Potenzreihen

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!}x^n = 1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots \quad \text{für} \quad x \in \mathbb{R}$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1} = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - + \cdots \quad \text{für} \quad x \in \mathbb{R}$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}x^{2n} = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - + \cdots \quad \text{für} \quad x \in \mathbb{R}$$

$$\sinh x = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!}x^{2n+1} = x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \cdots \quad \text{für} \quad x \in \mathbb{R}$$

$$\cosh x = \sum_{n=0}^{\infty} \frac{1}{(2n)!}x^{2n} = 1 + \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \cdots \quad \text{für} \quad x \in \mathbb{R}$$

$$\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}x^{2n+1} = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + - \cdots \quad \text{für} \quad |x| < 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}x^n = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + - \cdots \quad \text{für} \quad -1 < x \leq 1$$

$$\ln(1-x) = -\sum_{n=1}^{\infty} \frac{1}{n}x^n = -(x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \cdots) \quad \text{für} \quad -1 \leq x < 1$$

$$\sqrt{1+x} = \sum_{n=0}^{\infty} \binom{\frac{1}{2}}{n} x^n = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + - \cdots \quad \text{für} \quad |x| \leq 1$$

$$\frac{1}{\sqrt{1+x}} = \sum_{n=0}^{\infty} \binom{-\frac{1}{2}}{n} x^n = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \frac{35}{128}x^4 - + \cdots \quad \text{für} \quad |x| < 1$$

geometrische Reihe $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$, für $|x| < 1$

endliche geom. Reihe $\sum_{n=0}^{k} x^n = 1 + x + x^2 + \cdots + x^k = \frac{1-x^{k+1}}{1-x}$, für $x \neq 1$

harmonische Reihe $\sum_{n=1}^{\infty} \frac{1}{n^x} = 1 + \frac{1}{2^x} + \frac{1}{3^x} + \cdots$ konvergent $\iff x > 1$

binomische Reihe $\sum_{n=0}^{\infty} \binom{r}{n} x^n = 1 + rx + \binom{r}{2} x^2 + \binom{r}{3} x^3 + \cdots = (1+x)^r$, $|x| \leq 1,\ r > 0$; $|x| < 1,\ r < 0$

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots = \infty$$
$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + - \cdots = \ln 2$$
$$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots = e$$
$$1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + - \cdots = \frac{1}{e}$$
$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = 2$$
$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + - \cdots = \frac{\pi}{4}$$
$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$$
$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + - \cdots = \frac{\pi^2}{12}$$
$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots = \frac{\pi^2}{8}$$

wichtige Grenzwerte $(n \to \infty)$

$\sqrt[n]{a} \to 1$	$\left(\frac{n+1}{n}\right)^n \to e$	$\binom{a}{n} \to 0,\ a > -1$
$\sqrt[n]{n} \to 1$	$\left(1+\frac{1}{n}\right)^n \to e$	$\frac{a^n}{n!} \to 0$
$\sqrt[n]{n!} \to \infty$	$\left(1+\frac{x}{n}\right)^n \to e^x$	$\frac{n^n}{n!} \to \infty$
$\frac{1}{n}\sqrt[n]{n!} \to \frac{1}{e}$	$\left(1-\frac{x}{n}\right)^n \to e^{-x}$	$\frac{a^n}{n^k} \to \infty \begin{cases} a > 1 \\ k \text{ fest} \end{cases}$
		$a^n n^k \to 0 \begin{cases} \lvert a\rvert < 1 \\ k \text{ fest} \end{cases}$

Differentiations– und Integrationsregeln

Produktregel: $(u \cdot v)' = u' \cdot v + u \cdot v'$, $(uvw)' = u'vw + uv'w + uvw'$

partielle Integration: $\int u'v\,dx = uv - \int uv'\,dx$

Quotientenregel: $\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$

Vektorfunktionen

$$(\lambda\vec{u})' = \lambda'\vec{u} + \lambda\vec{u}'$$
$$(\vec{u} \cdot \vec{v})' = \vec{u}' \cdot \vec{v} + \vec{u} \cdot \vec{v}'$$
$$(\vec{u} \times \vec{v})' = \vec{u}' \times \vec{v} + \vec{u} \times \vec{v}'$$
$$(\vec{u}(\lambda(t)))' = \vec{u}'(\lambda(t)) \cdot \lambda'(t)$$

Kettenregel: $(y(x(t)))' = \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = y'(x(t)) \cdot x'(t)$

Substitutionsregel: $\int f(x)\,dx = \int f(g(t))\,g'(t)\,dt$, dabei ist $\begin{cases} x = g(t) \\ dx = g'(t)\,dt \end{cases}$

f	f'
x^n	nx^{n-1}
$\frac{1}{x^n}$	$\frac{-n}{x^{n+1}}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
$\sqrt[n]{x}$	$\frac{1}{n\sqrt[n]{x^{n-1}}}$
e^x	e^x
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$
$\log_a x$	$\frac{1}{\ln a}\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$\frac{-1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\operatorname{arccot} x$	$\frac{-1}{1+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sinh^2 x}$
$\operatorname{arsinh} x$	$\frac{1}{\sqrt{x^2+1}}$
$\operatorname{arcosh} x$	$\frac{1}{\sqrt{x^2-1}},\ x > 1$
$\operatorname{artanh} x$	$\frac{1}{1-x^2},\ \lvert x\rvert < 1$
$\operatorname{arcoth} x$	$\frac{1}{1-x^2},\ \lvert x\rvert > 1$
$\int g\,dx$	g

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad (n \neq -1) \qquad \int \frac{f'}{f}\,dx = \ln|f|$$

$$\int \frac{1}{x}\,dx = \ln|x|$$
$$\int \frac{dx}{x+a} = \ln|x+a|$$
$$\int \frac{dx}{(x+a)^2} = -\frac{1}{x+a}$$
$$\int \tan x\,dx = -\ln|\cos x|$$
$$\int \sin^2 ax\,dx = \frac{1}{2}x - \frac{1}{4a}\sin 2ax$$
$$\int \cos^2 ax\,dx = \frac{1}{2}x + \frac{1}{4a}\sin 2ax$$
$$\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x}$$
$$\int \frac{1}{\sqrt[3]{x}}\,dx = \frac{3}{2}\sqrt[3]{x^2}$$
$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$
$$\int xe^{ax}\,dx = \frac{ax-1}{a^2}e^{ax}$$
$$\int \ln x\,dx = x\ln x - x$$
$$\int x\ln x\,dx = x^2\left(\frac{\ln x}{2} - \frac{1}{4}\right)$$
$$\int \ln^2 x\,dx = x\ln^2 x - 2x\ln x + 2x$$
$$\int \sin ax \cos ax\,dx = \frac{1}{2a}\sin^2 ax$$
$$\int \frac{dx}{\sin ax \cos ax} = \frac{1}{a}\ln|\tan ax|$$
$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$
$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$
$$\int x\sin ax\,dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax$$
$$\int x\cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$$

Bezeichnungen: $X = ax^2 + bx + c$, $\Delta = 4ac - b^2$, $a \neq 0$

$$\int \frac{dx}{X} = \begin{cases} \frac{2}{\sqrt{\Delta}}\arctan\frac{2ax+b}{\sqrt{\Delta}} & (\Delta > 0) \\ \left.\begin{array}{l} \frac{-2}{\sqrt{-\Delta}}\operatorname{artanh}\frac{2ax+b}{\sqrt{-\Delta}} \\ \frac{1}{\sqrt{-\Delta}}\ln\frac{2ax+b-\sqrt{-\Delta}}{2ax+b+\sqrt{-\Delta}} \end{array}\right\} & (\Delta < 0) \\ \frac{-2}{2ax+b} & (\Delta = 0) \end{cases}$$

$$\int \frac{dx}{X^2} = \frac{2ax+b}{\Delta X} + \frac{2a}{\Delta}\int \frac{dx}{X}$$

$$\int \frac{x\,dx}{X} = \frac{1}{2a}\ln|X| - \frac{b}{2a}\int \frac{dx}{X}$$

$$\int \sqrt{x^2+a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2+a^2} + a^2\operatorname{arsinh}\frac{x}{a}\right) = \frac{1}{2}\left(x\sqrt{x^2+a^2} + a^2\ln(x+\sqrt{x^2+a^2})\right)$$
$$\int \sqrt{x^2-a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2-a^2} - a^2\operatorname{arcosh}\frac{x}{a}\right) = \frac{1}{2}\left(x\sqrt{x^2-a^2} - a^2\ln(x+\sqrt{x^2-a^2})\right)$$
$$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2-x^2} + a^2\arcsin\frac{x}{a}\right)$$

Trigonometrische Funktionen

	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$	π	$\frac{7}{6}\pi$	$\frac{5}{4}\pi$	$\frac{4}{3}\pi$	$\frac{3}{2}\pi$	$\frac{5}{3}\pi$	$\frac{7}{4}\pi$	$\frac{11}{6}\pi$	2π
	0^0	30^0	45^0	60^0	90^0	120^0	135^0	150^0	180^0	210^0	225^0	240^0	270^0	300^0	315^0	330^0	360^0
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
$\cot x$	$\pm\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$\pm\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$\pm\infty$

Additionstheoreme

$$\cos(x\pm y) = \cos x\cos y \mp \sin x \sin y$$
$$\sin(x\pm y) = \sin x\cos y \pm \cos x \sin y$$
$$\tan(x\pm y) = \frac{\tan x \pm \tan y}{1\mp \tan x \tan y}$$

doppelter Winkel

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$
$$\sin 2x = 2\sin x\cos x$$
$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$
$$\cot 2x = \frac{\cot^2 x - 1}{2\cot x}$$

halber Winkel

$$\cos\frac{x}{2} =^* \pm\sqrt{\frac{1}{2}(1+\cos x)}$$
$$\sin\frac{x}{2} =^* \pm\sqrt{\frac{1}{2}(1-\cos x)}$$
$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x} =^* \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$
$$\cot\frac{x}{2} = \frac{1+\cos x}{\sin x} = \frac{\sin x}{1-\cos x} =^* \pm\sqrt{\frac{1+\cos x}{1-\cos x}}$$

Symmetrie

$\cos(-x)$	$=$	$\cos x$	gerade Funktion
$\sin(-x)$	$=$	$-\sin x$	ungerade Funktion
$\tan(-x)$	$=$	$-\tan x$	ungerade Funktion
$\cot(-x)$	$=$	$-\cot x$	ungerade Funktion

$$\boxed{\cos^2 x + \sin^2 x = 1}$$

$$\cos^2 x = \frac{1}{2}(1+\cos 2x) \qquad \sin x =^* \frac{\tan x}{\pm\sqrt{1+\tan^2 x}}$$
$$\sin^2 x = \frac{1}{2}(1-\cos 2x) \qquad \cos x =^* \frac{1}{\pm\sqrt{1+\tan^2 x}}$$
$$\cos x = \sin\left(\frac{\pi}{2}\pm x\right) \qquad \tan x = \frac{\sin x}{\cos x}$$
$$\sin x = \cos\left(\frac{\pi}{2}-x\right) \qquad \cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$
$$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$$
$$\sin x\cdot\sin y = \frac{1}{2}\left(\cos(x-y) - \cos(x+y)\right)$$
$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$
$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$
$$\cos x\cdot\cos y = \frac{1}{2}\left(\cos(x-y) + \cos(x+y)\right)$$
$$\sin x\cdot\cos y = \frac{1}{2}\left(\sin(x-y) + \sin(x+y)\right)$$

* Vorzeichen je nach Quadranten!

Hyperbelfunktionen

$$\cosh x = \frac{1}{2}(e^x + e^{-x}) \qquad \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^{2x}-1}{e^{2x}+1} \qquad \cosh 0 = 1,\ \sinh 0 = 0,\ \tanh 0 = 0$$
$$\sinh x = \frac{1}{2}(e^x - e^{-x}) \qquad \tanh\frac{x}{2} = \frac{e^x-1}{e^x+1} \qquad \boxed{\cosh^2 x - \sinh^2 x = 1}$$

$$\cosh(-x) = \cosh x,\quad \sinh(-x) = -\sinh x,\quad \tanh(-x) = -\tanh x,\quad \coth(-x) = -\coth x$$

Additionstheoreme

$$\cosh(x\pm y) = \cosh x\cosh y \pm \sinh x \sinh y$$
$$\sinh(x\pm y) = \sinh x\cosh y \pm \cosh x \sinh y$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$\sinh 2x = 2\sinh x\cosh x$$

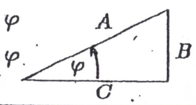
$$\cosh\frac{x}{2} = \sqrt{\frac{1}{2}(\cosh x + 1)}$$
$$\sinh\frac{x}{2} = \pm\sqrt{\frac{1}{2}(\cosh x - 1)},\quad \text{für } \begin{cases} x \geq 0 \\ x < 0 \end{cases}$$
$$\operatorname{arsinh} x = \ln(x + \sqrt{x^2+1})$$
$$\operatorname{arcosh} x = \ln(x + \sqrt{x^2-1}),\quad \text{für } x \geq 1$$

Überlagerung von Schwingungen

$$\boxed{A_1\sin(\omega t+\varphi_1) + A_2\sin(\omega t + \varphi_2) = A\sin(\omega t + \varphi)}$$
$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_1-\varphi_2)}$$
$$\tan\varphi = \frac{A_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2} \quad \text{(Quadranten beachten!)}$$

Spezialfall: $\boxed{B\cos\omega t + C\sin\omega t = A\sin(\omega t+\varphi)}$

$$B = A\sin\varphi \qquad C = A\cos\varphi$$



$$A = \sqrt{B^2 + C^2} \qquad \tan\varphi = \frac{B}{C}$$ Quadranten beachten!

Quadratische Gleichung

$$x^2 + px + q = 0$$
$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}\,.$$

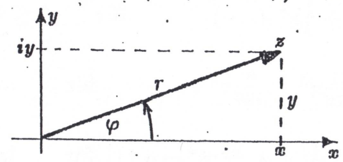
allgemeine Binomialkoeffizienten

$r \in \mathbb{R}$ und $k = 1, 2, \ldots$

$$\binom{r}{k} = \frac{r(r-1)\cdots(r-k+1)}{k!}$$
$$\binom{r}{0} = \binom{r}{r} = 1,\quad \binom{r}{1} = r$$

Polarkoordinaten

$$x = r\cos\varphi \qquad r = \sqrt{x^2+y^2}$$
$$y = r\sin\varphi \qquad \tan\varphi = \frac{y}{x} \ \text{Quadranten beachten!}$$
$$dF = r\,dr\,d\varphi$$
$$z = x + iy = r(\cos\varphi + i\sin\varphi) = re^{i\varphi}$$



Rechnen mit Potenzen und Logarithmen

a: Basis, mit $0 < a \neq 1$

$a^{x+y} = a^x a^y$	$\log_a xy = \log_a x + \log_a y$
$a^{-x} = \frac{1}{a^x}$	$\log_a \frac{1}{x} = -\log_a x$
$a^0 = 1$	$\log_a 1 = 0$
$(a^x)^r = a^{xr}$	$\log_a x^r = r\log_a x$

Logarithmen zu verschiedenen Basen:

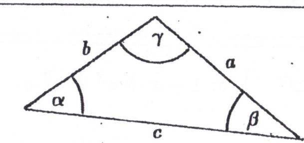
$$\log_a x = \frac{\log_b x}{\log_b a},\ \text{speziell: } \log_a x = \frac{\ln x}{\ln a}$$

Cosinussatz

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Pythagoras

$$c^2 = a^2 + b^2,\ \text{falls } \gamma = 90^0.$$

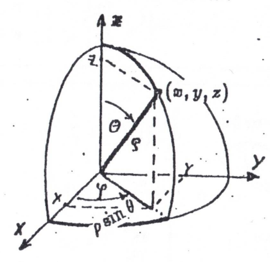


Sinussatz

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Kugelkoordinaten

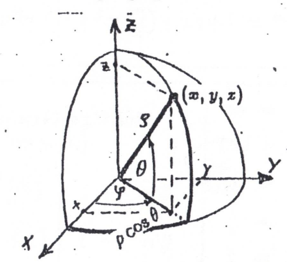
θ : Polabstand



$$x = \rho\sin\theta\cos\varphi$$
$$y = \rho\sin\theta\sin\varphi$$
$$z = \rho\cos\theta$$
$$dV = \rho^2\sin\theta\,d\rho\,d\theta\,d\varphi$$

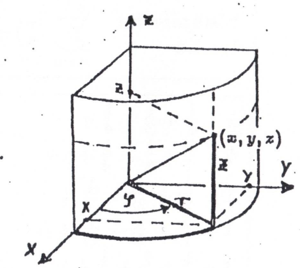
Kugelkoordinaten

θ : geographische Breite



$$x = \rho\cos\theta\cos\varphi$$
$$y = \rho\cos\theta\sin\varphi$$
$$z = \rho\sin\theta$$
$$dV = \rho^2\cos\theta\,d\rho\,d\theta\,d\varphi$$

Zylinderkoordinaten



$$x = r\cos\varphi$$
$$y = r\sin\varphi$$
$$z = z$$
$$dV = r\,dr\,d\varphi\,dz$$