Spontaneous Symmetry Breaking in the Standard Model

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Spontaneous Symmetry Breaking in the Standard Model

- The idea of spontaneous symmetry breaking
- Spontaneous breaking of discrete symmetries
- Spontaneous breaking of continuous symmetries
- Abelian Higgs model
- The Electroweak Theory of Leptons and the Higgs Mechanism
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The idea of spontaneous symmetry breaking

In Lagrangian field theory, a symmetry is said to be exact if it leaves the Lagrangian invariant: $\delta\mathcal{L} = 0$.

- Explicit SB: $\mathcal{L}$ is not invariant under the symmetry transformations.
- Spontaneous SB: $\mathcal{L}$ is exactly invariant, but its degenerate set of vacuum states are not.

In SSB, each vacuum state breaks the symmetry. Still arise exact conservation laws, but the symmetry is hidden in observable quantities.

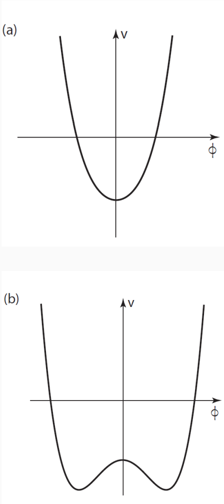
Spontaneous breaking of discrete symmetries

Real scalar field ϕ, potential $V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}|\lambda|\phi^4$ symmetric under parity.

- $\mu^2 > 0$: single minimum which preserves symmetry.

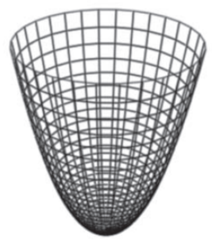
- $\mu^2 < 0$: degenerate minima at $\pm v = \pm\sqrt{-\mu^2/|\lambda|}$.

$\Rightarrow$ Choosing a vacuum breaks $\phi \to -\phi$ symmetry.

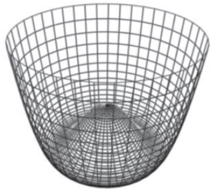


Spontaneous breaking of continuous symmetries

Two real fields: ϕ_1, ϕ_2, potential $V(\phi^2) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}|\lambda|(\phi^2)^2$ symmetric under SO(2) rotations.



- $\mu^2 > 0$: single minimum which preserves symmetry.



- $\mu^2 < 0$: degenerate minima on circle at $v^2 = \frac{-\mu^2}{|\lambda|}$.

Choosing a vacuum selects a preferred direction in the (ϕ_1, ϕ_2) space, breaking the SO(2) symmetry.

$\Rightarrow$ Let us select $\langle\phi\rangle_0 = \left(\begin{smallmatrix} v \\ 0 \end{smallmatrix}\right)$.

Spontaneous breaking of continuous symmetries

Expanding about the vacuum by $\phi' \equiv \phi - \langle\phi\rangle_0 \equiv \binom{\eta}{\zeta}$:

$$\mathcal{L}_{\text{so}} \propto \frac{1}{2}\left[(\partial_\mu\eta)(\partial^\mu\eta) + 2\mu^2\eta^2\right] + \frac{1}{2}\left[(\partial_\mu\zeta)(\partial^\mu\zeta)\right].$$

- η-particle, associated with radial oscillations, has a mass $m_\eta^2 = -2\mu^2$.
- ξ-particle (Nambu–Goldstone boson), associated with angular oscillations, is massless. This arises from the SO(2) invariance of $\mathcal{L}$.

Goldstone theorem: if a continuous global symmetry of $\mathcal{L}$ is not a symmetry of the physical vacuum, then exists one Goldstone boson for each generator of the broken symmetry.

Spontaneous breaking of a gauge symmetry

SSB of a gauge symmetry leads to the **Higgs mechanism**.

- global SB $\rightarrow$ massless Nambu–Goldstone bosons (Goldstone theorem).
- gauge SB $\rightarrow$ Nambu–Goldstone bosons absorbed by gauge field

$\Rightarrow$ gauge bosons acquire a mass.

Let us consider two cases: Abelian Higgs model and Electroweak SB (Standard Model).

Abelian Higgs Model

Lagrangian $\mathcal{L} = |(\partial_\mu + iqa_\mu)\phi|^2 - \mu^2|\phi|^2 - |\lambda|(\phi^*\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$.

Complex scalar field $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$, gauge boson a_μ.

$\mathcal{L}$ symmetric under global U(1) rotations and the local U(1) gauge transformations:

$$\phi(x) \to \phi'(x) = e^{iq\alpha(x)}\phi(x), \quad a_\mu(x) \to a'_\mu(x) = a_\mu(x) - \partial_\mu\alpha(x).$$

- $\mu^2 < 0$: degenerate minima at $\langle|\phi|^2\rangle_0 = \frac{-\mu^2}{2|\lambda|} \equiv \frac{v^2}{2}$.

Chosen $\langle\phi\rangle_0 = \frac{v}{\sqrt{2}}$, and shift the field as $\phi - \langle\phi\rangle_0 = \frac{1}{\sqrt{2}}e^{i\zeta/v}(v+\eta) \approx \frac{1}{\sqrt{2}}(v+\eta+i\zeta)$:

$$\mathcal{L}_{\text{so}} \propto \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) + \mu^2\eta^2 + \frac{1}{2}(\partial_\mu\zeta)(\partial^\mu\zeta) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + qva^\mu\partial_\mu\zeta + \frac{q^2v^2}{2}a^\mu a_\mu.$$

Abelian Higgs Model

From $\frac{q^2v^2}{2}\left(a_\mu + \frac{1}{qv}\partial_\mu\zeta\right)^2$, consider the gauge transformation:

$$a_\mu \to a'_\mu = a_\mu + \frac{1}{qv}\partial_\mu\zeta \quad \text{"unitary gauge"}.$$

In unitary gauge: $\mathcal{L}_{\text{so}} \propto \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) + \mu^2\eta^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2v^2}{2}a'^\mu a'_\mu$.

- A scalar field η, with mass $m_\eta^2 = -2\mu^2 = 2\lambda v^2$.
- A massive vector field a'_μ, with mass $m_a = qv$.
- No ζ field.

The formerly massless Nambu-goldstone boson ζ has become the longitudinal component of the **massive vector field** a'_μ.

The Electroweak Theory of Leptons and the Higgs Mechanism

Lepton sector of $\mathrm{SU(2)_L} \otimes \mathrm{U(1)_Y}$ Electroweak Theory.

- Left-handed weak isospin doublet and right-handed singlet:

$$L \equiv \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \ \nu_L = \frac{1}{2}(1-\gamma^5)\nu, \ e_L = \frac{1}{2}(1-\gamma^5)e; \quad R = e_R = \frac{1}{2}(1+\gamma^5)e.$$

- $\mathcal{L}$ bosons term:

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F^l_{\mu\nu}F^{l\mu\nu} - \frac{1}{4}f_{\mu\nu}f^{\mu\nu}, \quad F^l_{\mu\nu} = \partial_\mu b^l_\nu - \partial_\nu b^l_\mu + g\varepsilon^{ljk} b^j_\mu b^k_\nu.$$

- $\mathcal{L}$ matter term:

$$\mathcal{L}_{\text{leptons}} = \bar{R} i\gamma^\mu \left(\partial_\mu + i\frac{g'}{2}a_\mu Y\right) R + \bar{L} i\gamma^\mu \left(\partial_\mu + i\frac{g'}{2}a_\mu Y + i\frac{g}{2}\boldsymbol{\tau}\cdot\mathbf{b}_\mu\right) L.$$

The Electroweak Theory of Leptons and the Higgs Mechanism

As written, the theory has two main problems:

- An explicit $\frac{1}{2} m_b^2 \, b_\mu^A b^{A\mu}$ mass term would break the gauge invariance

$\Rightarrow$ The theory predicts the existence of four massless gauge bosons (b^1, b^2, b^3, and a), whereas experimentally only one massless gauge boson is observed, the photon.

- An explicit $-m_e \bar{e} e = -m_e(\bar{e}_R e_L + \bar{e}_L e_R)$ mass term would break the gauge invariance.

$\Rightarrow$ The theory predicts massless charged leptons

The Electroweak Theory of Leptons and the Higgs Mechanism

- Scalar doublet $\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$.
- Lagrangian: $\mathcal{L}_{\text{scalar}} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi^\dagger \phi)$,
  $D_\mu = \partial_\mu + i\frac{g'}{2} a_\mu Y + i\frac{g}{2}\boldsymbol{\tau} \cdot \mathbf{b}_\mu, \quad V(\phi^\dagger\phi) = \mu^2(\phi^\dagger\phi) + |\lambda|(\phi^\dagger\phi)^2.$
- Yukawa interaction term: $\mathcal{L}_{\text{Yukawa}} = -\lambda_e \left[\bar{R}(\phi^\dagger L) + (\bar{L}\phi)R\right]$.

For $\mu^2 < 0$, the vacuum state $\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$ breaks both symmetries:

$$\tau_1 \langle\phi\rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0, \quad \tau_2 \langle\phi\rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0,$$

$$\tau_3 \langle\phi\rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0, \quad Y \langle\phi\rangle_0 = Y_\phi \langle\phi\rangle_0 = +1 \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0.$$

The Electroweak Theory of Leptons and the Higgs Mechanism

Gell-Mann–Nishijima relation $Q = T_3 + \frac{1}{2}Y$, the electric charge operator leaves the vacuum invariant:

$$Q\langle\phi\rangle_0 = (T_3 + \frac{1}{2}Y)\langle\phi\rangle_0 = \frac{1}{2}\begin{pmatrix} Y_\phi + 1 & 0 \\ 0 & Y_\phi - 1 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

The subgroup $\mathrm{U}(1)_{\mathrm{EM}}$ remains unbroken after spontaneous symmetry breaking of the gauge group: $SU(2)_L \otimes U(1)_Y \to U(1)_{EM}$.

$\Rightarrow$ Three of the four original gauge bosons acquire mass: massive $W^\pm$ and Z^0 bosons.
$\Rightarrow$ Remaining massless gauge boson corresponds to the unbroken $U(1)_{EM}$: photon.

The Electroweak Theory of Leptons and the Higgs Mechanism

Expand about the vacuum in unitary gauge: $\phi(x) = \begin{pmatrix} 0 \\ \frac{v+\eta(x)}{\sqrt{2}} \end{pmatrix}$.

$$(D_\mu\phi)^\dagger(D^\mu\phi) = \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) + \frac{1}{8}g^2|b^1_\mu - ib^2_\mu|^2(v+\eta)^2 + \frac{1}{8}(g'a_\mu - gb^3_\mu)^2(v+\eta)^2.$$

- charged gauge fields $W^\pm_\mu = \frac{b^1_\mu \mp ib^2_\mu}{\sqrt{2}} \to M_{W^\pm} = \frac{gv}{2}$.
- $Z_\mu = \frac{-g'a_\mu + g\,b^3_\mu}{\sqrt{g^2+g'^2}}$, $A_\mu = \frac{ga_\mu + g'\,b^3_\mu}{\sqrt{g^2+g'^2}} \to M_Z = \frac{\sqrt{g^2+g'^2}\;v}{2}$.
- η degree of freedom (radial fluctuations of ϕ) corresponds to the physical Higgs boson $H \to m_H^2 = -2\mu^2$.

$\mathcal{L}_{\text{Yukawa}} = -\lambda_e \frac{(v+\eta)}{\sqrt{2}}\left(\bar{e}_R e_L + \bar{e}_L e_R\right)$:

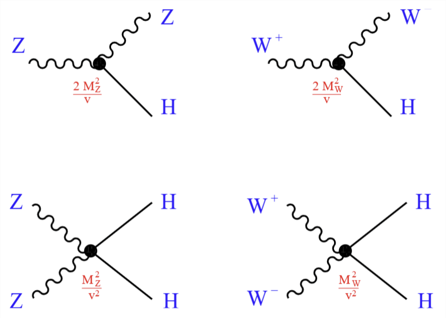
- $m_e = \frac{\lambda_e v}{\sqrt{2}}$, fermionic mass term without breaking gauge invariance.
- $-\frac{\lambda_e\eta}{\sqrt{2}}\bar{e}e$ fermionic coupling.

The Electroweak Theory of Leptons and the Higgs Mechanism

From $\mathcal{L}_{\text{scalar}}$, coupling terms between $W^{\pm}$, Z^0 and H:

- $\frac{1}{4}g_W^2 W_\mu^- W^{+\mu}(v+\eta)^2 = M_W^2 W_\mu^- W^{+\mu} + 2\frac{M_W^2}{v} W_\mu^- W^{+\mu}\eta + \frac{M_W^2}{v^2} W_\mu^- W^{+\mu}\eta^2$.
- $\frac{1}{8}(g^2+g'^2)Z_\mu Z^\mu(v+\eta)^2 = \frac{1}{2}M_Z^2 Z_\mu Z^\mu + \frac{M_Z^2}{v}Z_\mu Z^\mu\eta + \frac{1}{2}\frac{M_Z^2}{v^2}Z_\mu Z^\mu\eta^2$.

Triple coupling ($VV\eta$) and quartic coupling ($VV\eta\eta$), both proportional to M_V^2 and $1/v$.



The Electroweak Theory of Leptons and the Higgs Mechanism

From $\mathcal{L}_{\text{lepton}}$, coupling between $W^{\pm}$, Z^0, γ and leptons:

$$\frac{g}{2}\boldsymbol{\tau}\cdot\mathbf{b}_\mu = \frac{g}{2}\begin{pmatrix} b_\mu^3 & b_\mu^1 - ib_\mu^2 \\ b_\mu^1 + ib_\mu^2 & -b_\mu^3 \end{pmatrix}$$

$$= \frac{g}{2}\begin{pmatrix} \cos\theta_W Z_\mu + \sin\theta_W A_\mu & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & -\cos\theta_W Z_\mu - \sin\theta_W A_\mu \end{pmatrix}.$$

- $\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}}\left(\bar{\nu}\gamma^\mu(1-\gamma^5)eW_\mu^+ + \bar{e}\gamma^\mu(1-\gamma^5)\nu W_\mu^-\right) \Rightarrow \frac{g^2}{8} = G_F\frac{M_W^2}{\sqrt{2}}$ to match low-energy weak interaction phenomenology.
- $\mathcal{L}_{NC} = \frac{gg'}{\sqrt{g^2+g'^2}}\bar{e}\gamma^\mu eA_\mu - \frac{\sqrt{g^2+g'^2}}{2}\bar{\nu}_L\gamma^\mu\nu_L Z_\mu + \frac{1}{\sqrt{g^2+g'^2}}\left[-g'^2\bar{e}_R\gamma^\mu e_R + \frac{g^2-g'^2}{2}\bar{e}_L\gamma^\mu e_L\right]Z_\mu$
  $\Rightarrow \frac{gg'}{\sqrt{g^2+g'^2}} = e$ to identify A_μ as the photon from QED.

Experimental tests of compatibility of the Higgs couplings with the Standard Model predictions

CMS collaboration at the LHC, 2015: tests of the Higgs boson couplings as predicted by the SM, **no significant deviations were found**.

Introduced coupling modifiers: scale factors κ_i.

$$\kappa_i^2 = \frac{\sigma_i}{\sigma_i^{\mathrm{SM}}} \text{ (Higgs production)}, \quad \kappa_j^2 = \frac{\Gamma_{jj}}{\Gamma_{jj}^{\mathrm{SM}}} \text{ (Higgs decay)}, \quad \kappa_H^2 = \frac{\Gamma_{\mathrm{tot}}}{\Gamma_{\mathrm{tot}}^{\mathrm{SM}}} \text{(tot width)}.$$

Assumed the narrow-width approximation (NWA) holds, permitting its production and decay to be considered independently.

Test 1: W vs Z couplings

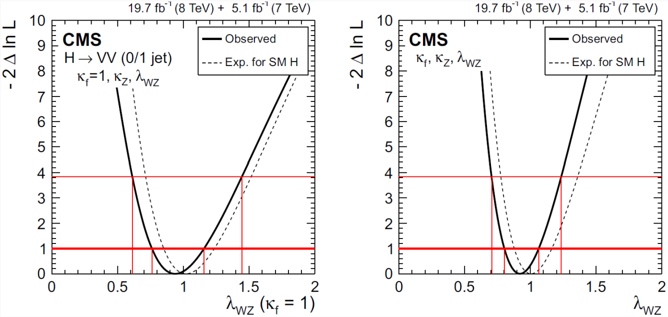
0-jet and 1-jet channels of $H \to WW \to \ell\nu\ell\nu$, untagged channels of $H \to ZZ \to 4\ell$: ggH dominant production mechanism $\Rightarrow$ their event ratio largely insensitive to the Higgs production model.

- Likelihood scan versus $\lambda_{WZ} = \kappa_W/\kappa_Z$ for $H \to VV$:

$$\lambda_{WZ} = 0.94^{+0.22}_{-0.18}$$

- Likelihood scan versus λ_{WZ} combining all channels, profiling coupling with fermions κ_f:

$$\lambda_{WZ} = 0.92^{+0.14}_{-0.12}$$

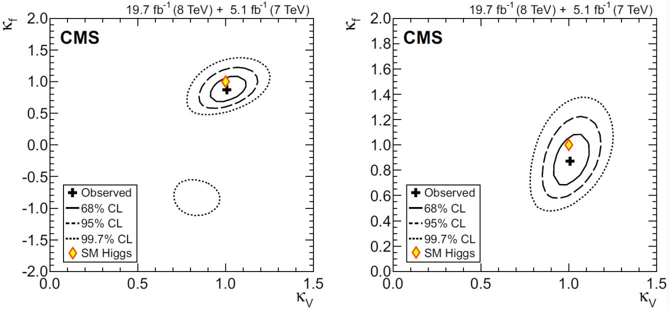


One can use a common factor κ_V.

Test 2: vector vs fermion couplings

$\Gamma_{\gamma\gamma}$ is the only partial width combined in the analyses to not scale either as κ_V^2 or κ_f^2 at LO, being induced via loops with virtual W bosons or top quarks:
$H \to \gamma\gamma$ channel is sensitive to the relative sign of κ_V and κ_f.

2D likelihood scan for κ_V and κ_f.
Data are compatible with the expected $(\kappa_V, \kappa_f) = (1, 1)$.



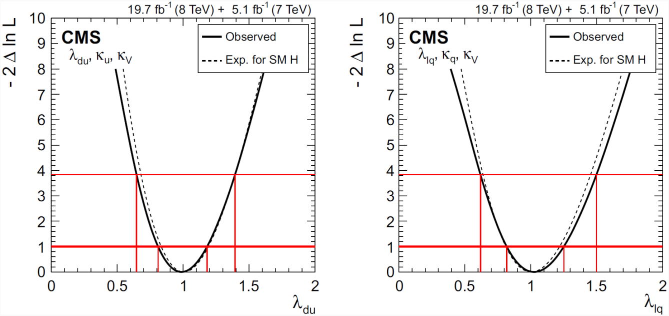
Test 3: leptons vs quarks couplings

- Likelihood scan versus the $\lambda_{du} = \kappa_d/\kappa_u$, with κ_V and κ_u profiled together:

$$\lambda_{du} \in [0.65,\ 1.39]$$

- Likelihood scan versus the $\lambda_{\ell q} = \kappa_l/\kappa_q$, with κ_V and κ_q profiled together:

$$\lambda_{\ell q} \in [0.62,\ 1.50]$$



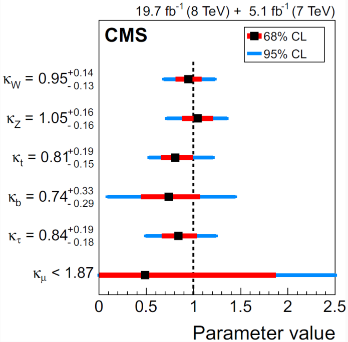
No evidence that different classes of fermions have different scaling factors.

Test 4: scaling of coupling with the mass

First, fit for deviations in κ_W, κ_Z, κ_b, κ_τ, κ_t, and κ_μ.

Model: the loop-induced processes (σ_{ggH}, Γ_{gg}, and $\Gamma_{\gamma\gamma}$) are expressed in terms of the above tree-level κ parameters.

Likelihood scan: no significant deviations from the SM expectation $\kappa_i = 1$.



Test 4: scaling of coupling with the mass

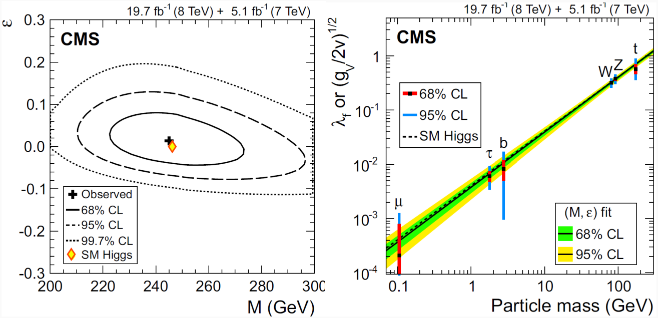
Parameterization relating m_f and m_V to the corresponding κ using two parameters, M and ϵ: $\kappa_f = v\, m_f^{\epsilon}/M^{1+\epsilon}$, $\kappa_V = v\, m_V^{2\epsilon}/M^{1+2\epsilon}$. SM expectation is $(M, \epsilon) = (v, 0)$.

- Likelihood scan for (M, ϵ):

  $$M \in [217,\ 279]\ \text{GeV}$$

  $$\epsilon \in [-0.054,\ 0.100]$$

- Recap: plot of $\lambda_f \sim \kappa_f m_f/v$ and $\sqrt{g_V/(2v)} = \kappa_V^{1/2} m_V/v$ (same mass dependence).



The Higgs couples differently to each particle, with couplings proportional to their mass.

Thank you for your attention