search, tackling first variable *opp*, finding opponents for each team (which is the most combinatorial subpart), then *per* (choosing the periods), then *h* (if home/away games).

Feasibility experiments

We show the different experiments tackled in order to find an efficient search for the variable *opp*. Once *opp* is fixed, the domains of *per* and *h* reduces, so we completed the search with:

$$\begin{aligned}&\texttt{int\_search(}per\texttt{, dom\_w\_deg(/f\_f)(/input\_order), indomain\_min)}\\&\texttt{bool\_search(}h\texttt{, input\_order, indomain\_min)}\end{aligned} \tag{19}$$

In the following tables the search techniques will refer to the decision variable *Opp*:

	Gecode NoSB				Gecode SB			
n	Base	DWD+Min	DWD+Rand	FF	Base	DWD+min	DWD+Rand	FF
2	0	0	0	0	0	0	0	0
4	UNS	UNS	UNS	UNS	UNS	UNS	UNS	UNS
6	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0
10	8	0	0	0	0	0	0	0
12	70	1	0	0	21	0	0	0
14	-	7	15	2	-	3	4	0
16	-	-	51	-	-	-	-	-

Table 1: Performance of Gecode in seconds. DWD=dom_w_deg, FF=first_fail

n	Chuffed						OR_tools					
	NoSB			SB			NoSB			SB		
	Base	FF	Rand	Base	FF	Rand	base	DWD	FF	base	DWD	FF
2	0	0	0	0	0	0	0	0	0	0	0	0
4	UNS	UNS	UNS	UNS	UNS	UNS	UNS	UNS	UNS	UNS	UNS	UNS
6	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	2	0	0	4	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	9	1	0	8	0	0
14	1	2	1	0	0	0	217	3	42	-	2	23
16	32	1	6	0	1	-	-	-	195	-	-	50
18	31	-	110	30	22	-	-	-	-	-	-	-

Table 2: Comparison of Chuffed and OR_tools with and without symmetry breaking (SB) in seconds. FF=first_fail, DWD=Dom_w_deg

The experimental data reveal two key patterns:

1. Symmetry breaking (SB) cuts the search space: by eliminating the mirror home/away schedule, SB reduces the search tree by orders of magnitude, cutting the run-time on most of the tested instances.

2. Heuristic choice is critical and depends on instance size and on solver. Overall, the *first-fail* variable ordering reaches solutions the fastest, quickly pruning infeasible branches early in the search. As the number of teams increases, however, the *dom_w_deg* heuristic particularly when combined with random value selection is also able to reach solutions pretty fast. Notably, for Gecode, even finding a solution at n=16 where FF fails. Overall, Chuffed is able to reach the maximum depth solution with n=18.

Optimization experiments

For optimization we mixed different type of searches (first_fail,dom_w_deg) with random or minimum value selection, also combined with Luby restarts to escape dead ends, showing that Luby is a key factor for improving the search escaping local maxima. We tried also relax and reconstruct on the opponent variable to diversify the search, but not seeing particular improvement from it. In all the model used SB constraints since they showed to greatly improve the performances, only 2 columns will be showed without them (indicated as +noSB) to have a comparison.

n	Gecode								Chuffed				Or-Tool	
	DWD	DMD+NoSB	DWD+luby	DWD+luby+nls	FF	FF+NoSB	FF+luby	FF+luby+lns	FF	FF+luby	Rand	Rand+luby	DWD	FF
2	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
4	UNS	UNS	UNS	UNS	UNS	UNS	UNS	UNS	UNS	UNS	UNS	UNS	UNS	UNS
6	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**
8	**6**	8	**6**	**6**	8	8	**6**	**6**	**6**	**6**	**6**	**6**	**6**	**6**
10	12	12	**8**	**8**	12	12	10	10	10	10	14	14	-	-
12	18	20	26	26	18	20	30	28	32	28	38	30	-	-
14	40	40	92	52	40	50	86	88	50	52	66	58	-	-
16 -	-	-	-	-	-	-	-	-	84	86	112	-	-	-
18 -	-	-	-	-	-	-	-	-	148	146	-	-	-	-

Table 3: Optimization

For the small instances ($n \leq 8$) almost every setting reaches the global optimum. The maximum n where we reached optimal objective is $n = 10$ where Gecode with `dom_w_deg` plus Luby restarts, optionally followed by an LNS phase, drives the objective down to the optimal, 8, efficiently escaping local minima. Beyond $n = 12$ objective values inevitably rise. Across all solvers the NoSB configurations never beat their SB counterparts, confirming that the search time saved by removing symmetric branches is recycled into a deeper optimisation pass. `First_fail` remains the strongest systematic order in Chuffed. `Dom_w_deg` responds well with restarts: the heuristic's bias toward high-pressure variables makes early commitments brittle, and periodic back-jumps let the search revisit those points under alternative value choices.

3 SAT model

For the SAT model, we focused only on the decision version, not optimized (instead tackled in CP,SMT,MILP).

Decision variables

We tackled the problem with propositional Boolean logic and relies on the following decision variables:

- $match_{w,i,j} \in \{\text{true}, \text{false}\} \quad w \in W,\ i, j \in T, i < j$, a Boolean variable indicating whether team i plays against team j in week w
- $home_{w,t} \in \{\text{true}, \text{false}\} \quad w \in W,\ t \in T$, a Boolean variable indicating whether team t plays at home in week w
- $period_{w,t,p} \in \{\text{true}, \text{false}\} \quad w \in W,\ t \in T,\ p \in P$, a Boolean variable indicating whether team t plays in period p during week w

Constraints

The following constraints model the SRR tournament problem using Boolean satisfiability:

- Each team plays exactly one match per week:

$$\forall w \in W, t \in T, \quad \texttt{ExactlyOne}\left(\{match_{w,i,j} : (i,j) \in T^2, i < j, t \in \{i,j\}\}\right) \tag{20}$$

- SRR constraints - one match against each other team

$$\forall i,j \in T, i < j, \quad \texttt{ExactlyOne}\left(\{match_{w,i,j} : w \in W\}\right) \tag{21}$$

- Each team plays in exactly one period per week:

$$\forall w \in W, t \in T, \quad \texttt{ExactlyOne}\left(\{period_{w,t,p} : p \in P\}\right) \tag{22}$$

- Each period contains exactly two teams (one match):

$$\forall w \in W, p \in P, \quad \sum_{t \in T} \texttt{If}(period_{w,t,p}, 1, 0) = 2 \tag{23}$$

- Period consistency, if 2 teams play each other, they must be in the same period:

$$\forall w \in W, i,j \in T, i < j, \quad match_{w,i,j} \Rightarrow \bigwedge_{p \in P} (period_{w,i,p} \Leftrightarrow period_{w,j,p}) \tag{24}$$

- Home/away complementarity - exactly one team plays at home in each match:

$$\forall w \in W, i,j \in T, i < j, \quad match_{w,i,j} \Rightarrow (home_{w,i} \oplus home_{w,j}) \tag{25}$$

- Period frequency constraint - each team appears at most twice in the same period:

$$\forall t \in T, p \in P, \quad \sum_{w \in W} \texttt{If}(period_{w,t,p}, 1, 0) \leq 2 \tag{26}$$

Symmetry breaking

The Boolean formulation enables symmetry breaking through fixed variable assignments:

- Lexicographic ordering of Team 0 opponents eliminates week permutation symmetry:

$$\forall w \in \{1, 2, \ldots, |W|-1\}, \quad \sum_{j=1}^{n-1} j \cdot match_{w,0,j} > \sum_{j=1}^{n-1} j \cdot match_{w-1,0,j} \tag{27}$$

Implied constraints

- We explicitly state that if team i play j, j plays i.

$$\forall w \in W, i, j \in T, i < j, \quad match_{w,i,j} = match_{w,i,j} \tag{28}$$

Validation experiments

For the SAT model, we conducted experiments using two configurations: a basic model with only the hard constraints and the implied and an enhanced model that included also symmetry-breaking constraints. The results, shown in Table 4, indicate that for most instances, adding symmetry-breaking constraints significantly reduces the time required to find a feasible solution. However, for the largest instance with 18 teams, the overhead of the additional constraints resulted in a longer runtime compared to the basic model.

n	**Basic**	**Basic + SB**
2	0	0
4	UNS	UNS
6	0	0
8	0	0
10	0	0
12	4	0
14	11	1
16	23	16
18	42	137

Table 4: SAT Solving Times (seconds)

4 SMT model

Decision variables

The SMT model uses the logic of quantifier-free linear integer arithmetic with arrays (QF_ALIA) and relies on the following decision variables:

- $opp_w[t] \in T \quad w \in W,\ t \in T$, an array representing the opponent of team t in week w
- $home_w[t] \in \{\text{true}, \text{false}\} \quad w \in W,\ t \in T$, a boolean array indicating whether team t plays at home in week w
- $period_w[t] \in P \quad w \in W,\ t \in T$, an array representing the period in which team t plays during week w

Objective functions

The objective function minimizes the total number of breaks across all teams. We define break integer variables as:

$$breaks_t \in \mathbb{Z}_{\geq 0}, \qquad t \in T \tag{29}$$

Each $breaks_t$ counts the H/A breaks for team t:

$$breaks_t = \sum_{w=1}^{n-2} \texttt{If}(home_{w-1}[t] = home_w[t], 1, 0) \tag{30}$$

The objective function is defined as:

$$\text{obj} = \min \sum_{t \in T} breaks_t \tag{31}$$

Constraints

In the following we lists the main constraints used to model the SMT satisfiability/opt models, that follows the logic explained in the introduction part and in the CP:

- Domain constraints for the opponent assignment:

$$\forall w \in W, t \in T, \quad 0 \leq opp_w[t] < n \wedge opp_w[t] \neq t \tag{32}$$

- Reciprocal opponent constraint ensuring bidirectional matching (uses array composition):

$$\forall w \in W, t \in T, \quad opp_w[opp_w[t]] = t \tag{33}$$

- Home/away complementarity constraint:

$$\forall w \in W, t \in T, \quad home_w[t] = \neg home_w[opp_w[t]] \tag{34}$$

- A team can appear maximum 2 times in the same period all over the tournament:

$$\forall t \in T, p \in P, \quad \sum_{w \in W} \texttt{If}(period_w[t] = p, 1, 0) \leq 2 \tag{35}$$

Symmetry breaking

The array-based formulation facilitates symmetry breaking by fixing specific array values:

- Fix the first match (it eliminates home/away symmetry):

$$opp_0[0] = 1 \land home_0[0] = \text{true} \tag{36}$$

  This constraint fixes team 0 to play against team 1 at home in the first week, breaking the symmetry between home and away assignments.

- Enforce lexicographic ordering of Team 0's schedule (it eliminates week permutation symmetry):

$$\forall w \in \{1, 2, \ldots, n-2\}, \quad opp_w[0] > opp_{w-1}[0] \tag{37}$$

  This constraint ensures Team 0's opponents appear in increasing order across weeks, eliminating equivalent solutions that differ only by week permutation.

Implied constraints

Explicit reciprocal implication:

$$\forall w \in W, i, j \in T, i \neq j, \quad opp_w[i] = j \Rightarrow opp_w[j] = i \tag{38}$$

This constraints showed to improve the results output.

Validation experiments

The SMT model was tested for both feasibility and optimization using the Z3 solver, with results presented for a basic configuration and one augmented with symmetry breaking (SB)

Feasibility Experiments

In the feasibility tests, the addition of symmetry-breaking constraints generally improved performance, leading to faster solutions for most tournament sizes, as detailed in Table 5. Interestingly, the trend reversed for n=14, where the basic model was faster, suggesting that for certain problem sizes, the overhead of SB constraints can outweigh their benefits in pruning the search space.

n	Basic	Basic + SB
2	0	0
4	UNS	UNS
6	0	0
8	1	0
10	7	5
12	27	19
14	120	200
16	-	-

Table 5: SMT Satisfiability, runtime in seconds.

Optimization Experiments

The results in Table 6 show that the symmetry-breaking model consistently found better or equal objective values compared to the basic model within the given timeframe. For example, at n=14 and n=16, the SB model achieved superior objective values of 36 and 58, respectively. This demonstrates that by reducing the search space, the solver can dedicate more time to finding higher-quality solutions.

n	**Basic**	**Basic + SB**
2	0	0
4	UNS	UNS
6	0	0
8	1	0
10	**8**	**8**
12	16	12
14	40	36
16	66	58

Table 6: SMT Optimization, best objective found.

5 MILP model

The Mixed-Integer Linear Programming (MILP) model uses binary variables to represent the scheduling choices and linear constraints to enforce the tournament rules. This formulation allowed us to leverage powerful MILP solvers like CBC, SCIP and HiGHS to find both satisfiable and optimal solutions.

Decision Variables

The model is formulated using three primary sets of binary variables, plus an additional set for the optimization version. The indices are 1-based for clarity ($t \in \{1, ..., n\},\ w \in \{1, ..., n-1\},\ p \in \{1, ..., n/2\}$).

- $opp_{w,t,j} \in \{0, 1\}$: 1 if team t plays against team j in week w, and 0 otherwise.
- $home_{w,t} \in \{0, 1\}$: 1 if team t plays at home in week w, and 0 otherwise.
- $period_{w,t,p} \in \{0, 1\}$: 1 if team t plays in period p of week w, and 0 otherwise.
- $break_{w,t} \in \{0, 1\}$: (For optimization) 1 if team t has a home/away break between week $w-1$ and w.

Objective Function

For the satisfiability version, the goal is simply to find a feasible solution, so no objective function is minimized. For the optimization model, the objective is to minimize the total number of breaks, which are consecutive home (HH) or away (AA) games for a team.

$$\min \sum_{t=1}^{n} \sum_{w=2}^{n-1} break_{w,t} \tag{39}$$

To strengthen the linear relaxation of the optimization model, a known lower bound on the objective is added as a constraint.

$$\sum_{t=1}^{n} \sum_{w=2}^{n-1} break_{w,t} \geq n-2 \tag{40}$$

The $break_{w,t}$ variables are defined by the constraints below.

Constraints

The tournament rules are enforced through the following linear constraints:

- Each team plays exactly one other team per week, and every pair of teams plays each other exactly once over the course of the tournament.

$$\forall w \in W,\ \forall t \in T: \quad \sum_{j \in T,\ j \neq t} opp_{w,t,j} = 1 \tag{41}$$

$$\forall i, j \in T,\ i < j: \quad \sum_{w \in W} opp_{w,i,j} = 1 \tag{42}$$

- Implied Constraint: A match is a two-way agreement; if team i plays team j, team j must also play team i.

$$\forall w \in W,\ \forall i, j \in T,\ i < j: \quad opp_{w,i,j} = opp_{w,j,i} \tag{43}$$

- Each team plays in exactly one period per week, and each period hosts exactly one match (two teams).

$$\forall w \in W,\ \forall t \in T: \quad \sum_{p \in P} period_{w,t,p} = 1 \tag{44}$$

$$\forall w \in W,\ \forall p \in P: \quad \sum_{t \in T} period_{w,t,p} = 2 \tag{45}$$

- If two teams play each other, they must be in the same period. This logical condition: if $opp_{w,i,j} = 1$, then $period_{w,i,p} = period_{w,j,p}$, is linearized as:

$$\forall w \in W,\ \forall i,j \in T,\ i \neq j,\ \forall p \in P: \quad period_{w,i,p} - period_{w,j,p} \leq 1 - opp_{w,i,j} \tag{46}$$

$$period_{w,j,p} - period_{w,i,p} \leq 1 - opp_{w,i,j} \tag{47}$$

- Each team can play at most twice in the same period slot over the entire tournament.

$$\forall t \in T,\ \forall p \in P: \quad \sum_{w \in W} period_{w,t,p} \leq 2 \tag{48}$$

- For any given match, one team plays at home and the other plays away. If $opp_{w,i,j} = 1$, then $home_{w,i} + home_{w,j} = 1$, linearized using the "big-M" method:

$$\forall w \in W,\ \forall i,j \in T,\ i < j: \quad home_{w,i} + home_{w,j} \geq 1 - (1 - opp_{w,i,j}) \tag{49}$$

$$home_{w,i} + home_{w,j} \leq 1 + (1 - opp_{w,i,j}) \tag{50}$$

- (For optimization) These constraints define the $break_{w,t}$ variables. A break occurs if $home_{w-1,t} = home_{w,t}$.

$$\forall t \in T,\ \forall w \in \{2,\dots,n-1\}: \quad break_{w,t} \geq home_{w-1,t} + home_{w,t} - 1 \quad \text{(for HH breaks)} \tag{51}$$

$$break_{w,t} \geq 1 - (home_{w-1,t} + home_{w,t}) \quad \text{(for AA breaks)} \tag{52}$$

Symmetry Breaking

To prune the search space, the following constraints are added:

- **Team 1 Home Opener:** Team 1 is fixed to play at home in the first week, breaking the symmetry of complementary home/away schedules.

$$home_{1,1} = 1 \tag{53}$$

- **Week Permutation:** To eliminate equivalent schedules that are just permutations of weeks, we require that the opponents of Team 1 appear in lexicographically increasing order:

$$\forall w \in \{2,\dots,n-1\}: \quad \sum_{j \in T} j \cdot opp_{w,1,j} > \sum_{j \in T} j \cdot opp_{w-1,1,j} \tag{54}$$

Validation Experiments

The MILP model was evaluated for both feasibility and optimization using three different solvers: CBC, SCIP, and HiGHS. We compared the performance of a basic model against one including symmetry-breaking (SB) constraints.

Feasibility Experiments

As shown in Table 7, the HiGHS solver consistently outperformed both CBC and SCIP, finding feasible solutions much faster across almost all instances. The inclusion of symmetry-breaking constraints provided a clear advantage, further reducing the runtime, especially for larger problems.

	Basic			**Basic + SB**		
n	**CBC**	**SCIP**	**HiGHS**	**CBC**	**SCIP**	**HiGHS**
2	0	0	0	0	0	0
4	UNS	UNS	UNS	UNS	UNS	UNS
6	0	0	0	0	0	0
8	15	0	0	15	0	0
10	27	3	8	29	3	8
12	-	-	80	-	-	75
14	-	-	265	-	-	262

Table 7: MILP Satisfiability, runtime in seconds.

Optimization Experiments

In the optimization phase, the objective was to minimize the total number of breaks. The results in Table 8 again highlight the superiority of the HiGHS solver, which found optimal or near-optimal solutions more rapidly than the others. The symmetry-breaking constraints proved crucial for optimization, enabling the solvers to find better solutions faster. For instance, at n=14, HiGHS with SB was the only approach able to find a solution.

	Basic			**Basic + SB**		
n	**CBC**	**SCIP**	**HiGHS**	**CBC**	**SCIP**	**HiGHS**
2	**0**	**0**	**0**	**0**	**0**	**0**
4	UNS	UNS	UNS	UNS	UNS	UNS
6	**4**	**4**	**4**	**4**	**4**	**4**
8	8	**6**	**6**	**6**	**6**	**6**
10	12	12	10	16	10	10
12	-	-	18	-	-	18
14	-	-	-	-	-	24

Table 8: MILP Optimization, objective found

6 Conclusions

In this project, we successfully tackled the NP-hard Single Round-Robin (SRR) tournament scheduling problem, focusing first on establishing feasibility and subsequently on optimizing schedules by minimizing the total number of breaks. We explored a comprehensive set of modeling paradigms, including Constraint Programming (CP), SAT, SMT, and Mixed-Integer Linear Programming (MILP), to compare their effectiveness.

A pivotal and universal finding across all methodologies was the profound impact of the objective functions and the symmetry-breaking (SB) constraints. By reducing the total number of breaks, strengthening the linear relaxation by adding a known upper and lower bound on the objective, eliminating redundant parts of the search space, such as permutations of weeks or complementary home/away patterns, the constraints consistently led to drastically reduced runtimes and superior objective values.

The performance comparison revealed distinct advantages for different approaches:

- The CP model, particularly when paired with the Chuffed solver, demonstrated good scalability in finding feasible solutions, successfully solving instances up to n=18. For optimization, Gecode combined with DWD and Luby restarts proved highly effective for smaller instances, finding the optimal objective value until n=10.

- SAT and SMT models proved to be effective declarative approaches for finding feasible schedules, with their performance also significantly enhanced by the application of symmetry breaking, SAT in particular reached satisfiable results for n=18 and SMT found solution for optimization until n=16.

- The MILP model emerged as a powerful tool, especially when using the HiGHS solver, which consistently outperformed other solvers in both feasibility and optimization tasks, reaching solutions until n=14.

In conclusion, this work, completed through a collaborative effort, highlights that no single modeling paradigm is universally superior. Instead, a trade-off exists between raw scalability and optimization quality.

Authenticity and Author Contribution Statement

All the experiments were completed limiting the core to 1 in sequential mode. The project was completed between the end of May and the beginning of July, both the partecipants worked on all the parts, in particular Lorenzo Venturi focused on CP,SAT and testing part, Chayan Talukder on SMT,MILP.

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