Additional Material to The Concentration Channel of the Minimum Wage

Salvatore Lo Bello
Bank of Italy

Lorenzo Pesaresi
Bank of Italy

1 Stylized Model

In the presence of a binding MW, firms choose employment by solving the following profit-maximization problem:

$$\begin{aligned} \max_{\ell} \quad & p(\ell)y - w(\ell)\ell - \kappa \\ \text{s.t.} \quad & y = z\ell, \;\; w(\ell) = \max\{\underline{w}, \ell^{\frac{1}{\eta}}\}, \;\; p(\ell) = y^{-\frac{1}{\epsilon(N)}}. \end{aligned}$$

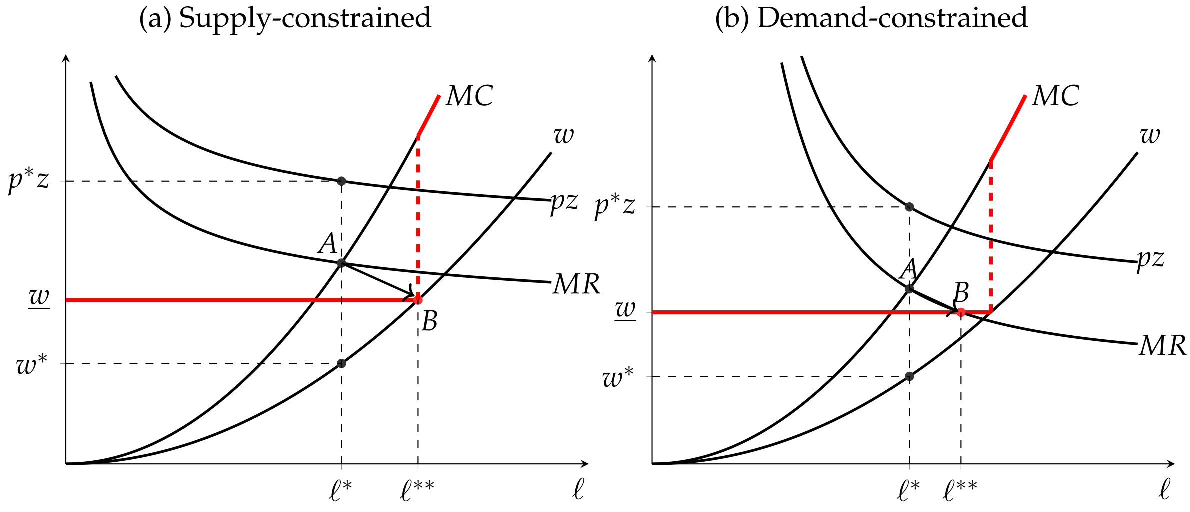
Firms are either constrained or unconstrained in their wage setting problem. Unconstrained firms set their optimal markup, $\mu = \frac{\epsilon}{\epsilon-1}$, and their optimal markdown, $\psi = \frac{1+\eta}{\eta}$. Hence, their employment, price and wage policies are as follows:

$$\begin{aligned} \ell(z) &= z^{\eta\frac{\epsilon-1}{\epsilon+\eta}}(\mu\psi)^{-\frac{\epsilon\eta}{\epsilon+\eta}}, \\ p(z) &= (z\ell(z))^{-\frac{1}{\epsilon}}, \\ w(z) &= \ell(z)^{\frac{1}{\eta}}, \end{aligned} \tag{1}$$

where the dependence of ϵ on N is left implicit. Unconstrained firms are affected by the MW only through the response of the elasticity of demand, mediated by the response of the number of active firms. Notice that the elasticity of demand affects output both through the shape of the demand curve and the optimal markup. Lemma 1 derives a parametric restriction such that firm-level output is increasing in the number of active firms.

Constrained firms set their optimal markup but are not able to set their optimal markdown (because the unconstrained optimal employment would be off the labor supply curve). Figure 1 shows that constrained firms can be further distinguished in two groups. Lower-productivity firms are demand-constrained, in that their employment is pinned down by product demand. Such firms are extracted their entire labor market power, that is, their markdown is compressed to one. Higher-productivity firms are supply-constrained, in that their employment is pinned down by labor supply. Such firms lose only part of their labor market power.

Figure 1: Effects of the MW on constrained firms



For supply-constrained firms, the constrained markdown is pinned down by the condition such that employment lies on the labor supply curve (left panel of Figure 1). Hence, the policy functions of supply-constrained firms are given by:

$$\ell(z) = z^{\eta\frac{\epsilon-1}{\epsilon+\eta}}(\mu\psi(z))^{-\frac{\epsilon\eta}{\epsilon+\eta}}, \tag{2}$$

$$p(z) = (z\ell(z))^{-\frac{1}{\epsilon}},$$

$$\underline{w} = \ell(z)^{\frac{1}{\eta}}. \tag{3}$$

From (2) and (3), it follows that the constrained markdown is given by:

$$\psi(z) = z^{\frac{\epsilon-1}{\epsilon}}\mu^{-1}\underline{w}^{-\frac{\epsilon+\eta}{\epsilon}} \in (1, \psi).$$

Since the constrained markdown is decreasing in the MW, all supply-constrained firms that would pay wages below the MW in the baseline equilibrium raise employment. Since employment is decreasing in the markup, all supply-constrained firms that would pay wages above the MW in the baseline equilibrium reduce employment. The least productive supply-constrained firm is defined by the following productivity level:

$$z^{ds}(\underline{w}) = \left(\mu\underline{w}^{\frac{\epsilon+\eta}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}.$$

For demand-constrained firms, the constrained markdown is pinned down by the condition such that employment lies on the marginal revenue curve (right panel of Figure 1). As a result, the markdown is always equal to one. Hence, the policy functions of demand-constrained firms are given by:

$$\ell(z) = z^{\epsilon-1}(\mu\underline{w})^{-\epsilon}, \tag{4}$$

$$p(z) = (z\ell(z))^{-\frac{1}{\epsilon}}.$$

Demand-constrained firms can either increase or decrease their employment with respect to the baseline equilibrium. For given elasticity of demand, we can define a productivity threshold z^c, such that all firms with higher (lower) productivity than z^c increase (decrease) employment, as follows:

$$z^c(\underline{w}) = (\mu^{\epsilon}\psi^{-\eta})^{\frac{1}{\epsilon-1}}\underline{w}^{\frac{\epsilon+\eta}{\epsilon-1}}.$$

Figure 1 in the main text reports the cross-sectional adjustment to a MW increase such that firms at the z^c cutoff make zero profits (with the MW in place), i.e., $\pi(z^c(\underline{w});\mu,\psi) = 0$. It follows that, for given markup, all the demand-constrained firms that would have scaled down in size following the MW exit the market.

Lemma 1 (Firm-level output response to the number of active firms)
If $\underline{z} > \left(\frac{\epsilon(N)}{\epsilon(N)-1}\frac{1+\eta}{\eta}\right)^{\frac{\eta}{1+\eta}} \exp\left\{-\frac{\epsilon(N)+\eta}{(1+\eta)(\epsilon(N)-1)}\right\}\ \forall N \in \mathbb{N}$, *then unconstrained and supply-constrained firms reduce their output following a reduction in the number of active firms.*

Let $\underline{z}(\underline{w})$ *be the least productive active firm when the MW is* $\underline{w}$. *If* $\underline{z}(\underline{w}) > \frac{\epsilon(N)}{\epsilon(N)-1}\exp\{-\frac{1}{\epsilon(N)-1}\}\underline{w}\ \forall N \in \mathbb{N}$, $\forall \underline{w} \in \mathbb{R}^+$, *then demand-constrained firms reduce their output following a reduction in the number of active firms.*

Proof. Differentiating optimal unconstrained employment (1) with respect to ϵ yields:

$$\frac{d\ln(\ell(z))}{d\epsilon} = \frac{1+\eta}{(\epsilon+\eta)^2}\eta\ln(z) - \frac{\eta^2}{(\epsilon+\eta)^2}\ln\left(\frac{\epsilon}{\epsilon-1}\psi\right) + \frac{\eta}{\epsilon+\eta}\frac{1}{\epsilon-1},$$

where the first two terms represent the employment response to a change in the elasticity of the demand curve for given markup (*elasticity effect*) and the last term the employment response to a change in markup for given elasticity of demand curve (*markup effect*). It holds that:

$$\frac{d\ln(\ell(z))}{d\epsilon} > 0 \iff z > \left(\frac{\epsilon}{\epsilon-1}\psi\right)^{\frac{\eta}{1+\eta}} \exp\left\{-\frac{\epsilon+\eta}{(1+\eta)(\epsilon-1)}\right\}.$$

Since $d\epsilon/dN > 0$ by assumption, if this condition is met by the least productive firms, then it holds for all the firms. Notice that optimal employment of supply-constrained firms (2) is the same as that of unconstrained firms up to a weakly lower markdown. Hence, the same condition guarantees that supply-constrained firms reduce their output as the number of active firms decreases.

Differentiating optimal employment of demand-constrained firms (4) with respect to ϵ yields:

$$\frac{d\ln(\ell(z))}{d\epsilon} = \ln\left(\frac{z}{\mu\underline{w}}\right) + \frac{1}{\epsilon-1},$$

where the first term represents the elasticity effect and the second term the markup effect. It holds that:

$$\frac{d\ln(\ell(z))}{d\epsilon} > 0 \iff z > \frac{\epsilon}{\epsilon-1}\exp\left\{-\frac{1}{\epsilon-1}\right\}\underline{w}.$$

Since $d\epsilon/dN > 0$ by assumption, if this condition is met by the least productive firms, then it holds for all the demand-constrained firms. ■

2 Quantitative Model

2.1 Matching Function Specification

In this section we motivate our functional form assumption on the matching function. The functional form of the matching function is relevant in our framework for two reasons. First, our baseline equilibrium features congestion externalities. Second, our counterfactual experiments point to a reduction in congestion externalities as the main driver of the positive response of aggregate consumption through lower misallocation. By governing the extent (as well as the scope for a reduction) of congestion externalities, the functional form of the matching function is critical for properly quantifying worker reallocation. In particular, when considering large MW reforms, it is crucial to account for potential nonlinearities in the matching function.

For this reason, we adopt a CES matching function, $\mathcal{M}(S,V) = \chi\left[\alpha V^{-\iota} + (1-\alpha)S^{-\iota}\right]^{-\frac{1}{\iota}}$, and calibrate the elasticity parameters ι and α according to the estimates of Şahin et al. (2014).[1] As pointed out by Şahin et al. (2014), the estimated parameters imply only a slight deviation from the standard Cobb-Douglas specification with vacancy elasticity of 0.5. However, we prefer the CES specification because it delivers an endogenous elasticity of firms' meeting rate with respect to labor market tightness. Formally,

$$\epsilon_{q(\theta),\theta} = -\frac{1}{1+\frac{\alpha}{1-\alpha}\theta^{-\iota}}.$$

From this formulation, it easy to see that the standard Cobb-Douglas specification – nested by setting $\iota = 0$ and $\alpha = 0.5$ – predicts a constant elasticity of 0.5. On the other hand, the CES specification implies that the elasticity is increasing in labor market tightness. As a result, the slacker the labor market is to start with, the lower the gain in terms of meeting rate of further reducing tightness. This implies that, as the MW gets larger and labor market tightness decreases, hiring cost savings from lower congestion effects progressively die out. It follows that our choice of a CES matching function is conservative in terms of positive response of value added to the MW.

2.2 Derivation of Stationary Distributions

Let $G_{a_i}(w)$ be the share of employed workers of skill a who earn wage rate equal or lower than w. This share evolves in response to inflows (unemployed workers who accept jobs with $w' \leq w$) and outflows (employed workers who accept jobs with $w' > w$ or lose their job)

[1] Since the estimated elasticities are based on US data, we internally estimate the efficiency of the matching function χ on our Italian data.

according to the following Kolmogorov forward equation:

$$\dot{G}_{a_i}(w) = \lambda(\theta_{a_i})u_{a_i}F_{a_i}(w) - \Big(\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})(1 - F_{a_i}(w))\Big)e_{a_i}G_{a_i}(w).$$

Similarly, the measures of employed e_{a_i} and unemployed u_{a_i} workers evolve according to labor market transitions governed by the separation and job-finding rates:

$$\dot{u}_{a_i} = \delta_{a_i}e_{a_i} - \lambda(\theta_{a_i})u_{a_i}, \quad \dot{e}_{a_i} = \lambda(\theta_{a_i})u_{a_i} - \delta_{a_i}e_{a_i}.$$

Under the assumption of constant population ($u_{a_i} + e_{a_i} = 1$), one can solve for these distributions in stationary equilibrium (i.e., setting $\dot{G}_{a_i}(w) = \dot{u}_{a_i} = \dot{e}_{a_i} = 0$):

$$\begin{aligned} G_{a_i}(w) &= \frac{\lambda(\theta_{a_i})F_{a_i}(w)}{\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})(1 - F_{a_i}(w))}\frac{u_{a_i}}{e_{a_i}}, \\ u_{a_i} &= \frac{\delta_{a_i}}{\delta_{a_i} + \lambda(\theta_{a_i})}. \end{aligned} \tag{5}$$

Equation 5 reveals the close connection between G_{a_i}, which corresponds to the observed employment wage distribution, and F_{a_i}, the wage offer distribution. In general, it can be shown that G_{a_i} diverges from F_{a_i} – with more and more mass being concentrated in the right part of the support of wages – when the frictional index $\chi_{a_i} \equiv \frac{\delta_{a_i}}{s_{a_i}\lambda(\theta_{a_i})}$ is low. As already noted by Burdett and Mortensen (1998), the frictional index is inversely related to the speed at which workers climb the job ladder in the model and represents a synthetic measure of the extent of labor market frictions in the economy.

We now investigate the relationship between G_{a_i} and F_{a_i}. Upon substituting the stationarity condition $\frac{u_{a_i}}{e_{a_i}} = \frac{\delta_{a_i}}{\lambda(\theta_{a_i})}$ into (5), we get

$$G_{a_i}(w) = \frac{\delta_{a_i}F_{a_i}(w)}{\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})\,(1 - F_{a_i}(w))} \implies \frac{G_{a_i}(w)}{F_{a_i}(w)} = \frac{\delta_{a_i}}{\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})(1 - F_{a_i}(w))}.$$

From the previous expression, it is clear that the G_{a_i} dominates (in a first-order stochastic dominance sense) the F_{a_i} distribution. Formally, $G_{a_i}(w) \leq F_{a_i}(w) \ \ \forall\, w$. It can also be noted that the parameter s_{a_i} regulates the distance between the G_{a_i} and the F_{a_i} distributions. In particular, as $s_{a_i} \to 0$ for a given δ_{a_i}, we note that $G_{a_i}(w) \to F_{a_i}(w)$. Conversely, the distance between the two distributions can be made arbitrarily large by setting large values of s_{a_i}. After some manipulation, the previous expression can be rearranged as:

$$G_{a_i}(w) = \frac{F_{a_i}(w)}{1 + \chi_{a_i}(\theta_{a_i})(1 - F_{a_i}(w))}. \tag{6}$$

This formulation makes clear that the relationship between the two distributions is fully determined by $\chi_{a_i}(\theta_{a_i})$. Intuitively, if employed workers find job offers at a much faster rate than they lose their job (i.e., fall off the ladder), then more mass will be placed to higher values in the support of the wage offer distribution. Finally, one can also note that the difference between the two distributions also depends on the rank of the firm in the wage offer distribution, $F_{a_i}(w)$. Intuitively, as $F_{a_i}(w) \to 1$, then $G_{a_i}(w) \to F_{a_i}(w) = 1$, and conversely $F_{a_i}(w) \to 0$, then $G_{a_i}(w) \to F_{a_i}(w) = 0$, i.e., the two distributions need to coincide at the upper and lower point of the wage offer distribution.

2.3 General Labor Supply Curve with Mass Points

In the main text we reported the labor supply curve faced by firms when the wage distribution is continuous, that is:

$$\ell_{a_i}(w,v) = \frac{v}{V_{a_i}} \lambda(\theta_{a_i}) \frac{u_{a_i} + s_{a_i} e_{a_i} G_{a_i}(w)}{\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})(1 - F_{a_i}(w))},$$

Using Equation 5, one can substitute G_{a_i} away and obtain:

$$\ell_{a_i}(w,v) = \frac{v}{V_{a_i}} u_{a_i} \lambda(\theta_{a_i}) \frac{\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})}{\left[\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})(1 - F_{a_i}(w))\right]^2}.$$

If the equilibrium wage distributions have mass points, one needs to discipline workers' transitions across equally-paying firms. Let q_p and q_ℓ denote the probability that a firm poaches a worker from an equally paying firm and that a worker accepts a wage offer from an equally paying firm, respectively. Stationary employment reads:

$$\ell_{a_i}(w,v;q_p,q_\ell) = \frac{v}{V_{a_i}} \lambda(\theta_{a_i}) \frac{u_{a_i} + s_{a_i} e_{a_i} [G_{a_i}(w^-) + q_p\,(G_{a_i}(w) - G_{a_i}(w^-))]}{\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})\,[1 - F_{a_i}(w) + q_\ell\,(F_{a_i}(w) - F_{a_i}(w^-))]}.$$

Substituting for (6) yields:

$$\ell_{a_i}(w,v;q_p,q_\ell) = \frac{v}{V_{a_i}} \lambda(\theta_{a_i}) u_{a_i} \frac{h_{a_i}(w^-)h_{a_i}(w) + s_{a_i}\lambda(\theta_{a_i})\left[F_{a_i}(w^-)h_{a_i}(w) + q_p\,(F_{a_i}(w)h_{a_i}(w^-) - F_{a_i}(w^-)h_{a_i}(w))\right]}{h_{a_i}(w^-)h_{a_i}(w)[\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})\,(1 - q_\ell F_{a_i}(w^-) - (1-q_\ell)F_{a_i}(w))]}, \tag{7}$$

where $h_{a_i}(w) \equiv \delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})(1 - F_{a_i}(w))$.

Letting $q_p = q_\ell$, we recover the familiar expression for stationary employment in the equilibrium search literature:

$$\ell_{a_i}(w,v) = \frac{v}{V_{a_i}} u_{a_i} \lambda(\theta_{a_i}) \frac{\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})}{\left[\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})(1 - F_{a_i}(w))\right]\left[\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})(1 - F_{a_i}(w^-))\right]}.$$

2.4 Algorithm to solve the model

Consistently with our estimation strategy, we solve the discretized version of our model with a continuum of productivity types. Hence, we let residual wage (and MRPL) dispersion vanish and work with one-to-one firms' policy functions (rather than set-valued).

The model is solved by guess and verify for the collection of MRPL functions $\tilde{z}_{k(a_i)}(z)$.

Although the same solution concept applies to any set of parameters, we will add some specific comments related to an increase in the MW parameter.

To initialize the solution routine, we make an initial guess on the collection of MRPL functions, $\tilde{z}^0_{k(a_i)}(z)$. If the MW is higher than some guessed $\tilde{z}$ values (with a positive number of firms), we adjust any such points upward until they all exceed the MW. This preliminary adjustment amounts to conjecturing that all the potential firms are active. In this respect, we leverage the theoretical insight that the presence of market power on both the labor and the product market guarantees that all firms make positive operating profits for *any* MW level. Since overhead costs do not affect firms' policy functions in any way, we can therefore find a candidate equilibrium for any set of active firms. To discipline the extensive margin of adjustment (firm exit) , we check ex-post whether the candidate equilibrium is sustainable, i.e., whether any firm is making negative profits at that equilibrium, and, if not, gradually removing firms from the market.

Upon initializing the solution routine as just described, we solve the model according to the $\tilde{z}^0(z)$ guess. This is performed in four steps:

1. Solve for the equilibrium in each labor market $a_i = 1, \ldots, IA$ as follows:

   – Compute the number of firms at each $\tilde{z}^0_{a_i}(z)$-value as a share of the number of active firms (which is set equal to the number of potential firms in the estimated model) to identify the discretized counterpart of the MRPL density $\phi_{a_i}(\tilde{z})$;

   – Guess the wage and wage offer distribution as a function of the guessed $\tilde{z}^0_{a_i}$ function, $w_{a_i}(\tilde{z}^0_{a_i})$ and $\mathcal{H}_{a_i}(\tilde{z}^0_{a_i})$. Apply a first-order Taylor expansion to the nonlinear system of differential equations for $\mathcal{H}'_{a_i}(\tilde{z})$ and $w'_{a_i}(\tilde{z})$ around the guessed wage and wage offer distribution functions to transform it into a linear system of differential equations. Then, use the Euler (finite difference) method to transform the linear system of differential equations into a linear system of difference equations with boundary conditions $w_{a_i}(\underline{\tilde{z}_{a_i}}) = R^u_{a_i}$ and $\mathcal{H}_{a_i}(\underline{\tilde{z}_{a_i}}) = 0$.[2];

   – Find the labor-market-specific equilibrium job finding rate by solving the linear system of difference equations: this can be performed efficiently by first identifying a lower bound and an upper bound to the equilibrium job finding rate corresponding to too much job creation, i.e., $\lim_{\tilde{z}_{a_i} \to \bar{\tilde{z}}_{a_i}} \mathcal{H}_{a_i}(\tilde{z}_{a_i}) > 1$, and too little job creation, i.e., $\lim_{\tilde{z}_{a_i} \to \bar{\tilde{z}}_{a_i}} \mathcal{H}_{a_i}(\tilde{z}_{a_i}) < 1$, respectively, and then applying a bisection algorithm within such bounds;

   – Compute the labor-market-specific policy functions:

$$\{w_{a_i}(\tilde{z}^0_{a_i}), \mathcal{H}_{a_i}(\tilde{z}^0_{a_i}), v_{a_i}(\tilde{z}^0_{a_i}), \ell_{a_i}(\tilde{z}^0_{a_i}), \psi_{a_i}(\tilde{z}^0_{a_i})\}$$

   by making use of the differential equations for $\mathcal{H}'_{a_i}(\tilde{z})$ and $w'_{a_i}(\tilde{z})$, the functional

[2] We check that results are unchanged by applying the Euler method directly to the nonlinear system of differential equations.

relationship between $\mathcal{H}'_{a_i}(\tilde{z})$ and $v_{a_i}(\tilde{z})$, the markdown definition, and the labor supply curve.

2. Solve for the equilibrium in each product market $k = 1, \ldots, K$ as follows:

   – Compute sectoral, industry, and aggregate output by aggregating up firm-specific output policies $y_{k(a_i)}(\tilde{z}^0_{k(a_i)}(z)) = az\ell_{k(a_i)}(\tilde{z}^0_{k(a_i)}(z))$ according to the aggregate CD aggregator, the industry-level CES aggregator, and the sector-level CES aggregator. To avoid that the number of sectors affects industry output, we remove love-of-variety effects from the discretized industry CES aggregator, that is, industry output reads:

$$Y_i^{\text{discr}} = K_i^{-\frac{1}{\rho-1}} \left(\sum_{k=1}^{K_{a_L i}} Y_{k(a_L i)}^{\frac{\rho-1}{\rho}} + \sum_{k=1}^{K_{a_H i}} Y_{k(a_H i)}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}};$$

   – Compute firm-specific price policies by making use of the demand constraint, as well as sectoral, industry, and aggregate price indices according to their definitions, by aggregating up firm-level prices;

   – Thanks to the oligopolistically-competitive product market structure and nested CES preferences, firm-level prices are sufficient for pinning down each firm's market share by

$$sh_{k(a_i)}(\tilde{z}^0_{k(a_i)}(z)) = \frac{p_{k(a_i)}\left(\tilde{z}^0_{k(a_i)}(z)\right)^{1-\sigma}}{N_{k(a_i)} \sum_{n=1}^{\mathcal{Z}_{k(a_i)}} p_{k(a_i)}\left(\tilde{z}^0_{k(a_i)}(z_n)\right)^{1-\sigma} \gamma_{k(a_i)}(z_n)},$$

   and the markup policy;

   – Compute firm-specific profits as:

$$\pi_{k(a_i)}\left(\tilde{z}^0_{k(a_i)}(z_n)\right) = p_{k(a_i)}\left(\tilde{z}^0_{k(a_i)}(z_n)\right) y_{k(a_i)}\left(\tilde{z}^0_{k(a_i)}(z_n)\right) - ac_{k(a_i)}\left(\tilde{z}^0_{k(a_i)}(z_n)\right) - a\kappa_{k(a_i)}.$$

   Compute the implied MRPLs, $\tilde{z}^{implied}_{k(a_i)}(z)$, by making use of the MRPL definition, i.e.,

$$\tilde{z}^{implied}_{k(a_i)}(z) \equiv \frac{p_{k(a_i)}\left(\tilde{z}^0_{k(a_i)}(z_n)\right)}{\mu_{k(a_i)}\left(\tilde{z}^0_{k(a_i)}(z_n)\right)} z.$$

3. Upon solving the model conditional on the $\tilde{z}^0_{k(a_i)}(z)$ guess, we proceed by verifying and potentially updating the guess. To do so, the solution algorithm goes through the following steps:

   – Let $\varepsilon^0 \equiv \mathbb{E}\left[|\tilde{z}^0_{a_i}(z) - \tilde{z}^{implied}_{a_i}(z)|\right]$ denote the average convergence error, and set a sensitivity $\bar{\varepsilon}$ ($= 10^{-4}$ in our simulations);[3]

 – If $\epsilon^0 > \bar{\epsilon}$, we update the initial $\tilde{z}^0$-guess via bisection, i.e. $\tilde{z}^1_{a_i}(z) = 0.5^*\tilde{z}^0_{a_i}(z) + 0.5^*\tilde{z}^{implied}_{a_i}(\tilde{z}^0_{a_i}(z))$. If the new guess $\tilde{z}^1$ features nonmonotonicity in some labor market, we reduce the weight on the implied MRPL until such nonmonotonicity disappears.[4] Repeat the steps 1-3 iteratively until finding a guess $\tilde{z}^n_{a_i}(z)$ at the $n-$th repetition such that $\epsilon^n < \bar{\epsilon}$;

 – Store the model solution for the $\tilde{z}^n_{a_i}(z)$ guess as candidate equilibrium.

4. Check that no firm makes negative profits in the candidate equilibrium. If it is the case, the candidate equilibrium is sustainable and the model is solved. Otherwise, an extensive margin adjustment needs to be enacted:

 – Following the equilibrium refinement device of Berry (1992), the firm making lowest (negative) profits is removed from the market. Since our model features identical sectors sourcing from the same labor market, we remove one of the worst loss-making firms in each identical sector at a time.[5] Then, the algorithm restarts from step 1.

2.5 Dynamic Firm's problem

The sequential firm's profit maximization problem reads:

$$\max_{w_t \geq \underline{w}/a, v_t \geq 0} \int_0^\infty e^{-rt}\, a \left[\left(p_{k(a_i)}(y_t) z - w_t \right) \ell_t - c_{a_i}(w_t, v_t) - \kappa_{a_i} \right] dt \tag{8}$$

$$\text{s.t. } \dot{\ell}_t = -\left(\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i t})[1 - F_{a_i t}(w_t)]\right)\ell_t + \frac{v_t}{V_{a_i t}}\lambda(\theta_{a_i t})[u_{a_i t} + s_{a_i}(1 - u_{a_i t})G_{a_i t}(w_t)], \tag{9}$$

$$p_{k(a_i)}(y_t) = {y_t}^{-\frac{1}{\sigma}}\, Y_{k_{a_i} t}(y_t)^{\frac{1}{\sigma} - \frac{1}{\rho}}\, {Y_{it}}^{\frac{1}{\rho} - 1} \alpha_i Y_t, \tag{10}$$

$$y_t = az\ell_t \tag{11}$$

We proceed by setting up the current value Hamiltonian and dropping time indices:

$$H(w, v; \ell) = \pi(w, v; \ell) + \xi \left(\frac{v}{V_{a_i}} \lambda(\theta_{a_i})[u_{a_i} + s_{a_i}(1 - u_{a_i})G_{a_i}(w)] - (\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})[1 - F_{a_i}(w)])\,\ell \right),$$

where $\pi(w, v; \ell) = \left(p_{k(a_i)}\left(y_{a_i}(\ell)\right) z - w \right) \ell - c_{a_i}(w, v)$ and ξ is the co-state variable.

[3] Our guess $\tilde{z}^0_{a_i}(z)$ is common across firms with the same productivity sourcing from the same labor market, i.e., $\tilde{z}^0_{k(a_i)}(z) = \tilde{z}^0_{a_i}(z)$. Due to integer constraints, sectors sourcing from the same labor market may not be exactly identical. In those cases, we compute the implied MRPL, $\tilde{z}^{implied}_{a_i}(z)$, as the sales-weighted average of implied MRPLs of firms with the same productivity.

[4] Because of the monotonicity restriction, this simple modified bisection algorithm is preferable to Jacobian-based methods, as well as more efficient.

[5] If the worst loss-making firms are more than 100 in each sector, we remove 10% of them to speed up the algorithm. We check that this shortcut has no bearing on the results.

The first-order conditions are:

$$\frac{\partial H}{\partial w} = 0 \iff \ell + \frac{\partial c_{a_i}(w,v)}{\partial w} = \xi\left(\frac{v}{V_{a_i}}\lambda(\theta_{a_i})s_{a_i}(1-u_{a_i})g_{a_i}(w) + s_{a_i}\lambda(\theta_{a_i})f_{a_i}(w)\ell\right) \tag{12}$$

$$\frac{\partial H}{\partial v} = 0 \iff \frac{\partial c_{a_i}(w,v)}{\partial v} = \xi\frac{\lambda(\theta_{a_i})[u_{a_i} + s_{a_i}(1-u_{a_i})G_{a_i}(w)]}{V_{a_i}} \tag{13}$$

$$\frac{\partial H}{\partial \ell} = r\xi - \dot{\xi} \iff \tilde{z}_{k(a_i)} - w - \xi\left(\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})[1-F_{a_i}(w)]\right) = r\xi - \dot{\xi} \tag{14}$$

$$\frac{\partial H}{\partial \xi} = \dot{\ell} \iff \dot{\ell}_t = -\left(\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})[1-F_{a_i}(w)]\right)\ell + \frac{v}{V_{a_i}}\lambda(\theta_{a_i})[u_{a_i} + s_{a_i}(1-u_{a_i})G_{a_i}(w)] \tag{15}$$

where $\tilde{z}_{k(a_i)} = \left(1 + \epsilon^{-1}_{k(a_i)}(\ell)\right) p_{k(a_i)} z$ denotes the MRPL. In stationary equilibrium, i.e., setting $\dot{\ell} = \dot{\xi} = 0$, the wage and vacancy policy functions read:

$$\tilde{z}_{k(a_i)} - w = \frac{r + \delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})[1-F_{a_i}(w)]}{2s_{a_i}\lambda(\theta_{a_i})f_{a_i}(w)}\left(1 - \phi\frac{s_{a_i}\lambda(\theta_{a_i})f_{a_i}(w)}{\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})[1-F_{a_i}(w)]}\frac{c_{a_i}(w,v)}{\ell_{a_i}(w,v)}\right), \tag{16}$$

$$\tilde{z}_{k(a_i)} - w = \frac{r + \delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})[1-F_{a_i}(w)]}{\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})[1-F_{a_i}(w)]}(1+\zeta)\frac{c_{a_i}(w,v)}{\ell_{a_i}(w,v)} \tag{17}$$

Equations 16-17 boil down to their stationary counterparts in the main text in the *timeless* limit, i.e., as $r \to 0$. By putting together the two FOCs, we can compute the dynamically consistent equilibrium markdown function as follows:

$$\psi_{a_i}(\tilde{z}) = 1 + \frac{r/(s_{a_i}\lambda(\theta_{a_i})) + \chi_{a_i}(\theta_{a_i}) + [1 - F_{a_i}(w_{a_i}(\tilde{z}))]}{2\nu f_{a_i}(w_{a_i}(\tilde{z}))w_{a_i}(\tilde{z})}. \tag{18}$$

Positive time discounting raises equilibrium markdowns, without changing any qualitative results.

3 Efficiency Properties of Baseline Equilibrium

In this section we characterize the constrained efficient allocation of our quantitative model. To do so, we solve the problem of a social planner that aims to maximize aggregate consumption in steady state: [6]

$$\mathcal{W} = \max_{\substack{\vec{v}^j_{a_i}(z_n)\geq 0, \\ \vec{q}_{1,a_i}(n,n')\in[0,1], \\ \vec{q}_{2,a_i}(j,j',n)\in[0,1]}} Y - \sum_{i=1}^{I} \Xi(i) \int aM_{a_i} \left(\sum_{n}^{\mathcal{Z}_i} \int_0^1 c^j_{a_i} \left(\vec{q}_{1,a_i}, \vec{q}_{2,a_i}, V_{a_i}, v_{a_i}(z_n)\right) dj \; \gamma_{a_i}(z_n) \right) d\Omega_i(a),$$

$$\text{s.t.} \quad Y = \prod_{i=1}^{I} Y_i^{\alpha_i}, \quad Y_i = \left(\int \int_0^{K_{a_i}} Y_{k(a_i)}^{\frac{\rho-1}{\rho}} dk_{a_i} \, da_i \right)^{\frac{\rho}{\rho-1}}, \tag{19}$$

$$Y_{k(a_i)} = \left(N_{k(a_i)} \sum_{n=1}^{\mathcal{Z}_i} \int_0^1 y^j_{k(a_i)}(z_n)^{\frac{\sigma-1}{\sigma}} dj \; \gamma_{k(a_i)}(z_n) \right)^{\frac{\sigma}{\sigma-1}}, \tag{20}$$

$$y^j_{k(a_i)}(z_n) = a z_n \ell^j_{a_i} \left(\vec{q}_{a_i}(z_n, z_{-n}), V_{a_i}, v_{a_i}(z_n) \right), \tag{21}$$

$$\ell^j_{a_i} \left(\vec{q}_{a_i}, V_{a_i}, \vec{v}_{a_i}(z) \right) = \frac{v^j_{a_i}(z_n)}{V_{a_i}} \lambda_{a_i} u_{a_i} \frac{h^j_{a_i}(1,n) h^j_{a_i}(0,n) + s_{a_i}\lambda_{a_i} \left[(1 - \bar{\mathcal{H}}^j_{2,a_i}(n)) h^j_{a_i}(1,n) + \ldots \right.}{h^j_{a_i}(1,n) \; h^j_{a_i}(0,n) \; h^j_{a_i}(q^j_2(n), n)} \tag{22}$$

$$\frac{\left. \ldots (1 - q^j_2(n)) \left((1 - \bar{\mathcal{H}}^j_{1,a_i}(n)) h^j_{a_i}(0,n) - (1 - \bar{\mathcal{H}}^j_{2,a_i}(n)) h^j_{a_i}(1,n) \right) \right]}{}, \tag{23}$$

$$u_{a_i}(\lambda_{a_i}) = \frac{\delta_{a_i}}{\delta_{a_i} + \lambda_{a_i}}, \tag{24}$$

$$\lambda_{a_i}\left(V_{a_i}\right) = \mathcal{M}\left(1, \frac{V_{a_i}}{S_{a_i}(\lambda_{a_i}(V_{a_i}))}\right), \tag{25}$$

$$h^j_{a_i}(p,n) = \delta_{a_i} + s_{a_i}\lambda_{a_i} \left[1 - p\left(1 - \bar{\mathcal{H}}^j_{1,a_i}(n)\right) + (1-p)\left(1 - \bar{\mathcal{H}}^j_{2,a_i}(n)\right) \right], \tag{26}$$

$$\bar{\mathcal{H}}^j_{1,a_i}\left(\vec{q}_{1,a_i}, \vec{q}_{2,a_i}, V_{a_i}, \vec{v}_{a_i}(z_{n'})\right) = M_{a_i} \sum_{n' \neq n} q_{1,a_i}(n,n') \frac{\bar{v}_{a_i}(z_{n'})}{V_{a_i}} \gamma_{a_i}(z_{n'}) + M_{a_i} \frac{\bar{v}_{a_i}(z_n)}{V_{a_i}} \gamma_{a_i}(z_n) q^j_2(n), \tag{27}$$

$$\bar{\mathcal{H}}^j_{2,a_i}\left(\vec{q}_{1,a_i}, \vec{q}_{2,a_i}, V_{a_i}, \vec{v}_{a_i}(z_{n'})\right) = M_{a_i} \sum_{n' \neq n} q_{1,a_i}(n,n') \frac{\bar{v}_{a_i}(z_{n'})}{V_{a_i}} \gamma_{a_i}(z_{n'}), \tag{28}$$

$$q^j_2(n) = \int_0^1 q_{2,a_i}(j,j',n) \frac{v^{j'}_{a_i}(z_n)}{V_{a_i}(z_n)} dj', \tag{29}$$

$$q_{1,a_i}(n,n') + q_{a_i}(n',n) \leq 1, \; \forall n,n', \quad q_{2,a_i}(j,j',n) + q_{2,a_i}(j',j,n) \leq 1, \; \forall j,j',n, \tag{30}$$

where $V_{a_i} = M_{a_i} \sum_n \bar{v}_{a_i}(z_n)\gamma_{a_i}(z_n)$, $S_{a_i}\left(\lambda_{a_i}\right) = u_{a_i}(\lambda_{a_i}) + s_{a_i}\left(1 - u_{a_i}(\lambda_{a_i})\right)$, and $c^j_{a_i}(z_n) = \bar{c}_{a_i} h_{a_i}(0,z_n)^{\phi} \frac{v^j_{a_i}(z_n)^{1+\zeta}}{1+\zeta}$.

The social planner seeks to maximize aggregate consumption by leveraging two sets

[6] Hence, we abstract both from distributional concerns and elastic labor supply.

of control variables. First, the planner mandates each firm j with productivity z and skill requirement a operating in industry i to post a mass of vacancies $v^j_{a_i}(z)$. Second, the planner chooses the probability $q_{1,a_i}(n,n')$ with which a worker with skill a in industry i employed in a firm with productivity z_n transitions into a firm with productivity $z_{n'}$ if given the chance, and the probability $q_{2,a_i}(j,j',n)$ with which a worker with skill a in industry i employed in a firm with productivity z_n transitions into a firm with the same productivity if given the chance. These transition probabilities give rise to a a *shadow wage offer distribution* $\bar{\mathcal{H}}^j_{a_i}\left(\vec{q}_{1,a_i},\vec{q}_{2,a_i},V_{a_k},\vec{v}^{j'}_{a_i}(z_{n'})\right)$ as defined by (27).[7] Exactly as the equilibrium wage offer distribution, the shadow wage offer distribution plays an allocative role. Intuitively, firms' ranking in the $\mathcal{H}$ distribution determines the direction of worker reallocation. The problem setup allows for residual dispersion in policy functions if identical firms are dictated different reallocation patterns, i.e., if $q^j_2(n) \neq q^{j'}_2(n)$. The stationary employment constraint (22) equals the flow-consistent firm-level employment for a generic residual transition probability $q^j_2(n)$ among equally productive firms in the presence of finite productivity types (see (7) for its equilibrium counterpart). This expression has the property that $M_{a_i}\sum_n \bar{\ell}_{a_i}(z_n) = 1 - u_{a_i}(\lambda_{a_i})$, thus effectively representing a (frictional) labor resource constraint.

We now substitute for the tightness constraint (25) into the Beveridge curve (24) and plug the latter into the objective function. The Lagrangian associated to the social planner problem reads:

$$
\begin{aligned}
\mathcal{L}(\vec{v}^j_{a_i}(z),\vec{q}_{1,a_i},\vec{q}_{2,a_i}) &= Y\left(\vec{\ell}^j_{a_i}\left(\vec{q}_{1,a_i},\vec{q}_{2,a_i},V_{a_i},\vec{v}^j_{a_i}(z)\right)\right) - \sum_{i=1}^{I}\Xi(i)\int a\Bigg[M_{a_i}\Bigg(\sum_n^{\mathcal{Z}_i}\int_0^1 c^j_{a_i}\left(\vec{q}_{1,a_i},\vec{q}_{2,a_i},V_{a_i},\vec{v}^j_{a_i}(z_n)\right)dj \\
&\gamma_{a_i}(z_n)\Bigg) - \sum_n\sum_{n'}\xi_{1,a_i}(n,n')[1-q_{1,a_i}(n,n')-q_{a_i}(n',n)] - \sum_n\int_0^1\int_0^1 \xi_{2,a_i}(j,j',n)[1-q_{2,a_i}(j,j',n)-q_{a_i}(j',j,n)] \\
&+\int_0^1 \xi^j_{v,a_i}(z_n)v^j(z_n)dj\Bigg]d\Omega_i(a),
\end{aligned}
$$

where $\xi^j_{v,a_i}(z_n) \geq 0$ is the Kuhn Tucker multiplier attached to the non-negativity constraint on vacancies, while $\xi_{1,a_i}(n,n')$ and $\xi_{2,a_i}(j,j',n)$ are the Lagrange multiplier attached to the adding-up constraints on the transition probabilities.

The efficient transition probability from two firms j and j' with the same productivity z_n solves:

$$
\begin{aligned}
\frac{\partial\mathcal{L}}{\partial q_{2,a_i}(j,j',n)} &= \left[mp^j_{a_i}(z_n)\frac{\partial\ell^j_{a_i}(z_n)}{\partial q_{2,a_i}(j,j',n)} - \frac{\partial c^j_{a_i}}{\partial q_{2,a_i}(j,j',n)}\right] - \left[mp^{j'}_{a_i}(z_n)\frac{\partial\ell^{j'}_{a_i}(z_n)}{\partial q_{2,a_i}(j',j,n)} - \frac{\partial c^{j'}_{a_i}}{\partial q_{2,a_i}(j',j,n)}\right] \\
&\geq 0,
\end{aligned}
\tag{31}
$$

where $mp^j_{a_i}(z_n) \equiv \frac{\partial Y}{\partial\ell^j_{a_i}(z_n)}$ is the marginal product of labor of firm j with productivity z_n. Condition 31 is obviously satisfied with equality if $q^j_{2,a_i} = \frac{1}{2}\ \forall j$, so that all the firms with a certain productivity in a given labor market have the same vacancy yield. In principle, the planner

[7] Notice that (28) differs from (27) in that it ignores transitions between equally productive firms.

would be indifferent among any potentially nondegenerate distributions of q^j_{2,a_i} consistent with (31) holding as equality. Since firms are structurally identical, candidate distributions should have the property that $\frac{\partial}{\partial q^j_{2,a_i}}\left[mp^j_{a_i}(z_n)\frac{\partial \ell^j_{a_i}(z_n)}{\partial q_{2,a_i}(j,j',n)}-\frac{\partial c^j_{a_i}}{\partial q_{2,a_i}(j,j',n)}\right]=0$. However, no such distributions generally exist.[8] Hence, we can specialize on the problem setup with no residual dispersion, thus dropping the j index henceforth. As a result, the stationary employment constraint (22) simplifies to its familiar expression:

$$\ell_{a_i}\left(\vec{q}_{a_i}, V_{a_i}, \vec{v}_{a_i}(z)\right) = \frac{v_{a_i}(z_n)}{V_{a_i}}\lambda_{a_i}u_{a_i}\frac{\delta_{a_i}+s_{a_i}\lambda_{a_i}}{h_{a_i}(1,n)h_{a_i}(0,n)}.$$

The efficient transition probability from a firm with productivity z_n to a firm with productivity $z_{n'}$ in labor market a_i solves:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial q_{1,a_i}(n,n')} &= \left[mp_{a_i}(z_n)\frac{\partial \ell_{a_i}(z_n)}{\partial q_{1,a_i}(n,n')}-\frac{\partial c_{a_i}}{\partial q_{1,a_i}(n,n')}\right]\gamma_{a_i}(z_n)-\left[mp_{a_i}(z_{n'})\frac{\partial \ell_{a_i}(z_{n'})}{\partial q_{1,a_i}(n',n)}-\frac{\partial c_{a_i}(z_{n'})}{\partial q_{1,a_i}(n',n)}\right]\gamma_{a_i}(z_{n'})\\ &= \left[mp_{a_i}(z_n)\left(\frac{d\tilde{\ell}_{a_i}(z_n)}{d\bar{\mathcal{H}}_{1,a_i}(n)}+\frac{d\tilde{\ell}_{a_i}(z_n)}{d\bar{\mathcal{H}}_{2,a_i}(n)}\right)-mp_{a_i}(z_{n'})\left(\frac{d\tilde{\ell}_{a_i}(z_{n'})}{d\bar{\mathcal{H}}_{1,a_i}(n')}+\frac{d\tilde{\ell}_{a_i}(z_{n'})}{d\bar{\mathcal{H}}_{2,a_i}(n')}\right)\right]\\ &\quad+\frac{\phi}{1+\zeta}\bar{c}_{a_i}\left[\frac{v_{a_i}(z_{n'})^{\zeta}}{h_{a_i}(n')^{1-\phi}}-\frac{v_{a_i}(z_n)^{\zeta}}{h_{a_i}(n)^{1-\phi}}\right]\geq 0,\end{aligned} \tag{32}$$

where $\tilde{\ell}=\ell/v$ is the vacancy yield (independent of v) and the second line follows from simplifying for $\frac{\partial \bar{\mathcal{H}}_{s,a_i}(n)}{\partial q_{1,a_i}(n,n')}=\frac{\partial \bar{\mathcal{H}}_{s,a_i}(n)}{\partial q_{1,a_i}(n,n')}=\frac{v_{a_i}(z_{n'})}{V_{a_i}}\gamma_{a_i}(z_{n'})$, $s=1,2$, and the functional form of hiring costs. Notice that (32) describes a knife-edge condition. Specifically, it holds as equality if hiring rates, marginal products, and vacancies are all equalized across all firms. However, if hiring rates and vacancies are equalized, all firms have the same employment according to (22). If firms have the same employment, marginal products are strictly increasing in productivity. Therefore, equation 32 needs to hold as inequality, thus describing a corner solution. Formally, $q_{1,a_i}(n,n')\in\{0,1\}\ \forall n\neq n'$. We proceed by guessing and verifying that the planner chooses to reallocate workers towards higher-productivity firms whenever an opportunity arises, that is, $q_{a_i}(n,n')=1 \iff z_n<z_{n'}$. Hence, the shadow wage offer distribution equals the (complement to 1 of the) vacancy-weighted productivity distribution, i.e., $\bar{\mathcal{H}}_{a_i}\left(V_{a_i},\vec{v}_{a_i}(z_{n'})\right)=\sum_{n'=n+1}^{\mathcal{Z}_i}\frac{\bar{v}_{a_i}(z_{n'})}{V_{a_i}}\gamma_{a_i}(z_{n'})$.

Efficient vacancy posting by a firm with productivity z operating in labor market a_i meets the following conditions:

[8] Assume firms with identical productivity z_n are assigned a signal $\varrho\sim\Phi_n$. Hence, the planner can condition the choice of the residual transition probability on ϱ, e.g., $q_{2,a_i}(\varrho,\varrho',n)=\mathcal{F}(\varrho/\varrho')$. If the planner chooses to do so, then $\frac{\partial q^{\varrho}_{2,a_i}}{\partial \varrho}\neq 0$. For this strategy to be efficient, it must be that $\frac{\partial}{\partial \varrho}\left[mp^{\varrho}_{a_i}(z_n)\frac{\partial \ell^{\varrho}_{a_i}(z_n)}{\partial q_{2,a_i}(\varrho,\varrho',n)}-\frac{\partial c^{\varrho}_{a_i}}{\partial q_{2,a_i}(\varrho,\varrho',n)}\right]=0$. However, since efficient vacancies are increasing in vacancy yield and marginal product of labor, the derivative will be generally positive.

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial v_{a_i}(z_n)} = & \left[mp_{a_i}(z_n) \frac{\partial \ell_{a_i}(z_n)}{\partial v_{a_i}(z_n)} - \frac{\partial c_{a_i}(z_n)}{\partial v_{a_i}(z_n)} \right] M_{a_i} \gamma_{a_i}(z_n) \\ & + \sum_{n' \leq n}^{\mathcal{Z}_i} \left[mp_{a_i}(z_{n'}) \frac{\partial \ell_{a_i}(z_{n'})}{\partial \bar{\mathcal{H}}_{a_i}(n')} - \frac{\partial c_{a_i}(z_{n'})}{\partial \bar{\mathcal{H}}_{a_i}(n')} \right] \frac{\partial \bar{\mathcal{H}}_{a_i}(n')}{\partial v_{a_i}(z_n)} M_{a_i} \gamma_{a_i}(z_{n'}) \\ & + \sum_{n'=1}^{\mathcal{Z}_i} \left[mp_{a_i}(z_{n'}) \left(\frac{\partial \ell_{a_i}(z_{n'})}{\partial V_{a_i}} + \frac{\partial \ell_{a_i}(z_{n'})}{\partial \lambda_{a_i}} \frac{\partial \lambda_{a_i}}{\partial V_{a_i}} + \frac{\partial \ell_{a_i}(z_{n'})}{\partial \bar{\mathcal{H}}_{a_i}(n')} \frac{\partial \bar{\mathcal{H}}_{a_i}(n')}{\partial V_{a_i}} \right) \right. \\ & \left. - \frac{\partial c_{a_i}(z_{n'})}{\partial \bar{\mathcal{H}}_{a_i}(n')} \frac{\partial \bar{\mathcal{H}}_{a_i}(n')}{\partial V_{a_i}} \right] \frac{\partial V_{a_i}}{\partial v_{a_i}(z_n)} M_{a_i} \gamma_{a_i}(z_{n'}) \geq 0, \end{aligned} \tag{33}$$

$$\xi_{v,a_i}(z_n) v_{a_i}(z_n) = 0, \tag{34}$$

where (33) is the FOC with respect to vacancies, and (34) the respective complementary slackness condition.

The first line of (33) represents the *direct effect* of $v_{a_i}(z_n)$ on the firm's value added, that is, the excess marginal product of labor induced by the marginal vacancy over the marginal hiring cost.

The second line of (33) represents the *business-stealing effects* of $v_{a_i}(z)$ on the value added of lower-productivity firms operating in the same labor market. Specifically, the constrained social planner internalizes how vacancy posting by firms with higher productivity reduces the employment of firms with lower productivity, i.e., how the pace of worker reallocation affects the cross-sectional distribution of employment.[9]

The third and fourth line of (33) represent the *congestion effects* of $v_{a_i}(z_n)$ on the value added produced by workers operating in the same labor market. Specifically, the social planner internalizes that vacancy posting by some firm affects the meeting rate of all the firms via the induced change in aggregate vacancies and, in turn, labor market tightness. These negative (thin-market) externalities exerted on other firms are weighed against the positive (thick-market) externalities exerted on workers, that is, the marginal effect of $v_{a_i}(z_n)$ on the job finding rate – again mediated by the induced change in aggregate vacancies and labor market tightness.

Since both the shadow wage offer distribution and aggregate vacancies are linear in firm-level vacancies, equation 33 carries two important insights. First, conditional on a given rank in the productivity distribution, the business-stealing effects of vacancy posting by firm z_n in labor market a_i are independent of the mass of vacancies it posts. Second, the congestion effects of vacancy posting by firm z in labor market a_i are independent of the mass of vacancies it posts.

Let $E^{bs}_{a_i}(n)$ and $E^{c}_{a_i}$ denote the business-stealing effects and congestion effects of $v_{a_i}(z_n)$ on value added, respectively. It follows that vacancy posting of active firms in the efficient

[9]Intuitively, the higher the share of aggregate vacancies accounted for by firms with higher productivity, the lower the net poaching rate of firms with lower productivity.

allocation solves:

$$mp_{a_i}(z_n)\frac{\partial \ell_{a_i}(z_n)}{\partial v_{a_i}(z_n)} = \frac{\partial c_{a_i}(z_n)}{\partial v_{a_i}(z_n)} + E^{bs}_{a_i}(n) + E^{c}_{a_i}. \tag{35}$$

In words, the social planner equalizes the social marginal benefit of vacancy posting by firm z_n (left-hand side) to its social marginal cost (right-hand side). The latter is composed by the marginal vacancy posting cost and the external effects.

Notice that the social marginal benefit of vacancy posting equals the product between marginal product of labor and vacancy yield. Since the marginal product of labor is increasing in productivity, efficient vacancies are supermodular in productivity and vacancy yield. This property (coupled with the supermodularity of the output function in productivity and size) allows verifying our guess that the planner reallocates workers towards higher-productivity firms when it is given the chance, as an application of Topkis's theorem.

Relation with equilibrium allocation. In equilibrium, wages – rather than marginal products – play an allocative role via worker reallocation. Hence, for the efficient allocation to be sustainable in equilibrium, wages need to be increasing in productivity, i.e., $w'_{a_i}(z) > 0$. Notice that the social planner is indifferent between the composition of marginal costs between wages and markdowns.[10] However, comparing (35) with its equilibrium counterpart, it is apparent that both feature an "additive markdown", i.e., the difference between marginal (revenue) product and wage, defined as $\frac{\partial c_{a_i}(z)/\partial v_{a_i}(z)}{\partial \ell_{a_i}(z)/\partial v_{a_i}(z)}$. Hence, we proceed by defining the *shadow wage* as $w^{\star\star}_{a_i}(z) \equiv \frac{E^{bs}_{a_i}(n)+E^{c}_{a_i}}{\partial \ell_{a_i}(z)/\partial v_{a_i}(z)}$ and the *shadow markdown* as $\psi^{\star\star}_{a_i}(z) \equiv 1 + \frac{\partial c_{a_i}(z)/\partial v_{a_i}(z)}{w^{\star\star}_{a_i}(z)\partial \ell_{a_i}(z)/\partial v_{a_i}(z)}$. It follows that equilibrium markdowns are efficient if and only if wages are efficient. The efficient allocation would decentralize by dictating:

$$\mu_{a_i}(z_n) = 1, \tag{36}$$

$$\psi_{a_i}(z_n) = 1 + \frac{(1+\zeta)c_{a_i}(z_n)}{w_{a_i}(z_n)\ell_{a_i}(z_n)}, \tag{37}$$

$$w_{a_i}(z_n) = \frac{E^{bs}_{a_i}(n) + E^{c}_{a_i}}{\partial \ell_{a_i}(z_n)/\partial v_{a_i}(z_n)}. \tag{38}$$

Condition 36 makes sure that posted prices are efficient, i.e., equalizing the marginal product to the marginal *revenue* product of labor, condition 37 makes sure that markdowns are efficient for given wage policy, condition 38 makes sure that the wage policy induces efficient vacancy posting, i.e., equalizing social and private marginal cost.

How does the equilibrium wage function compare to its efficient counterpart (38)? First of all, we notice that residual wage (and marginal product) dispersion is inefficient. Yet, residual wage dispersion is needed to sustain the unique (mixed-strategy) Nash equilibrium in the wage-posting game with finite number of productivity types and continuous wage

[10] Since labor supply is inelastic, it is not influenced by the composition of income between wages and profits.

distributions. This allows establishing our first lemma:

Lemma 2 (Efficiency requires infinite productivity types)
If the number of productivity types is finite, the wage-posting equilibrium with a continuous wage offer distribution is inefficient.

Hence, we proceed by comparing efficient and equilibrium allocation in the limit case of our economy as $\mathcal{Z}_i \to \infty$. For consistency with the social planner problem setup, we further assume that workers set their reservation wage statically under risk-neutrality, so that the equilibrium reservation wage boils down to the *static* reservation wage $\hat{R}^u_{a_i} : a\hat{R}^u_{a_i} + \Pi(a\hat{R}^u_{a_i}) + T = ab_{a_i}$. We carry out the comparison in two steps. First, we show that the equilibrium wage function internalizes the business-stealing effects of vacancy posting if the static reservation wage is zero, absent any additional source of inefficiency (i.e., provided that the marginal revenue product of labor equals its marginal product and congestion effects are zero). Second, we claim that equilibrium wages are generally inefficient for given marginal product function due to congestion effects.

Lemma 3 (Wage posting internalizes business-stealing effects)
For given marginal product function $mp_{a_i}(z)$ and a zero static reservation wage ($\hat{R}^u_{a_i} = 0$), the equilibrium wage function internalizes the business-stealing effects of vacancy posting. Formally,

$$v_{a_i}(z) = v^\star_{a_i}(z) \implies mp_{a_i}(z)\frac{\partial \ell_{a_i}(z)}{\partial v_{a_i}(z)} = \frac{\partial c_{a_i}(z)}{\partial v_{a_i}(z)} + E^{bs}_{a_i}(\mathcal{H}_{a_i}(z)),$$

where $v^\star_{a_i}(z)$ is the equilibrium vacancy policy function and $\mathcal{H}_{a_i}(z)$ is the equilibrium wage offer distribution.

In general, equilibrium vacancies and wages are inefficient for three reasons: static reservation wages are positive, markup rates are positive, and congestion effects are different from zero. [11] Indeed, evaluating (33) at the equilibrium solution yields:

$$\frac{\partial \mathcal{L}}{\partial v_{a_i}(z)}\Big|_{v_{a_i}(z)=v^\star_{a_i}(z)} = \frac{\mu_{a_i}(z)-1}{\mu_{a_i}(z)} mp_{a_i}(z)\frac{\partial \ell_{a_i}(z)}{\partial v_{a_i}(z)} + (1-\mathcal{P}_{a_i}(z))\frac{\partial \ell_{a_i}(z)}{\partial v_{a_i}(z)}\hat{R}^u_{a_i} - E^c_{a_i} \lesseqgtr 0,$$

where $\mathcal{P}_{a_i}(z) \equiv 1 - e^{-\int_{\underline{z}}^{z} x_{a_i}(\hat{z})d\hat{z}} \in (0,1]$ and $x_{a_i}(z) \equiv \frac{2\nu s_{a_i}\lambda(\theta_{a_i})\mathcal{H}'_{a_i}(z)}{\delta_{a_i}+s_{a_i}\lambda(\theta_{a_i})[1-\mathcal{H}_{a_i}(z)]}$. Notice that the sign of the congestion effects at the equilibrium solution is a priori ambiguous, though generally different from zero. If congestion effects are positive, i.e., $E^c_{a_i} > 0$, *ceteris paribus* equilibrium marginal costs (wages times markdowns) are lower than efficient. Since congestion effects show up as an additive wedge in the first-order condition 35 and wages are increasing in productivity, lower-productivity firms are farther away from their efficient marginal cost

[11] Equilibrium wages and markdowns would be the constrained efficient outcome of the wage-posting game if job creation were exogenous and firm size were pinned down by search frictions only (Moscarini and Postel-Vinay, 2013). Intuitively, firm size would be determined by workers' efficient turnover. However, insofar as firms optimize their employment size because of decreasing returns (in production or in revenues) and/or endogenous vacancy posting, equilibrium markdowns are generally inefficient.

in relative terms. As a result, low-productivity firms typically post an inefficient mass of vacancies, leading to labor misallocation as in Shimer and Smith (2001) and Acemoglu (2001).

We conclude this section by singling out the sources of inefficiency featured by the baseline equilibrium and how the introduction of a minimum wage may affect them. To do so, let $\Delta_{a_i}(z)$ denote the firm-specific *labor wedge*, that is, the ratio between social marginal benefit (left-hand side of 35) and social marginal cost (right-hand side of 35) of vacancy posting by a firm with productivity z_n operating in labor market a_i. In the efficient allocation, $\Delta_{a_i}(z) = 1 \; \forall z, \; \forall a_i$. In the baseline equilibrium it equals:

$$\Delta_{a_i}(z) = \frac{\mu_{a_i}(z)}{1 + \frac{E^c_{a_i} - D_{a_i}(z)\hat{R}^u_{a_i}}{MCL_{a_i}(z)}}, \tag{39}$$

where $D_{a_i}(z) \equiv (1 - \mathcal{P}_{a_i}(z))\partial \ell_{a_i}(z)/\partial v_{a_i}(z)$, $\partial D_{a_i}(z)/\partial z \geq 0$ and $MCL_{a_i}(z) \equiv \psi_{a_i}(z) w_{a_i}(z)$. The baseline equilibrium is inefficient for two reasons. First, positive markup rates (due to imperfect substitutability across firm-level varieties) and distorted markdowns (due to congestion effects and positive static reservation wages) make the labor wedge generally differ from one in equilibrium. Positive markup rates push the labor wedge to be higher than one. Hence, if the net external effects in wage setting at the equilibrium solution are positive, equilibrium aggregate employment is inefficiently low. This means that value added would increase if all the firms posted more vacancies.

Second, heterogeneous markups (due to firms' granularity in their product market) and heterogeneous markdown distortions (due to heterogeneous impact of congestion effects and static reservation wages in the cross section) make the labor wedge generally differ across firms in equilibrium. This means that the economy features *misallocation* of labor across firms. Misallocation entails that the economy could produce the same amount of final output with lower aggregate hiring costs. It follows that value added would increase by reallocating the equilibrium mass of equilibrium vacancies across firms. Hence, with a large number of productivity types, decentralizing the efficient allocation requires three policy instruments: (i) a size-dependent subsidy to neutralize markup distortions (Edmond et al., 2023), (ii) a linear vacancy posting tax (or subsidy) to neutralize markdown distortions due to congestion effects, and (iii) a firm-specific wage subsidy to neutralize positive static reservation wages.

However, the amount of information required to implement such first-best policies is arguably beyond the possibilities of policymakers. On the other hand, policymakers can directly control the static reservation wage through MW setting. Since $\partial D_{a_i}(z)/\partial z \geq 0$, the MW is expected to affect more the labor wedge of low-productivity firms. Since, for standard parametrizations, congestion effects are positive and markups are increasing in productivity, low-productivity firms typically post more vacancies than efficient. By constraining their optimal vacancy posting, a higher minimum wage is likely to reduce labor misallocation. In this sense, a minimum wage can be thought of as a second-best policy when policymakers do not have access to more targeted policy instruments.

Proof Lemma 3 To establish the claim, we define a function $\mathcal{F}_{a_i}(v_{a_i}(z)) \equiv mp_{a_i}(z)\frac{\partial \ell_{a_i}(z)}{\partial v_{a_i}(z)} - \frac{\partial c_{a_i}(z)}{\partial v_{a_i}(z)} - E^{bs}_{a_i}\left(\mathcal{H}_{a_i}(z)\right)$, which equals zero at the efficient solution, i.e., $\mathcal{F}_{a_i}(v^{\star\star}_{a_i}(z)) = 0$. In equilibrium, optimal vacancy posting solves $mp_{a_i}(z)\frac{\partial \ell_{a_i}(z)}{\partial v_{a_i}(z)} - \frac{\partial c_{a_i}(z)}{\partial v_{a_i}(z)} - w_{a_i}(z)\frac{\partial \ell_{a_i}(z)}{\partial v_{a_i}(z)} = 0$. First of all, we notice that the business-stealing effects induced by the lowest-productivity firms are zero, i.e., $E^{bs}_{a_i}\left(\mathcal{H}_{a_i}(\underline{z}_{a_i})\right) = 0$. On the other hand, the lowest-productivity firms pay the reservation wage in equilibrium. As long as the reservation wage is positive, $\mathcal{F}_{a_i}(v^{\star}_{a_i}(\underline{z}_{a_i})) \neq 0$. Hence, equilibrium vacancy posting cannot be efficient.

We proceed by rearranging the expression for the business-stealing effects of vacancy posting as follows:

$$
\begin{aligned}
E^{bs}(\mathcal{H}_{a_i}(z)) &\equiv -\frac{1}{M_{a_i}\gamma_{a_i}(z)}\int_{\underline{z}_{a_i}}^{z}\left[mp_{a_i}(\hat{z})\frac{\partial \ell_{a_i}(\hat{z})}{\partial \bar{\mathcal{H}}_{a_i}(\hat{z})} - \frac{\partial c_{a_i}(\hat{z})}{\partial \bar{\mathcal{H}}_{a_i}(\hat{z})}\right]\frac{\partial \bar{\mathcal{H}}_{a_i}(\hat{z})}{\partial v_{a_i}(z)}M_{a_i}\gamma_{a_i}(\hat{z})d\hat{z} \\
&= \int_{\underline{z}_{a_i}}^{z}\left[mp_{a_i}(\hat{z})\frac{2s_{a_i}\lambda_{a_i}\bar{\mathcal{H}}'_{a_i}(\hat{z})}{\delta_{a_i}+s_{a_i}\lambda_{a_i}\bar{\mathcal{H}}_{a_i}(\hat{z})}\frac{\ell_{a_i}(\hat{z})}{v_{a_i}(\hat{z})} + \phi\frac{s_{a_i}\lambda_{a_i}\bar{\mathcal{H}}'_{a_i}(\hat{z})}{\delta_{a_i}+s_{a_i}\lambda_{a_i}\bar{\mathcal{H}}_{a_i}(\hat{z})}\frac{c_{a_i}(\hat{z})}{v_{a_i}(\hat{z})}\right]d\hat{z} \\
&= \int_{\underline{z}_{a_i}}^{z}\left[\frac{1}{\nu}mp_{a_i}(\hat{z})\frac{\ell_{a_i}(\hat{z})}{v_{a_i}(\hat{z})} + \frac{\phi}{2\nu}\frac{c_{a_i}(\hat{z})}{v_{a_i}(\hat{z})}\right]x_{a_i}(\hat{z})d\hat{z} \\
&= \int_{\underline{z}_{a_i}}^{z}\left[\frac{1}{\nu}mp_{a_i}(\hat{z}) + \left(1-\frac{1}{\nu}\right)\frac{\partial c_{a_i}(\hat{z})/\partial v_{a_i}(\hat{z})}{\partial \ell_{a_i}(\hat{z})/\partial v_{a_i}(\hat{z})}\right]\frac{\partial \ell_{a_i}(\hat{z})}{\partial v_{a_i}(\hat{z})}x_{a_i}(\hat{z})d\hat{z} \\
&= \int_{\underline{z}_{a_i}}^{z}\left[\frac{1}{\nu}mp_{a_i}(\hat{z}) + \left(1-\frac{1}{\nu}\right)\left(mp_{a_i}(\hat{z}) - w_{a_i}(\hat{z})\right)\right]\frac{\partial \ell_{a_i}(\hat{z})}{\partial v_{a_i}(\hat{z})}x_{a_i}(\hat{z})d\hat{z},
\end{aligned}
$$

where we made use of the following relationships and equilibrium conditions: $\frac{\partial \ell_{a_i}(\hat{z})}{\partial \bar{\mathcal{H}}_{a_i}(\hat{z})} = -\frac{2s_{a_i}\lambda_{a_i}}{\delta_{a_i}+s_{a_i}\lambda_{a_i}\bar{\mathcal{H}}_{a_i}(\hat{z})}\ell_{a_i}(\hat{z})$, $\frac{\partial \bar{\mathcal{H}}_{a_i}(\hat{z})}{\partial v_{a_i}(z)} = \frac{M_{a_i}}{V_{a_i}}\gamma_{a_i}(z)$, $\frac{\partial c_{a_i}(\hat{z})}{\partial \bar{\mathcal{H}}_{a_i}(\hat{z})} = \phi\frac{s_{a_i}\lambda_{a_i}}{\delta_{a_i}+s_{a_i}\lambda_{a_i}\bar{\mathcal{H}}_{a_i}(\hat{z})}c_{a_i}(\hat{z})$, $\frac{\partial c_{a_i}(\hat{z})}{\partial v_{a_i}(\hat{z})} = (1+\zeta)\frac{c_{a_i}(\hat{z})}{v_{a_i}(\hat{z})}$, $\bar{\mathcal{H}}'_{a_i}(\hat{z}) = \frac{M_{a_i}}{V_{a_i}}v_{a_i}(\hat{z})\gamma_{a_i}(\hat{z})$, and $\frac{\phi}{2(1+\zeta)\nu} = 1-\frac{1}{\nu}$.

Substituting for the equilibrium condition and the expression for business-stealing effects into $\mathcal{F}_{a_i}(v_{a_i}(z))$ yields:

$$
\mathcal{F}_{a_i}(z) = w_{a_i}(mp_{a_i}(z))\frac{\partial \ell_{a_i}(z)}{\partial v_{a_i}(z)} - \int_{\underline{z}_{a_i}}^{z}\left[\frac{1}{\nu}mp_{a_i}(\hat{z}) + \left(1-\frac{1}{\nu}\right)\left(mp_{a_i}(\hat{z}) - w_{a_i}(\hat{z})\right)\right]\frac{\partial \ell_{a_i}(\hat{z})}{\partial v_{a_i}(\hat{z})}x_{a_i}(\hat{z})d\hat{z}.
$$

Substituting for $\frac{\partial \ell_{a_i}(\hat{z})}{\partial v_{a_i}(\hat{z})}$ and collecting common terms, the function depends on the following three terms:

$$
\begin{aligned}
\mathcal{F}_{a_i}(z) \propto \frac{w_{a_i}(mp_{a_i}(z))}{h_{a_i}(z)^2} &- \int_{\underline{z}_{a_i}}^{z}\frac{w_{a_i}(\hat{z})}{h_{a_i}(\hat{z})^2}\frac{x_{a_i}(\hat{z})}{\nu}d\hat{z} \\
&- \int_{\underline{z}_{a_i}}^{z}\frac{\left(mp_{a_i}(\hat{z}) - w_{a_i}(\hat{z})\right)}{h_{a_i}(\hat{z})^2}x_{a_i}(\hat{z})d\hat{z},
\end{aligned}
$$

where $h_{a_i}(z) \equiv \delta_{a_i} + s_{a_i}\lambda_{a_i}(1-\mathcal{H}_{a_i}(z))$ is the hiring rate and $x_{a_i}(z) \equiv \frac{2\nu s_{a_i}\lambda_{a_i}\mathcal{H}'_{a_i}(z)}{h_{a_i}(z)}$.

We now focus on the second term and apply integration by parts:

$$\int_{\underline{z}_{a_i}}^{z} \frac{w_{a_i}(\hat{z})}{h_{a_i}(\hat{z})^2}\frac{x_{a_i}(\hat{z})}{\nu}d\hat{z} = \int_{\underline{z}_{a_i}}^{z} w_{a_i}(\hat{z})\frac{\partial h_{a_i}(\hat{z})^{-2}}{\partial \hat{z}}d\hat{z} = \frac{w_{a_i}(mp_{a_i}(z))}{h_{a_i}(z)^2} - \int_{\underline{z}_{a_i}}^{z} \frac{(mp_{a_i}(\hat{z}) - w_{a_i}(\hat{z}))}{h_{a_i}(\hat{z})^2}x_{a_i}(\hat{z})d\hat{z},$$

where the last equality follows from the slope of the equilibrium wage function. Substituting back into the previous expression allows establishing the claim:

$$v_{a_i}(z) = v^{\star}_{a_i}(z) \implies \mathcal{F}_{a_i}(z) = 0.$$

■

4 Structural Estimation

4.1 Details of cleaning wage distributions

In this section, we report the algorithms we use to clean the observed wage distributions.

Admissibility condition. For each labor market, we check the admissibility condition, and find out whether there are points of the empirical wage distributions for which this condition is not met. Essentially, the condition fails when the wage density grows *too quickly*, which in some cases happens in our data in the left part of the distributions. We work around this by applying the simple following algorithm. Let $G^D_{a_i}(w)$ be the discretized version of our wage distributions, that takes values on grid points $[w_1, w_2, ...w_N]$. We proceed as follows:

1. We identify the first grid point i^Y for which the admissibility condition holds, in the left part of the distribution;
2. Starting from w_1, we move mass from the lowest grid points to the $i^Y - 2$ grid point, until the condition for $i^Y - 1$ is met. If during the process the mass of w_1 runs out, then we move to w_2;
3. When the condition for $i^Y - 1$ is met, then we turn to move mass towards $i^Y - 3$;
4. We stop when the first grid point for which the admissibility condition holds is the second one with non-zero mass;
5. At the end of this, we check again the condition over the whole distribution. For all points for which this is not verified, we progressively add mass to the previous grid points, removing it from all other points of the distribution.

In fact, the adjustment takes place exclusively in the left part of the distributions, where rapidly growing density functions cannot be generated by the model. Overall, the distance between the left tails of the empirical (trimmed) wage distributions and the ones generated by the model is negligible. Hence, we claim that the failure of the admissibility conditions for some data points does not represent an issue for our analysis.

Replicate actual number of firms. Due to the granular product market structure of our model, the number of firms is an important empirical moment to match. To map the measure of firms populating our labor markets to their actual number, we propose the following normalization: we let the number of firms in the MRPL with lowest weighted density be equal to 1. We find this normalization the most sensible to introduce an integer constraint in the number of firms per MRPL level. To make sure that our model replicates the actual number of firms, we implement the following algorithm:

1. Compute $M_{a_i}\varphi_{a_i}(\tilde{z})\ \forall a_i$ by inverting the (trimmed) empirical wage distributions according to the structure of the model (see next paragraph);

2. Let $\underline{\ell}$ and $\underline{\tilde{z}}$ denote the labor market and MRPL level corresponding to $\min\{M_{a_i}\varphi_{a_i}(\tilde{z})\}$. Apply the normalization: $N_{\underline{\ell}}(\underline{\tilde{z}}) = 1$.

3. Compute the number of firms in all the labor markets and MRPL values by rounding the expression $N_{a_i}(\tilde{z}) = \frac{M_{a_i}\varphi_{a_i}(\tilde{z})}{\min\{M_{a_i}\varphi_{a_i}(\tilde{z})\}} \forall a_i, \forall \tilde{z}$.

4. If the total number of firms in the model exceeds the actual number of firms with employees in Italy in 2019 ($1,555,543$), transfer the mass corresponding to the MRPL grid point with lowest number of firms to the closest, nonzero grid point to the left in the same labor market. To preserve the shape of the MRPL distribution, constrain the maximum mass to transfer to half the difference between the receiving grid point and the preceding nonzero grid point. If exceeding, split the mass over multiple grid points;

5. Iterate this procedure until the number of firms in the model is (weakly) lower than in the data. If it is lower, assess whether the error is higher in the last or second-to-last iteration step and choose the most precise one.

Overall, the distance between the right tails of the empirical (trimmed) wage distributions and the ones generated by the model is negligible. Hence, we claim that the integer constraint does not represent an issue for our analysis.

5 Equilibrium Effects of the Minimum Wage

Table 1 reports the effects on the main aggregate variables of the two policy experiments considered in the main text: the welfare-maximizing MW equal to the 10^{th} percentile of the original wage distribution (70% Kaitz index), and the consumption-maximizing MW equal to the 40^{th} percentile of the initial distribution (92% Kaitz index).

Table 1: Policy experiments

Variable	Baseline	70% Kaitz index	92% Kaitz index
Panel a. Aggregate statistics			
Value Added	1.000	1.014	1.031
Gross output	1.000	1.000	1.002
Labor share	0.733	0.745	0.753
Aggregate welfare	1.000	1.005	0.987
Unemployment rate	0.135	0.155	0.203
Labor productivity	1.000	1.024	1.087
Average wage	1.000	1.055	1.149
Average firm size	8.737	10.32	15.00
Panel b. Market power statistics			
Aggregate markup	1.139	1.140	1.143
Aggregate markdown	1.471	1.427	1.385
Panel c. Labor market transitions			
Job finding rate	0.161	0.136	0.098
Job separation rate	0.025	0.025	0.025

Source: Model. *Note*: The variables Value added, Gross output, Aggregate Welfare, Labor productivity, and Average wage are normalized to 1 in the baseline equilibrium.

Table 2: Behavior vs. selection: decomposition of main aggregate effects

Variable	Percent change	Behavioral share	Compositional share
Average wage	14.85 %	43.59 %	56.41 %
Average labor share	2.33 %	114.66 %	-14.66 %
Aggregate markup rate	3.21 %	55.66 %	44.34 %
Aggregate markdown rate	-18.19 %	109.32 %	-9.32 %

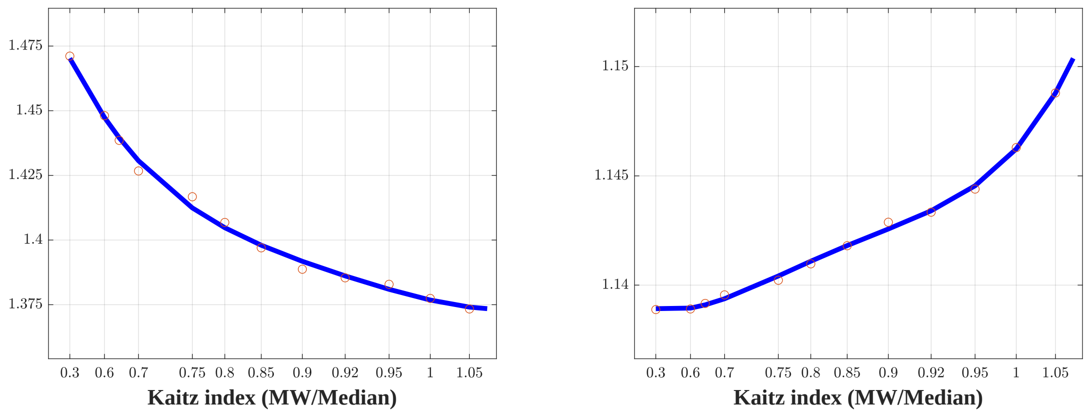
Source: Model. *Note*: The table reports percentage changes between the baseline equilibrium (no MW) and the consumption-maximizing MW (92% Kaitz index). Each aggregate variable X corresponds to a weighted average of the firm-specific variable $x(i)$ $X = \int x(j)\,\hat{w}(j)dj$, where i denotes a specific firm and $\hat{w}(j)$ is the firm-specific weight according to the aggregation results reported in Supplemental Appendix A.6. The change in the variable X can be decomposed as follows: $\Delta X = \int \Delta x(j)\,\hat{w}'(j)dj + \int x(j)\,\Delta\hat{w}(j)dj$, where $\hat{w}'(j)$ denotes the weight after the change. The first term represents the behavioral effect, the second represents the compositional effect.

5.1 Robustness: Nominal Minimum Wage

In the main text, we analyze the response of our economy to *real* MWs, i.e., set in terms of final good. However, in the reality, mandated MWs are *nominal*, i.e., set in monetary terms. As a robustness check, we repeat our policy experiments in an economy where money, rather than the final good, is the numeraire. To do so, we keep money supply equal to the nominal

Figure 2: Market power indices

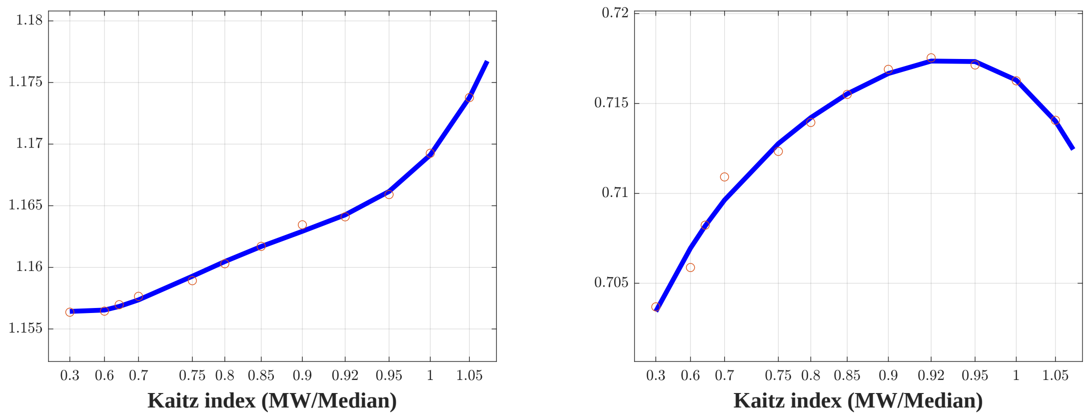
(a) Aggregate markdown response (b) Aggregate markup response



Source: Model. *Note*: Aggregate markdown and markup are cost-weighted averages of the respective firm-level variables, where weights are given by the wage bill. The x-axis is scaled so as to reflect the share of directly affected workers. The red circular markers represent the simulated data, the blue solid lines their 4^{th}-order polynomial fit.

Figure 3: Markup and labor share in most concentrated industry (manufacturing)

(a) Manufacturing markup response (b) Manufacturing labor share response



Source: Model. *Note*: Manufacturing markup is the cost-weighted average of the firm-level markup in the manufacturing industry, where weights are given by the wage bill. The x-axis is scaled so as to reflect the share of directly affected workers. The red circular markers represent the simulated data, the blue solid lines their 4^{th}-order polynomial fit.

output of the baseline economy, i.e., $M^s = PY$, and let the aggregate price level adjust in response to a monetary MW.

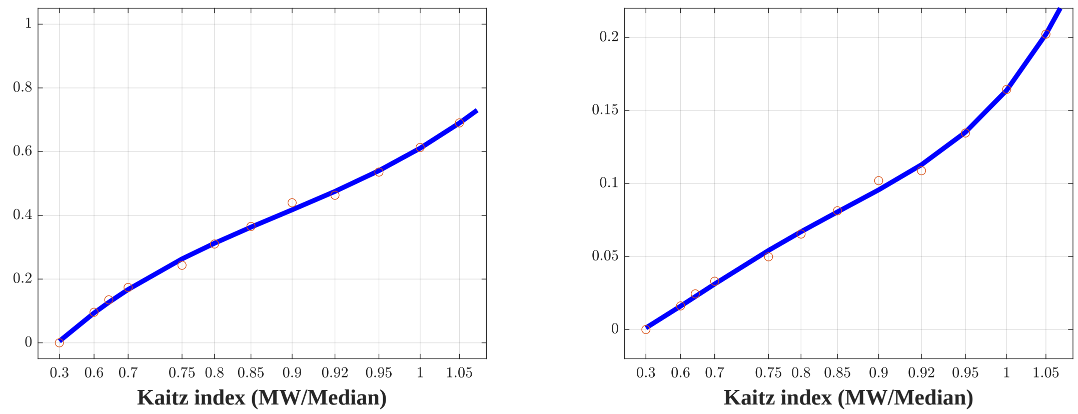
In Figure 7 we compare the aggregate response of the economy where the MW is set in nominal terms, money is the numeraire, and money supply is fixed, to our baseline. As apparent, setting the MW in real terms or in nominal terms does not make any significant difference.

6 Welfare Analysis

Following Floden (2001), utilitarian welfare changes from MW reforms can be decomposed into level, uncertainty, and distributional effects. Let ω_U be the consumption-equivalent

Figure 4: Firm exit effects of the MW

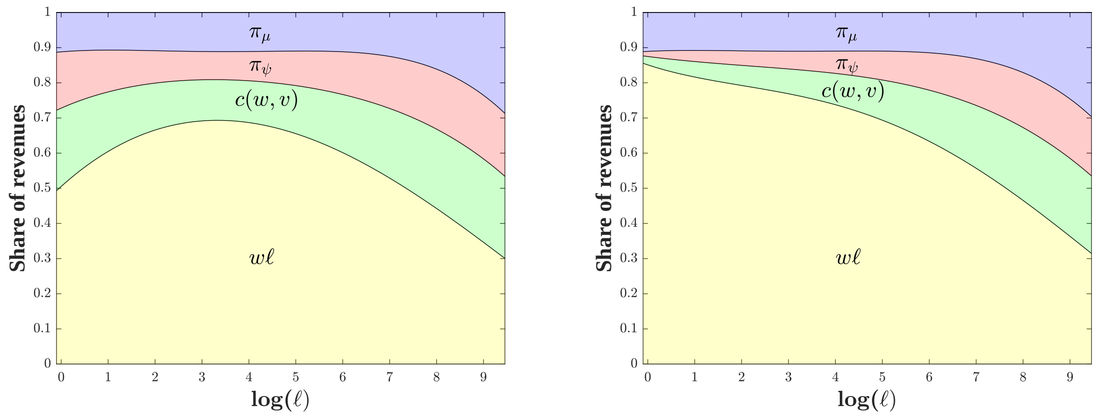
(a) Share of exiting firms, headcount (b) Share of exiting firms, employment share



Source: Model. *Note*: The *share of exiting firms firms, headcount* (Panel A) is the ratio between the number of exiting firms after a MW reform relative to the baseline economy; The *share of exiting firms firms, employment share* (Panel A) is the ratio between the baseline employment of firms exiting after a MW reform relative to total employment in the baseline economy. The x-axis is scaled so as to reflect the share of directly affected workers. The red circular markers represent the simulated data, the blue solid lines their 4^{th}-order polynomial fit.

Figure 5: Revenue shares response by firm size

(a) Baseline (b) 100% Kaitz index



Source: Model.

welfare change from raising the MW from its baseline level ($\underline{\mathbf{w}}$) to $\underline{\mathbf{w}}'$, that is,

$$\omega_U = \left(\frac{\mathcal{W}(\underline{\mathbf{w}}')}{\mathcal{W}(\underline{\mathbf{w}})}\right)^{\frac{1}{1-\vartheta}} - 1,$$

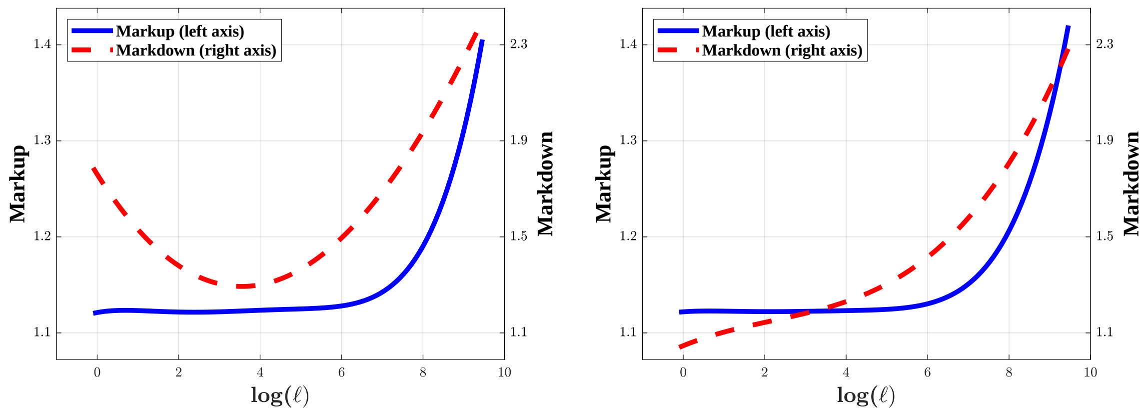
where $\mathcal{W}$ denotes aggregate utilitarian welfare.

Floden (2001) shows that utilitarian welfare admits the following multiplicative decomposition:

$$\omega_U = (1+\omega_{lev})(1+\omega_{unc})(1+\omega_{distr}) - 1. \tag{40}$$

Figure 6: Market power response by firm size

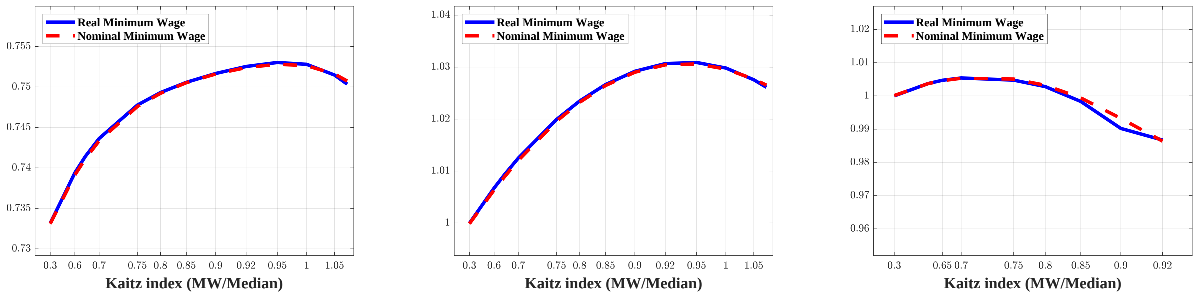
(a) Baseline (b) 100% Kaitz index



Source: Model.

Figure 7: Aggregate response: Real vs nominal Minimum Wage

(a) Aggregate labor share (b) Aggregate value added (c) Aggregate welfare



Source: Model. *Note*: The solid blue line represents the response of our baseline economy to a real minimum wage, while the dashed red line represents the response of our baseline economy to a nominal minimum wage. The x-axis is scaled so as to reflect the share of directly affected workers.

We now derive the three components one at a time. In our context, the *level effect* simply boils down to the aggregate consumption response:

$$\omega_{lev} = \frac{C(\underline{\mathbf{w}}')}{C(\underline{\mathbf{w}})} - 1. \tag{41}$$

Express the value of unemployment and employment at wage w in sequence form as:

$$\begin{aligned} U_{a_i} &= \mathbb{E}_t \left[\mathcal{U}_{a_i}(b_{a_i}, \{c_s\}_{s=t+1}^{\infty}) \right] \\ W_{a_i}(w) &= \mathbb{E}_t \left[\mathcal{U}_{a_i}(w, \{c_s\}_{s=t+1}^{\infty}) \right], \end{aligned}$$

where $\mathcal{U}$ is the lifetime utility function associated with our CRRA instantaneous utility function. Hence, we can compute the certainty-equivalent consumption, C^e, for each expected consumption stream starting from any labor market state as:

$$\begin{aligned} C^e(U_{a_i}) &= \mathcal{U}_{a_i}^{-1} \left(\mathbb{E}_t \left[\mathcal{U}_{a_i}(b_{a_i}, \{c_s\}_{s=t+1}^{\infty}) \right] \right) \\ C^e(W_{a_i}(w)) &= \mathcal{U}_{a_i}^{-1} \left(\mathbb{E}_t \left[\mathcal{U}_{a_i}(w, \{c_s\}_{s=t+1}^{\infty}) \right] \right). \end{aligned}$$

Let $C^e(\underline{\mathbf{w}})$ denote the aggregate certainty-equivalent consumption, that is,

$$C^e(\underline{\mathbf{w}}) = \sum_{i=1}^{I} \Xi(i) \int \left[u_{a_i}(\underline{w}) C^e\left(U_{a_i}(\underline{w})\right) + \left(1 - u_{a_i}(\underline{w})\right) \int C^e\left(W_{a_i}(aw;\underline{w})\right) \, dG_{a_i}(w;\underline{w}) \right] d\Omega_i(a).$$

In turn, let C^d be the aggregate consumption-equivalent utility of certainty-equivalent consumption, that is,

$$C^d(\underline{\mathbf{w}}) = \left(\sum_{i=1}^{I} \Xi(i) \int \left[u_{a_i}(\underline{w}) C^e\left(U_{a_i}(\underline{w})\right)^{1-\vartheta} + \left(1 - u_{a_i}(\underline{w})\right) \int C^e\left(W_{a_i}(aw;\underline{w})\right)^{1-\vartheta} \, dG_{a_i}(w;\underline{w}) \right] d\Omega_i(a) \right)^{\frac{1}{1-\vartheta}}.$$

The *uncertainty effect* is the change in the relative aggregate certainty-equivalent consumption with respect to aggregate consumption:

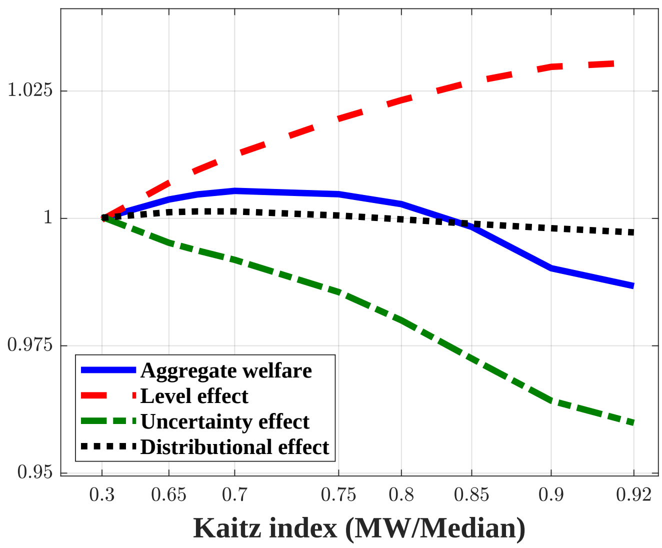
$$\omega_{unc} = \frac{C^e(\underline{\mathbf{w}}')/C(\underline{\mathbf{w}}')}{C^e(\underline{\mathbf{w}})/C(\underline{\mathbf{w}})} - 1. \tag{42}$$

Finally, the *distributional effect* is the change in the aggregate consumption-equivalent utility of certainty-equivalent consumption with respect to certainty-equivalent consumption:

$$\omega_{distr} = \frac{C^d(\underline{\mathbf{w}}')/C^e(\underline{\mathbf{w}}')}{C^d(\underline{\mathbf{w}})/C^e(\underline{\mathbf{w}})} - 1. \tag{43}$$

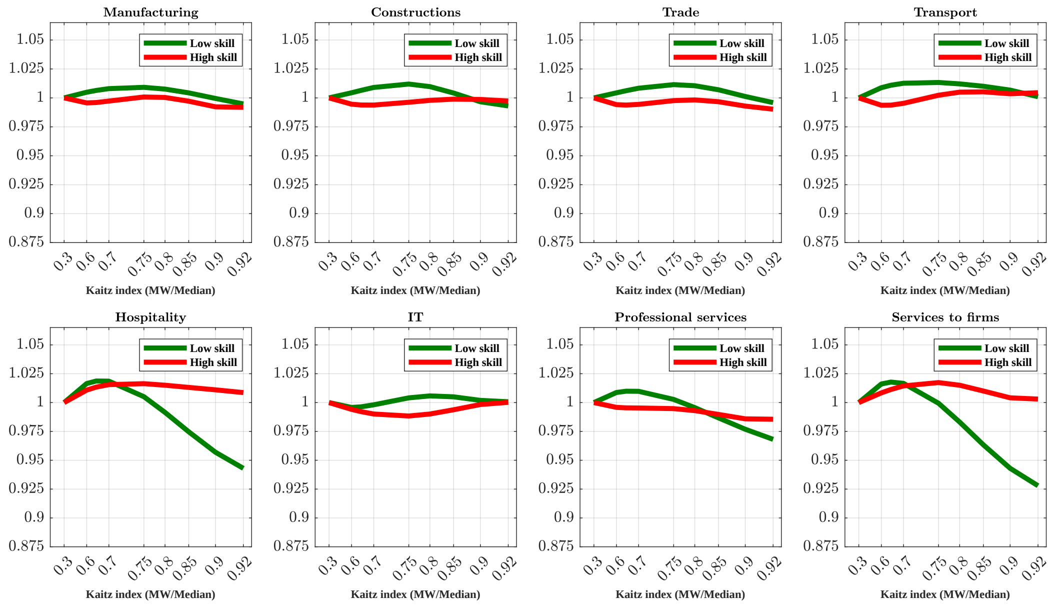
See Dávila and Schaab (2022) for further details on welfare assessments with heterogeneous agents.

Figure 8: Welfare effects, decomposition



Source: Model. *Note*: The figure decomposes the response of aggregate utilitarian welfare into level, uncertainty, and distributional effects as defined in (41)-(43).

Figure 9: Breaking down the distributional impact of the MW



Source: Model. *Note*: The graph reports the utilitarian welfare change for workers operating in each industry. The green lines represent low-skilled workers, the red lines high-skilled workers.

7 Empirical Validation

7.1 Institutional background

The collective bargaining system in Italy consists of a large number of contracts negotiated between trade unions and employers' associations (see Boeri (2012) for the economic implications of this system). Beyond regulating other aspects of labor contracts (maximum number of hours of work, number of days off, rules for promotions and training), these agreements set wage floors that are sector and skill-specific, and typically have a duration of 3 years (2 years prior to 2009). Importantly, these contracts have a virtually universal coverage – i.e., their validity extends *erga omnes* – and are generally used by labor courts and labor inspectors as a reference for a "fair wage". As a consequence, non-compliance to contractual wages in Italy is extremely rare (Adamopoulou and Villanueva, 2022). Moreover, bargaining at the firm level is also very unusual, with the exception of a few large firms. Therefore, agreed wages represent a very important component of worker pay in the Italian economy.[12] For a more detailed description of the institutional framework see D'Amuri and Nizzi (2018).

[12]Indeed, D'Amuri and Nizzi (2018) document that in the period 2005-2016 contractual wages defined at national level accounted for about 88% of overall total gross earnings. Moreover, a number of studies demonstrates the important role of collective bargaining for downward wage rigidity (Devicienti et al., 2007), wage inequality (Erikson and Ichino, 1994; Manacorda, 2004; Devicienti et al., 2019; Leonardi et al., 2019) and regional differences in employment (Boeri et al., 2021) in Italy.

7.2 Determinants of the labor share response

Several adjustment mechanisms at the firm level may bring about changes in the measured labor share. The data at our disposal allow us to dig deeper into the nature of these adjustments. Therefore, in Table 3 we run the regression model 21 in the main text for different dependent variables: log average wage, log size, log value added and log profits. Our estimates show that the average effect on wages does not depend on the level of concentration. Instead, firm size and value added respond differently depending on the HHI. In particular, firms in more concentrated sectors grow by less (or even shrink) following the wage increase. The dynamics of the value added qualitatively follows the one of firm size, but it is much less pronounced. As a result, profits in high-concentration sectors *increase* with the wage floor, in stark contrast with what happens in low-concentration sectors. Taken together, these findings show that the differential reaction of the labor share is strictly associated with opposite dynamics of the profit shares at the firm level.

Table 3: The Effect of Minimum Wages on the Determinants of the Labor Share

	(1) Log avg wage	(2) Log size	(3) Log value added	(4) Log profits
Log wage floor	0.369***	0.952*	0.293	-1.158
	(0.068)	(0.392)	(0.396)	(0.779)
Log wage floor × HHI (4-digit)	-0.030	-1.414	-0.634	1.487
	(0.192)	(1.412)	(0.532)	(1.318)
Time × Industry (2-digit) FE	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
N	6,155,191	6,158,901	5,774,132	4,915,300
R^2	0.932	0.988	0.978	0.952

Clustered standard errors at the firm-level in parentheses.

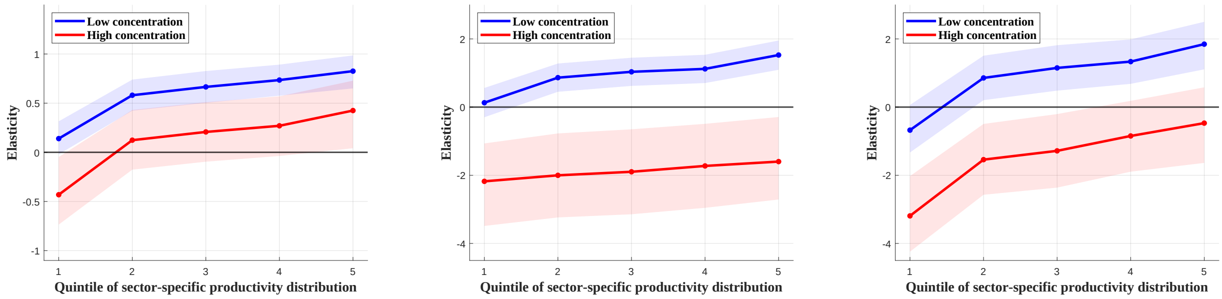
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Source: CERVED (2005-2020), INPS and Istat data. *Note*: Linear regressions of log average wage, log firm size, log value added and log profits (EBITDA). Regression models in all columns also control for the industry-specific level of HHI. Observations are weighted by firm employment.

We proceed by breaking down the cross-sectional labor share response into its components (Figure 10). We uncover a positive gradient by firm productivity in the response of both firm size and value added. However, the change in value added is larger than the one of firm size for low-productivity firms, causing a drop in their profits. Instead, more productive firms experience a less than proportional variation in value added relative to the change in firm size. As a consequence, profits tend to *rise* in the upper part of the productivity distribution. If we compare these patterns by the level of concentration, we find that the wage response is relatively similar, whereas large differences arise in the reaction of firm size and value added. In particular, it is especially in high-concentration sectors that the value added response is detached from the response of firm size, determining the largest increase in profits. Note that high productivity firms in highly concentrated sectors experience a large rise in profits in the face of a reduction in value added. This implies that their profit share is unambiguously increasing.

Figure 10: Heterogeneous Response of Labor Share Determinants

(a) Average wage (b) Employment (c) Value added



Source: CERVED (2005-2020), INPS and Istat data. *Note*: linear regressions of log average wage, log firm size, and log value added. All panels plot the estimated coefficients of interaction terms between the quintile of the sector-specific productivity distributions and the natural logarithm of wage floor. Observations are weighted by firm employment. Standard errors are clustered at the firm-level. Shaded areas represent 90% confidence intervals.

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