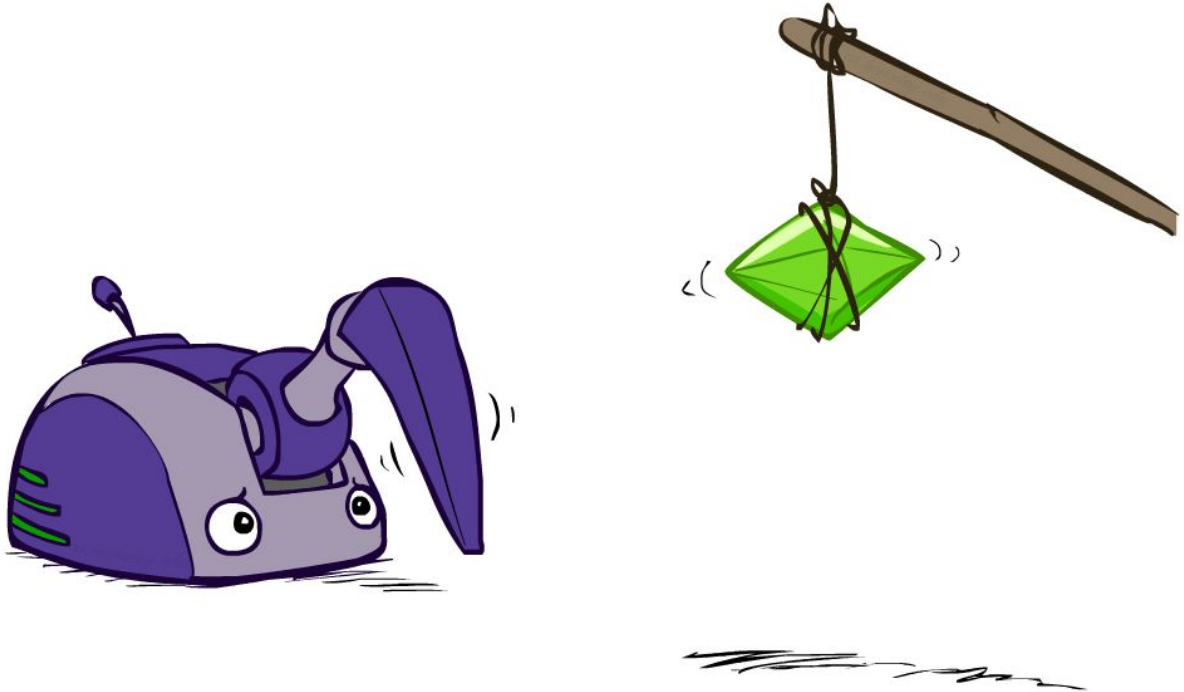
Most materials in this chapter are based on materials from http://aima.cs.berkeley.edu/ and http://ai.berkeley.edu

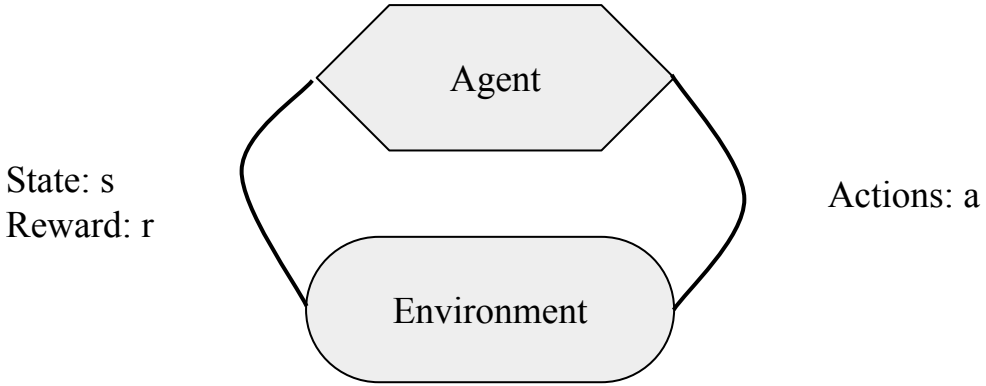
# 3. Reinforcement Learning

COMP 3270
Artificial Intelligence

Dirk Schnieders



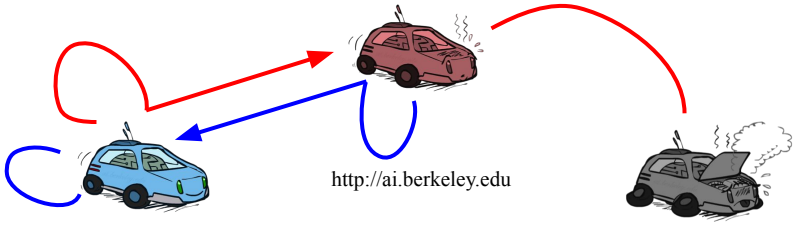
# Reinforcement Learning - Idea




- Basic idea:
  - Receive feedback in the form of rewards
  - Agent's utility is defined by the reward function
  - Must (learn to) act so as to maximize expected rewards
  - All learning is based on observed samples of outcomes!

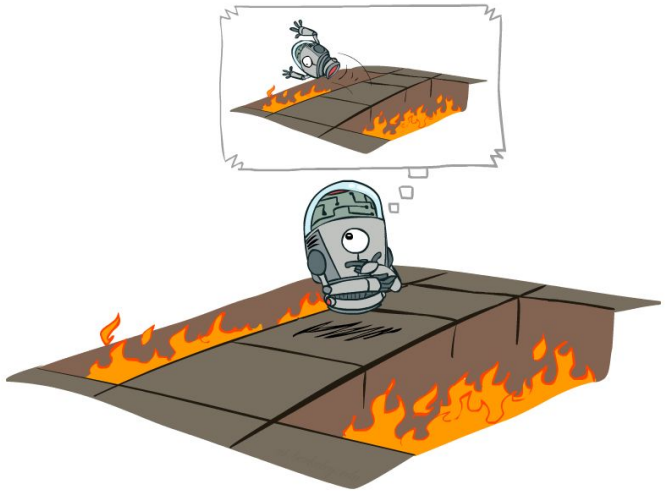
# Reinforcement Learning

- It's just the same ...
  - Still assume a Markov decision process (MDP):
    - A set of states s ∈ S
    - A set of actions (per state) A
    - A model T(s,a,s')
    - A reward function R(s,a,s')
  - Still looking for a policy **π**(s)




- New
  - Don't know T or R
    - I.e., we don't know what the actions do
  - Must actually try actions and states out to learn

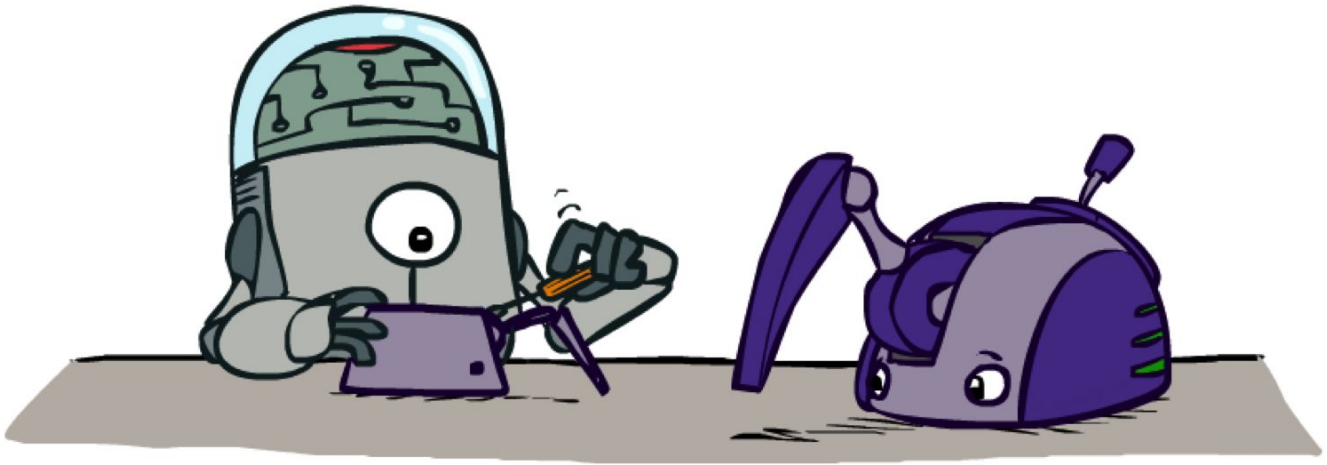
# Offline (MDPs) vs. Online (RL)



Offline

Online

# Model-Based Learning



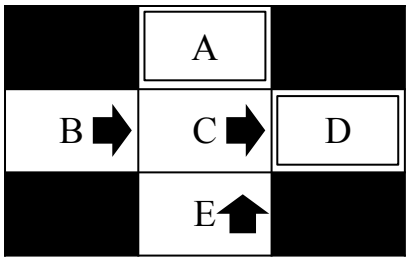
# Model-Based Learning

- Model-Based Idea:
  - Learn an approximate model based on experiences
  - Solve for values as if the learned model were correct
  - Step 1: Learn empirical MDP model
    - Count outcomes s' for each s, a
    - Normalize to give an estimate of $\hat{T}(s, a, s')$
    - Discover each $\hat{R}(s, a, s')$ when we experience (s, a, s')
  - Step 2: Solve the learned MDP
    - For example, use policy iteration, as before

$$V_{k+1}^{\pi_i}(s) \leftarrow \sum_{s'} \hat{T}(s, \pi_i(s), s')[\hat{R}(s, \pi_i(s), s') + \gamma V_k^{\pi_i}(s')]$$

# Example: Model-Based Learning

Given: $\pi$




$\gamma = 1.0$

Episode 1:
B, east, C, -1
C, east, D, -1
D, exit, , +10

Episode 2:
B, east, C, -1
C, east, D, -1
D, exit, , +10

T^(s, a, s'):
T^(B, east, C) = 1.00
T^(C, east, D) = 0.75
T^(C, east, A) = 0.25
...

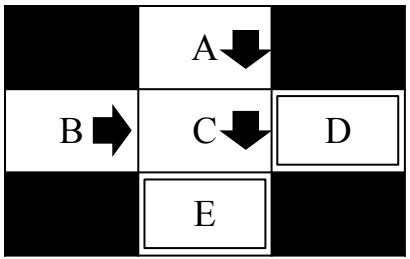
Episode 3:
B, east, C, -1
C, east, D, -1
D, exit, , +10

Episode 4:
E, north, C, -1
C, east, A, -1
A, exit, , -10

R^(s, a, s'):
R^(B, east, C) = -1
R^(C, east, D) = -1
R^(C, east, A) = -1
...

# Quiz - Model-Based Learning

Given: $\pi$




$\gamma = 1.0$

Episode 1:
A, south, C, -1
C, south, E, -1
E, exit, , 10

Episode 2:
B, east, C, -1
C, south, D, -1
D, exit, , -10

Episode 3:
B, east, C, -1
C, south, E, -1
E, exit, , 10

Episode 4:
A, south, C, -1
C, south, E, -1
E, exit, , 10

$\hat{T}(s, a, s')$:
$\hat{T}(A, south, C) = ?$
$\hat{T}(B, east, C) = ?$
$\hat{T}(C, south, E) = ?$
$\hat{T}(C, south, D) = ?$

# Example: Expected Age

- Goal: Compute expected age of comp3270 students

Known P(a)

$$E[A] = \sum_a P(a) \cdot a \quad = 0.35 \times 20 + \ldots$$

Unknown P(a): Model based

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

$$E[A] \approx \sum_a \hat{P}(a) \cdot a$$

Unknown P(a): Model free

$$E[A] \approx \frac{1}{N} \sum_i a_i$$

# Quiz - Model-Based Learning

- Consider an MDP with 3 states, A, B and C; and 2 actions CW and CCW
- We are given samples of what an agent experiences
- In this problem, we will first estimate the model
  - i.e., the transition function and the reward function
- Consider the samples (shown on the next slide) that the agent encountered

# Quiz (continued) - Model-Based Learning

| s | a | s' | r |
|---|---|---|---|
| A | CW | C | -9.0 |
| A | CW | C | -9.0 |
| A | CW | B | 0.0 |
| A | CW | B | 0.0 |
| A | CW | B | 0.0 |
| A | CCW | B | 0.0 |
| A | CCW | B | 0.0 |
| A | CCW | B | 0.0 |
| A | CCW | C | -3.0 |
| A | CCW | B | 0.0 |

| s | a | s' | r |
|---|---|---|---|
| B | CW | A | 2.0 |
| B | CW | A | 2.0 |
| B | CW | A | 2.0 |
| B | CW | A | 2.0 |
| B | CW | A | 2.0 |
| B | CCW | A | -6.0 |
| B | CCW | A | -6.0 |
| B | CCW | C | 5.0 |
| B | CCW | C | 5.0 |
| B | CCW | C | 5.0 |

| s | a | s' | r |
|---|---|---|---|
| C | CW | A | 3.0 |
| C | CW | A | 3.0 |
| C | CW | A | 3.0 |
| C | CW | A | 3.0 |
| C | CW | B | -4.0 |
| C | CCW | A | 0.0 |
| C | CCW | A | 0.0 |
| C | CCW | A | 0.0 |
| C | CCW | A | 0.0 |
| C | CCW | A | 0.0 |

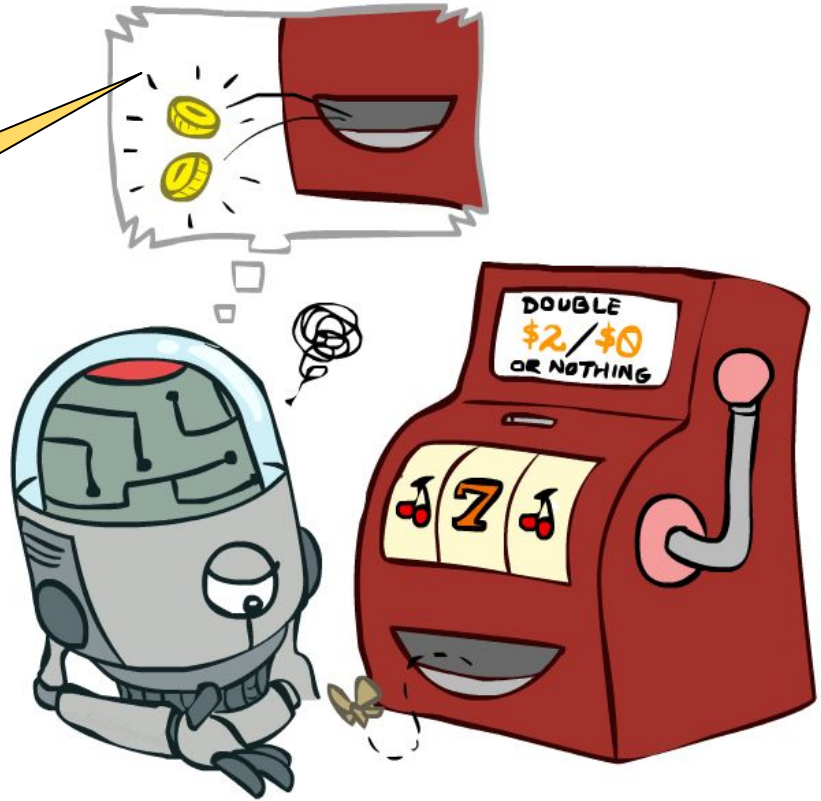
# Quiz (continued) - Model-Based Learning

- Estimating the transition function, T(s,a,s') and reward function R(s,a,s') for this MDP
- Fill in the missing values in the table for T(s,a,s') and R(s,a,s’)

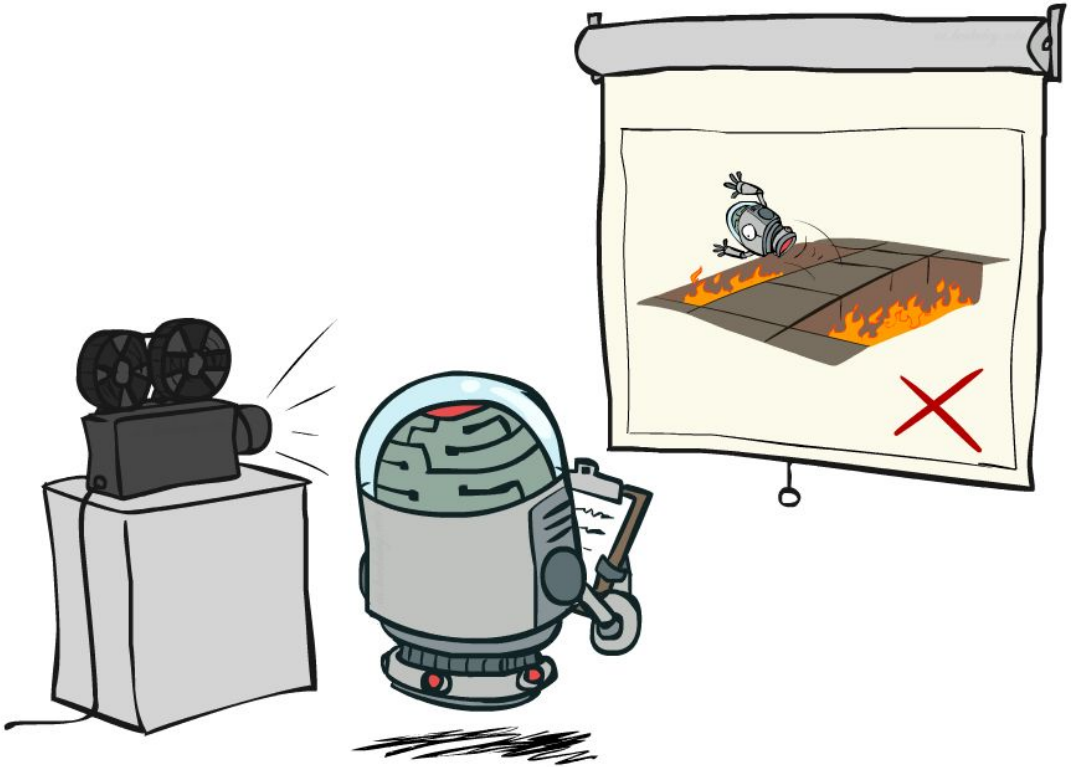
| s | a | s' | T(s,a,s') | R(s,a,s') |
|---|---|---|---|---|
| A | CW | B | | |
| A | CW | C | | |
| A | CCW | B | 0.8 | 0.0 |
| A | CCW | C | 0.2 | -3.0 |
| B | CW | A | 1.0 | 2.0 |
| B | CCW | A | 0.4 | -6.0 |
| B | CCW | C | 0.6 | 5.0 |
| C | CW | A | 0.8 | 3.0 |
| C | CW | B | 0.2 | -4.0 |
| C | CCW | A | 1.0 | 0.0 |

# Model-Free Learning

In model-free learning you compare what you want vs. what you got



# Passive Reinforcement Learning



# Passive Reinforcement Learning

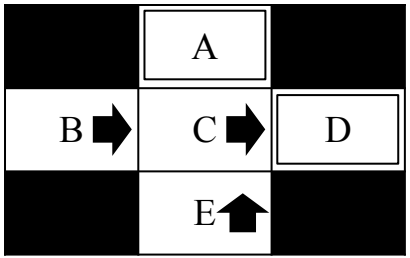
- Simplified task: policy evaluation
  - Input: a fixed policy $\pi(s)$
  - You don't know the transitions T(s,a,s')
  - You don't know the rewards R(s,a,s')
  - Goal: learn the state values
- In this case:
  - Learner is "along for the ride"
  - No choice about what actions to take
  - Just execute the policy and learn from experience
  - NOT offline planning! You actually take actions in the world

# Direct Evaluation

- Goal: Compute values for each state under $\pi$
- Idea: Average together observed sample values
  - Act according to $\pi$
  - Every time you visit a state, write down what the sum of discounted rewards turned out to be
  - Average those samples
- This is called direct evaluation

# Example: Direct Evaluation

Given: $\pi$

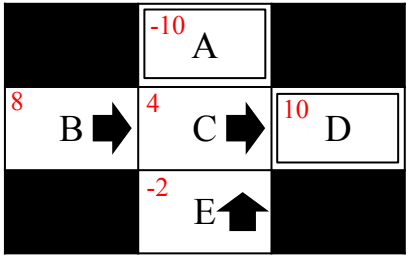



$\gamma = 1.0$

Episode 1:
B, east, C, -1
C, east, D, -1
D, exit, , +10

Episode 2:
B, east, C, -1
C, east, D, -1
D, exit, , +10

Output: Values

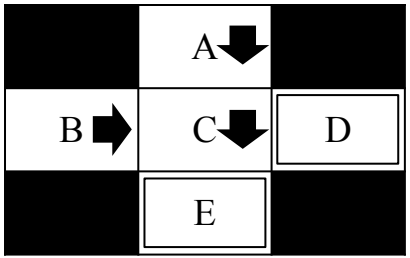



Episode 3:
E, north, C, -1
C, east, D, -1
D, exit, , +10

Episode 4:
E, north, C, -1
C, east, A, -1
A, exit, , -10

# Quiz - Direct Evaluation

Given: $\pi$

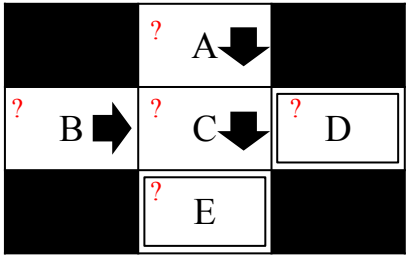



$\gamma = 1.0$

Episode 1:
A, south, C, -1
C, south, E, -1
E, exit, , +10

Episode 2:
B, east, C, -1
C, south, D, -1
D, exit, , -10

Output: Values



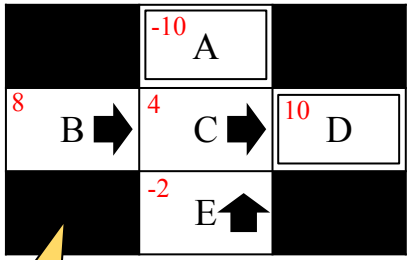

Episode 3:
B, east, C, -1
C, south, E, -1
E, exit, , +10

Episode 4:
A, south, C, -1
C, south, E, -1
E, exit, , +10

# Problems with Direct Evaluation

- Positives
  - Easy to understand
  - Knowledge of T and R not required
  - Eventually computes the correct values
- Negatives
  - State connections are not considered
  - Each state must be learned separately
  - Takes a long time to learn

Output: Values




B and E both go to C under this policy. However, their values are very different ...

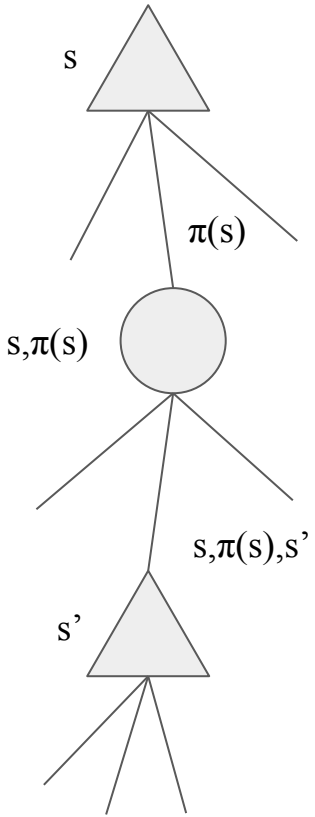
# Why Not Use Policy Evaluation?

- Can we use policy evaluation?
  - Each round, replace V with a one-step-look-ahead layer over V

$$V_0^\pi(s) = 0$$

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^\pi(s')]$$

- This approach fully exploits the connections
  - Unfortunately, we need T and R to do it



# Sample-Based Policy Evaluation?

- We want to improve our estimate of V by computing these averages:

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$

- Idea: Take samples of outcomes s' (by doing the action!) and average

$$sample_1 = R(s, \pi(s), s'_1) + \gamma V_k^{\pi}(s'_1)$$
$$sample_2 = R(s, \pi(s), s'_2) + \gamma V_k^{\pi}(s'_2)$$
$$\cdots$$
$$sample_n = R(s, \pi(s), s'_n) + \gamma V_k^{\pi}(s'_n)$$

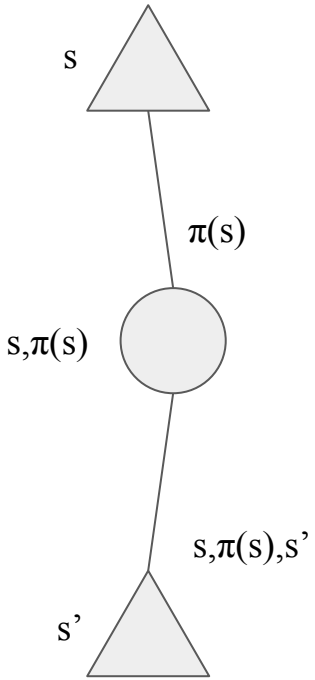
$$V_{k+1}^{\pi}(s) \leftarrow \frac{1}{n} \sum_i sample_i$$

Will this work?

Almost, however we cannot rewind time to get sample after sample from state s

# Temporal Difference Learning

- Big idea: learn from every experience!
  - Update V(s) each time we experience a transition (s, a, s', r)
  - Likely outcomes s' will contribute updates more often
- Temporal difference learning of values
  - Policy still fixed, still doing evaluation!
  - Move values toward value of whatever successor occurs: running average




Sample of V(s): $sample = R(s, \pi(s), s') + \gamma V^{\pi}(s')$

Update to V(s): $V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + (\alpha)sample$

Same update: $V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha(sample - V^{\pi}(s))$

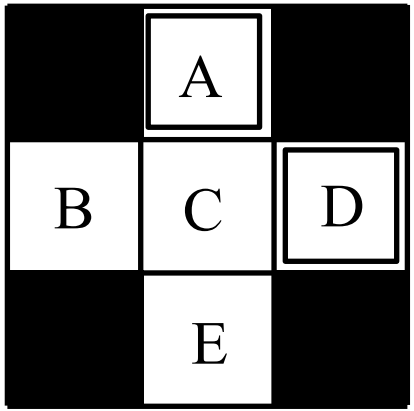
# Temporal Difference Learning

- Exponential Moving Average
  - The running interpolation update makes recent samples more important
  - Forgets about the past (distant past values were wrong anyway)
- Decreasing learning rate (alpha) can give converging averages

# Demo

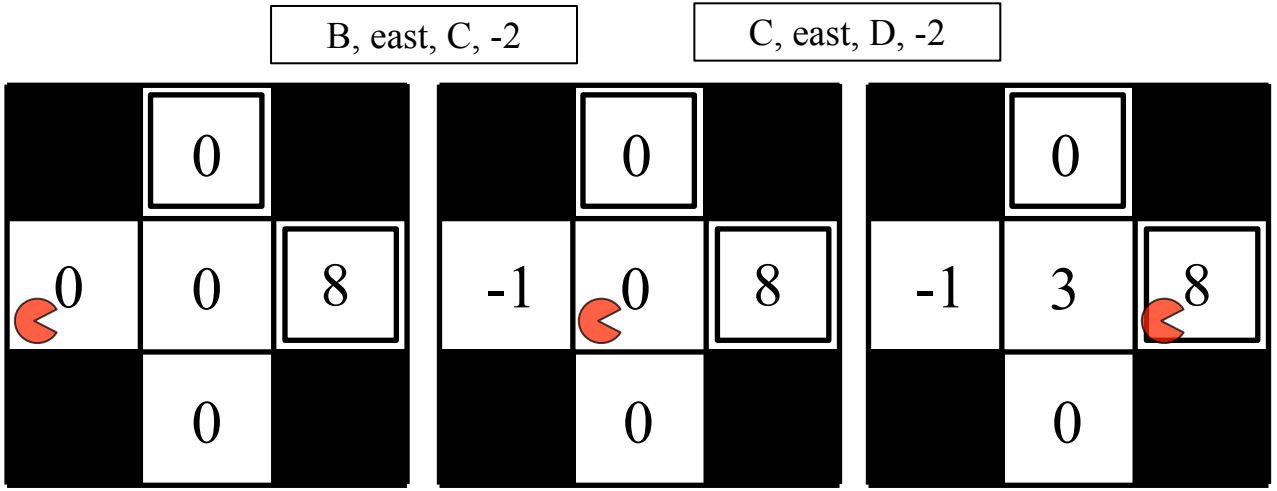
# Another Example

States




α = 0.5, γ = 1.0

Observed Transitions

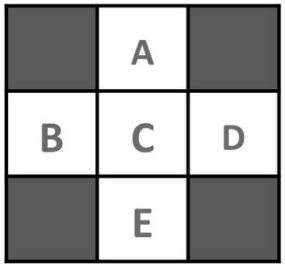



$$V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + \alpha\left[R(s,\pi(s),s') + \gamma V^{\pi}(s')\right]$$

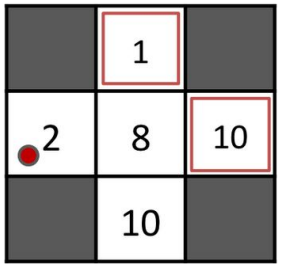
# Quiz - Temporal Difference Learning

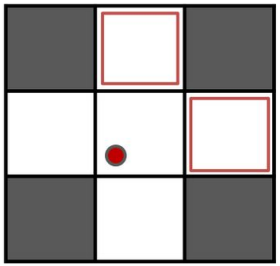
- Consider the grid world shown below
- The left panel shows the name of each state A through E. The middle panel shows the current estimate of the value function $V^{\pi}$ for each state
- A transition is observed, that takes the agent from state B through taking action east into state C, and the agent receives a reward of -2.
- What are the value estimates after the TD learning update?

States




Observed Transition: B, east, C, -2



*Assume:* $\gamma = 1$, $\alpha = 1/2$

$$V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + \alpha\left[R(s,\pi(s),s') + \gamma V^{\pi}(s')\right]$$

# TD Value Learning

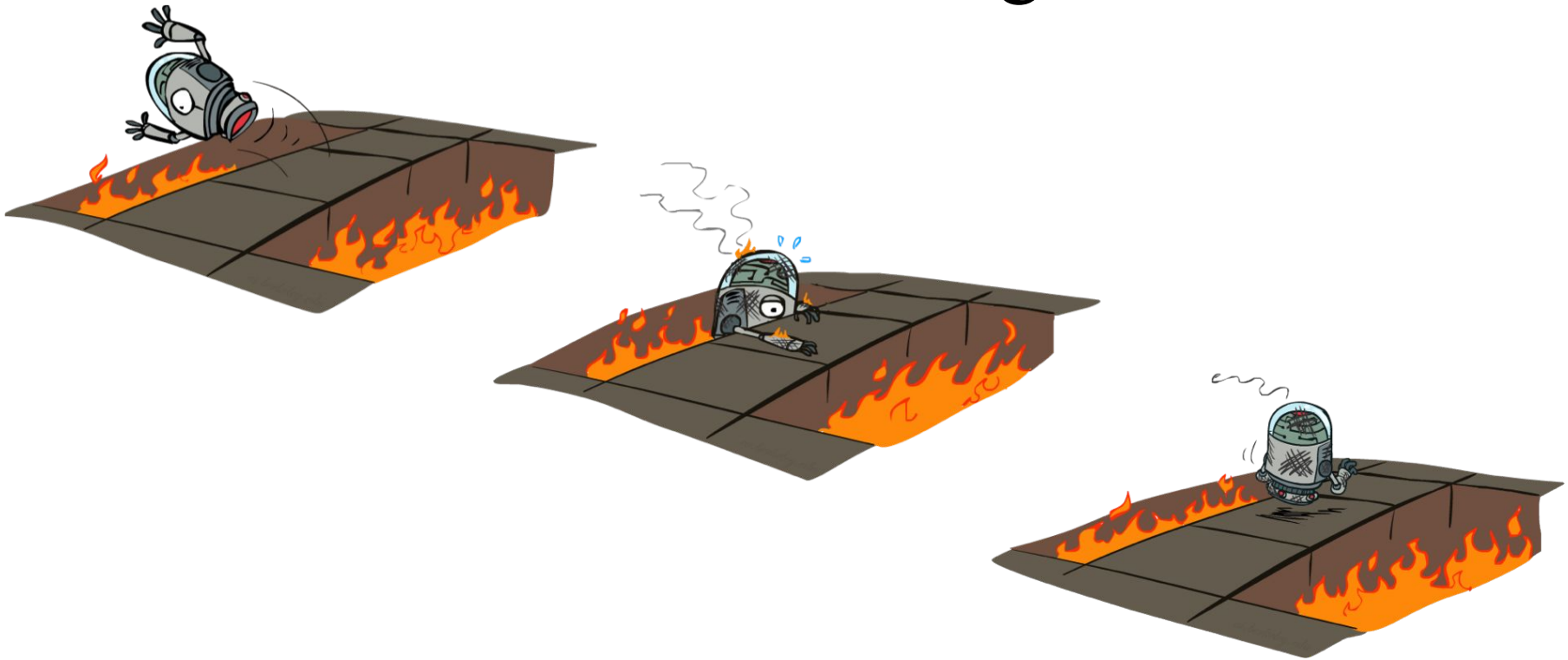
- Model-free way to do policy evaluation, mimicking Bellman updates with running sample averages
- However, we cannot turn values into a new policy

$$\pi(s) = \arg\max_a Q(s,a)$$

$$Q(s,a) = \sum_{s'} T(s,a,s') \left[ R(s,a,s') + \gamma V(s') \right]$$

- Idea
  - Learn Q-values, not values
  - Makes action selection model-free

# Active Reinforcement Learning



# Detour: Q-Value Iteration

- Value iteration: find successive (depth-limited) values
  - Start with $V_0(s) = 0$, which we know is right
  - Given $V_k$, calculate the depth k+1 values for all states:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[ R(s,a,s') + \gamma V_k(s') \right]$$

- But Q-values are more useful, so compute them instead
  - Start with $Q_0(s,a) = 0$, which we know is right
  - Given $Q_k$, calculate the depth k+1 q-values for all q-states:

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[ R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$$

# Q-Learning

- Q-Learning: sample-based Q-value iteration

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[ R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$$

- Learn Q(s,a) values as you go
  - Receive a sample (s,a,s',r)
  - Consider your old estimate: $Q(s,a)$
  - Consider your new sample estimate:
    $sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$
  - Incorporate the new estimate into a running average:
    $Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\,[sample]$

# Demo

# Quiz - Q-Learning

- Consider an MDP with 3 states, A, B and C; and 2 actions CW and CCW
- In this quiz, instead of first estimating the transition and reward functions, we will directly estimate the Q function using Q-learning
- Assume, the discount factor is 0.5 and the step size for Q-learning is 0.5
- Let the current Q function, Q(s,a), be

| | A | B | C |
|---|---|---|---|
| CW | -2 | 2 | 1 |
| CCW | 0.5 | 2 | 10 |

- The agent encounters the following samples

| s | a | s' | r |
|---|---|---|---|
| A | CW | B | 0.0 |
| B | CW | A | 2.0 |

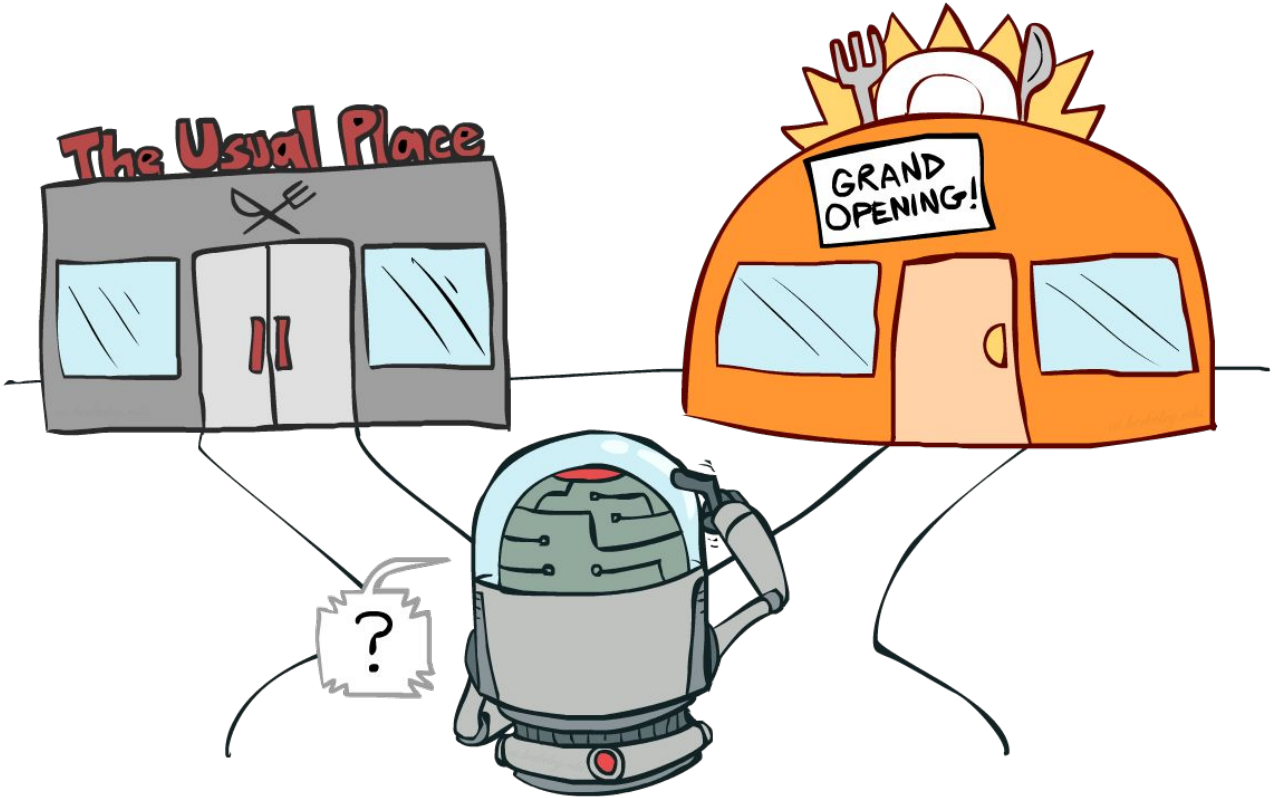
- Process the samples given above and fill in the Q-values after both samples have been accounted for

| | A | B | C |
|---|---|---|---|
| CW | | | |
| CCW | | | |

# Q-Learning Properties

- Amazing result
  - Q-learning converges to optimal policy -- even if you're acting suboptimally!
  - This is called off-policy learning
- Limitations:
  - You have to explore enough
  - You have to eventually make the learning rate small enough
    - … but not decrease it too quickly
  - Basically, in the limit, it doesn't matter how you select actions

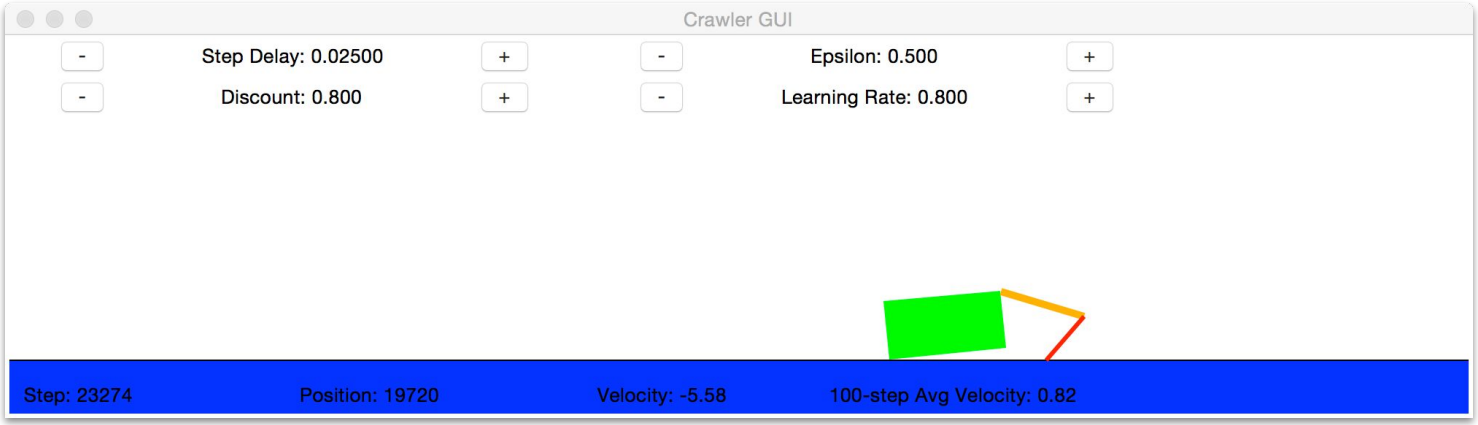
# Exploration vs. Exploitation



# How to Explore?

- Several schemes for forcing exploration
- Simplest: random actions (ε-greedy)
  - Every time step, flip a coin
    - With (small) probability ε, act randomly
    - With (large) probability 1-ε, act on current policy
  - Problems with random actions?
    - You do eventually explore the space, but keep thrashing around once learning is done
  - Solution
    - Lower ε over time
    - Exploration functions

# Crawler in Assignment 3



# Exploration Functions

- When to explore?
  - Random actions: explore a fixed amount
  - Better idea: explore areas whose badness is not (yet) established, eventually stop exploring
- Exploration function
  - Takes a value estimate u and a visit count n, and returns an optimistic utility, e.g. $f(u, n) = u + k/n$

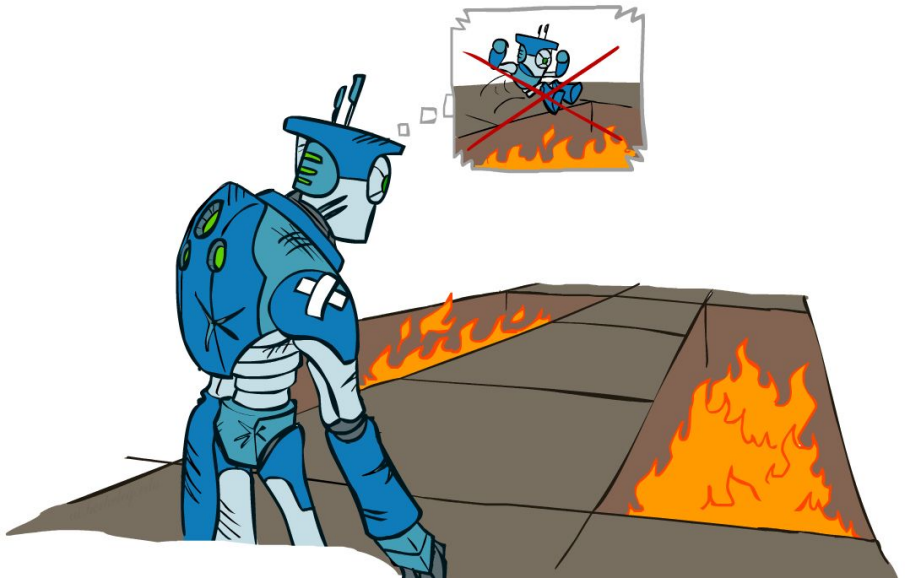
$$Q(s,a) \leftarrow_\alpha R(s,a,s') + \gamma \max_{a'} Q(s',a')$$

$$Q(s,a) \leftarrow_\alpha R(s,a,s') + \gamma \max_{a'} f(Q(s',a'), N(s',a'))$$

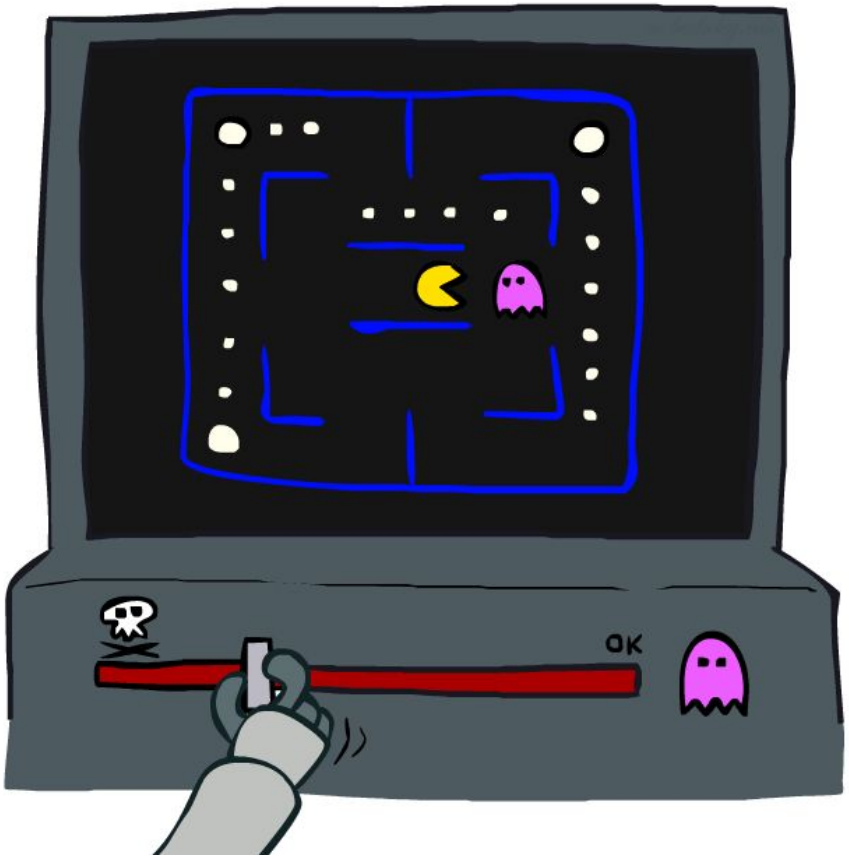
- Note: this propagates the “bonus” (k) back to states that lead to unknown states as well!

# Regret

- Even if you learn the optimal policy, you still make mistakes along the way!
- Regret is a measure of your total mistake cost: the difference between your (expected) rewards, including youthful suboptimality, and optimal (expected) rewards
- Minimizing regret goes beyond learning to be optimal – it requires optimally learning to be optimal
- Example: random exploration and exploration functions both end up optimal, but random exploration has higher regret

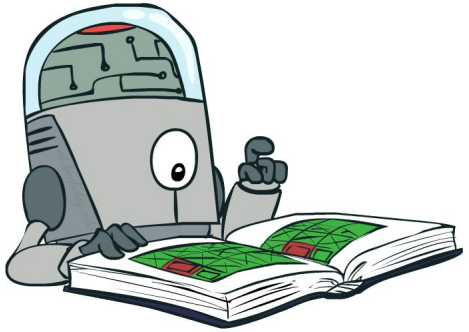


# Approximate Q-Learning



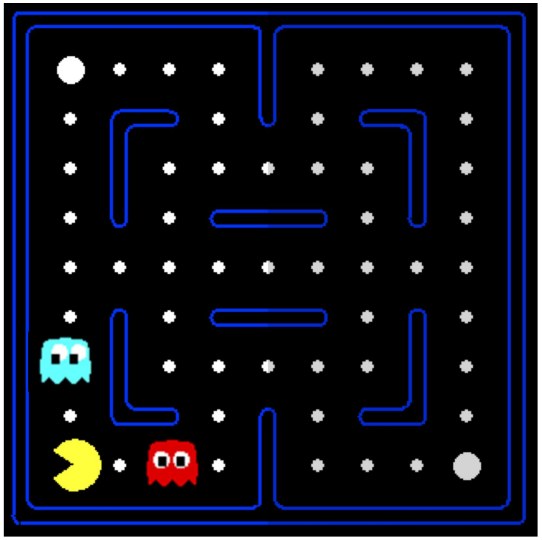
# Generalizing Across States

- Basic Q-Learning keeps a table of all q-values
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory
- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar situations
  - This is a fundamental idea in machine learning

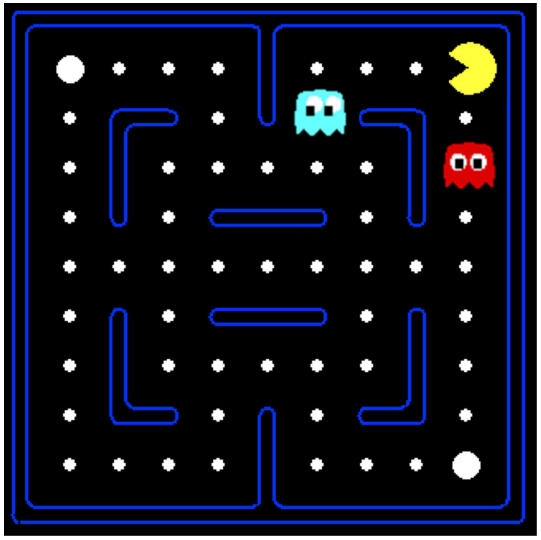


# Example: Pacman

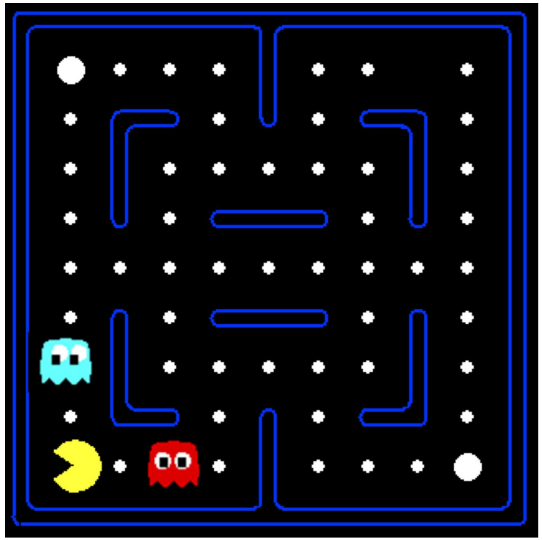
Let's say we discover through experience that this state is bad



In naive q-learning, we know nothing about this state

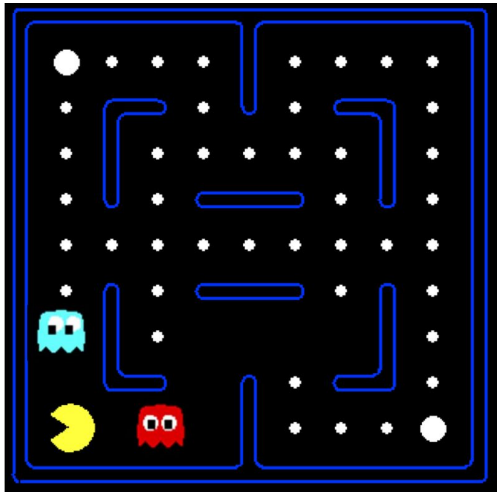


Or even this one!



# Feature-Based Representations

- Solution: describe a state using a vector of features (properties)
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - 1 / (dist to dot)$^2$
    - Is Pacman in a tunnel? (0/1)
    - …… etc.
    - Is it the exact state on this slide?



- Can also describe a q-state (s, a) with features (e.g. action moves closer to food)

# Linear Value / Q Functions

- Using a feature representation, we can write a Q-function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Advantage
  - Our experience is summed up in a few powerful numbers
- Disadvantage
  - States may share features but actually be very different in value

# Approximate Q-Learning

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

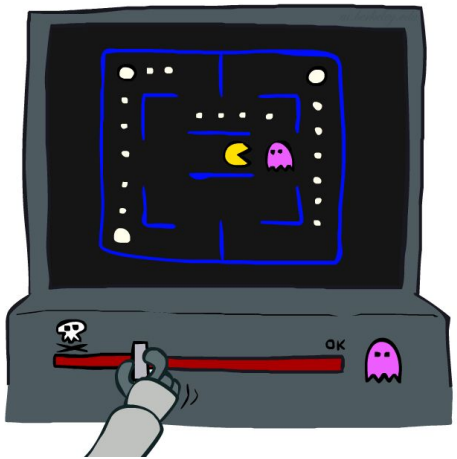
- Q-learning with linear Q-functions

$$\text{transition} = (s, a, r, s')$$

$$\text{difference} = \left[r + \gamma \max_{a'} Q(s', a')\right] - Q(s,a)$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha\,[\text{difference}] \qquad \text{Exact Q's}$$

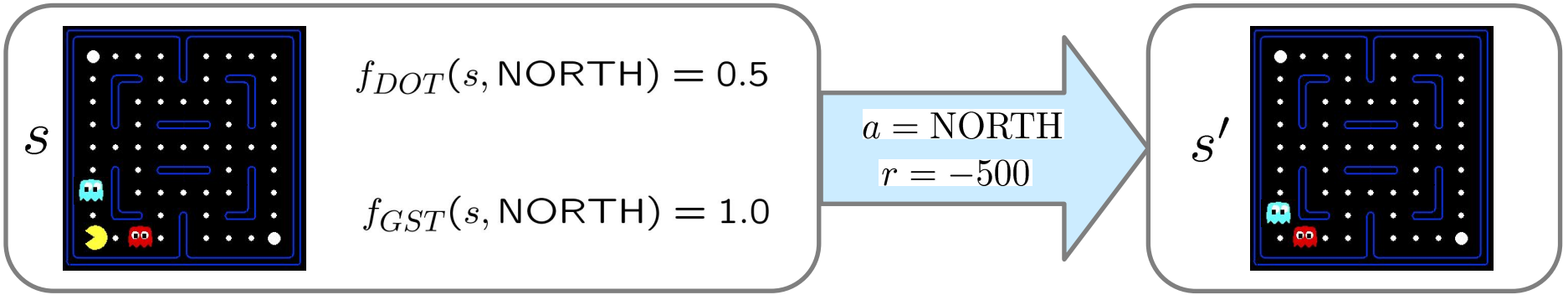
$$w_i \leftarrow w_i + \alpha\,[\text{difference}]\, f_i(s,a) \qquad \text{Approximate Q's}$$




- Intuitive interpretation:
  - Adjust weights of active features
  - E.g., if something unexpectedly bad happens, blame the features that were on
    - disprefer all states with that state's features

# Example: Q-Pacman

$$Q(s,a) = 4.0 f_{DOT}(s,a) - 1.0 f_{GST}(s,a)$$




$Q(s, \text{NORTH}) = +1$

$Q(s', \cdot) = 0$

$$r + \gamma \max_{a'} Q(s', a') = -500 + 0$$

$\text{difference} = -501$

$$w_{DOT} \leftarrow 4.0 + \alpha\,[-501]\,0.5$$
$$w_{GST} \leftarrow -1.0 + \alpha\,[-501]\,1.0$$

$$Q(s,a) = 3.0 f_{DOT}(s,a) - 3.0 f_{GST}(s,a)$$

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$



# Quiz: Approx. Q-Learning

$$\text{difference} = \left[ r + \gamma \max_{a'} Q(s', a') \right] - Q(s,a)$$

$$w_i \leftarrow w_i + \alpha \, [\text{difference}] \, f_i(s,a)$$

- Consider an unknown MDP where there are three states [A, B, C] and two actions [1, 2]
- We are given the following samples generated from taking actions
- Assume $\gamma = 1$ and $\alpha = 0.5$ and the following two features
- $f_1(s, a) = 1$ and $f_2(s, a) = (-1)^a$
- Starting from initial weights of 0, compute the updated weights after observing the following two samples

| s | a | s' | r |
|---|---|---|---|
| A | 2 | B | 4 |
| B | 1 | A | 0 |

- What are the weights after the first update?
- What are the weights after the second update?