Introduction to Software-Defined Radio

Analog demodulation of signals using GNURadio

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I. Presentation of the acquisition device

This part will mainly be dedicated to considering the problem from a mathematical point of view.

We know that the emitted signal can be written:

$$s_{RF}(t) = A(t) \cdot \cos(2 \cdot \pi \cdot f_0 \cdot t + \varphi(t)),\ t \in \mathbb{R} \quad (1)$$

We decided to note $s_R(t)$ the channel in phase and $s_I(t)$ the channel in quadrature so that:

- $s_R(t) = A(t) \cdot \cos(\varphi(t)),\ t \in \mathbb{R}$ (2)
- $s_I(t) = A(t) \cdot \sin(\varphi(t)),\ t \in \mathbb{R}$ (3)

Thus, we can rewrite $s_{RF}(t) = s_R(t) \cdot \cos(2 \cdot \pi \cdot f_0 \cdot t) - s_I(t) \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t),\ t \in \mathbb{R}$ (4)

The signal arriving to the receiver is named $r_{RF}(t)$. The receiver is described more precisely in Figure 1.

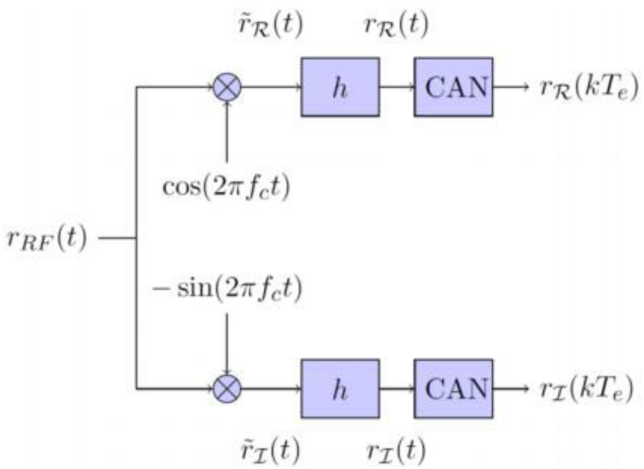

Figure 1 : Block diagram of the receiver

Question 1

According to Fig. 1, considering that the received signal is similar to the transmitted one ($r_{RF}(t) = s_{RF}(t)$) and using (4) and trigonometric formulas, express the signal $\tilde{r}_R(t)$ and $\tilde{r}_I(t)$ in function of $s_R(t)$, $s_I(t)$, f_0 and f_c.

$r_{RF}(t) = s_{RF}(t) = s_R(t) \cdot \cos(2 \cdot \pi \cdot f_0 \cdot t) - s_I(t) \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t),\ t \in \mathbb{R}$

We know that $\mathbf{\tilde{r}_R(t)} = r_{RF}(t) \cdot \cos(2 \cdot \pi \cdot f_c \cdot t)$

$$= [s_R(t) \cdot \cos(2 \cdot \pi \cdot f_0 \cdot t) - s_I(t) \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t)] \cdot \cos(2 \cdot \pi \cdot f_c \cdot t)$$

$$= s_R(t) \cdot \cos(2 \cdot \pi \cdot f_0 \cdot t) \cdot \cos(2 \cdot \pi \cdot f_c \cdot t) - s_I(t) \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t) \cdot \cos(2 \cdot \pi \cdot f_c \cdot t)$$

$$= \frac{s_R(t)}{2} \cdot [\cos(2 \cdot \pi \cdot f_0 \cdot t + 2 \cdot \pi \cdot f_c \cdot t) + \cos(2 \cdot \pi \cdot f_0 \cdot t - 2 \cdot \pi \cdot f_c \cdot t)]$$

$$- \frac{s_I(t)}{2} \cdot [\sin(2 \cdot \pi \cdot f_0 \cdot t + 2 \cdot \pi \cdot f_c \cdot t) + \sin(2 \cdot \pi \cdot f_0 \cdot t - 2 \cdot \pi \cdot f_c \cdot t)]$$

$$\boxed{\begin{aligned} &= \frac{s_R(t)}{2} \cdot [\cos(2 \cdot \pi \cdot (f_0 + f_c) \cdot t) + \cos(2 \cdot \pi \cdot (f_0 - f_c) \cdot t)] \\ &- \frac{s_I(t)}{2} \cdot [\sin(2 \cdot \pi \cdot (f_0 + f_c) \cdot t) + \sin(2 \cdot \pi \cdot (f_0 - f_c) \cdot t)] \end{aligned}}$$

We know that $\mathbf{\tilde{r}_I(t)}$ = $r_{RF}(t) \cdot (-\sin(2 \cdot \pi \cdot f_c \cdot t))$

$$= [s_R(t) \cdot \cos(2 \cdot \pi \cdot f_0 \cdot t) - s_I(t) \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t)] \cdot (-\sin(2 \cdot \pi \cdot f_c \cdot t))$$

$$= s_R(t) \cdot \cos(2 \cdot \pi \cdot f_0 \cdot t) \cdot (-\sin(2 \cdot \pi \cdot f_c \cdot t)) - s_I(t) \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t) \cdot (-\sin(2 \cdot \pi \cdot f_c \cdot t))$$

$$= \frac{s_I(t)}{2} \cdot [\cos(2 \cdot \pi \cdot f_0 \cdot t - 2 \cdot \pi \cdot f_c \cdot t) - \cos(2 \cdot \pi \cdot f_0 \cdot t + 2 \cdot \pi \cdot f_c \cdot t)]$$

$$- \frac{s_R(t)}{2} \cdot [\sin(2 \cdot \pi \cdot f_0 \cdot t + 2 \cdot \pi \cdot f_c \cdot t) - \sin(2 \cdot \pi \cdot f_0 \cdot t - 2 \cdot \pi \cdot f_c \cdot t)]$$

$$\boxed{\begin{aligned} &= \frac{s_I(t)}{2} \cdot [\cos(2 \cdot \pi \cdot (f_0 - f_c) \cdot t) - \cos(2 \cdot \pi \cdot (f_0 + f_c) \cdot t)] \\ &- \frac{s_R(t)}{2} \cdot [\sin(2 \cdot \pi \cdot (f_0 + f_c) \cdot t) - \sin(2 \cdot \pi \cdot (f_0 - f_c) \cdot t)] \end{aligned}}$$

Question 2

If we take $f_c = f_0$ -translation in the baseband by heterodyning - , what should be the characteristics of the h filters to get $r_R(t) = s_R(t)$ and $r_I(t) = s_I(t)$?

The role of the low pass filter h is to cut the peak at a certain frequency without modifying the signals $s_R(t)$ and $s_I(t)$. We know that $r_R(t) = \tilde{r}_R(t) \cdot h$. In order to do that we have to make a representation of $\tilde{R}_R(f)$ and $\tilde{R}_I(f)$, after using the Fourier transforms.

Knowing that $f_c = f_0$, we have the following equations for $\tilde{r}_R(t)$ and $\tilde{r}_I(t)$:

- $\tilde{r}_R(t) = \frac{s_R(t)}{2} \cdot [\cos(2 \cdot \pi \cdot (2 \cdot f_0) \cdot t) + 1] - \frac{s_I(t)}{2} \cdot [\sin(2 \cdot \pi \cdot (2 \cdot f_0) \cdot t]$
- $\tilde{r}_I(t) = \frac{s_I(t)}{2} \cdot [1 - \cos(2 \cdot \pi \cdot (2 \cdot f_0) \cdot t)] - \frac{s_R(t)}{2} \cdot [\sin(2 \cdot \pi \cdot (2 \cdot f_0) \cdot t]$

We apply the Fourier transform on those two equations as the frequency spectrum will help in determining which frequency range needs to be filtered to retrieve the information:

- $\tilde{R}_R(f) = \frac{1}{4} \cdot [2 \cdot S_R(f) + S_R \cdot (f - 2 \cdot f_0) + S_R \cdot (f + 2 \cdot f_0) + j \cdot S_I \cdot (f - 2 \cdot f_0) - j \cdot S_I \cdot (f + 2 \cdot f_0)]$
- $\tilde{R}_I(f) = \frac{1}{4} \cdot [2 \cdot S_I(f) + S_I \cdot (f - 2 \cdot f_0) + S_I \cdot (f + 2 \cdot f_0) + j \cdot S_R \cdot (f - 2 \cdot f_0) - j \cdot S_R \cdot (f + 2 \cdot f_0)]$

We obtain the following graphic:

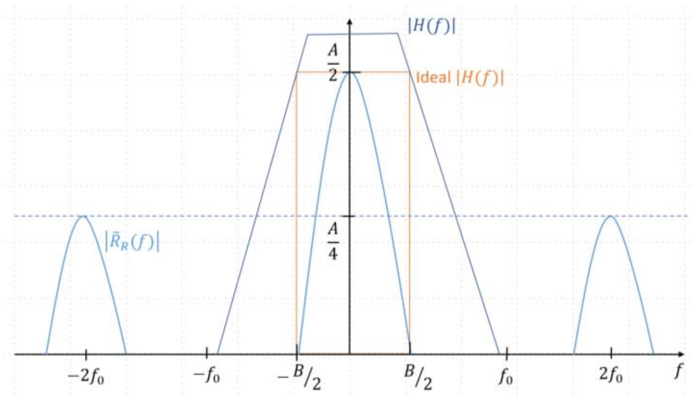

Figure 2: Representation of the received signal with the filter in the frequency domain

From Figure 2, we can conclude that the filter requires two characteristics:

$$\boxed{|H(f)| = 2}$$

$$\boxed{\frac{B}{2} < f_{cut} < 2f_0 - \frac{B}{2}}$$

Question 3

Can the receiver presented in Figure 1 work with wide-band signals?

If the received signal was a wide band one, it would mean that $f_0 < \frac{B}{2}$. This would cause an overlap between the main signal and the ones centred on $2f_0$ and $-2f_0$, as you can see on the figure below. This would make it impossible for the filter to keep the main signal without also keeping part of the signal that needs to be filtered.

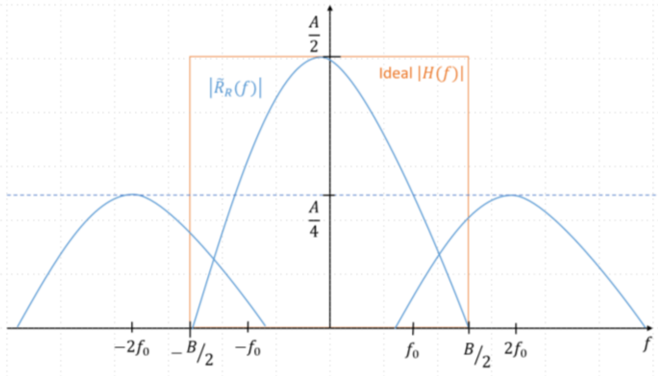

Figure 3: Representation of the wide band signal on the frequency domain

Question 4

How must the sampling period Te (with $Te = {}^1/_{fe}$) be chosen in order to recover $r_R(t), t\epsilon\ \mathbb{R}$ from $r_R(k \cdot \mathrm{Te}), k\epsilon\mathbb{Z}$?

To define the sampling period Te we use the Nyquist–Shannon sampling theorem. It establishes a sufficient condition for a sample rate that permits a discrete sequence of samples to capture all the information from a continuous-time signal of finite bandwidth. According to this theorem: $f_e > 2f_{max}$

We defined earlier that $f_{max} = \frac{B}{2}$. From this information we can easily conclude that:

$$\boxed{Te < \frac{1}{B}}$$

Question 5

Why do not we interchange the stages of frequency transposition and analog to digital conversion?

Even though nothing prevents us from inverting the two stages, it would require for the sampling frequency to be much higher in order to respect the Shannon's theorem. This, even though it is not an issue theoretically, can represent a limitation for the ADC. The common ones do not have the capacity to define a sampling rate that high. However, there are ADC's that handle these kinds of frequencies, but they are usually way more expensive. Transposing the signal before the conversion is an easy fix to this issue.

Question 6

Supposing a real narrow-band signal

$s_{RF}(t)=A(t)\cdot\cos(2\cdot\pi\cdot f_0\cdot t+\varphi(t))$

$s_{RF}(t)=s_R(t)\cdot\cos(2\cdot\pi\cdot f_0\cdot t)-s_I(t)\cdot\sin(2\cdot\pi\cdot f_0\cdot t)$, $t\in\mathbb{R}$.

Express -in frequential then in temporal- its analytic signal and its complex envelop in function of f_0, knowing that $S_{RF}(f)=S_{RF}{*}(-f)$ and that the analytic signal can be modelled by:

$S_a(f)$ = $\mathcal{F}\{s_a\}$(f) = $S_{RF}(f)+j\cdot\mathcal{H}\{S_{RF}(f)\}$ = $S_{RF}(f)+j\cdot(-j\cdot sgn(f)\cdot S_{RF}(f))$ with $\mathcal{H}\{\ \}$ *the Hilbert transform.*

Using the Fourier transform we can know that:

$$S_{RF}(f) = \frac{1}{2}\cdot(S_R(f-f_0)+S_R(f+f_0)+j\cdot S_I(f-f_0)-j\cdot S_I(f+f_0))$$

We obtain the following representation of $S_a(f)$:

$$S_a(f) = 2\cdot S_{RF}$$

$$S_a(f) = S_R\left(f-f_0\right)+j\cdot S_I\left(f-f_0\right)$$

$$S_a(f) = (S_R(f)+j\cdot S_I(f))\cdot\delta(f-f_0)$$

$$s_a(t) = (s_R(t)+j\cdot s_I(t))\cdot e^{-j2\pi f_0 t}$$

Moreover, we know that:

$$s(t) = s_a(t)\cdot e^{j2\pi f_0 t}$$

$$S(f) = S_a(f+f_0)$$

Based on those equations we obtain the following result in temporal and frequency domain:

$$\boxed{s(t) = s_R(t)+j\cdot s_I(t)}$$

$$\boxed{S(f) = S_R(f)+j\cdot S_I(f)}$$

II. Reception of frequency modulation (FM) broadcasting

In this second part of the lab, the objective is to use GNU Radio Companion to analyse a file containing FM broadcasting. In this file, the signal recorded has the following characteristics:

- A center frequency f_c = 99.5 MHz
- A sampling frequency F_e = 1.5 MHz

1. Frequency analysis of the recording

GNU Companion has a visual interface as it works with "blocks" linked to each other. To exploit the recording, we implemented the chain presented in Figure 4 below:

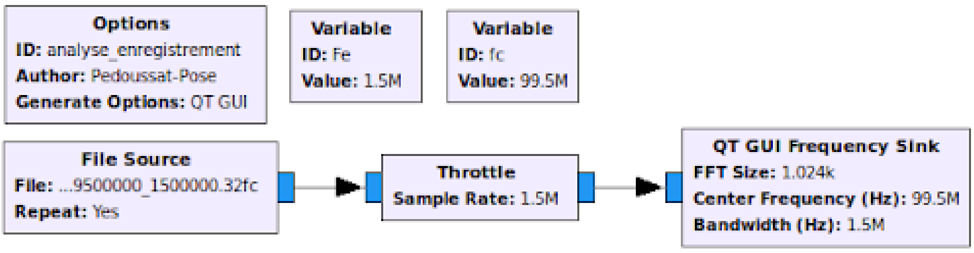

Figure 4: Our frequency analysis processing chain

Question 7

Variable: Declaration of a variable by associating an ID and its value. Thus, Fe will be considered as 1.5M and fc as 99.5M in the different blocks of the chain.

File Source: This block retrieves the data from the file *fm_99500000_1500000.32fc*. As the option Repeat is set to yes, the signal will loop infinitely.

Throttle: This block reads the data sent by "File Source" block at a defined speed (= Sample Rate). In our case, the Sample Rate is 1.5 M samples per second.

QT GUI Frequency Sink: This block displays the FFT (Fast Fourier Transform) in real time of the signal. We let the FFT Size at 1024 samples and we entered our signal characteristics Fe and fc.

Question 8

Specify the values of the missing variables in the characteristics function of the recording file.

The missing variables are visible in Figure 4:

- Fe = 1.5M
- Fc = 99.5 M
- Sample Rate = 1.5 M
- Center Frequency = 99.5 M
- Bandwidth = 1.5 M

Question 9

How many frequency channels –to be noted L- do you observe? According to the allocation of frequencies in the FM band near Toulouse, which stations are observed?

The visible signal visible at the end of the processing chain is displayed in Figure 5:

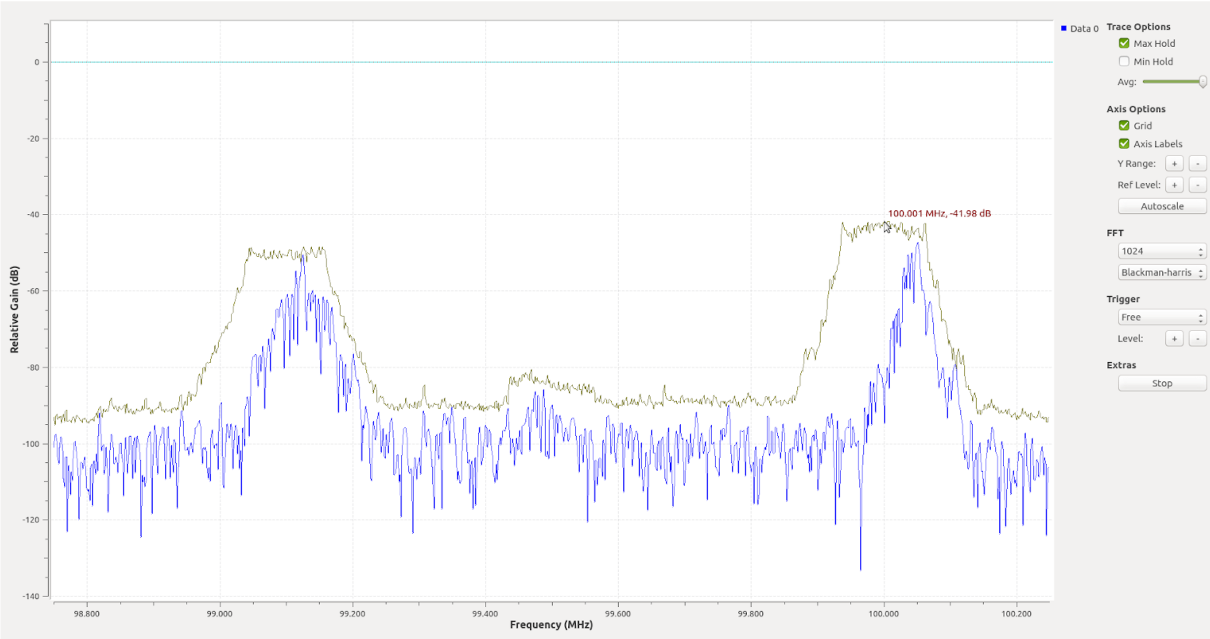

Figure 5: Signal plotted at the end of the signal processing chain

In this graph, we can clearly see two distinct peaks around 99.1 MHz and 100 MHz corresponding respectively to RFM Toulouse and Skyrock radio stations. In addition, a smaller peak is centered in 99.5 MHz is detectable. It corresponds to Nostalgie Toulouse.

Question 10

What is the measured signal-to-noise ratio in decibel? Do you think that is enough to be able to demodulate the signal?

For measuring the signal-to-noise ration (SNR), we took the maximum value of the graph (in green on Figure 5). We evaluate the maximum of noise level around -90dB. Thus, we have:

- $\mathrm{SNR}_{\mathrm{RFM}}$ = -50 – (-90) = 40 dB
- $\mathrm{SNR}_{\mathrm{Nostalgie}}$ = -82.5 – (-90) = 7.5 dB
- $\mathrm{SNR}_{\mathrm{Skyrock}}$ = -41.8 – (-90) = 48.2 dB

It seems that the signal coming from Nostalgie will be too weak to be demodulated as it is too much hidden in the noise. However, it seems possible to demodulate signals from RFM and Skyrock.

Question 11

What is the approximate bandwidth of a channel?

Using once again the Figure 5, we have measured graphically the following bandwidth:

- $\mathrm{BW_{RFM}}$ = (99.240 – 98.950)*10^6 = 290 kHz
- $\mathrm{BW_{Nostalgie}}$ = (99.567 – 99.435)*10^6 = 132 kHz
- $\mathrm{BW_{Skyrock}}$ = (100.136 – 99.862)*10^6 = 274 kHz

2. Channel extraction by frequency transposition and low-pass filtering

In the previous step, we have identified the different radio stations on the recording. Now, we want to receive the signal of one station at the time. The first step will consist in a frequency transposition to center the signal to listen. The second step will be the filtering around the useful signal to attenuate the out-of-band noise.

Question 12

What are the frequency offsets needed to center each channel?

The idea is to put the frequencies corresponding to the radio stations we want to listen in the center. In this respect, we will make the corresponding frequency transposition (f_c - $f_{channel}$):

- $\mathrm{Offset_{RFM}}$ = 99.5 MHz – 99.1 MHz = 400 kHz
- $\mathrm{Offset_{Nostalgie}}$ = 99.5 MHz – 99.5 MHz = 0
- $\mathrm{Offset_{Skyrock}}$ = 99.5 MHz – 100.0 MHz = -500 kHz

In the interface, we have added a cursor so that the user can select the desired radio station manually by centering the frequency required. The frequency shift is done by multiplying the complex envelope of the signal with a complex exponential $e^{-j2\pi f_{offset}t}$.

Question 13

What happens if the frequency offset is higher than the sampling frequency F_e ?

When the frequency offset is equal to F_e, it is equal to a zero-offset signal. Indeed, if we consider this issue on a mathematical point of view, it corresponds to multiplying a $e^{j2\pi F_e t}$ (complex envelope) with $e^{-j2\pi F_e t}$ (when f_{offset} = F_e). We clearly see that the periodicity of the complex exponential lead to zero effect on our signal. Thus, each frequency offset will be applied with a modulo F_e.

Question 14

What are the low-pass filter parameters, as well as those of the frequency analyser at the output of the filter?

Once the selected channel is centred, we want to filter it. To do that we use a Low Pass Filter with the following parameters:

- Decimation: 6, in order to lighten the computational load
- Gain: 1
- Sample rate: Fe = 1,5MHz
- Cut off frequency: 215kHz, approximately half the bandwidth of the signal (our bandwidth revolved around 130kHz and 290kHz)
- Transition width: 20KHz, around 10% of the cut off frequency

When using the frequency analyser at the output of the filter, it was necessary to divide the sample frequency by 6 (250kHz) because of the factor 6 used in the decimation parameter of the filter. This sample frequency is the one used in all the blocks that come after the filter.

3. Frequency demodulation and restitution

Question 15

Using the Carson rule, check that the bandwidth of the channel measured in the previous part confirms the theory.

In telecommunication, Carson's bandwidth rule defines the approximate bandwidth requirements of communications system components for a carrier signal that is frequency modulated by a continuous or broad spectrum of frequencies rather than a single frequency. The Carson rule is the following:

$$B_{FM} = 2 \cdot (\Delta f + f_m)$$

With Δf the maximum frequency excursion of the modulation, fixed at 75 kHz in the present case and f_m the maximum frequency of the composite signal m(t), fixed at 53kHz for stereophonic composite signals. We obtain a bandwidth requirement of:

$$2 \cdot (75 \cdot 10^3 + 53 \cdot 10^3) = 256kHz$$

The bandwidth measured previously confirm the results, with bandwidth revolving around 230 and 290kHz. The Nostalgie's bandwidth is lower than expected but we can consider that the signal was not strong enough (low amplitude).

Question 16

From the expression (5) of the transmitted signal and the affected processes until now (frequency transposition and low-pass filtering), show that the signals $y_l[k]$ can be noted:

$$y_l[k] = A \cdot e^{jk_f \sum_{i=0}^{k} m[i]} + b[k]$$

Define the value of k_f, with $b[k]$ a complex noise term introduced by the propagation channel as well as by the transceiver itself.

$$S_{RF}(t) = A \cdot cos\left(2 \cdot \pi \cdot f_0 \cdot t + \frac{\Delta f}{max(|m(t)|)} \cdot \int_{-\infty}^{t} m(u).du\right) \quad (5)$$

We know that:

$$S_{RF}(t) = A(t) \cdot cos\big(2 \cdot \pi \cdot f_0 \cdot t + \varphi(t)\big), t \in \mathbb{R}$$

By identification, we can assume that:

$$\varphi(t) = \frac{\Delta f}{max(|m(t)|)} \cdot \int_{-\infty}^{t} m(u).du$$

Moreover, we also know that:

$$s(t) = s_R(t) + j \cdot s_I(t)$$

$$s_R(t) = A(t) \cdot \cos\big(\varphi(t)\big)\ and\ s_I(t) = A(t) sin\big(\varphi(t)\big)$$

$$s_R(t) = A(t) \cdot e^{j\varphi(t)}$$

From these equations and by adding the noise b[k] in the discrete domain (after sampling at f_e frequency), we obtain the following result:

$$\boxed{s_R(t) = A[\frac{k}{f_e}] \cdot e^{j \cdot \frac{\Delta f}{max(|m(t)|)} \cdot \int_{-\infty}^{t} m(u).du} + b[k]}$$

From (5), we can identify k_f as:

$$\boxed{k_f = \frac{\Delta f}{max(|m(t)|)}}$$

Question 17

Plot the spectrum of the demodulated channel and compare with the Figure 6.

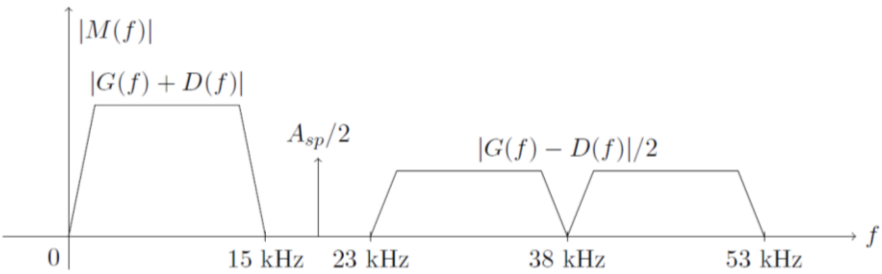

Figure 6: Stereophonic composite signal before the frequency modulation

After filtering the signal and in order to be able to decode it, we first need to demodulate the signal, with a WBFM Receive block. Obtaining, a signal similar to the one in Figure 6, as shown in the figure below. The one below represents also the symmetric signal.

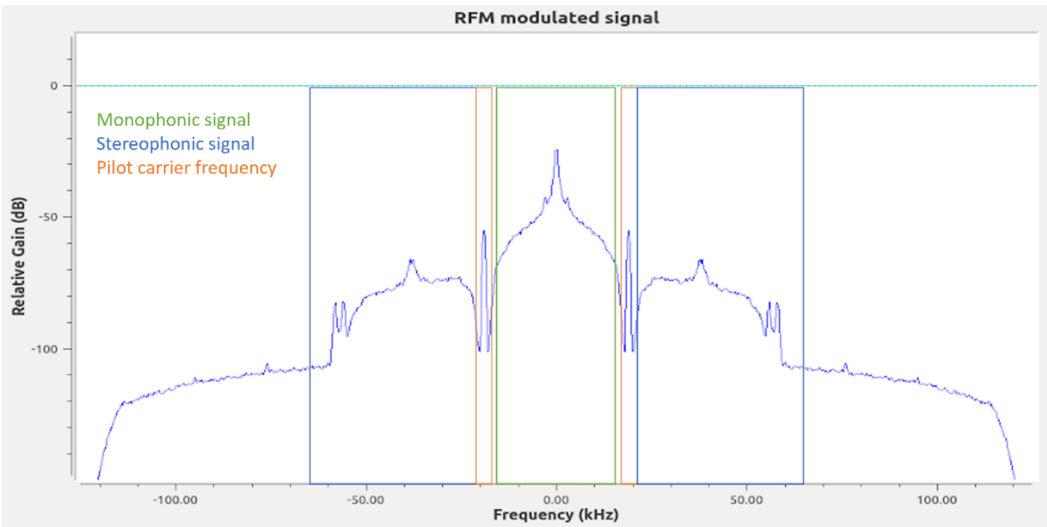

Figure 7: Demodulated signal of a radio channel

Question 18

Who won the Sam Smith album? What do we listen to on other stations?

With an Audio Sink block and after keeping only the monophonic signal thanks to a filter with a cut off frequency of 18kHz, we were able to listen to the three radios:

- RFM: Jordi won the Sam Smith album
- Nostalgie: played the YMCA song
- Skyrock: played Counting Stars by One Republic

4. Real time implementation with an USRP receiver

Finally, we changed our file source by the output of a SDR transceiver with a UHD USRP Source block, obtaining the real time signal, making it possible to listen to the different FM radios in real time. In order to obtain these results, we implemented the following chain:

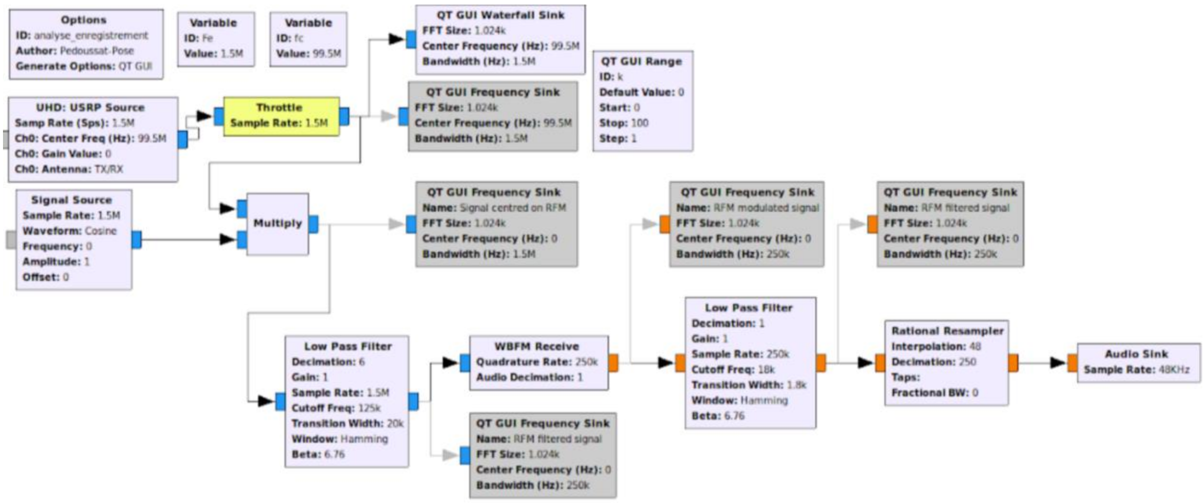

Figure 8: Implementation of the created chain on gnuradio-companion