The Concentration Channel of the Minimum Wage*

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Abstract

We study the equilibrium and welfare effects of the minimum wage when product market power is endogenous and varies with market competition. A higher minimum wage reallocates workers from small to large firms. Large firms gain market share and increase their price markups. We call this mechanism *concentration channel* of the minimum wage. We contribute an equilibrium model with frictional labor markets and oligopolistic product markets. We estimate the model on Italian administrative data, replicating the structure of detailed labor and product markets. We find that both the aggregate labor share and consumption are hump-shaped in the minimum wage, due to the opposing responses of price markups and wage markdowns. The optimal minimum wage affects 10% of workers. Endogenous product market power lowers the optimal minimum wage. We provide empirical evidence supporting the concentration channel.

1 Introduction

In recent years, there has been a renewed interest in the minimum wage (MW). Its traditional goal of reducing in-work poverty has given way to more ambitious macroeconomic objectives, such as reducing income inequality and curbing firms' market power. This shift in focus reflects in policy proposals advocating for raising the MW to unprecedented levels in many countries. Several studies suggest that raising the MW is an effective policy for countering firms' labor market power. When firms have high labor market power, a higher MW tends to reallocate workers to more productive firms, with small effects on unemployment (Dustmann et al., 2022; Azar et al., 2024). However, little is known about how the effects of the MW

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interact with product market power – that is, the ability of dominant firms to charge prices above marginal costs.[1] This gap is surprising, as high product market power may undermine the beneficial effects of the MW on both efficiency, by limiting reallocation, and redistribution, by benefiting firms with low labor share.

In this paper we study the equilibrium and welfare effects of the MW when product market power is endogenous and varies with market competition. First, we build a stylized model with imperfect competition both on the labor and the product market. This model advances the existing MW literature by allowing firms to increase their product market power as the number of competitors decreases – echoing the very premise of antitrust laws. In this context, we show that raising the MW reduces labor market power of low-productivity firms, but increases product market power of high-productivity firms. For low-productivity firms, raising the MW makes labor supply more elastic, which reduces their wage markdown – an index of labor market power. On the other hand, raising the MW also induces selection on the firm side, forcing low-productivity firms to either downsize or exit the market altogether. As a result, workers reallocate towards high-productivity firms, which increase their market share. A higher market share allows high-productivity firms to raise their price markup – an index of product market power. We call this novel mechanism *concentration channel* of the MW.[2] We show that the concentration channel affects the response of the aggregate labor share and output to the MW – both qualitatively and quantitatively.

To quantify the effects of the concentration channel, we build a novel quantitative model where heterogeneous workers and firms interact in frictional labor markets (Engbom and Moser, 2022) and oligopolistic product markets (Atkeson and Burstein, 2008). Labor markets are segmented by worker skills and industries (e.g., low-skilled in manufacturing). Product markets are populated by a finite number of firms with the same skill requirements and industry (e.g., socks manufacturing sector). Since labor markets are frictional, workers are partially locked in at their current workplace because they receive job offers only from time to time. Firms profit from this lack of outside options by marking down the wage of their workers relative to the marginal revenue product. Moreover, the allocation of workers across firms is generally inefficient since firms with different productivities compete for the same workers. Since product markets are oligopolistic, large firms leverage their dominant position to mark up their price relative to the marginal cost. Hence, both price markups and wage markdowns are endogenous and react to changes in the competitive environment induced by policy reforms, such as the MW.

We estimate the model using administrative data from the Italian Social Security and Institute of Statistics. Upon running a two-way fixed effect regression á la Abowd et al. (1999), we distinguish two worker skill types (high- and low-skilled) based on the median worker

[1] Two exceptions are Harasztosi and Lindner (2019) and Link (2024), which document that firms affected by the MW are more likely to pass the cost increase onto consumer prices when product market competition is low.

[2] We see the concentration channel as a plausible explanation for the vocal support (and active lobbying) of big companies in the US for a drastic increase in the federal minimum wage (e.g., see `https://www.aboutamazon.com/news/policy-news-views/its-time-to-raise-the-federal-minimum-wage` for Amazon's official position).

fixed effect. Labor markets are defined by the skills of workers and the 1-digit industry codes of the firms they work for. Product markets are defined by 4-digit industry codes. We address the vast cross-sectional heterogeneity in market structures by replicating the observed wage distributions in each labor market and the number of competing firms in each product market. We estimate the key parameters governing the elasticities of demand and labor supply via the Simulated Method of Moments (SMM), by targeting the degree of concentration of 4-digit product markets and the average correlation between labor share and market share in each such markets. Within the SMM routine, we use the structure of our model to estimate the firms' productivity distribution that rationalizes the observed wage distributions. As a result, the estimated model replicates the wage distributions for each industry and worker type, which is key to assess the actual impact of MW reforms and the scope for worker reallocation.

We use the estimated model to simulate the effects of MW reforms in the Italian economy. According to our results, aggregate labor market power decreases with the MW, driven by low-productivity firms compressing profit margins. On the contrary, aggregate product market power increases with the MW, driven by the concentration channel. The rise in aggregate product market power is relatively modest (up to 1.5pp), occurs primarily for high MWs, and is driven by industries with high baseline concentration. Still, for sufficiently high MWs, the product market power response leads to an increase in *aggregate* market power with the MW. Mirroring the aggregate market power response, both the aggregate labor share and consumption are hump-shaped in the MW size. The increase in aggregate consumption is driven by productivity gains from worker reallocation (due to lower misallocation). However, the reallocation process produces some excess unemployment. It follows that the MW has the potential to raise aggregate consumption, but at the cost of reducing employment.

To strike a balance between these opposing forces, we resort to a utilitarian social welfare function as a normative criterion. The optimal MW equals 70% of the current median wage (10% of workers directly affected), with associated welfare gains of 0.5% in consumption equivalent units. The welfare effects of the MW are heterogeneous across worker skill types, which generates winners and losers. Welfare of low-skilled workers increases with the MW up to 65% of the current median wage. As the MW increases further, the high-skilled workers are the main beneficiaries. Hence, our results express a word of caution about the idea of raising the MW to unprecedented levels to reduce (welfare) inequality.

Then, we zoom into the role of endogenous markups in shaping the equilibrium and welfare effects of the MW. To do so, we replicate the same counterfactual experiments in an observationally equivalent economy with monopolistic competition in product markets, which features exogenous markups. Ignoring endogenous markups would result in a monotonically increasing labor share, as well as to higher consumption-maximizing and optimal MWs (from 40 to 50%, and from 10 to 17% of workers directly affected, respectively). Welfare gains from implementing the optimal MW would be nearly 50% larger. We conclude that endogenous product market power is an important determinant of the equilibrium and welfare effects of the MW.

Finally, we provide empirical validation of the concentration channel on Italian firms' balance sheet data, leveraging the variation induced by industry-specific contractual wage floors. We document two sets of reduced-form evidence. First, higher wage floors induce reallocation of workers towards larger and more productive firms, which increases product market concentration and average labor productivity. Second, the firm-level labor share response decreases with the concentration of the (4-digit) product market in which the firm operates. As predicted by the concentration channel, this effect is driven by the positive response of profits for high-productivity firms in highly concentrated product markets.[3]

This paper is related to three main strands of literature. First, it contributes to the literature on structural modelling of the equilibrium effects of the MW. Hurst et al. (2023) and Drechsel-Grau (2023) propose search models to study the distributional impact of the MW across heterogeneous households, while Ahlfeldt et al. (2022) and Karabarbounis et al. (2022) focus on the spatial dimension and entry decisions, respectively. The closest papers to ours in this literature are Engbom and Moser (2022) and Berger et al. (2025). Engbom and Moser (2022) uses a quantitative wage-posting model to study the effect of the MW on earnings inequality. Our model extends Engbom and Moser (2022)'s by considering a generalized hiring cost function and adding product market power, as well as characterizing the optimal MW. Berger et al. (2025) studies the welfare effects of the MW in the oligopsonistic-competition model of Berger et al. (2022), enriched by worker heterogeneity in wealth and productivity. Unlike Berger et al. (2025)'s, our framework features inefficient (frictional) unemployment, endogenous hiring costs and endogenous product market power, which react to the introduction of the MW.

Second, this paper speaks to the macroeconomic literature on oligopolistic competition in product markets (Atkeson and Burstein, 2008; Edmond et al., 2015; De Loecker et al., 2021, 2020; Burstein et al., 2025; Edmond et al., 2023). The closest papers in this literature to ours are MacKenzie (2020), Deb et al. (2022, 2023) and Firooz (2023), which propose models of oligopolistic competition in sectoral product markets and oligopsonistic competition (or wage bargaining) in local labor markets.Our model is the first to combine oligopolistic competition in product markets with wage posting in labor markets, and can be used to study spillover effects between product and labor markets induced by a variety of policies – not limited to the MW.

Finally, this paper contributes to the literature on wage-posting models, whose foundations are laid down by Burdett and Mortensen (1998), van den Berg and Ridder (1998), Bontemps et al. (1999) and Bontemps et al. (2000). Recent advances related to our framework include Engbom and Moser (2022), Gouin-Bonenfant (2020), Bilal and Lhuillier (2021), and Gottfries and Jarosch (2023). We contribute to this literature by characterizing the wage-posting equilibrium with decreasing returns to scale in the revenues, endogenous job creation, and a finite number of productivity types.

[3]In line with the existing literature, the concentration of local labor markets affects the labor share response *positively*. This reinforces our hypothesis that the negative effect of product market concentration on the firm-level labor share reflects an increase in price markups – as opposed to wage markdowns.

2 Stylized Model

In this section we develop a stylized model to (i) formalize the necessary conditions behind the concentration channel of the MW and (ii) highlight how the equilibrium effects of the MW depend on the direction of the market power response, i.e., the net effect of the response of labor and product market power. The model is intentionally parsimonious and reduced-form. In the next section, we will incorporate these qualitative insights into a fully-fledged, microfounded quantitative model.

Baseline Equilibrium. We consider an economy populated by a finite number N of firms with heterogeneous productivity z. Firms' productivity follows some distribution $\Gamma(z)$ with support $[\underline{z}, \bar{z}]$. Let $\hat{\gamma}(z)$ be the probability mass function of realized productivity. Both the labor and the product market are imperfectly competitive. Firms choose their labor demand by solving the following profit maximization problem:

$$\max_{\ell} \; p(y)y - w(\ell)\ell - \kappa \;\; \text{s.t.} \;\; y = z\ell, \quad w(\ell) = \ell^{\frac{1}{\eta}}, \quad p(y) = y^{-\frac{1}{\epsilon(N)}}.$$

Firms seek to maximize current profits, which equal the difference between revenues, py, and costs. In turn, costs consist of the wage bill, $w\ell$, and some overhead costs, κ.[4] Firms operate a linear technology in labor. Labor market power stems from an upward-sloping labor supply curve with elasticity of labor supply $\eta \in (0, \infty)$. Since wages are increasing in employment, the marginal cost exceeds the wage. Formally, $MCL(\ell) = w(\ell) + w'(\ell)\ell$. Similarly, product market power stems from a downward-sloping demand curve with elasticity of demand $\epsilon \in (1, \infty)$. We let the elasticity of demand be an increasing function of the number of active firms, i.e., $d\epsilon/dN > 0$ (Jaimovich, 2007). It follows that firms have more product market power the fewer the number of competitors they face – echoing the very premise of antitrust laws. Hence, we interpret N as an inverse index of market *concentration*. Since prices are decreasing in output, the marginal revenue product of labor falls short of its marginal product, $p(y)z$. Formally, $MRPL(y) = p(y)z + p'(y)zy$.

The solution to the profit maximization consists of a double wedge between the optimal price and (productivity-adjusted) wage:

$$p(z) = \underbrace{\frac{\epsilon(N)}{\epsilon(N) - 1}}_{\mu(N)} \underbrace{\frac{1+\eta}{\eta}}_{\psi} \frac{w(z)}{z}. \tag{1}$$

Equation (1) allows us to single out two different sources of market power. Let μ denote the price markup over marginal costs that firms optimally charge, i.e., $\mu(N) \equiv \frac{p(z)z}{MCL(z)} = \frac{\epsilon(N)}{\epsilon(N)-1} \geq 1$. The price markup is an inverse function of the elasticity of demand $\epsilon(N)$, thus representing an index of product market power. Similarly, let ψ denote the wage markdown

[4] Overhead costs are needed to induce some exit of low-productivity firms upon introducing the MW, in line with the empirical evidence (Draca et al., 2011; Dustmann et al., 2022).

that firms optimally charge, i.e., $\psi \equiv \frac{MRPL(z)}{w(z)} = \frac{1+\eta}{\eta} \geq 1$. The wage markdown is an inverse function of the elasticity of labor supply η, thus representing an index of labor market power.[5] Henceforth, we will refer to the price markup and wage markdown simply as markup and markdown, respectively. The optimality condition (1), along with the labor supply and product demand constraints, also characterizes the firm-specific labor share in the operative revenues, $LS(z)$, and output, $y(z)$:

$$LS(z) \equiv \frac{w(z)\ell(z)}{p(z)z\ell(z)} = \frac{1}{\mu(N)\psi}, \quad y(z) = z^{\epsilon\frac{1+\eta}{\epsilon+\eta}} \left(\frac{1}{\mu(N)\psi}\right)^{\frac{\eta\epsilon}{\epsilon+\eta}}.$$

Notice that both labor share and output are decreasing in the firm's market power, that is, the product between markup and markdown. The labor share is constant across the productivity distribution, while output tracks productivity differences, with higher-productivity firms having higher output, as well.

We conclude the characterization of the baseline equilibrium by defining two key aggregate variables, that is, the aggregate labor share in the operative revenues, LS, and the aggregate output, Y:

$$LS \equiv \sum_{\underline{z}}^{\bar{z}} LS(z)\hat{\gamma}(z) = \frac{1}{\mu(N)\psi}, \quad Y \equiv N \sum_{\underline{z}}^{\bar{z}} y(z)\hat{\gamma}(z) = NZ^{\epsilon\frac{1+\eta}{\epsilon+\eta}} \left(\frac{1}{\mu(N)\psi}\right)^{\frac{\eta\epsilon}{\epsilon+\eta}},$$

where $Z \equiv \left(\sum_{\underline{z}}^{\bar{z}} z^{\epsilon\frac{1+\eta}{\epsilon+\eta}}\hat{\gamma}(z)\right)^{\frac{\epsilon+\eta}{\epsilon(1+\eta)}}$ is an aggregate productivity index. Hence, aggregate market power acts as a uniform tax on aggregate output.

MW Equilibrium. We now study the response of this economy to the introduction of a binding minimum wage $\underline{w}$. Formally, we assume that the labor supply curve faced by firms is now given by $w(\ell) = \max\left\{\underline{w}, \ell^{\frac{1}{\eta}}\right\}$. Hence, the labor supply curve is now piecewise: it is perfectly flat for low enough employment and increasing thereafter. This means that firms with low employment are effectively wage-taker, as if they operated in perfect competition.

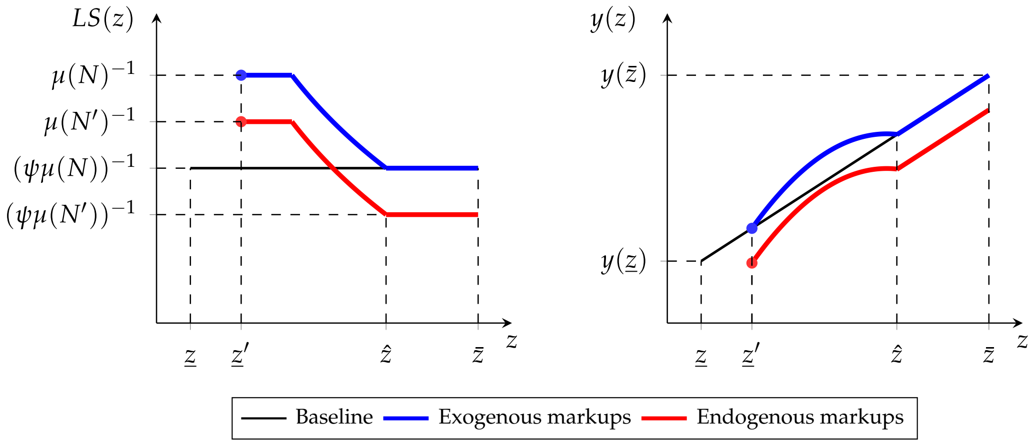
In response to the MW, some low-productivity firms exit the market, as they can no longer break even overhead costs with operating profits. Surviving firms can be ranked in two groups based on their productivity. Relatively low-productivity firms – which end up paying the MW after the reform – are *constrained* by the MW in their wage setting. Relatively high-productivity firms – which pay higher wages than the MW – are *unconstrained*. Let $\hat{z}(\underline{w})$ denote the highest-productivity firm that is constrained by the MW.

To single out the role of the endogenous markup response, we carry out our analysis in two steps. First, we study the effects of the MW with exogenous markups, i.e., $d\mu/dN = 0$, as

[5] The existing literature uses the term *markdown* either to denote the ratio between MCL and wage – e.g., Gottfries and Jarosch (2023) – or the ratio between wage and MCL – e.g., Berger et al. (2025). For ease of interpretation, we adopt the former formulation that grants a direct link between the notions of markdown and labor market power.

in the existing literature (Clemens, 2021). Figure (1) displays in blue the adjustment of the firm-level labor share and output to the MW across the productivity distribution when markups are exogenous.[6] In this case, the effects of the MW are shaped by the labor market power response only. A binding MW pushes constrained firms to pay higher wages by compressing

Figure 1: Cross-sectional impact of the MW



their markdowns (as their labor supply is more elastic). In turn, the markdown reduction increases both their labor share (left panel) and their output (right panel). Unconstrained firms are unaffected by the MW reform. Hence, if markups are exogenous, a higher MW raises the aggregate labor share. Moreover, if output gains from surviving firms exceed the output losses from exiting firms, the aggregate output increases with the MW, as well.

Second, we factor in the endogenous markup response. Figure (1) displays in red the adjustment to the MW when markups are endogenous. In this case, the effects of the MW are shaped by the direction of the market power response, i.e., the net effect of the response of labor and product market power. Because of the exit of some low-productivity firms, all the surviving firms face a flatter demand curve, i.e., $\epsilon(N') < \epsilon(N)$. Hence, they find it optimal to exercise their higher product market power by raising their price markups, i.e., $\mu(N') > \mu(N)$, and reducing their output. Since the positive markup response is mediated by heightened concentration (lower N) in the product market, we call this mechanism *concentration channel* of the MW.[7] Depending on whether firms are constrained by the MW or not, the labor share and output responses are different. In particular, the responses of constrained firms are qualitatively ambiguous. On the one hand, they lose (some or all) their labor market power, showing up as a markdown reduction. On the other hand, they gain product market power, showing up as a markup increase. Instead, unconstrained firms end up having *more* market power than before, due to the increase in product market power. Therefore, both their labor

[6]For simplicity, in Figure (1) we plot a MW reform such that all the constrained firms that would scale down their employment exit the market and all firms that would (weakly) scale up their employment survive.

[7]Models of oligopsonistic competition, such as Berger et al. (2022), would predict a further "concentration channel" operating through a positive markdown response of unconstrained firms. This intuition may be formalized in reduced form by making the elasticity of labor supply, η, depend on N positively, as well. This addition would increase the magnitude of the concentration channel. However, Section 8 provides evidence that the concentration channel occurs primarily through the product market.

share and output fall with respect to the baseline equilibrium.

Overall, both the aggregate labor share and output response are qualitatively ambiguous and crucially depend on the direction of the market power response – that is, whether the reduction in the aggregate markdown outweighs the increase in the aggregate markup, or vice versa.

3 Quantitative Model

In this section, we develop a fully-fledged quantitative model to assess the equilibrium and welfare effects of the MW when product market power is endogenous and varies with market competition. To do so, we develop a novel quantitative model putting together a wage-posting model, that generalizes Engbom and Moser (2022), with the workhorse price-setting model of Atkeson and Burstein (2008). We consider segmented labor markets by worker skills and industry, characterized by search-and-matching frictions and on-the-job search. The source of labor market power is matching frictions, which capture geographical distance and incomplete information.[8] We consider a continuum of product markets, in which a finite number of firms engage in oligopolistic competition. The source of product market power is the imperfect substitutability across firms' varieties and the granularity of firms in their product market.

3.1 Environment

We study a continuous-time, stationary economy populated by a measure 1 of workers, a measure M of potential firms producing intermediate goods, and a continuum of final good producers. The economy is organized in $I \in \mathbb{N}$ industries. For each industry i, we consider a continuum of labor markets indexed by a_i. For each labor market, we consider a continuum of identical product markets, or *sectors*, indexed by $k(a_i)$.

Workers. Workers are hand-to-mouth agents with preferences exhibiting constant relative risk aversion (CRRA) over a consumption good. Workers differ in their permanent skill type a and industry i.[9] Let $\Xi(i)$ denote the share of workers operating in industry i and $\Omega_i(a)$ denote the industry-specific skill distribution. Workers own the firms and get rebated a share of aggregate profits proportional to their relative wage.

[8]Endogenous markdowns can stem either from idiosyncratic preferences for workplaces, when firms are granular in their labor market, or search-and-matching frictions (or, most likely, from both). According to Berger et al. (2023), search-and-matching frictions are the quantitatively most relevant determinant of markdowns. Unlike in frictionless models such as Berger et al. (2025), the MW induces reallocation of workers to more productive firms not only because of their markdown response, but also by alleviating congestion effects in hiring.

[9]We think of skill types as reflecting both observable and unobservable invariant heterogeneity across workers. We model skills as industry-specific to reflect educational choices prior to labor market entry.

Final good producers. Final good producers operate a Cobb-Douglas (CD) production function that aggregates up industry goods into a homogeneous consumption good. In turn, each industry good is a double-nested aggregator with constant elasticity of substitution (CES) over firm-level varieties. Let ρ denote the elasticity of substitution across sectoral goods, and σ denote the elasticity of substitution across firm-level varieties within sectors. We assume that $\rho > 1$ and $\sigma > \rho$, meaning that goods are more substitutable within than across sectors. The final good market is perfectly competitive.

Intermediate firms. Intermediate firms differ in their physical productivity z, skill requirement $\hat{a}$, and industry i. We model a firm's skill requirement, $\hat{a}$, as the (only) worker skill type that fits its production process.[10] All firms operate a linear technology in labor. Following Engbom and Moser (2022), labor productivity is a composite of worker skill and firm physical productivity, combining in a multiplicative fashion. Firms maximize profits by posting vacancies v and wage piece rates w per skill unit in the labor market corresponding to their skill requirement. Henceforth, we will refer to intermediate firms simply as *firms*.

Labor market structure. Labor markets are segmented by worker skill a and industry i. Both unemployed and employed workers randomly search for vacancies, each offering some wage piece rate w per skill unit.[11] As in Burdett and Mortensen (1998), offered wage piece rates follows a continuous wage *offer* distribution $F_{a_i}(w)$ – an equilibrium object that is determined by firms' optimization. Let $G_{a_i}(w)$ be the resulting employment wage distribution, i.e., the cumulative density of employed workers who earn piece rates equal or lower than w. On-the-job search restrains firms' labor market power: the harder employed workers search for job offers, the more compressed the wage distribution is around the competitive wage (i.e., the marginal revenue product of labor). Each labor market a_i is characterized by search-and-matching frictions, namely an exogenous separation rate δ_{a_i}, an endogenous job finding rate per unit of search effort λ_{a_i}, and an exogenous on-the-job search effort s_{a_i} – with the search effort of unemployed workers normalized to one. Aggregate search effort is equal to $S_{a_i} = u_{a_i} + s_{a_i}(1 - u_{a_i})$, where u_{a_i} denotes the mass of unemployed workers. The total number of meetings occurring at any point in time is regulated by a constant return to scale (CRS) matching function $\mathcal{M}(S_{a_i}, V_{a_i})$, where V_{a_i} is the mass of outstanding vacancies. Define labor market tightness as $\theta_{a_i} = V_{a_i}/S_{a_i}$. By virtue of CRS, the job finding rate per unit of search effort equals $\lambda_{a_i} = \lambda(\theta_{a_i}) = \mathcal{M}(1, \theta_{a_i})$.

Product market structure. Each (intermediate) product market $k(a_i)$ is populated by a finite number of potential firms $\bar{N}_{k(a_i)}$ with the same industry i and skill requirement $\hat{a}$. Firms operating in the same product market source workers from one labor market but sell their

[10] Formally, the effective productivity of a firm with physical productivity z and skill requirement $\hat{a}$ hiring workers of skill a is given by $z_{\hat{a}} = z\, \mathbb{1}\{a = \hat{a}\}$.

[11] We model the search process as random, instead of directed, for equilibrium markdowns to be inefficient. Hence, as in our stylized model, a higher MW can increase value added. On the contrary, markdowns are always efficient if search is directed (absent other distortions).

good to all consumers (via final good producers). In equilibrium, a subset $N_{k(a_i)} \subseteq \bar{N}_{k(a_i)}$ of potential firms is operating. Let $\Gamma_{k(a_i)}(z)$ denote the realized productivity distribution of potential firms. To ensure equilibrium uniqueness, we assume that all the product markets sourcing from the same labor market are identical.[12] Given that $N_{k(a_i)} < \infty$, firms are granular in their product market, i.e., they account for a positive share of sectoral production, and engage in oligopolistic competition á la Cournot.

3.2 Worker's problem

Let U_{a_i} and $W_{a_i}(w)$ be the value of unemployment and employment at piece rate w for a worker of skill type a in industry i. These values are defined recursively by the following Hamilton-Jacobi-Bellman (HJB) equations:

$$rU_{a_i} = \frac{(ab_{a_i})^{1-\vartheta}}{1-\vartheta} + \lambda(\theta_{a_i}) \int_{\underline{w}_{a_i}}^{\bar{w}_{a_i}} \{W_{a_i}(w) - U_{a_i}\}^+ \, dF_{a_i}(w), \tag{2}$$

$$rW_{a_i}(w) = \frac{(e_a(w))^{1-\vartheta}}{1-\vartheta} + s_{a_i}\lambda(\theta_{a_i}) \int \{W_{a_i}(w') - W_{a_i}(w)\}^+ \, dF_{a_i}(w') + \delta_{a_i}\left(U_{a_i} - W_{a_i}(w)\right), \tag{3}$$

where r is the instantaneous interest rate, ϑ is the CRRA coefficient, $e_a(w) = aw + \Pi(aw) + T$ represents the employed worker's earnings, and $\{x\}^+ \equiv \max\{x, 0\}$.

An unemployed worker of skill type a in industry i receives ab_{a_i} units of consumption good as unemployment benefits and finds a job offer at rate $\lambda(\theta_{a_i})$, that she accepts if it provides a higher value than unemployment. An employed worker of skill type a earns labor earnings aw, and receives transfers T and her share of aggregate profits $\Pi(aw)$. She finds another job opportunity at rate $s_{a_i}\lambda(\theta_{a_i})$, which she accepts if it pays better than the one she currently holds. Finally, her job is destroyed at rate δ_{a_i}, in which case she transitions into unemployment. Unemployment benefits and lump-sum transfers are financed by a proportional profit tax with rate $\tau \in [0,1]$. Formally, $T(1-u) + \sum_i \int u_{a_i} ab_{a_i} d\Omega_i(a)\Xi(i) = \tau\bar{\Pi}$, where $\bar{\Pi}$ denotes aggregate profits. As in Kaplan et al. (2018), employed workers are further rebated a share $\mathcal{S}(G(aw)) \in [0,1]$ of after-tax aggregate profits proportional to their relative wage, where $G(aw)$ is the economy-wide wage distribution. Formally, $\Pi(aw) = \mathcal{S}(G(aw))(1-\tau)\bar{\Pi}$. We interpret these profits as the profit-sharing component of worker compensation.

[12]Formally, they share the same number of potential firms, $\bar{N}_{k(a_i)}$, and the same (realized) productivity distribution of potential firms, $\Gamma_{k(a_i)}(z)$.

3.3 Final good producer's problem

Final good producers solve the following profit maximization problem:

$$\max_{\{y_{jk_i}\}} PY - \sum_{i=1}^{I} \int \int_0^{K_{a_i}} \sum_{j=1}^{N_{k(a_i)}} p_{jk(a_i)} y_{jk(a_i)} \, dk(a_i) \, d\Omega_i(a), \tag{4}$$

$$\text{s.t.} \quad Y = \prod_{i=1}^{I} Y_i^{\alpha_i}, \tag{5}$$

$$Y_i = \left(\int \int_0^{K_{a_i}} Y_{k(a_i)}^{\frac{\rho-1}{\rho}} \, dk(a_i) \, d\Omega_i(a) \right)^{\frac{\rho}{\rho-1}}, \tag{6}$$

$$Y_{k(a_i)} = \left(\sum_{j=1}^{N_{k(a_i)}} y_{jk(a_i)}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \tag{7}$$

The final good is a CD aggregator of industry goods Y_i, with CD weights $\alpha_i \in [0,1], \sum_{i=1}^{I} \alpha_i = 1$. As apparent from the specification of the CES aggregators (6)-(7), we assume that, for each industry i and skill requirement a, there is a measure K_{a_i} of identical product markets (sectors), indexed by $k(a_i)$, within which a finite number $N_{k(a_i)}$ of firms compete.[13]

3.4 Firm's problem

Following the equilibrium search literature, we reduce firms' dynamic problem into a static profit maximization problem by assuming steady-state behavior (Burdett and Mortensen, 1998; Bontemps et al., 1999; Engbom and Moser, 2022).[14] A firm with productivity z in industry i with skill requirement a solves the following profit maximization problem:

$$\pi_{a_i}(z) = \max_{w \geq \underline{\mathbf{w}}/a, v} \; a \left[p_{k(a_i)}(\ell) z - w \right] \ell_{a_i}(w,v) - a c_{a_i}(w,v) - a \kappa_{a_i} \tag{8}$$

$$\text{s.t.} \quad \ell_{a_i}(w,v) = \frac{v}{V_{a_i}} \frac{\lambda(\theta_{a_i})[u_{a_i} + s_{a_i}(1-u_{a_i}) G_{a_i}(w)]}{\delta_{a_i} + s_{a_i} \lambda(\theta_{a_i})[1 - F_{a_i}(w)]}, \tag{9}$$

$$p_{k(a_i)}(\ell) = (az\ell)^{-\frac{1}{\sigma}} Y_{k(a_i)}(\ell)^{\frac{1}{\sigma} - \frac{1}{\rho}} Y_i^{\frac{1}{\rho} - 1} \alpha_i Y, \tag{10}$$

$$c_{a_i}(w,v) = \bar{c}_{a_i} h_{a_i}(w)^{\phi} \frac{v^{1+\zeta}}{1+\zeta}, \tag{11}$$

where $h_{a_i}(w) \equiv \delta_{a_i} + s_{a_i} \lambda(\theta_{a_i})[1 - F_{a_i}(w)]$ is the hiring rate, i.e., the ratio between new hires and stationary employment.

Firms maximize stationary profits by leveraging two control variables, namely a wage

[13] The measure K_{a_i} of product markets is pinned down so as to replicate a measure M_{a_i} of firms for each (industry -skill requirement) pair.

[14] In ONLINE Appendix, we develop the dynamically consistent version of the firm's problem and show that the two problem setups yield the same results in the *timeless* limit, i.e., when $r \to 0$.

piece rate w – subject to the MW constraint – and a mass of vacancies v. Profits equal revenues net of labor costs, $aw\ell_{a_i}(w,v)$, hiring costs, $ac_{a_i}(w,v)$, and overhead costs, $a\kappa_{a_i}$. Firms face both an upward-sloping labor supply curve and a downward-sloping product demand curve. The profit maximization problem shares many similarities with that analyzed in the stylized model, as well as three key differences. First, following Engbom and Moser (2022), all the relevant variables scale with worker skill a. This is the simplest way to generate wage dispersion due to worker and firm heterogeneity separately. Second, firms face search frictions in hiring. Hence, to grow larger in size, they need to post costly vacancies v subject to the hiring cost function (11). Third, and most importantly, the elasticities of labor supply and product demand are no longer parametric.

The labor supply curve (9) is determined by the stationary employment of a firm posting a wage piece rate w and vacancies v. Stationary employment balances workers' inflows to and outflows from the firm. Workers' inflows equal the probability that firm's vacancies get sampled by searching workers, v/V_{a_i}, times the measure of searching workers who meet a vacancy and would accept the firm's wage offers, $\lambda(\theta_{a_i})[u_{a_i} + s_{a_i}(1-u_{a_i})G_{a_i}(w)]$, i.e., all unemployed workers and employed workers at lower-paying firms. Workers' outflows equal the quit rate, $\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})[1-F_{a_i}(w)]$, i.e., the probability that an employee separates from the firm (be it due to quits or poaching from higher-paying firms), times the stationary employment, $\ell_{a_i}(w,v)$. Crucially, the firm can actively influence its stationary employment through the wage (and vacancy) policy: the higher the posted wage, the higher the filling rate and the lower the quit rate, that is, the higher the stationary employment.

The product demand curve (10) is determined by the demand function of final good producers. Since firms are granular in their product market $k(a_i)$, they internalize not only the direct effect of their employment on their price, with elasticity $-\frac{1}{\sigma}$, but also the indirect effect operating through the sectoral output, with elasticity $\frac{1}{\sigma} - \frac{1}{\rho}$.

The hiring cost (piece rate) function (11) is increasing in the hiring rate $h_{a_i}(w)$, with elasticity ϕ, and increasing and convex in vacancies, with elasticity $1+\zeta$. Intuitively, hiring costs comprise training costs and vacancy posting costs. Training costs are captured by the hiring rate component: the higher the share of new hires relative to employment, the more costly their training is – as documented in the literature (Merz and Yashiv, 2007; Mongey and Violante, 2019). Vacancy posting costs are convex for a wage posting equilibrium with endogenous job creation to be well-defined.[15]

Limit cases. The firm's problem (8) nests two popular models as limit cases. On the one hand, it nests the wage-posting model of Engbom and Moser (2022) for $p_{k(a_i)} = 1$, $\phi = 0$ and $\kappa_{a_i} = 0$, i.e., if firms have no product market power, hiring costs are independent of the hiring rate, and there are no overhead costs.[16] On the other hand, it nests the price-setting model

[15] If vacancy posting costs were linear, firms would not find it optimal to use their wage policy for hiring.

[16] In turn, by further letting $v = \bar{v}$, i.e., assuming exogenous job creation, the problem setup boils down to traditional wage posting-models with heterogeneous productivity (Burdett and Mortensen, 1998; Bontemps et al., 1999).

of Atkeson and Burstein (2008) for $\bar{c}_{a_i} = 0$ (or, interchangeably, for an infinitely efficient matching function, i.e., $\mathcal{M}(S_{a_i}, V_{a_i}) \to \infty$, $\forall a_i$) and $\kappa_{a_i} = 0$, i.e., if there are no search frictions and overhead costs.

3.5 Equilibrium characterization

Since all the product markets sourcing from the same labor market are identical, in each labor market there are finite productivity types. Even if the productivity distributions are not continuous, we now define and prove existence of a stationary equilibrium with continuous wage offer distributions. Such an equilibrium is characterized by residual dispersion of wages and output, i.e., firms with the same productivity pay different wages and produce different quantities.

Definition (Stationary equilibrium)
For given number of potential firms $\bar{N}_{k(a_i)}$ per product market $k(a_i)$, a stationary equilibrium of the economy consists of:

- *Reservation wages $\{R^u_{a_i}, R^e_{a_i}(w)\}, \forall a_i$, for both unemployed and employed workers, that maximize the value of unemployment (2) and employment (3);*
- *Output demand functions $y_{k(a_i)}(p_{k(a_i)}), \forall a_i$, that solve the final good producers' problem (4);*
- *Continuous residual wage offer distributions $F_{a_i}(w;z), \forall a_i$, such that firms with the same productivity z make the same profits, and associated continuous marginal revenue product distributions $\Phi_{a_i}(\tilde{z};z), \forall a_i$, such that $F_{a_i}(w;z)$ is consistent with firms' optimization;*
- *Wage and vacancy policies $w_{a_i}(\tilde{z}), v_{a_i}(\tilde{z})$, $\forall \tilde{z} \in [\underline{\tilde{z}}_{a_i}(z), \bar{\tilde{z}}_{a_i}(z)), \forall z, \forall a_i$, that solve the firms' problem (8) for given residual marginal revenue product distributions;*
- *Productivity thresholds $\left\{\underline{z}_{k(a_i)}\right\}$ that determine the marginally active firm in each product market $k(a_i)$, $\forall a_i$;*
- *Measures $\{F_{a_i}(w), G_{a_i}(w), \Phi_{a_i}(\tilde{z}), u_{a_i}, V_{a_i}\}$ and market tightness $\theta_{a_i}, \forall a_i$, that are consistent with firms' vacancy policies, balance of flows, and the meeting technology;*
- *Intermediate product market clearing conditions ensuring that supply and demand for each firm-level variety coincide: $y_{k(a_i)}\left(p_{k(a_i)}(\tilde{z},z)\right) = az\ell_{a_i}(\tilde{z}) \quad \forall \tilde{z} \in [\underline{\tilde{z}}_{a_i}(z), \bar{\tilde{z}}_{a_i}(z)), \forall z, \forall a_i$;*
- *Final product market clearing condition ensuring that the aggregate resource constraint holds, that is, aggregate consumption equals aggregate value added:*[17]

$$C = Y - \sum_{i=1}^{I} \int_a \int_{\tilde{z}} M_{a_i}\, ac_{a_i}(\tilde{z})\, d\Phi_{a_i}(\tilde{z})\, d\Omega_i(a) - \sum_{i=1}^{I} \int_a M_{a_i}\, a\kappa_{a_i}\, d\Omega_i(a),$$

[17] We assume that all the potential firms pay the overhead cost and operate only if they can (weakly) break even, e.g., because of limited liability. This implies that the expansionary effects of the MW on aggregate consumption due to firm selection are entirely driven by worker reallocation – not by the opportunity cost of firm entry.

where $C = \sum_{i=1}^{I} \int_a \left[\int_{\underline{w}_{a_i}(\tilde{z})}^{\bar{w}_{a_i}(\tilde{z})} a w_{a_i}(\tilde{z}) \ell_{a_i}(\tilde{z}) \, d\Phi_{a_i}(\tilde{z}) + M_{a_i} \sum_{j=1}^{\mathcal{Z}_{a_i}} \pi_{k(a_i)}(z_j) \right] d\Omega_i(a)$ *denotes aggregate consumption, and* Y *is the CD aggregator defined in (5).*

Existence of a stationary equilibrium requires two mild parametric assumptions. First, the elasticity of substitution across sectors (ρ) is assumed to be high enough such that the revenue function is supermodular in size and productivity. Second, the unemployment benefits (b_{a_i}) are assumed to be low enough such that profits of the lowest-ranked firm in each labor market – which pays the reservation wage – are positive and increasing in the rank of the wage offer distribution ($F_{a_i}(w)$).

Proposition 1 (Existence and uniqueness of stationary equilibrium with exit ordering based on losses)
There exists a symmetric stationary equilibrium with continuous wage offer distributions and exit ordering based on losses. The equilibrium is unique within the set of equilibria with nondegenerate wage distributions.

We now inspect the policy functions of the agents in our economy one at a time.

Worker's policy. Worker's problem is solved by setting a state-specific reservation wage, $R^s_{a_i}$, $s = \{u, e\}$, such that all offers equal or higher than that are accepted. The reservation wage solves $W_{a_i}(R^u_{a_i}) = U_{a_i}$ for unemployed and is equal to the current wage for employed, i.e., $R^e_{a_i}(w) = w$.

Final good producer's policy. Solving the profits maximization problem (4) yields the following demand function for each firm's variety j in product market $k(a_i)$:

$$y_{jk(a_i)} = \left(\frac{p_{jk(a_i)}}{P_{k(a_i)}} \right)^{-\sigma} \left(\frac{P_{k(a_i)}}{P_i} \right)^{-\rho} \left(\frac{P_i}{P} \right)^{-1} \alpha_i Y \quad \forall j = 1, \dots, N_{k(a_i)}, \forall a_i. \tag{12}$$

Reflecting the triple-nested production function (5), the demand for the good produced by firm j in sector $k(a_i)$ negatively depends on its price vis-à-vis the price index of the sector, on the price index of the sector vis-à-vis that of the industry, and on the price index of the industry vis-à-vis the aggregate price index. Henceforth, we use the consumption good as numeraire and normalize its price to 1, i.e., $P = 1$. Notice that the CD aggregator of industry-specific output prevents any reallocation of demand across industries following an asymmetric shock, such as the introduction of a MW.

Firm's policy. The solution to the firm's profit maximization problem (8) consists of the following system of first-order conditions (FOCs):

$$\ell_{a_i}(w,v) = \left[\left(1+\epsilon^{-1}_{k(a_i)}(\ell)\right) p_{k(a_i)}(\ell)\, z - w\right] \frac{\partial \ell_{a_i}(w,v)}{\partial w} - \frac{\partial c_{a_i}(w,v)}{\partial w}, \tag{13}$$

$$\frac{\partial c_{a_i}(w,v)}{\partial v} = \left[\left(1+\epsilon^{-1}_{k(a_i)}(\ell)\right) p_{k(a_i)}(\ell)\, z - w\right] \frac{\partial \ell_{a_i}(w,v)}{\partial v}, \tag{14}$$

where $\epsilon_{k(a_i)}(\ell)$ denotes the endogenous elasticity of demand.[18] The FOC with respect to the wage (13) equalizes the marginal increase in the wage bill (left-hand side) to the marginal increase in profits by raising employment through a wage rise (right-hand side). In turn, the latter is composed by the profit margin earned on new employees net of the induced price response ($\epsilon^{-1}_{k(a_i)}(\ell) < 0$), and by the marginal hiring cost saving from reducing worker turnover through a higher wage ($\partial c_{a_i}(w,v)/\partial w < 0$). The FOC with respect to vacancies (14) equalizes the marginal increase in the hiring cost (left-hand side) to the marginal increase in profits by raising employment through an increase in vacancies (right-hand side). Importantly, firms internalize that growing larger in size, i.e., selling larger quantities, entails cutting their price. Hence, product market power restrains optimal employment.

After some manipulation of the system of first-order conditions, we recover the familiar double-wedge representation of the optimality condition between price and (productivity-adjusted) wage:

$$p_{a_i}(z) = \underbrace{\frac{\epsilon_{a_i}(z)}{\epsilon_{a_i}(z)-1}}_{\mu_{a_i}(z)} \underbrace{\frac{1+\nu\eta_{a_i}(z)}{\nu\eta_{a_i}(z)}}_{\psi_{a_i}(z)} \frac{w_{a_i}(z)}{z}, \tag{15}$$

where η_{a_i} denotes the endogenous elasticity of labor supply. [19] Equation (15) shares the same structure as its counterpart in the stylized model (1), but features two key differences. First, both the elasticity of demand, ϵ, and the elasticity of labor supply, η, are endogenous. This means that the degree of market power is differentiated both across markets and across firms within a given market. Second, the elasticity of labor supply is multiplied by the hiring cost correction term $\nu \equiv 1 + \frac{\phi}{2(1+\zeta)}$, which depends on the ratio between the elasticity of hiring costs with respect to the hiring rate, ϕ, and with respect to vacancies, $1+\zeta$. Intuitively, this term captures the net reduction in hiring costs by growing larger in employment, so it is absent from any model where hiring costs are size-independent.

Finally, let $\tilde{z}_{a_i}(z) \equiv p_{a_i}(z)z/\mu_{a_i}(z)$ denote the equilibrium MRPL(s) of a firm with productivity z operating in labor market a_i. In Appendix (B.7), we show that the wage policy

[18] Notice that (13) holds only where the wage offer distribution is differentiable, i.e., $suppF_{a_i}(w) \backslash \left\{ \{\underline{w}_{a_i}(z_n)\}_{n=1}^{\mathcal{Z}_{a_i}} \cup \bar{w}_{a_i}(z_{\mathcal{Z}_{a_i}}) \right\}$.

[19] Notice that all the policy functions with respect to productivity z are set-valued due to residual wage dispersion. In other words, firms with same productivity pay different wages in equilibrium, so the wage policy function is set-valued. The price, markup, and markdown policy functions are one-to-one for given productivity *and* wage.

function equals the expected outside option of a searching worker:

$$w_{a_i}(\tilde{z}) = (1 - \mathcal{P}_{a_i}(\tilde{z}))\, R^u_{a_i} + \mathcal{P}_{a_i}(\tilde{z})\mathbb{E}_{\max \mathrm{MRPL}|\mathrm{MRPL}<\tilde{z}, m_{a_i}(\tilde{z})=1}\left[\mathrm{MRPL}\right], \tag{16}$$

where $\mathcal{P}_{a_i}(\tilde{z}) \equiv \Pr\left(m_{a_i}(\tilde{z}) = 1\right)$ and $m_{a_i}(\tilde{z}) \equiv \mathbb{1}\{\text{meet at least one firm w/ MRPL } \leq \tilde{z}\}$ is an indicator of alternative job offers. Equation (16) is best understood in terms of static allocation of workers to firms. Accordingly, wage offers sampled by employed workers over the course of their tenure at a given firm are interpreted as the (appropriately discounted) pool of alternative job offers from which a worker can statically choose. With probability $1 - \mathcal{P}_{a_i}(\tilde{z})$, the worker does not meet any firm with MRPL lower than $\tilde{z}$, in which case the worker's outside option when meeting a firm with MRPL $\tilde{z}$ boils down to her reservation wage. With complementary probability, the worker meets at least one firm with lower MRPL than $\tilde{z}$, in which case the worker's outside option when meeting a firm with MRPL $\tilde{z}$ equals the maximum wage the best of such firms can afford paying, that is, the highest expected MRPL among firms with lower MRPL than $\tilde{z}$.

3.6 Inspecting Markups and Markdowns

We now turn to our main objects of interest, that is, the equilibrium markups and markdowns. In particular, our focus will be on understanding how they would react to the introduction of a MW.

Equilibrium Markups. Let $\mathcal{C}_{k(a_i)}(z) \equiv \psi_{a_i}(z) w_{a_i}(z)$ denotes the equilibrium MCL(s) of a firm with productivity z operating in product market $k(a_i)$. The double-nested CES demand structure within each industry, as well as the granularity of firms in their own product market, entail that the elasticity of demand faced by a firm with productivity z and marginal cost $\mathcal{C}$ operating in sectoral market $k(a_i)$ reads:

$$\epsilon_{k(a_i)}(\mathcal{C}, z) = \left[-\frac{1}{\rho} sh_{k(a_i)}(\mathcal{C}, z) - \frac{1}{\sigma}(1 - sh_{k(a_i)}(\mathcal{C}, z))\right]^{-1}, \tag{17}$$

where $sh_{k(a_i)}(\mathcal{C}, z) \equiv p_{k(a_i)}(\mathcal{C}, z) y_{k(a_i)}(\mathcal{C}, z) / \sum_{j=1}^{N_{k(a_i)}} p_{jk(a_i)} y_{jk(a_i)}$ is the sectoral market share of the firm. Hence, the firm-level elasticity of demand depends on how large the firm is relative to its product market in terms of revenues. Formally, it equals a harmonic average between the elasticity of demand across sectors, $-1/\rho$, and the elasticity of demand within sectors, $-1/\sigma$, weighted by the market share and its complement to one, respectively (Atkeson and Burstein, 2008). Equation (17) provides a microfoundation to the positive dependence of the elasticity of demand on the number of active firms assumed in the stylized model in Section (2): for given productivity distribution, the larger the number of firms, the lower the firm's market share and the higher the elasticity of demand (in magnitude).

From Equation (17) and the markup definition (15), the equilibrium markup $\mu_{k(a_i)}(\mathcal{C}, z)$ of a firm with productivity z and marginal cost $\mathcal{C}$ operating in product market a_i is given by:

$$\mu_{k(a_i)}(\mathcal{C}, z) = \frac{\sigma}{(\sigma - 1)\left[1 - \frac{\sigma/\rho - 1}{\sigma - 1} sh_{k(a_i)}(\mathcal{C}, z)\right]}. \tag{18}$$

The equilibrium markup is increasing in the firm's market share, as well as moving between two intuitive bounds. When the firm is small relative to its product market, i.e., $sh_{k(a_i)}(\mathcal{C}, z) \to 0$, the markup approaches $\frac{\sigma}{\sigma-1}$, as if the firm were operating in monopolistic competition within the sector. On the contrary, when the firm is large relative to its sector, i.e., $sh_{k(a_i)}(\mathcal{C}, z) \to 1$, then the markup approaches $\frac{\rho}{\rho-1}$, as if the firm were operating in monopolistic competition against other sectors.

A higher MW is expected to induce some exit (or layoffs) among low-productivity firms. In the presence of labor market frictions, displaced workers from low-productivity firms will (imperfectly) reallocate towards higher-productivity and larger firms, which have higher baseline markups. Moreover, the reallocation process will make large firms gain additional market shares, leading to higher markups through the concentration channel. Overall, the markup distribution is expected to shift rightward following the introduction of a MW, due to such reinforcing compositional and behavioral effects.

Equilibrium Markdowns. Let $\tilde{z}_{a_i}(z) \equiv p_{a_i}(z)z/\mu_{a_i}(z)$ denote the equilibrium MRPL(s) of a firm with productivity z operating in labor market a_i. Workers' search behavior and the labor market structure jointly entail that the labor supply elasticity faced by a firm with marginal revenue product $\tilde{z}$ operating in labor market a_i reads:

$$\eta_{a_i}(\tilde{z}) = \frac{2 f_{a_i}(w_{a_i}(\tilde{z})) w_{a_i}(\tilde{z})}{\chi_{a_i}(\theta_{a_i}) + [1 - F_{a_i}(w_{a_i}(\tilde{z}))]} = \frac{w_{a_i}(\tilde{z}) \ell_{a_i}(\tilde{z})}{(1+\zeta) c_{a_i}(\tilde{z})},, \tag{19}$$

where the two alternative formulations follow from Equations (13) and (14), respectively.

The first formulation focuses on the role of on-the-job search and the shape of the wage offer distribution. The elasticity of labor supply equals twice the quit rate elasticity.[20] The quit rate elasticity depends on a common component across all firms operating in a labor market and a firm-specific component. The common component is represented by the frictional index $\chi_{a_i}(\theta_{a_i}) \equiv \frac{\delta_{a_i}}{s_{a_i}\lambda(\theta_{a_i})}$, which is an inverse measure of the speed at which employed workers climb the job ladder. The larger the frictional index, the lower the quit rate elasticity, as the competitive pressure exerted by on-the-job search is dampened. The firm-specific component makes the quit rate elasticity positively depend on the firm's wage policy. For given rank in the wage distribution and posted wage, the quit rate elasticity is increasing in the extent of *local* competition the firm is facing, as summarized by the wage offer density. Intuitively, the higher the density of the wage distribution, the larger the pool of competitors that could poach workers from the firm if it were to marginally reduce its offered wage. The second formulation

[20] Since the quit rate elasticity equals the hiring rate elasticity in stationary equilibrium, the elasticity of labor supply amounts to twice the quit rate elasticity (Manning, 2003).

zooms into the role of hiring costs in the elasticity of labor supply. As apparent, the elasticity of labor supply is decreasing in hiring costs. If hiring costs tend to zero, then $\eta_{a_i}(\tilde{z})$ approaches infinity, i.e., the perfect competition benchmark. This is because in our model hiring costs are the source of search frictions, which in turn are the structural determinant of the upward-sloping labor supply curve faced by firms. Equation (19) endogenizes the elasticity of labor supply taken as exogenous in the stylized model in Section (2): depending on the specific extent of hiring frictions, firms face different elasticities of labor supply.

From Equation (19) and the markdown definition (15), the equilibrium markdown, $\psi_{a_i}(\tilde{z})$, of a firm with marginal revenue product $\tilde{z}$ operating in labor market a_i is given by:

$$\psi_{a_i}(\tilde{z}) = 1 + \frac{\chi_{a_i}(\theta_{a_i}) + [1 - F_{a_i}(w_{a_i}(\tilde{z}))]}{2\nu f_{a_i}(w_{a_i}(\tilde{z}))\, w_{a_i}(\tilde{z})} = 1 + \frac{(1+\zeta)c_{a_i}(\tilde{z})}{w_{a_i}(\tilde{z})\ell_{a_i}(\tilde{z})}. \tag{20}$$

Owing to the two alternative formulations of the equilibrium elasticity of labor supply, the equilibrium markdown itself can be expressed in two different ways. The first formulation shows that equilibrium *net* markdowns, i.e., $\psi_{a_i}(\tilde{z}) - 1$, equal the inverse of twice the quit rate elasticity normalized by the hiring cost correction term $\nu > 1$. The more sensitive hiring costs are to the hiring rate, the lower equilibrium markdowns are. Hence, according to our model, standard markdown estimates based on doubling the quit rate elasticity are generally upward biased. The second formulation shows that the equilibrium markdown is increasing in the ratio between hiring costs and wage bill, reflecting the key role of search frictions.

As in our stylized model, a higher MW would reduce the markdowns of constrained firms by raising their (effective) reservation wage. As in Burdett and Mortensen (1998), some unconstrained firms will find it optimal to compress their markdowns, as well, to preserve their rank in the equilibrium wage offer distribution.[21] Hence, these behavioral effects will push the markdown distribution to shift leftward. Depending on the convexity of the hiring cost function, excess hiring at higher-paying firms will partially offset job losses at low-paying firms, though aggregate vacancies are expected to drop and the frictional index to rise. Moreover, depending on the shape of the baseline markdown distribution, worker reallocation may bring about a further compositional effect, presumably raising the aggregate markdown. Overall, the response of the markdown distribution to the MW is a priori ambiguous.

3.7 Efficiency Properties of Baseline Equilibrium

We conclude this section by commenting on the efficiency properties of the baseline equilibrium and how they are expected to react to the introduction of a MW. The interested reader can find all the details and formal derivations underlying our statements in the companion ONLINE APPENDIX.

As in stationary equilibrium, a constrained social planner would reallocate workers

[21] This spillover effect can be interpreted as a local increase in the wage offer density through the lens of Equation (20).

towards higher-productivity firms whenever an opportunity arises. However, the social planner would dictate a common hiring rate across firms with the same productivity. Hence, residual dispersion in marginal products is inefficient, as in Menzio (2024).

To study the role of the MW, let $\Delta_{a_i}(z)$ denote the *labor wedge* at the firm level, i.e., the ratio between social marginal benefit and marginal cost of vacancy posting (Hsieh and Klenow, 2009). In equilibrium, $\Delta_{a_i}(z) = \mu_{a_i}(z) \Big/ \left(1 + \frac{D_{a_i}(z) R^u_{a_i} - E^c_{a_i}}{MCL_{a_i}(z)}\right)$, where $MCL_{a_i}(z) \equiv \psi_{a_i}(z) w_{a_i}(z)$ is the equilibrium marginal cost, $E^c_{a_i} \lesseqgtr 0$ equals the congestion effects, and $D_{a_i}(z) > 0, D'_{a_i}(z) \geq 0$, is some multiplier. The MW influences the labor wedge by increasing the reservation wage $R^u_{a_i}$.

On top of residual dispersion in marginal products, the stationary equilibrium features two sources of inefficiency. First, the existence of positive markup rates (due to imperfect substitutability across firm-level varieties) and distorted markdowns (due to congestion effects and distorted reservation wages) make the average labor wedge generally differ from one. If the MW pushes the average labor wedge closer (farther) to one, this inefficiency will ameliorate (worsen) by showing up as an increase (decrease) in aggregate vacancies and employment. Second, heterogeneity in markups (due to firms' granularity in their product market) and markdown distortions (due to firm-specific impact of congestion effects and reservation wages) across firms entails that the allocation of labor is inefficient, because it does not minimize aggregate hiring costs for given final output. Intuitively, misallocation arises from the fact that the labor wedge is heterogeneous across firms. If the MW decreases (increases) the productivity-adjusted dispersion of the firm-level labor wedge, this inefficiency will ameliorate (worsen) by showing up as a decrease (increase) in aggregate hiring costs per unit of final good, which we will adopt as our misallocation index.

4 Structural Estimation

In this section we outline how we estimate the parameters of our structural model using data from the Italian National Institute for Social Security (INPS) and the Italian Institute of Statistics (Istat). First, we discuss how to link the model-based notions of labor and product markets into their (closest) real-world counterparts. Then, we present our estimation strategy and results.

Market definition. Estimating firms' market power involves taking a stance on the definition of labor and product markets. This is a challenging task as industrial organization and geographical scope vary significantly across markets (Rossi-Hansberg et al., 2021; Eeckhout, 2020). Aware of the inevitable simplifications a macroeconomic perspective entails, we propose market definitions based on workers' heterogeneity for labor markets and goods differentiation for product markets.

First, we notice that model-based labor markets are segmented by workers' skills and industry, as well as populated by a measure of firms – none of them being large relative to the market. Moreover, matching frictions within each labor market capture the average effect of geographical distance and incomplete information on the job finding rate.[22] Therefore, we run an Abowd-Kramarz-Margolis (AKM) regression on our matched employer-employee data from INPS spanning the longest time period available to us (1990-2018) and compute worker fixed effects (Abowd et al., 1999). Conceptually, AKM worker fixed effects capture permanent heterogeneity in skills that reflects in wage differences, thus representing an intuitive empirical counterpart of the skill type a in the model. We proceed by splitting workers into two skill types (high- and low-skilled) based on the median AKM worker fixed effect. Moreover, we assign each worker to an industry based on the ATECO 1-digit industry code of the firm in which he or she has been most frequently employed between 2016 and 2018.[23] Hence, we define labor markets as segmented by AKM worker fixed effects and 1-digit industry. The idea is that individual ability and educational choices prior to labor market entry constitute hard constraints to one's labor market, while matching frictions prevents workers from receiving job offers from firms located in distant places. It follows that the continuum of labor markets in the model maps into 16 segmented labor markets in the data. Average wages are increasing in our skill measure conditional on each industry, and exhibit substantial heterogeneity across industries (see Figure (A.2a)). Hence, our labor market definition seems to capture relevant structural differences.

Second, we observe that model-based product markets are populated by a finite number of firms that compete oligopolistically. Conceptually, they should be mapped to relatively narrowly defined products. Following the literature (Autor et al., 2020), we therefore think of product market boundaries in the model as comparable to the ATECO 4-digit industry codes – the most granular level we have access to.[24] It follows that the average number of firms within a 4-digit sector is $2,963$ – a large number in international comparison, explained by the disproportionate incidence of small firms in the Italian economy. Moreover, the average number of firms per sector masks substantial heterogeneity across 1-digit industries (see Figure (A.2b)) – an important element we will take care of in our structural estimation.

Estimation Strategy. We estimate the model by replicating a stationary equilibrium for the Italian economy, targeting empirical moments from the period 2016-2019. First, we externally set structural parameters with direct counterparts in the data. Then, we jointly estimate the remaining parameters, which are related to some informative data moments by

[22] If each labor market were composed by a collection of identical local labor markets with granular firms, matching frictions would capture (in reduced-form) the extent of geographical mismatch between job seekers and vacancies. We see the investigation of spatial heterogeneity as a promising avenue for future research, but beyond the scope of the paper.

[23] ATECO industry codes complies with the European NACE industry classification standards. Some industries in which the number of observations is too small are merged with the closest industry with enough observations. Specifically, "mining" is merged with "manufacturing", "other services" with "services to firms", "real estate" with "professional services", and "energy" and "water and disposal" with "construction". Results are unchanged if we simply drop such observations.

[24] Our product market definition has two main limitations. First, the geographical scope of product markets can vary significantly across industries (tradable vs nontradable). Second, multi-product firms operate in multiple product markets, as well, so it is unclear what market share is relevant for pricing purposes.

some equilibrium conditions, using the Simulated Method of Moments (SMM). An original contribution of our strategy is the non-parametric estimate of firms' productivity from the wage distribution in the presence of both endogenous markups and markdowns.

Externally set parameters. We externally set the parameters governing market structure, workers' preferences, labor market frictions, and profits rebates. We pin down the relative size of worker population across labor markets by replicating the employment composition by industry and skill from INPS data (2016-2018). We consider $K = 567$ product markets, which equals the number of 4-digit sectors we observe in our Istat data. We account for the heterogeneous market structure across industries by assigning a number of potential (and operating) firms to a product market $k(a_i)$ equal to the average in the respective industry i, $\bar{N}_{k(a_i)} = \bar{N}_i$. We pick the separation rates δ_{a_i} to replicate the EN rate separately by skill type. We set the instantaneous interest rate r to replicate an annual interest rate of 4%. Following the macro literature, we set the coefficient of relative risk aversion, ϑ, to 1. To model labor market frictions, we adopt a CES matching function $\mathcal{M}(S, V) = \chi \left[\beta V^{-\iota} + (1-\beta) S^{-\iota}\right]^{-\frac{1}{\iota}}$ with substitution parameter ι, vacancy share α, and meeting efficiency χ.[25] We externally set the values of ι and β from Şahin et al. (2014) and internally estimate χ to capture the meeting efficiency of the Italian economy.

Next, we proceed by discretizing the profits rebate function $\mathcal{S}(G(aw))$ over a 10-point grid. It follows that workers of the same decile in the wage distribution get rebated the same amount of profits. This allows us to identify the share of aggregate profits accruing to each decile of the wage distribution, $\mathcal{S}_d$, with the empirical share of non-labor earnings across the income distribution reported by the Survey on Household Income and Wealth (SHIW).Finally, we pin down the profits tax rate, τ, to replicate the ratio of non-labor earnings to labor income in the top decile of the income distribution. Table (1) Panel A summarizes our externally set parameters.

Internally estimated parameters. The remaining parameters are estimated internally via SMM. We normalize the skill parameter of low-skilled workers, a_L, to 1 and estimate the skill parameter of high-skilled workers, a_H, internally. We leave the shape of the productivity distributions unrestricted and identify them from the labor-market-specific wage distributions. Hence, while being jointly determined with the other parameters in the SMM routine, the productivity distributions are identified non-parametrically. We clean the raw wage distributions as follows. We trim the lowest 1% of each distribution to guard against outliers, as well as picking the lowest wage of the left-trimmed distribution as reservation wage. Next, we treat the share tr of wages to right-trim as a parameter to estimate.[26] To let

[25] We choose this more general functional form rather than the standard CD specification for conservativeness about potential nonlinearities in the elasticity of firms' meeting rate to labor market tightness. It follows that the scope for productivity gains from worker reallocation is *more limited* than in a model with CD matching function.

[26] The structure of our model predicts that firms commit to a wage rate for all their employees and that larger firms pay higher wages: it follows that a small share of high observed wages (e.g., of top managers) would be

the data inform our choice, we internally estimate the share of right trimming by targeting the employment share of top employers, i.e., firms with more than 250 employees. Finally, we map the measure of firms in the labor markets to the finite number of firms in the product markets by normalizing to 1 the number of firms at the lowest value of the (discretized) weighted MRPL densities inferred from the observed wage distributions. See Appendix D.4 for further details.

Let Θ be the vector of parameters still to be determined: $\Theta = \left\{\bar{c}_{a_i}, \zeta, a_H, \chi, \phi, s_{a_i}, \sigma, \rho, tr\right\}$. We choose parameter values that minimize the sum of weighted squared percentage deviations between a set of moments estimated in actual data, μ_m, and those generated by the model. Formally, $\hat{\mu}_m$: $\Theta^* = \arg\min_{\Theta \in \overline{\Theta}} \sum_{m \in \mathcal{M}} \left(\frac{\hat{\mu}_m(\Theta) - \mu_m}{\mu_m}\right)^2$, where $\overline{\Theta}$ denotes the parameter space and $\mathcal{M}$ is the set of targeted moments.

As in Engbom and Moser (2022), we estimate the scalar and convexity of the hiring cost function, $\bar{c}_{a_i}$ and ζ, to replicate the average observed NE rate in all labor markets and the employment share of firms with more than 50 employees, respectively. Similarly, we pin down the on-the-job search intensity, s_{a_i}, to replicate the EE rate for each skill type. Turning to our original targets, we first pin down the skill parameter of high-skilled workers, a_H, by replicating the share of log-wage variance accounted for by worker-firm sorting, i.e., twice the covariance between worker and firm fixed effects, in the AKM regression.[27] Then, we identify the meeting efficiency, χ, by replicating the vacancy share, i.e., the ratio between unfilled jobs and total (filled and unfilled) jobs. The hiring rate elasticity of hiring costs, ϕ, is informed by the average Herfindahl-Hirschman index (HHI) in 4-digit sectors, weighted by the sectoral value added. The higher ϕ, the higher the hiring cost correction term in the equilibrium markdown (20). A higher hiring cost correction term compresses the markdowns of high-productivity firms disproportionately more. Hence, for given wage distribution, the revenue productivity distribution is more skewed to the right, thus raising the weighted average HHI.

For product market parameters, we identify the sectoral elasticity of demand, σ, by targeting an estimate of aggregate markup in Italy (Ciapanna et al., 2022). The larger the elasticity of demand, the lower the markup of all firms. Next, we estimate the elasticity of demand across sectors, ρ – or, more precisely, the difference between σ and ρ – by targeting the semi-elasticity of firms' labor share to their market share in the 4-digit sector where they operate (Edmond et al., 2015). Intuitively, the higher ρ (with respect to σ), the higher the markup of large firms and the more negative the labor share gradient with respect to the market share.

associated with extremely big firms, whose size would be unrealistic. This observation suggests to remove some wages at the top of the distribution.

[27]The AKM regression has no structural interpretation in our model due to differential labor market power across labor markets and no skill heterogeneity within each firm. Still, we proceed by simulating labor market histories generated by the stationary equilibrium of our model. Then, we assign the same identifier to firms with the same MRPL – the sufficient statistic for wage posting decisions in our model. Finally, we run the same AKM regression in the data and in the model. As a result, the sorting share of log-wage variance constitutes a legitimate reduced-form target.

Our estimation strategy delivers some implied parameters. Unemployment benefits b_{a_i} are identified by the labor-market-specific reservation wage, the overhead costs κ_{a_i} are set marginally lower than the operating profits generated in equilibrium by the smallest (observed) firm in each labor market (for entry of additional firms to be unprofitable), and the CD industry weights are implied by the industry revenue shares consistent with the wage distributions. Table (1) Panel B summarizes our internally estimated parameters.

Table 1: Parameter Estimates

Parameter	Description	Value	Target (Source)	Data	Model
		A. Externally set parameters			
Market structure					
I	Number of industries	8	ATECO 1-digit industries (ISTAT)		
A	Number of skill types	2	Median AKM worker FE (INPS)		
$d(i)\omega_i(a)$	Worker population, by labor	Tab. A.1	Employment composition, by industry and skill (INPS)		
K	Number of product markets	567	ATECO 4-digit sectors (ISTAT)		
M_{a_i}	Firm-to-worker population ratio, by skill	Tab. A.1	Average firm size (ISTAT)		
$\bar{N}_i$	Number of potential firms in sector, by industry	Fig. (A.2b)	Avg number of firms in 4-digit sectors, by 1-digit industry (ISTAT)		
Workers' preferences					
r	Discount rate	0.004	4% annualized int. rate		
ϑ	Coeff relative risk aversion	1.000	Macro literature		
Labor market frictions					
ι	CES subst parameter	0.152	Şahin et al. (2014)		
β	Vacancy share	0.576	Şahin et al. (2014)		
δ_a	Separation rate, by skill	[0.029,0.021]	EN rate by skill		
Profits rebate					
τ	Profits tax rate	0.654	Non-labor earnings to labor income, top income decile (SHIW)		
$\mathcal{S}_{d=1}^{10}$	Profits rebate, by wage decile	Fig. (A.2c)	Share non-labor earnings by income decile (SHIW)		
		B. Internally estimated parameters			
Labor market parameters					
$\bar{c}_{a_i}$	Scalar hiring costs	Fig. (A.4a)	NE rate (INPS)	0.161	0.161
ζ	Vacancy convexity hiring costs	0.772	Employment share 50+ firms (ISTAT)	0.427	0.441
s_{a_i}	On-the-job search intensity	Fig. (A.4b)	EE rate, by skill (INPS)	[0.013, 0.010]	[0.013, 0.010]
a_H	Skill parameter, high skills	1.320	AKM sorting share of wage variance (INPS)	-0.066	-0.066
χ	Meeting efficiency	1.465	Vacancy rate (ISTAT)	0.011	0.011
ϕ	Hiring rate elasticity hiring costs	1.715	Weighted avg HHI, 4-digit (ISTAT)	0.077	0.077
tr	Right trimming wage distributions	0.087	Employment share 250+ firms (ISTAT)	0.262	0.255
Product market parameters					
σ	Elast. of subst. within sectors	9.171	Aggregate markup rate (Ciapanna et al., 2022)	0.139	0.139
ρ	Elast. of subst. across sectors	3.049	Semi-elasticity labor share-market share (ISTAT)	-0.450	-0.449
Γ_{k_i}	Productivity distribution, by product market	Fig. (A.5a)	Labor-market-specific wage distributions (INPS)		
Implied parameters					
b_{a_i}	Home production	Fig. (A.4c)	Reservation wage		
κ_{a_i}	Overhead costs	Fig. (A.4d)	90% smallest operating profits		
α_i	CD industry weights	Tab. (A.2a)	Industry revenue share		

Source: Authors' calculations, model, INPS, Istat, and SHIW. *Note*: Labor market transition rates, wage distributions, and industrial and skill composition of wage distributions are computed on matched employer-employee data (2016-2018) from INPS. Average firm size, the average number of firms in 4-digit Ateco sectors, the share of employment in firms with more than 50/250 employees, and the vacancy rate are taken from Istat data (2019). The HHI of 4-digit ATECO sectors and the semi-elasticity of the labor share with respect to the market share at 4-digit level are computed on the Structural Business Statistics dataset from Istat (2019). The share of non-labor earnings by income decile is taken from the 2016 wave of the Survey on Household Income and Wealth (SHIW) administered by the Bank of Italy.

Productivity estimation. We now detail our strategy to estimate firms' productivity distributions from the (trimmed) observed wage distributions within the SMM routine. First of all, we discretize the wage distributions on a 125-point, equally-spaced grid. We then assume that residual wage dispersion is small enough relative to the distance between grid points, so that each wage grid point corresponds to a productivity type.[28]

Then, we follow a two-step estimation procedure. In the first stage, we notice that the observed wage distributions, hiring cost correction terms, and worker transition rates are

[28]Technically, we estimate the discretized version of the limit case of our baseline equilibrium for a large number of productivity types.

sufficient statistics to identify the equilibrium markdowns per wage level in each labor market according to (20), i.e., $\psi_{a_i}(w)$. It follows that the marginal cost of firms posting wage w in labor market a_i equals $MCL_{a_i}(w) = \psi_{a_i}(w)w$. Since, in equilibrium, firms equalize MCL to MRPL, this strategy allows us to characterize the model-consistent MRPL distribution in each labor market non-parametrically, that is, $\tilde{z} \sim \Phi_{a_i}(\psi_{a_i}(w)w)$ (Bontemps et al., 2000). Within each labor market, we consider a number of product markets such that each of them is populated by the average number of competing firms across 4-digit sectors in each industry. Next, we assign firms to product markets so that each product market sourcing from the same labor market has the same MRPL distribution.

In the second stage, we notice that, for given CES elasticities of demand and MRPL distribution in each product market, there exists a unique combination of prices, markups, CD industry weights, and physical productivities consistent with our product market structure (see the system of equations (77) in Appendix). In this way, we can finally back out the productivity distributions that rationalize the observed wage distributions through the lens of our model.

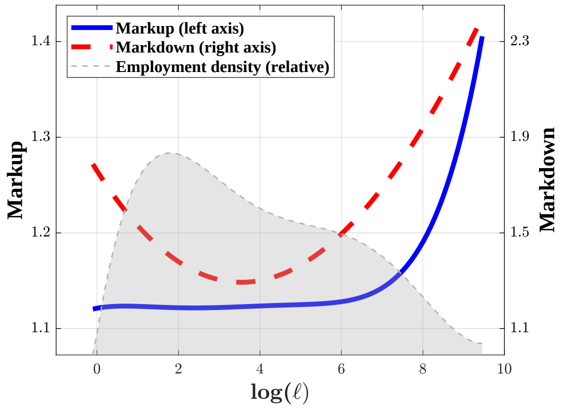
This estimation strategy is specifically geared towards guaranteeing the best possible fit to the labor-market-specific wage distributions, whose shape is critical for both the bite of the MW reforms and the scope for worker reallocation.

Parameter estimates and model fit. The last two columns of Table (1) compare the estimation targets in the data with those implied by the estimated model. Our estimated model is successful at replicating all the targeted moments accurately. Note that the implied hiring cost correction term equals 1.48, thus representing an important determinant of equilibrium markdowns. The estimated skill parameter for high-skilled workers is 1.32, indicating that nearly all of the variation in average wages across labor markets can be attributed to worker heterogeneity, rather than productivity heterogeneity among firms operating in different markets. [29] The parameters governing the elasticity of demand are estimated to be $\sigma = 9.17$ and $\rho = 3.05$. In the light of the low level of concentration of the Italian product markets, our estimated elasticity of substitution across sectors is remarkably high. This suggests that markups of Italian firms are not as sensitive to product market concentration in international comparisons (Edmond et al., 2015; Atkin et al., 2015; Edmond et al., 2023; Burstein et al., 2025; De Loecker et al., 2021). Figures (A.5a) report the estimated productivity distributions by industry. In spite of substantial variation across industries, the productivity distributions generally appear more right-skewed than log-normal. Our estimation strategy grants a virtually perfect fit to the observed wage distributions in each labor market, which allows us to replicate the actual bite of MW reforms (Figure A.3b).

Finally, we focus on the estimated distributions of markups and markdowns by firm size, reported in Figure (2). The cross-sectional distribution of market power carries two main insights. First, the largest firms ($> 1,000$ employees) are estimated to have both higher

[29] Indeed, wages in high-skilled labor markets are on average exactly 32% higher than in low-skilled labor markets. Within labor markets, all wage dispersion is due to productivity differences across firms.

Figure 2: Market power distribution, by firm size



Source: Model. *Note*: The figure reports the average markup (in solid blue) and markdown (in dashed red) for given firm employment size in the baseline equilibrium of the estimated model. The area in gray represents a linear transformation of the density of employment across firm employment size such that the relative density is preserved.

markups and markdowns than their competitors. Second, markdowns are estimated to be U-shaped in firm size, with small firms extracting more rents than mid-sized ones in the labor market. This is a distinctive prediction of our model that aligns well with the robust (and untargeted) empirical pattern that, within a 1-digit industry, the average labor share in the first tercile of employment is lower than that in the second tercile (see Tab. (A.2b)). [30] In terms of magnitudes, the aggregate markdown exceeds the aggregate markup (1.47 versus 1.14) – even though the former does not translate one-to-one into pure profits due to hiring costs. Two remarks are in order. First, our estimate of the aggregate markdown is remarkably close to those of Berger et al. (2022) and Yeh et al. (2022) for the US. Second, both our empirical setting and estimation strategy are likely to identify a lower bound for the role of endogenous markups in shaping the effects of the MW. Indeed, the Italian economy features low product market power in international comparisons.[31] Moreover, we estimate the elasticity of demand across sectors – which determines a monopolist's markup – by matching the *average* semi-elasticity of firms' labor share to their market share. Coupled with the functional form of equilibrium markups (18), this choice implies that our estimated markups for the largest firms are bounded at 1.4, while the 99^{th} percentile of the markup distribution in the data is estimated at 1.9 (Ciapanna et al., 2022).

[30] Models of oligopolistic and oligopsonistic competition where product and labor market boundaries coincide, such as Deb et al. (2023), imply identical cross-sectional behavior of markups and markdowns.

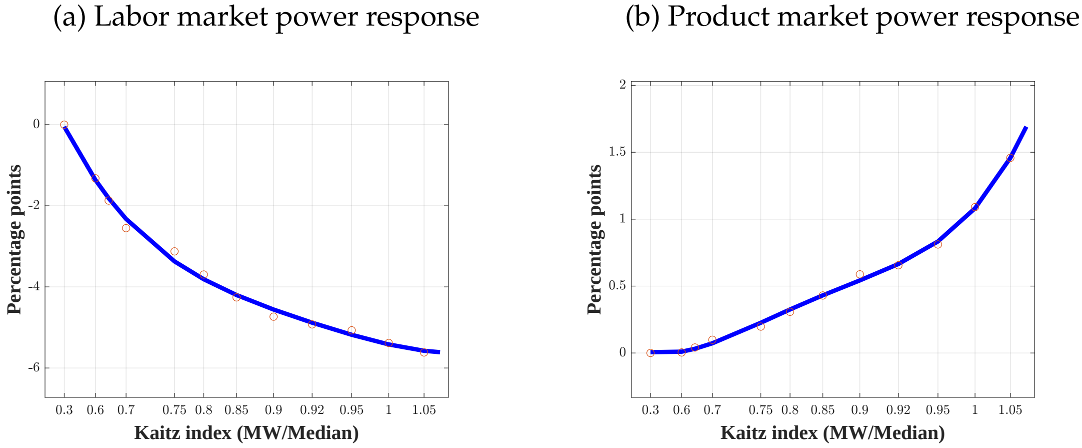
[31] According to Ciapanna et al. (2022), the aggregate markup in Italy (1.14) is comparable to that of France, and lower than that of Germany (1.16), Spain (1.18), and the US (1.20). These estimates are more comparable to our model-consistent notion of aggregate markup than those in De Loecker and Eeckhout (2018) as they consider the universe of firms (rather than focusing on public listed companies) and do not restrict firms to share a common production function.

5 Equilibrium Effects of the Minimum Wage

In this section we study the equilibrium effects of setting a nationwide MW in the Italian economy (where none is currently in place). To do so, we make use of the estimated model as a laboratory to simulate the response of our stationary equilibrium to varying the MW parameter $\underline{\mathbf{w}}$.[32] Following the literature, we measure MW reforms through their *Kaitz index*, i.e., the ratio between the MW and the pre-reform median wage. Since the Kaitz index is not a sufficient statistic for the MW bite, we couple this statistic with the share of workers directly affected. As a benchmark, the 2015 MW reform in Germany directly affected 15% of workers (Dustmann et al., 2022), which would correspond to a Kaitz index of approximately 75% in our model economy (see Figure (A.6a)).

We start by assessing how market power reacts to the MW. Figure (3a) shows that labor

Figure 3: Market power effects



Source: Model. *Note*: The labor market power response corresponds to the aggregate markdown response weighted for the baseline markup net of a hiring cost sensitivity term. The product market power response corresponds to the aggregate markup response weighted for the baseline markdown. See (78) for the formal definition. The x-axis is scaled so as to reflect the share of directly affected workers. The red circular markers represent the simulated data, the blue solid lines their 4^{th}-order polynomial fit.

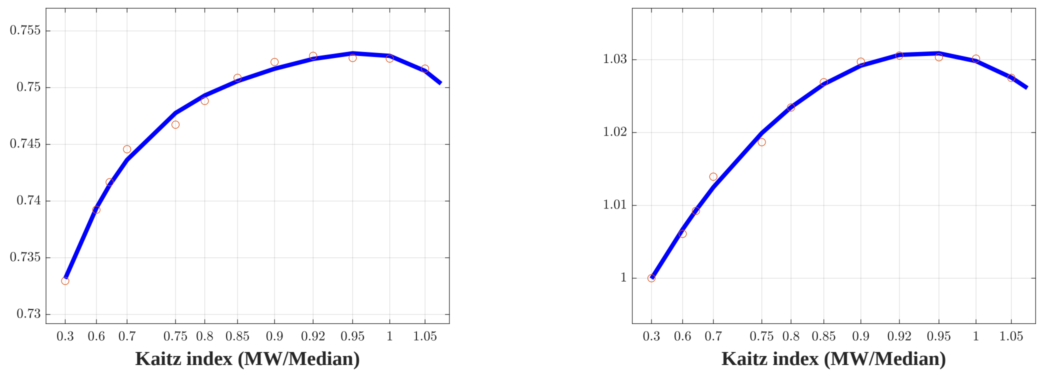
market power is decreasing in the MW at a progressively slower rate. This size-dependent pattern results from the net effect of two opposing forces. On the one hand, firms that are affected by the MW – either directly (constrained) or indirectly (due to spillover effects) – reduces their markdowns. This behavioral effect drives down the aggregate markdown. On the other hand, displaced workers from the most affected firms reallocate towards larger firms, which have higher baseline markdown on average. This compositional effect progressively slows down the reduction of the aggregate markdown as the MW increases. Overall, labor market power decreases by 5.5pp from the baseline equilibrium to the highest MW considered, where it troughs. Figure (3b) shows that product market power is increasing in the MW at a progressively faster rate. This reflects both the concentration channel of the MW (behavioral effect) and worker reallocation towards higher-markup firms (compositional

[32] This comparative statics exercise is informative about the effects of the MW in the medium term, while being silent on the transitional dynamics of the labor market towards the new equilibrium (short-term adjustment), as well as on the potential substitution away from labor, automation, and entry decisions induced by the MW (long-term adjustment).

effect).[33] Overall, product market power increases by 1.5pp from the baseline equilibrium to the highest MW considered, where it peaks. The product market power response is driven by product markets with the highest baseline concentration, as measured by their HHI. We estimate that an increase in the baseline HHI of one standard deviation (starting from the mean) raises the elasticity of the sectoral markup rate to the MW by 24%. Overall, the product market power response accounts for a quarter of the (gross) aggregate market power response. However, the importance of the product market power response is strongly MW-size-dependent. For Kaitz indices until 80%, the labor market power response dominates the product market power response. For larger MW reforms, the two responses are of the same order of magnitude – with the product market power response even dominating for the highest MW reforms.

Figure 4: Aggregate effects of the MW

(a) Aggregate labor share response (b) Aggregate consumption response



Source: Model. *Note*: Aggregate labor share is defined as $LS \equiv \frac{WL}{C}$ and aggregate consumption is defined as $C = Y - \mathcal{HC} - \mathcal{FC}$, where $\mathcal{HC}$ is total hiring costs and $\mathcal{FC}$ total overhead costs. See Appendix (B.9) for the model-consistent aggregation. The x-axis is scaled so as to reflect the share of directly affected workers. The red circular markers represent the simulated data, the blue solid lines their 4^{th}-order polynomial fit.

We now focus on the aggregate effects of the MW. Figure (4a) reports the aggregate labor share response to the MW. We uncover a hump-shaped response of the aggregate labor share to the MW, peaking at a Kaitz index of 92%. As in the stylized model, the aggregate labor share response is driven by the net market power response. On the one hand, the behavioral effect restraining the aggregate markdown is the dominating force behind the initial increase of the aggregate labor share. On the other hand, the flattening and eventual decline of the aggregate labor share is driven by the concentration channel raising the aggregate markup. Overall, the aggregate labor share increases by 1.9pp from the baseline equilibrium to the highest MW considered, with an increase by 2.0pp at peak in correspondence to a Kaitz index of 92%. In more concentrated sectors, where the concentration channel is quantitatively more relevant, the labor share response is more muted and the humped shape is more pronounced. We estimate that an increase in the baseline HHI of one standard deviation (starting from the mean) reduces the elasticity of the sectoral labor share to the MW by 22%. Figure (4b)

[33] Notice that the product market power response is estimated conservatively. This is because aggregate profits reduce with the MW (see Fig. (A.9b)). Thus, a model where the mass of potential firms were pinned down by a free entry condition would entail more firm exit than ours.

reports the aggregate consumption response to the MW. Just like the labor share, aggregate consumption is hump-shaped in the MW, with a peak at a Kaitz index of 92%. Overall, aggregate consumption increases by 2.8% from the baseline equilibrium to the highest MW considered, with an increase by 3.1% at peak in correspondence to a Kaitz index of 92%.

Next, we explore the determinants of the positive consumption response. Figures (A.9a)-(A.10d) in Appendix show that the reduction in labor misallocation is the main driver behind the positive consumption response – aggregate output being virtually unaffected by the MW. In turn, we break down the output response into employment and productivity effects. Figures (A.10a)-(A.10c) in Appendix show that output remains stable because productivity gains from worker reallocation offsets a reduction in headcount employment. The estimated magnitude of the employment effects aligns well with reduced-form estimates from other countries.[34]

6 Welfare Analysis

In the previous section we have shown that introducing a MW generally raise both aggregate consumption and the unemployment rate. Hence, the welfare effects of the MW are a priori ambiguous. On the one hand, lifetime consumption is higher (*level effect*). On the other hand, higher unemployment risk entails higher consumption volatility (*uncertainty effect*).[35] Moreover, the MW is likely to induce some redistribution across worker types (*distributional effect*). To strike a balance between these forces, we resort to a utilitarian social welfare function, which provides us with a normative criterion to solve the trade-off optimally. Consistently with the steady-state behavior adopted by firms in our model economy, we focus on steady-state comparisons.

Figure (5a) shows that aggregate utilitarian welfare is maximized in correspondence to a Kaitz index of 70%. Henceforth, we refer to the welfare-maximizing MW, $\underline{\mathbf{w}}^{\star}$, as *optimal* MW. The optimal MW brings about a welfare gain of 0.5% in consumption equivalent units, in the face of a 1.4% increase in aggregate consumption. Figure (A.15) in Appendix shows that, for Kaitz indices up to 75%, the increase in aggregate consumption (level effect) dominates the increase in consumption volatility due to heightened unemployment risk (uncertainty effect) in welfare terms. Still, welfare gains from redistribution to poorer workers (distributional effect) are maximized at a Kaitz index of 65%. The optimal MW balances efficiency and redistribution.

To investigate the determinants of the distributional effect, Figure (5b) reports the welfare

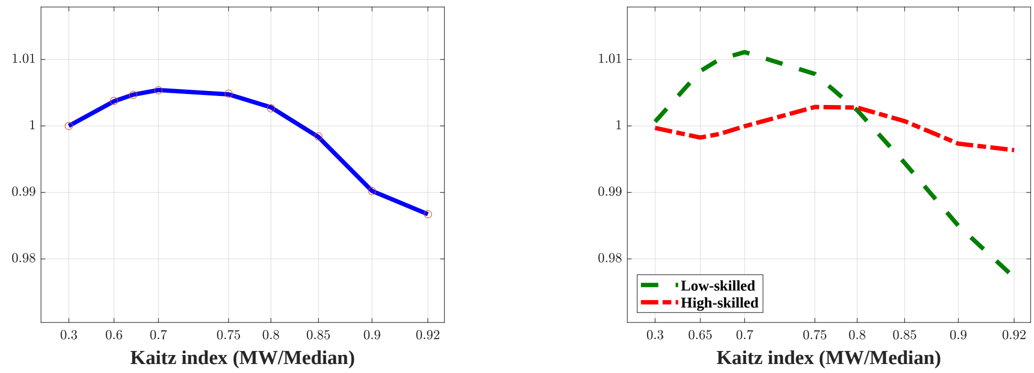
[34]To assess the (dis)employment effects of the MW, we compute the semi-elasticity of the unemployment rate to the MW among among two commonly used – and model-consistent – categories of *minimum wage workers*, that is, low-skilled workers and low-skilled workers in hospitality. We find that, when raising the MW from 60 to 70% of the current median wage (arguably the most comparable reform to those studied in the existing literature), the semi-elasticity equals $-0.10(-0.14)$ for low-skilled workers (low-skilled workers in hospitality). This figure is in line with existing reduced-form estimates (Belman and Wolfson, 2019; Dube and Lindner, 2024).

[35]As our model abstracts from intertemporal consumption-savings decisions, workers have no means to mitigate consumption volatility.

Figure 5: Welfare effects of the MW

(a) Aggregate welfare

(b) Welfare effects, by skill



Source: Model. *Note*: Panel (a) reports the consumption-equivalent percentage variation of the aggregate utilitarian welfare across MW reforms in solid blue. Aggregate utilitarian welfare is defined as: $\sum_{i=1}^{I} \int \left[u_{a_i}(\underline{w}) U_{a_i}(\underline{w}) + (1 - u_{a_i}(\underline{w})) \int W_{a_i}(aw; \underline{w})\, dG_{a_i}(w; \underline{w}) \right] d\Omega_i(a)\, \Xi(i)$, where U_{a_i} and $W_{a_i}(w)$ are the labor-market-specific asset values of unemployment and employment at wage aw, respectively, defined in (2)-(3). Panel b) decomposes the utilitarian welfare response into low-skilled and high-skilled welfare response. The x-axis is scaled so as to reflect the share of directly affected workers.

effects of the MW across skill types. Welfare of low-skilled workers is maximized at a Kaitz index of 65%. Welfare of high-skilled workers is maximized at a Kaitz index of 75%. Kaitz indices up to 65% redistribute consumption shares towards low-skilled workers. As a result, welfare inequality drops, which reflects in a positive distributional effect. On the contrary, larger Kaitz indices hit significantly harder low-skilled than high-skilled workers due to larger job losses, which reflects in a negative distributional effect. Therefore, our results express a word of caution about the idea of raising the MW to unprecedented levels in order to reduce inequality, as in MaCurdy (2015). In fact, Kaitz indices higher than 65% would end up *increasing* welfare inequality.

7 The Role of Endogenous Markups

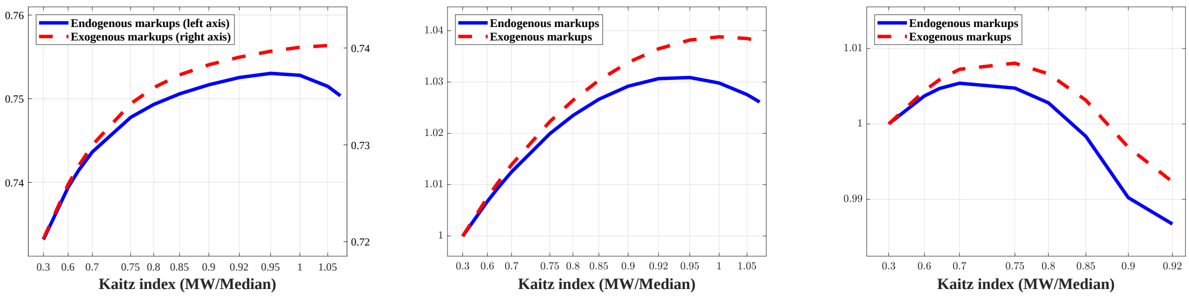
In the previous two sections we have tackled positive and normative questions related the introduction of a MW in an economy with endogenous markups. In this section, we aim to single out the role of endogenous markups in driving our results. With this goal in mind, we repeat the same policy experiments of Section (5) in an observationally equivalent economy – that is, matching the same empirical moments as our baseline – with monopolistic competition in sectoral markets, which implies exogenous and identical markups across firms.[36] See Appendix G for further details on the alternative economy.

Figure (6) reports the response of the aggregate labor share, consumption, and welfare across the two economies. Our baseline economy with endogenous markups is depicted in solid blue, the alternative economy with exogenous markups in dashed red. From the response of aggregate variables, we draw three main takeaways. First, the aggregate labor share would be monotonically increasing in the MW if markups were exogenous. Second,

[36]We interpret differential results across the baseline and the alternative economy as *structural* counterfactuals, i.e., what would happen by ignoring endogenous markups when estimating the equilibrium effects of the MW.

Figure 6: Aggregate response: Endogenous vs exogenous markups

(a) Aggregate labor share (b) Aggregate consumption (c) Aggregate welfare



Source: Model. *Note*: The *endogenous markup* economy in solid blue is our baseline economy with oligopolistic competition in sectoral markets, while the *exogenous markup* economy in dashed red is an observationally equivalent economy to our baseline with monopolistic competition in sectoral markets. The x-axis is scaled so as to reflect the share of directly affected workers. Panel (a) compares the aggregate labor share response in levels, while Panels (b) and (c) compare the percentage change of aggregate consumption and aggregate welfare.

aggregate consumption would peak at a Kaitz index of 100% (instead of 92%) if markups were exogenous. Aggregate consumption is higher in the economy with exogenous markups across the entire spectrum of MW reforms. The consumption-maximizing MW brings about higher value added gains than in the baseline economy by 0.9pp (28%). Finally, the economy with exogenous markups calls for a higher optimal MW. The optimal MW equals a Kaitz index of 75% (rather than 70%), affecting 17% of workers (rather than 10%), with associated welfare gain of 0.8% (rather than 0.5%) consumption-equivalent units. Hence, if markups were exogenous, the optimal MW would yield nearly 50% larger welfare gains. Overall, we conclude that endogenous markups matters for quantifying the equilibrium effects of MW reforms.

8 Empirical Validation

In this section we provide empirical validation to the concentration channel of the MW in a model-free setting. The concentration channel predicts that (i) a higher MW reallocates market share to high-productivity firms, and (ii) high-productivity firms raise markups, especially in already concentrated product markets.

We test these predictions by leveraging Italian firms' balance sheet data from CERVED and exploiting the variation induced by industry-specific contractual wage floors published by Istat.[37] Absent a nationwide MW, the Italian wage setting system is based on contractual wage floors agreed through a centralized collective bargaining process, which apply to all private-sector employees. These contractual wages are set at the national level at predetermined dates. Hence, they represent an important source of variation of labor costs to firms, just like the introduction of a MW. Differently from the latter, however, collective bargaining agreements set an entire array of contractual wage floors that vary by job title. As long as the effects are heterogeneous across firms, the concentration channel is still expected to operate

[37] CERVED is an Italian business information agency, which provides data on the universe of incorporated Italian firms.

in the same way, though.[38] On the one hand, endogeneity concerns may arise because the size of the adjustment may be correlated with the business performance of the respective industry. On the other hand, a key advantage of using industry-specific contractual wage floors is that they allow us to control for aggregate shocks, due to the staggered nature of the collective bargaining agreements across different industries.

Overall, our estimating sample from CERVED includes about 6.2 million yearly observations, corresponding to more than 20% of total private-sector firms and 63 to 77% of total private-sector employees (Devicienti and Fanfani, 2021). Hence, our data captures the bulk of employment in the Italian private-sector economy.

Reallocation effects. The first prediction of concentration channel is that a higher MW reallocates market share to high-productivity firms. Therefore, a testable implication is that the firm-level value added response is increasing in productivity. To bring this testable implication to the data, we first assign firms to their most frequent quintile of the 4-digit-sector-year distribution of log value added per worker – a proxy of productivity. We then estimate the following regression model:

$$\log VA_{i,t} = \sum_{q=1}^{5} \beta_q \log MW_{s_3(i),t} \cdot \mathbb{1}_{\{Q(i)=q\}} + \gamma_{s_2(i)} \cdot \phi_t + \alpha_i + \epsilon_{i,t}, \tag{21}$$

where $\log VA_{i,t}$ is the natural logarithm of value added of firm i at time t, $\log MW_{s_3(i),t}$ is the natural logarithm of the contractual wage floor of the 3-digit industry in which firm i operates, $\gamma_{s_2(i)} \cdot \phi_t$ are 2-digit-industry-specific time trends, α_i are firm fixed effects, and $\epsilon_{i,t}$ is an idiosyncratic error term. The interaction term between the contractual wage floor and the quintile indicators allows for heterogeneous value added response by productivity bin. We find that the firm-level elasticity of value added to the wage floor varies strongly across productivity, ranging from -1.5 for the first bin to above 1 for last bin (Figure A.17a), implying reallocation of market share towards high-productivity firms. These findings are consistent with a large number of empirical studies (Draca et al., 2011; Aaronson et al., 2018; Dustmann et al., 2022; Devicienti and Fanfani, 2021). A mechanical implication of the heterogeneous value added response by productivity is that sectoral concentration and measured labor productivity both rise, driven by compositional effects.

Concentration effects. The second prediction of the concentration channel is that high-productivity firms raise markups, especially in already concentrated product markets. Absent price data, we focus on the firm-level labor share response as a proxy for the market power response to guard against potential bias in estimated markups (Bond et al., 2021).[39]

[38] If high-productivity firms are over-represented in employers' associations, collective bargaining agreements may even be used strategically to reduce competition (Haucap et al., 2001).

[39] Thanks to product-level custom data, Dodini et al. (2023) documents a reduction in markdowns and an increase in markups by large firms in response to an exogenous shift in union density, i.e. the firm-level share of unionized workers. Consistently with the concentration channel, the markup response is present only in sectors

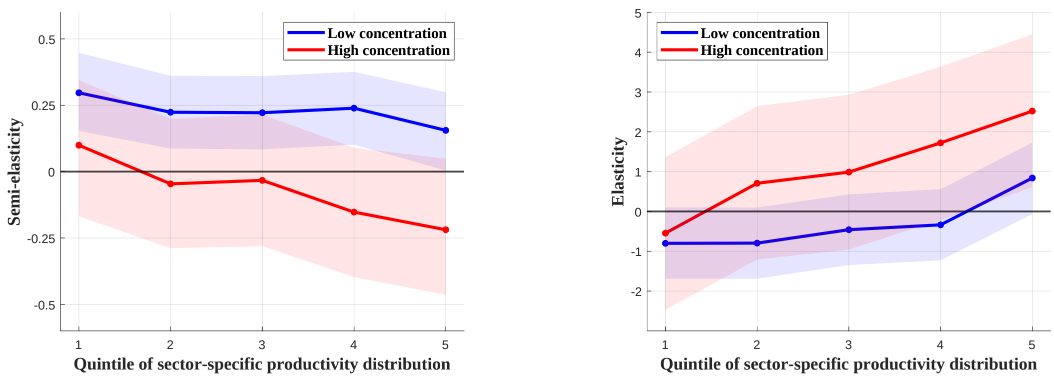
To study heterogeneous effects by productivity and product market concentration, we estimate the model (21) interacted with a dummy indicating whether the HHI of concentration in the sector in which firm i operates is above the weighted average of the economy:[40]

$$Y_{i,t} = \sum_{q=1}^{5} (\beta_q + \beta_q' D_i) \log MW_{s_3(i),t} \mathbb{1}_{\{Q(i)=q\}} + \left(\gamma_{s_2(i)} + \gamma_{s_2(i)}' D_i\right)\phi_t + \alpha_i + \epsilon_{i,t}, \tag{22}$$

where $D_i \equiv \mathbb{1}_{\{HHI_{s_4(i)} > \overline{HHI}_{s_4}\}}$. The outcome variables, $Y_{i,t}$, are either the labor share of firm i at time t – defined as the ratio between total labor costs and value added – or accounting profits (EBITDA). Standard errors are clustered at the firm level.

Figure 7: Firm-level response by concentration and productivity

(a) Labor share (b) Profits (EBITDA)



Source: CERVED (2005-2020), INPS and Istat data. *Note*: The graph reports the average semi-elasticity of firm-level labor share and the elasticity of firm-level profits (EBITDA) to the MW by productivity bins, computed from the regression model (22). Standard errors are clustered at the firm-level. Shaded areas represent 90% confidence intervals.

The regression results are plotted in Figure (7), which uncovers three important results. First, the labor share response is decreasing in productivity. Second, the labor share response is higher in low concentrated markets than in highly concentrated markets across the entire productivity distribution. Hence, the labor share response is decreasing in product market concentration. Third, the labor share response of high-productivity firms in highly concentrated product markets is negative, driven by a positive profits response.[41] The increase in profits hints at a positive markup response – rather than capital-labor substitution – as the main driver of the negative labor share response. Estimated markups on revenue data are consistent with this interpretation (Figure (A.17b)). In Appendix H.5 we repeat the same analysis using common proxies of labor market concentration and show that labor market power is not a likely confounder of our results. In fact, we find that the labor share response is higher in *highly* concentrated labor markets than in low concentrated labor markets across the entire productivity distribution (see Figure (A.19)) – in line with existing studies (Azar et al.,

where significant market shares are reallocated from small to large firms.

[40] We measure the HHI of value added in 2019 in our Istat data, which are representative of the universe of Italian firms with employees.

[41] Only the profits response is statistically different from zero with clustered standard errors. Results are unchanged if we look at the elasticity of the labor share to the MW, i.e., if we use log labor share as outcome variable.

2024; Popp, 2023; Azkarate-Askasua and Zerecero, 2023). We conclude that our reduced-form estimates are supportive of a significant role of the concentration channel of the MW.

9 Conclusions

This paper studies the equilibrium and welfare effects of the minimum wage when product market power is endogenous and varies with market competition. We make two main original contributions to the literature. First, we are the first to formalize and empirically validate the concentration channel of the MW. A higher minimum wage reallocates workers from small to large firms. If product market power is endogenous, large firms gain market share and increase their price markups. Second, we develop and estimate a new structural model combining oligopolistic competition in product markets with wage posting in frictional labor markets, which can be used to study spillover effects between product and labor markets induced by a variety of policies, such as the MW.

The main takeaway from this paper is that the optimal minimum wage depends on the structure of product markets. It follows that setting a high minimum wage may not be advisable if product markets feature a few dominant firms. An interesting avenue for future research is assessing the welfare effects of the transitional dynamics to the new stationary equilibrium following the introduction of a MW.

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Appendix

A Stylized Model

In the presence of a binding MW, firms choose employment by solving the following profit-maximization problem:

$$\begin{aligned} &\max_{\ell} \;\; p(\ell)y - w(\ell)\ell - \kappa \\ &\text{s.t.} \;\; y = z\ell, \;\; w(\ell) = \max\{\underline{w}, \ell^{\frac{1}{\eta}}\}, \;\; p(\ell) = y^{-\frac{1}{\epsilon(N)}}. \end{aligned}$$

As discussed in the main text, firms are either constrained or unconstrained in their wage setting problem.

Unconstrained firms set their optimal markup, $\mu = \frac{\epsilon}{\epsilon-1}$, and their optimal markdown, $\psi = \frac{1+\eta}{\eta}$. Hence, their employment, price and wage policies are as follows:

$$\ell(z) = z^{\eta\frac{\epsilon-1}{\epsilon+\eta}} (\mu\psi)^{-\frac{\epsilon\eta}{\epsilon+\eta}}, \tag{23}$$

$$p(z) = (z\ell(z))^{-\frac{1}{\epsilon}}, \tag{24}$$

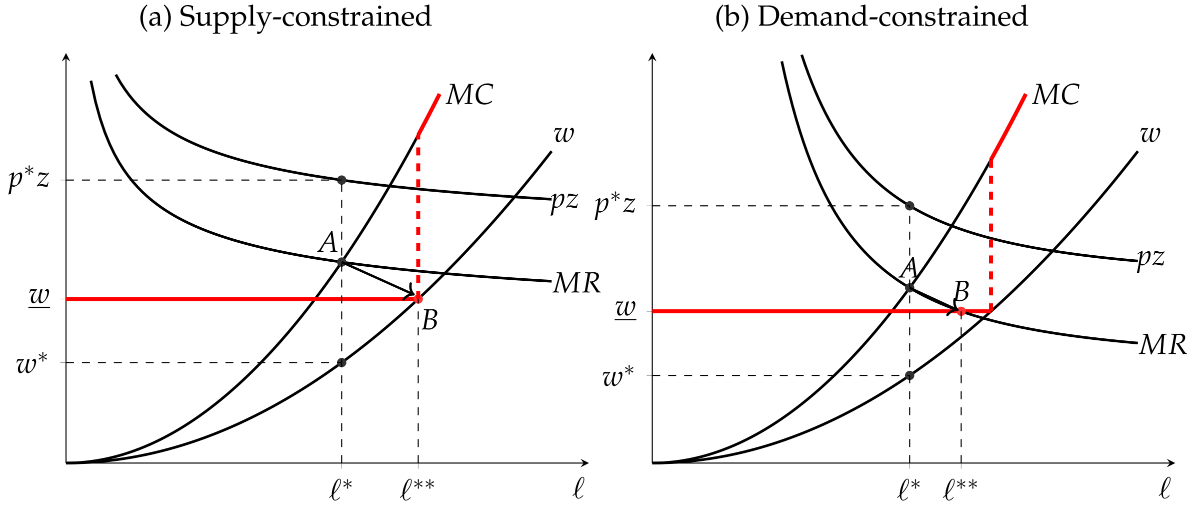
$$w(z) = \ell(z)^{\frac{1}{\eta}}, \tag{25}$$

where the dependence of ϵ on N is left implicit. Unconstrained firms are affected by the MW only through the response of the elasticity of demand, mediated by the response of the number of active firms. Notice that the elasticity of demand affects output both through the shape of the demand curve and the optimal markup. Lemma A.1 derives a parametric restriction such that firm-level output is increasing in the number of active firms.

Constrained firms set their optimal markup but are not able to set their optimal markdown (because the unconstrained optimal employment would be off the labor supply curve). Figure A.1 shows that constrained firms can be further distinguished in two groups. Lower-productivity firms are demand-constrained, in that their employment is pinned down by product demand. Such firms are extracted their entire labor market power, that is, their markdown is compressed to one. Higher-productivity firms are supply-constrained, in that their employment is pinned down by labor supply. Such firms lose only part of their labor market power.

For supply-constrained firms, the constrained markdown is pinned down by the condition such that employment lies on the labor supply curve (left panel of Figure A.1). Hence, the

Figure A.1: Effects of the MW on constrained firms



policy functions of supply-constrained firms are given by:

$$\ell(z) = z^{\eta \frac{\epsilon-1}{\epsilon+\eta}} (\mu \psi(z))^{-\frac{\epsilon\eta}{\epsilon+\eta}}, \tag{26}$$

$$p(z) = (z\ell(z))^{-\frac{1}{\epsilon}},$$

$$\underline{w} = \ell(z)^{\frac{1}{\eta}}. \tag{27}$$

From (26) and (27), it follows that the constrained markdown is given by:

$$\psi(z) = z^{\frac{\epsilon-1}{\epsilon}} \mu^{-1} \underline{w}^{-\frac{\epsilon+\eta}{\epsilon}} \in (1, \psi).$$

Since the constrained markdown is decreasing in the MW, all supply-constrained firms that would pay wages below the MW in the baseline equilibrium raise employment. Since employment is decreasing in the markup, all supply-constrained firms that would pay wages above the MW in the baseline equilibrium reduce employment. The least productive supply-constrained firm is defined by the following productivity level:

$$z^{ds}(\underline{w}) = \left(\mu \underline{w}^{\frac{\epsilon+\eta}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}.$$

For demand-constrained firms, the constrained markdown is pinned down by the condition such that employment lies on the marginal revenue curve (right panel of Figure A.1). As a result, the markdown is always equal to one. Hence, the policy functions of demand-constrained firms are given by:

$$\ell(z) = z^{\epsilon-1} (\mu \underline{w})^{-\epsilon}, \tag{28}$$

$$p(z) = (z\ell(z))^{-\frac{1}{\epsilon}}.$$

Demand-constrained firms can either increase or decrease their employment with respect to the baseline equilibrium. For given elasticity of demand, we can define a productivity threshold z^c, such that all firms with higher (lower) productivity than z^c increase (decrease) employment, as follows:

$$z^c(\underline{w}) = (\mu^\epsilon \psi^{-\eta})^{\frac{1}{\epsilon-1}} \underline{w}^{\frac{\epsilon+\eta}{\epsilon-1}}.$$

Figure 1 in the main text report the cross-sectional adjustment to a MW increase such that firms at the z^c cutoff make zero profits (with the MW in place), i.e., $\pi(z^c(\underline{w}); \mu, \psi) = 0$. It follows that, for given markup, all the demand-constrained firms that would have scaled down in size following the MW exit the market.

Lemma A.1 (Firm-level output response to the number of active firms)
If $\underline{z} > \left(\frac{\epsilon(N)}{\epsilon(N)-1}\frac{1+\eta}{\eta}\right)^{\frac{\eta}{1+\eta}} \exp\left\{-\frac{\epsilon(N)+\eta}{(1+\eta)(\epsilon(N)-1)}\right\}$ $\forall N \in \mathbb{N}$, *then unconstrained and supply-constrained firms reduce their output following a reduction in the number of active firms.*

Let $\underline{z}(\underline{w})$ *be the least productive active firm when the MW is* $\underline{w}$. *If* $\underline{z}(\underline{w}) > \frac{\epsilon(N)}{\epsilon(N)-1}\exp\{-\frac{1}{\epsilon(N)-1}\}\underline{w}$ $\forall N \in \mathbb{N}$, $\forall \underline{w} \in \mathbb{W}$, *then demand-constrained firms reduce their output following a reduction in the number of active firms.*

Proof of A.1. Differentiating optimal unconstrained employment (23) with respect to ϵ yields:

$$\frac{d\ln(\ell(z))}{d\epsilon} = \frac{1+\eta}{(\epsilon+\eta)^2}\eta\ln(z) - \frac{\eta^2}{(\epsilon+\eta)^2}\ln\left(\frac{\epsilon}{\epsilon-1}\psi\right) + \frac{\eta}{\epsilon+\eta}\frac{1}{\epsilon-1},$$

where the first two terms represent the employment response to a change in the elasticity of the demand curve for given markup (*elasticity effect*) and the last term the employment response to change in markup for given elasticity of demand curve (*markup effect*). It holds that:

$$\frac{d\ln(\ell(z))}{d\epsilon} > 0 \iff z > \left(\frac{\epsilon}{\epsilon-1}\psi\right)^{\frac{\eta}{1+\eta}} \exp\left\{-\frac{\epsilon+\eta}{(1+\eta)(\epsilon-1)}\right\}.$$

Since $d\epsilon/dN > 0$ by assumption, if this condition is met by the least productive firms, then it holds for all the firms. Notice that optimal employment of supply-constrained firms (26) is the same as that of unconstrained firms up to a weakly lower markdown. Hence, the same condition guarantees that supply-constrained firms reduce their output as the number of active firms decreases.

Differentiating optimal employment of demand-constrained firms (28) with respect to ϵ yields:

$$\frac{d\ln(\ell(z))}{d\epsilon} = \ln(\frac{z}{\mu\underline{w}}) + \frac{1}{\epsilon-1},$$

where the first term represents the elasticity effect and the second term the markup effect. It holds that:

$$\frac{d\ln(\ell(z))}{d\epsilon} > 0 \iff z > \frac{\epsilon}{\epsilon-1}\exp\left\{-\frac{1}{\epsilon-1}\right\}\underline{w}.$$

Since $d\epsilon/dN > 0$ by assumption, if this condition is met by the least productive firms, then it holds for all the demand-constrained firms. ■

B Quantitative Model

B.1 Matching Function Specification

In this Section we motivate our functional form assumption on the matching function. As explained in Section (3.7), our baseline equilibrium features congestion externalities. Moreover, our counterfactual experiments in Section (5) point to a reduction in congestion externalities as the main driver of the positive response of value added through lower misallocation. By governing the extent (as well as the scope for a reduction) of congestion externalities, the functional form of the matching function is relevant in our framework. In particular, when considering large MW reforms, it is crucial to account for potential nonlinearities in the matching function.

For this reason, we adopt a CES matching function, $\mathcal{M}(S,V) = \chi\left[\alpha V^{-\iota} + (1-\alpha)S^{-\iota}\right]^{-\frac{1}{\iota}}$, and calibrate the elasticity parameters ι and α according to the estimates of Şahin et al. (2014).[42] As pointed out by Şahin et al. (2014), the estimated parameters imply only a slight deviation from the standard Cobb-Douglas specification with vacancy elasticity of 0.5. However, we prefer the CES specification because it delivers an endogenous elasticity of firms' meeting rate with respect to labor market tightness. Formally,

$$\epsilon_{q(\theta),\theta} = -\frac{1}{1+\frac{\alpha}{1-\alpha}\theta^{-\iota}}.$$

From this formulation, it easy to see that the standard Cobb-Douglas specification – nested by setting $\iota = 0$ and $\alpha = 0.5$ – predicts a constant elasticity of 0.5. On the other hand, the CES specification implies that the elasticity is increasing in labor market tightness. As a result, the slacker the labor market is to start with, the lower the gain in terms of meeting rate of further reducing tightness. This implies that, as the MW gets larger and labor market tightness decreases, hiring cost savings from lower congestion effects progressively die out. It follows that our choice of a CES matching function is conservative in terms of positive response of value added to the MW.

B.2 Derivation of Stationary Distributions

Let $G_{a_i}(w)$ be the share of employed workers of skill a who earn wage rate equal or lower than w. This share evolves in response to inflows (unemployed workers who accept jobs with $w' \leq w$) and outflows (employed workers who accept jobs with $w' > w$ or lose their job) according to the following Kolmogorov forward equation:

$$\dot{G}_{a_i}(w) = \lambda(\theta_{a_i})u_{a_i}F_{a_i}(w) - \Big(\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})(1-F_{a_i}(w))\Big)e_{a_i}G_{a_i}(w). \tag{29}$$

[42] Since the estimated elasticities are based on US data, we internally estimate the efficiency of the matching function χ on our Italian data.

Similarly, the measures of employed e_{a_i} and unemployed u_{a_i} workers evolve according to labor market transitions governed by the separation and job-finding rates:

$$\dot{u}_{a_i} = \delta_{a_i} e_{a_i} - \lambda(\theta_{a_i}) u_{a_i}. \tag{30}$$

$$\dot{e}_{a_i} = \lambda(\theta_{a_i}) u_{a_i} - \delta_{a_i} e_{a_i}. \tag{31}$$

Under the assumption of constant population ($u_{a_i} + e_{a_i} = 1$), one can solve for these distributions in stationary equilibrium (i.e., setting $\dot{G}_{a_i}(w) = \dot{u}_{a_i} = \dot{e}_{a_i} = 0$):

$$G_{a_i}(w) = \frac{\lambda(\theta_{a_i}) F_{a_i}(w)}{\delta_{a_i} + s_{a_i} \lambda(\theta_{a_i})(1 - F_{a_i}(w))} \frac{u_{a_i}}{e_{a_i}}, \tag{32}$$

$$u_{a_i} = \frac{\delta_{a_i}}{\delta_{a_i} + \lambda(\theta_{a_i})}. \tag{33}$$

Equation (32) reveals the close connection between G_{a_i}, which corresponds to the observed employment wage distribution, and F_{a_i}, the wage offer distribution. In general, it can be shown that G_{a_i} diverges from F_{a_i}, with more and more mass being concentrated in the right part of the support of wages, when the frictional index $\chi_{a_i} \equiv \frac{\delta_{a_i}}{s_{a_i} \lambda(\theta_{a_i})}$ is low. As already noted by Burdett and Mortensen (1998), the frictional index is inversely related to the speed at which workers climb the job ladder in the model and represents a synthetic measure of the extent of labor market frictions in the economy.

We now investigate the relationship between G_{a_i} and F_{a_i}. Upon substituting the stationarity condition $\frac{u_{a_i}}{e_{a_i}} = \frac{\delta_{a_i}}{\lambda(\theta_{a_i})}$ into (32), we get

$$G_{a_i}(w) = \frac{\delta_{a_i} F_{a_i}(w)}{\delta_{a_i} + s_{a_i} \lambda(\theta_{a_i}) \left(1 - F_{a_i}(w)\right)} \implies \frac{G_{a_i}(w)}{F_{a_i}(w)} = \frac{\delta_{a_i}}{\delta_{a_i} + s_{a_i} \lambda(\theta_{a_i})(1 - F_{a_i}(w))}.$$

From the previous expression, it is clear that the G_{a_i} dominates in a first-order stochastic dominance sense the F_{a_i} distribution. Formally, $G_{a_i}(w) \leq F_{a_i}(w) \ \forall\, w$. It can also be noted that the parameter s_{a_i} regulates the distance between the G_{a_i} and the F_{a_i} distributions. In particular, as $s_{a_i} \to 0$ for a given δ_{a_i}, we note that $G_{a_i}(w) \to F_{a_i}(w)$. Conversely, the distance between the two distributions can be made arbitrarily large by setting large values of s_{a_i}. After some manipulation, the previous expression can be rearranged as:

$$G_{a_i}(w) = \frac{F_{a_i}(w)}{1 + \chi_{a_i}(\theta_{a_i})(1 - F_{a_i}(w))}. \tag{34}$$

This formulation makes clear that the relationship between the two distributions is fully determined by $\chi_{a_i}(\theta_{a_i})$. Intuitively, if employed workers find job offers at a much faster rate than they lose their job (i.e., fall off the ladder), then more mass will be placed to higher values in the support of the wage offer distribution. Finally, one can also note that the difference between the two distributions also depends on the rank of the firm in the wage offer distribution, $F_{a_i}(w)$. Intuitively, as $F_{a_i}(w) \to 1$, then $G_{a_i}(w) \to F_{a_i}(w) = 1$, and conversely $F_{a_i}(w) \to 0$, then $G_{a_i}(w) \to F_{a_i}(w) = 0$, i.e., the two distributions need to coincide at the upper and lower point of the wage offer distribution.

B.3 Derivation of Labor Supply Curve

The evolution of employment $\ell_{a_i}(w,v)$ of a firm hiring from market a, posting a piece rate w and v vacancies is governed by the following differential equation:

$$\dot{\ell}_{a_i}(w,v) = v\, q(\theta_{a_i}) \left(\frac{u_{a_i} + s_{a_i} e_{a_i} G_{a_i}(w)}{S_{a_i}} \right) - \Big(\delta_{a_i} + s_{a_i} \lambda(\theta_{a_i})(1 - F_{a_i}(w)) \Big) \ell_{a_i}(w,v),$$

where the first term represents the inflow and the second term represents the outflow pf workers. By evaluating the equation in stationary equilibrium, i.e., setting $\dot{\ell} = 0$, and using the fact that $q(\theta_{a_i}) = \frac{\lambda(\theta_{a_i})}{\theta_{a_i}} = \lambda(\theta_{a_i}) \frac{S_{a_i}}{V_{a_i}}$, stationary employment equals the labor supply curve reported in the main text (9):

$$\ell_{a_i}(w,v) = \frac{v}{V_{a_i}} \lambda(\theta_{a_i}) \frac{u_{a_i} + s_{a_i} e_{a_i} G_{a_i}(w)}{\delta_{a_i} + s_{a_i} \lambda(\theta_{a_i})(1 - F_{a_i}(w))},$$

that is the labor supply curve shown in the firms' problem in the main text. According to standard results in the equilibrium search literature, the employment wage distribution $G_{a_i}(w)$ is related to the wage offer distribution by the following relationship: $G_{a_i}(w) = \frac{\lambda(\theta_{a_i}) F_{a_i}(w)}{\delta_{a_i} + s_{a_i} \lambda(\theta_{a_i})(1 - F_{a_i}(w))} \frac{u_{a_i}}{e_{a_i}}$. Hence, one can substitute G_{a_i} away and obtain:

$$\ell_{a_i}(w,v) = \frac{v}{V_{a_i}} u_{a_i} \lambda(\theta_{a_i}) \frac{\delta_{a_i} + s_{a_i} \lambda(\theta_{a_i})}{\left[\delta_{a_i} + s_{a_i} \lambda(\theta_{a_i})(1 - F_{a_i}(w)) \right]^2}.$$

By using the stationarity condition for u_a and the definition of V_a, it is possible to further simplify the expression into

$$\ell_a(w,v) = v \, \frac{\delta_a \, \lambda(\theta_a)}{\left(\delta_a + s_a \lambda(\theta_a)(1 - F_a(w)) \right)^2}.$$

If the equilibrium wage distributions have mass points, one needs to discipline workers' transitions across equally-paying firms. Let q_p and q_ℓ denote the probability that a firm poaches a worker from an equally paying firm and that a worker accepts a wage offer from an equally paying firm, respectively. Stationary employment reads:

$$\ell_{a_i}(w,v;q_p,q_\ell) = \frac{v}{V_{a_i}} \lambda(\theta_{a_i}) \frac{u_{a_i} + s_{a_i} e_{a_i} [G_{a_i}(w^-) + q_p \left(G_{a_i}(w) - G_{a_i}(w^-) \right)]}{\delta_{a_i} + s_{a_i} \lambda(\theta_{a_i}) \left[1 - F_{a_i}(w) + q_\ell \left(F_{a_i}(w) - F_{a_i}(w^-) \right) \right]}.$$

Substituting for (34) yields:

$$\ell_{a_i}(w,v;q_p,q_\ell) = \frac{v}{V_{a_i}}\lambda(\theta_{a_i})u_{a_i}\frac{h_{a_i}(w^-)h_{a_i}(w) + s_{a_i}\lambda(\theta_{a_i})\left[F_{a_i}(w^-)h_{a_i}(w) + q_p\left(F_{a_i}(w)h_{a_i}(w^-) - F_{a_i}(w^-)h_{a_i}(w)\right)\right]}{h_{a_i}(w^-)h_{a_i}(w)[\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})\left(1 - q_\ell F_{a_i}(w^-) - (1-q_\ell)F_{a_i}(w)\right)]}, \tag{35}$$

where $h_{a_i}(w) \equiv \delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})(1 - F_{a_i}(w))$.

Letting $q_p = q_\ell$, we recover the familiar expression for stationary employment in the equilibrium search literature:

$$\ell_{a_i}(w,v) = \frac{v}{V_{a_i}}u_{a_i}\lambda(\theta_{a_i})\frac{\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})}{\left[\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})(1 - F_{a_i}(w))\right]\left[\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})(1 - F_{a_i}(w^-))\right]}.$$

B.4 Derivation of Product Demand Curve

The Lagrangean associated to the profit maximization problem of a final good producer (4) is the following:

$$\mathcal{L} = P\prod_{i=1}^{I}\left(\int\int_0^{K_{a_i}} Y_{k(a_i)}^{\frac{\rho-1}{\rho}}\, dk(a_i)\, d\Omega_i(a)\right)^{\alpha_i\frac{\rho}{\rho-1}} - \sum_{i=1}^{I}\int\int_0^{K_{a_i}}\sum_{j=1}^{N_{k(a_i)}} p_{jk(a_i)}y_{jk(a_i)}\, dk(a_i)\, d\Omega_i(a).$$

Taking the FOC with respect to a generic variety j in sector $k(a_i)$, we retrieve the (inverse) product demand function reported in the main text (10). Similarly, the FOCs with respect to (fictitious) sectoral and industry good aggregators read: $P_{k(a_i)} = P\left(\frac{Y_{k(a_i)}}{Y_i}\right)^{-\frac{1}{\rho}}\left(\frac{Y_i}{\alpha_i Y}\right)^{-1}$, $P_i = P\left(\frac{Y_i}{\alpha_i Y}\right)^{-1}$. Substituting for $Y_{k(a_i)}$ and Y_i into (10) yields the (direct) product demand curve reported in the main text (12). The price indices at different level of aggregation follow from consistency and zero-profit conditions:

$$P_{k(a_i)} = \left(\sum_{j=1}^{N_{k(a_i)}} p_{jk(a_i)}^{1-\sigma}\right)^{\frac{1}{1-\sigma}},\ P_i = \left(\int\int_0^{K_{a_i}} P_{k(a_i)}^{1-\rho}dk(a_i)d\Omega_i(a)\right)^{\frac{1}{1-\rho}},\ P = \left(\prod_{i=1}^{I}\left(\frac{\alpha_i}{P_i}\right)^{\alpha_i}\right)^{-1}.$$

Finally, we use the final good as numeraire and normalize the aggregate price index P to 1.

B.5 Derivation of Reservation Wage

Employed workers find it optimal to accept all job offers with a piece rate that exceeds the one of their current job, i.e., $R^e_{a_i}(w) = w$. By setting $W_{a_i}(aR^u_{a_i}) = U_{a_i}$ and solving for $R^u_{a_i}$, we can characterize the reservation wage for the unemployed workers as follows:

$$
\begin{aligned}
aR^u_{a_i} + \Pi(aR^u_{a_i}) + T &= \left[(ab_{a_i})^{1-\vartheta} + (1-\vartheta)(1-s_{a_i})\lambda(\theta_{a_i}) \int_{aR^u_{a_i}}^{a\bar{w}_{a_i}} \left(W_{a_i}(aw') - U_{a_i}\right) dF_{a_i}(w') \right]^{\frac{1}{1-\vartheta}} \\
&= \Bigg[(ab_{a_i} + T)^{1-\vartheta} + (1-\vartheta)(1-s_{a_i})\lambda(\theta_{a_i}) \int_{R^u_{a_i}}^{\bar{w}_{a_i}} \frac{1 - F_{a_i}(w)}{r + \delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})(1 - F_{a_i}(w))} \\
&\qquad \frac{1 + \Pi'(aw)}{(aw + \Pi(aw) + T)^{\vartheta}} \, dw \Bigg]^{\frac{1}{1-\vartheta}}.
\end{aligned}
$$

Since $\Pi' \geq 0$, the left-hand side is increasing in $R^u_{a_i}$ and the right-hand side is decreasing in $R^u_{a_i}$. Hence, the reservation wage for unemployed workers is unique.

B.6 Existence and Uniqueness of Stationary Equilibrium

In this section, we develop the proof of Proposition 1, claiming the existence and uniqueness of our symmetric stationary equilibrium with continuous wage offer distributions and exit ordering based on losses.

On each labor market, firms engage in a wage-posting game with a finite number of productivity types. Existence and uniqueness of a continuous equilibrium wage offer distribution have been established for the case of CRS revenue function and exogenous job creation by Mortensen (1988) and Burdett and Mortensen (1998). We now generalize the result to the case of DRS revenue function and endogenous job creation. Assume there are $n = 1, \ldots, \mathcal{Z}$ productivity types. Let γ_n denote the probability mass function of productivity. To simplify notation, we omit the a_i and k_{a_i} subscripts throughout. Consider the following problem:

$$
\begin{aligned}
\pi(z_n) &= \max_{w,v} \, (pz_n - w)v\tilde{\ell}(F(w)) - c(F(w), v) - \kappa \\
\text{s.t.} \quad \tilde{\ell}(F(w)) &= \frac{1}{V}\frac{\chi_0}{1+\chi_0}\frac{1+\chi_1}{[1+\chi_1(1-F(w^-))][1+\chi_1(1-F(w))]}, \; p = p(v\tilde{\ell}; z_n),
\end{aligned}
$$

where $\tilde{\ell}$ denotes the vacancy yield, i.e., stationary employment per vacancy, and $\chi_0 \equiv \frac{\lambda}{\delta}$, $\chi_1 \equiv \frac{s\lambda}{\delta}$ are (inverse) frictional indices. Since the number of productivity types is finite, no continuous equilibrium wage offer distribution exists in pure strategies. Hence, we proceed to characterize a mixed strategy Nash equilibrium (MSNE). The MSNE features residual wage dispersion for each productivity type, represented by type-specific wage offer distributions. Formally, the wage offer distribution of firms of type n, $F_n(w)$, is defined by:

$$
\begin{aligned}
(p_n z_n - w_n)v_n\tilde{\ell}_n - c_n - \kappa &= \pi_n \quad w_n \in \text{supp}F_n(w) = [\underline{w}_n, \bar{w}_n), \\
(p_n z_n - w_n)v_n\tilde{\ell}_n - c_n - \kappa &\leq \pi_n \quad w_n \notin \text{supp}F_n(w) = [\underline{w}_n, \bar{w}_n).
\end{aligned}
$$

For each wage level, optimal vacancy posting $v_n(w_n)$ solves:

$$
\frac{\partial c(F(w_n), v)}{\partial v} = \left(\frac{p_n(v\tilde{\ell}_n)}{\mu_n(v\tilde{\ell}_n)} z_n - w_n \right) \tilde{\ell}(F(w_n)),
$$

where $\frac{\partial c(F(w_n),v)}{\partial v \partial w_n} < 0$ and $\frac{\partial \tilde{\ell}(F(w_n))}{\partial w_n} > 0$. Notice that residual dispersion in wages is associated one-to-one with residual dispersion in marginal revenue products $\tilde{z}$ (that is, firms with the same productivity have different marginal revenue products in equilibrium). Specifically, wages need to be increasing in MRPL.[43] Hence, we can express wages and vacancies as a function of $\tilde{z}$. In other words, there exists a unique type-specific distribution of $\tilde{z}$ such that firm-level optimization is consistent with the equilibrium wage offer distribution. Monotonicity of wages in $\tilde{z}$ implies that $F_n(w_n(\tilde{z})) = \int_{\underline{\tilde{z}}}^{\tilde{z}} \frac{v_n(\tilde{z})}{\bar{v}_n} \varphi_n(\tilde{z}) d\tilde{z}$. It follows that the type-specific density of $\tilde{z}$ solves $\varphi_n(\tilde{z}) = \frac{\bar{v}_n}{v_n(w_n(\tilde{z}))} w_n'(\tilde{z}) f_n(w_n(\tilde{z}))$, where $f_n(w_n) \equiv F_n'(w_n)$ and $\bar{v}_n = \int_{\underline{\tilde{z}}_n}^{\bar{\tilde{z}}_n} v_n(\tilde{z}) \varphi_n(\tilde{z}) d\tilde{z}$.

We now guess that firms with higher productivity pay higher wages, that is, $\underline{w}_n \geq \bar{w}_{n-1}$. It follows that the vacancy yield $\tilde{\ell}$ is increasing in productivity. If the elasticity of substitution across sector-level output, ρ, is high enough, the revenue function is strictly supermodular, i.e., $\frac{\partial p(v\tilde{\ell}_n) z_n v \tilde{\ell}_n}{\partial (v\tilde{\ell}_n) \partial z_n} = \frac{\partial p(v\tilde{\ell}_n)/\mu(v\tilde{\ell}_n) z_n}{\partial z_n} > 0$.[44] Since the wage offer distribution is continuous and the productivity distribution has mass points, the marginal benefit of vacancy posting jumps upward moving to a higher productivity type, whereas the marginal cost increases continuously. Hence, more productive firms find it optimal to post more vacancies. From the MSNE definition, it follows that:

$$\begin{aligned}
\pi_n &= (p_n z_n - w_n) v_n \tilde{\ell}_n - c_n - \kappa \\
&\geq (p_{n-1} z_n - w_{n-1}) v_{n-1} \tilde{\ell}_{n-1} - c_{n-1} - \kappa \\
&> (p_{n-1} z_{n-1} - w_{n-1}) v_{n-1} \tilde{\ell}_{n-1} - c_{n-1} - \kappa = \pi_{n-1} \\
&\geq (p_{n-1} z_{n-1} - w_n) v_n \tilde{\ell}_n - c_n - \kappa,
\end{aligned}$$

where the first and third inequalities follow from firm's optimization. Therefore, $(p_n z_n - p_{n-1} z_{n-1}) v_n \tilde{\ell}_n \geq (p_n z_n - p_{n-1} z_{n-1}) v_{n-1} \tilde{\ell}_{n-1} \iff v_n \tilde{\ell}_n > v_{n-1} \tilde{\ell}_{n-1}$. Since $v_n > v_{n-1}$ and $\tilde{\ell}_n > \tilde{\ell}_{n-1}$, our guess is verified. Hence, if $\rho > \underline{\rho}$, the wage ranking equals the productivity ranking. Let $\pi_n(w, F(w))$ denotes profits of a firm of the n-th productivity type posting wage w and the corresponding optimal vacancies. Since firms with higher productivity pay higher

[43] If it were not the case, either wage dispersion would be inconsistent with firm's optimization ($w'(\tilde{z}) = 0$) or with equal profits on the support of the wage distribution ($w'(\tilde{z}) < 0$). Notice that monotonicity of vacancies in MRPL is not required for this argument.

[44] Since product markets sourcing from the same labor market are perfectly symmetric, for given wage, the marginal revenue product is a one-to-one correspondence with productivity. Any revenue function stemming from oligopolistic competition á la Cournot with CES demand system exhibits strict supermodularity if the elasticity of substitution across sector-level output is close enough to that across firm-level output. Formally, $\frac{\partial p(y) y}{\partial \ell \partial z} = p(y) \frac{1 - \mu'(y) y}{\mu^2(y)}$. Since $\lim_{\rho \to \sigma} \mu'(y) = 0$, there always exists a threshold $\underline{\rho}$ such that $\frac{\partial p(y) y}{\partial \ell \partial z} > 0 \; \forall y$ if $\rho > \underline{\rho}$.

wages, the equilibrium wage offer distribution is characterized by the following conditions:

$$
\begin{aligned}
&\underline{w}_1 = R, \quad \underline{w}_n = \bar{w}_{n-1}, \quad \forall n > 1, \\
&\bar{w}_n : \pi_n \left(\bar{w}_n, F(\underline{w}_n) + \gamma_n \frac{V_n}{V} \right) = \pi_n(\underline{w}_n, F(\underline{w}_n)) \\
&F_n(w) : \pi_n \left(w, F(\underline{w}_n) + \gamma_n \frac{V_n}{V} F_n(w) \right) = \pi_n(\underline{w}_n, F(\underline{w}_n)), \quad w \in [\underline{w}_n, \bar{w}_n).
\end{aligned}
$$

In words, the infimum of the support of the type-specific wage offer distribution is the workers' reservation wage for the lowest-productivity type and the supremum of the support of the wage offer distribution of the lower-productivity type for the others. The supremum of the support of the type-specific wage offer distributions is pinned down by the indifference condition between posting the lowest wage ($F_n(\underline{w}_n) = 0$) and the highest wage ($F_n(\bar{w}_n) = 1$). Similarly, the shape of the wage offer distribution is pinned down by the indifference condition among any wage in its support. The intuition is that, in equilibrium, firms of the same type need to be indifferent between extracting higher margins from a lower mass of workers or extracting lower margins from a larger mass of workers. Hence, the necessary condition for existence of the wage-posting equilibrium is that $\frac{\partial \pi(w,F(w);z_1)}{\partial F(w)}\Big|_{w=R} > 0$.[45] Under this necessary condition, the wage offer distribution of the lowest productivity type is non-degenerate. Given that the revenue function is supermodular in size and productivity and the market wage offer distribution is continuous, the same condition, i.e., $\frac{\partial \pi_n(w,F(w))}{\partial F(w)}\Big|_{w=\underline{w}_n} > 0$, holds for every productivity type, as well. Let $\varphi(\tilde{z})$ denote the market density of MRPL, which is the mixture of the type-specific MRPL distributions, i.e., $\varphi(\tilde{z}) \equiv \sum_{n=1}^{\mathcal{Z}} \varphi_n(\tilde{z})\gamma_n$. In turn, the market wage offer distribution is the mixture:

$$
F(w) = M \sum_{n=1}^{\mathcal{Z}} \gamma_n \frac{\bar{v}_n}{V} F_n(w) = M \int_{\underline{\tilde{z}}}^{\bar{\tilde{z}}} \frac{v(\tilde{z})}{V} \varphi(\tilde{z}) d\tilde{z},
$$

where $V = M \sum_{n=1}^{\mathcal{Z}} \bar{v}_n \gamma_n = M \int_{\underline{\tilde{z}}}^{\bar{\tilde{z}}} v(\tilde{z})\varphi(\tilde{z}) d\tilde{z}$ and $\bar{v}_n = \int v_n(\tilde{z})\varphi_n(\tilde{z}) d\tilde{z}$. Finally, for given market wage offer distribution, the reservation wage is unique (see equation (**??**)). Hence, we have constructed the unique wage-posting equilibrium with a continuous wage offer distribution.[46] Finally, notice that the price function in our model is sector-specific. Even though sectors sourcing from the same labor market are structurally identical, residual

[45]Since the reservation wage R is increasing in unemployment benefits b and $\frac{\partial \pi(w,F(w);z_1)}{\partial F(w)}\Big|_{w=0} > 0$ since revenues are increasing in size, there always exists a threshold $\bar{b}$ such that a wage-posting equilibrium exists if $b < \bar{b}$.

[46]As firstly pointed out by Mortensen and Vishwanath (1991), in the presence of DRS in revenues, the wage-posting equilibrium described so far may coexist with a competitive equilibrium featuring a degenerate wage offer distribution. In such an equilibrium, the unique wage clears the labor market by equalizing aggregate labor demand, stemming from the optimal vacancy condition, and aggregate (wage-insensitive) labor supply, coming from frictions. By allowing for degenerate equilibrium wage offer distributions, we conjecture that the nature of the best equilibrium (i.e., granting highest profits) depends on the value of the frictional index χ_1: if frictions are low enough, the equilibrium wage offer distribution can either have a density and a mass point or degenerate on a mass point, as described in Menzio (2024) for the case of frictional product markets.

wage dispersion would make them differ in terms of wage/MRPL distribution.[47] Hence, we assume that the demand curve firms internalize depends on the expected – rather than the realized – sectoral output, which is constant. As the number of productivity types grows larger, i.e., $\mathcal{Z} \to \infty$, our wage-posting equilibrium *purifies* in the sense that the type-specific wage offer distributions $F_n(w)$ degenerate into mass points, thus removing all the residual wage dispersion.

On each product market, a finite number of firms engage in a Cournot game on quantities. Due to strategic interaction, multiple equilibria with different sets of active firms may arise. Hence, to determine which firms from the set of potential firms are active, we need to specify an equilibrium selection device. To this purpose, we consider a refinement of the Nash equilibrium of a simultaneous entry game first introduced by Berry (1992) and used by De Loecker et al. (2021). Specifically, we first assume that all the potential firms operate and compute the general equilibrium accordingly. Upon computing equilibrium profits, we proceed by removing the firm which makes the highest losses, if any.[48] Iterating this procedure until all active firms make non-negative profits, we recover the equilibrium with the largest number of active firms. Hence, our equilibrium selection device constitutes the most conservative choice in terms of concentration response following shocks, such as the introduction of a MW.

B.7 Derivation of Firms' Policy Functions

We derive firm's policy functions in three steps. First, we characterize labor market policies as a function of the marginal revenue product of labor. Second, we characterize product market policies as a function of marginal costs. Third, we characterize firm's policy functions by setting marginal revenue product equal to marginal cost.

First, for given residual distributions of MRPL in labor market a_i for each productivity type, $\Phi_{a_i}(\tilde{z}; z)$, we solve for the firm's labor market policies as a function of its MRPL. We proceed by guessing (and verifying) that optimal wages are increasing in the MRPL. This allows us to express the wage offer distribution F_{a_i} in terms of vacancy policies as follows:

$$F_{a_i}\left(w_{a_i}(\tilde{z})\right) = \frac{M_{a_i}}{V_{a_i}} \int_{\underline{\tilde{z}}_{a_i}}^{\tilde{z}} v(z')\, d\Phi_{a_i}(z'), \tag{36}$$

where $\Phi_{a_i}(\tilde{z})$ is the MRPL distribution of potential firms in labor market a_i, and V_{a_i}/M_{a_i} denotes average vacancies per potential firm in labor market a_i.[49] Intuitively, the equilibrium wage offer distribution weights firms by their vacancy posting, which determines their *visibility*. It follows that the likelihood of a searching worker sampling a job offer up to a

[47]This is the case because, even if there is a continuum of sectors sourcing from the same labor market, each of them is populated by a finite number of firms.

[48]Since product markets sourcing from each labor market are perfectly symmetric, we remove the corresponding firm in all product markets at a time.

[49]See Appendix (B.6) for how to recover the unique market MRPL distribution $\Phi_{a_i}(\tilde{z})$ from the type-specific wage offer distributions.

given wage equals the relative mass of vacancies posted by lower-paying firms. Equation (36), provides a key to express the first-order conditions (13)-(14) as a system of differential equations pinning down the wage policy function. [50]

Upon taking the derivative of equation (36) with respect to $\tilde{z}$, we find out the mapping between the density of the wage offer distribution and that of the MRPL distribution at each wage level:

$$f_{a_i}(w_{a_i}(\tilde{z}))w'_{a_i}(\tilde{z}) = \frac{M_{a_i}}{V_{a_i}} v_{a_i}(\tilde{z})\varphi_{a_i}(\tilde{z}). \tag{37}$$

Let $\mathcal{H}_{a_i}(\tilde{z}) = F_{a_i}(w_{a_i}(\tilde{z}))$ be the wage offer distribution as a function of the MRPL. This implies that $f_{a_i}(w_{a_i}(\tilde{z})) = \frac{\mathcal{H}'_{a_i}(\tilde{z})}{w'_{a_i}(\tilde{z})}$. Combining this with equation (37) delivers the mass of vacancies per MRPL consistent with the equilibrium wage offer distribution:

$$\mathcal{H}'_{a_i}(\tilde{z}) = \frac{M_{a_i}}{V_{a_i}} v_{a_i}(\tilde{z})\varphi_{a_i}(\tilde{z}) \implies v_{a_i}(\tilde{z}) = \frac{V_{a_i}}{M_{a_i}} \frac{\mathcal{H}'_{a_i}(\tilde{z})}{\varphi_{a_i}(\tilde{z})}. \tag{38}$$

Replacing equation (37) and (38) into the FOCs of the firm's problem yields the following system of differential equations:

$$(v): \quad \mathcal{H}'_{a_i}(\tilde{z}) = \frac{M_{a_i}}{V_{a_i}} \varphi_{a_i}(\tilde{z}) \left(\frac{\tilde{z} - w_{a_i}(\tilde{z})}{\bar{c}_{a_i}} \frac{\lambda(\theta_{a_i})u_{a_i}(\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i}))/V_{a_i}}{(\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})\left[1 - \mathcal{H}_{a_i}(\tilde{z})\right])^{2+\phi}} \right)^{\frac{1}{\zeta}}, \tag{39}$$

$$(w): \quad w'_{a_i}(\tilde{z}) = (\tilde{z} - w_{a_i}(\tilde{z})) \frac{2\nu s_{a_i}\lambda(\theta_{a_i})\mathcal{H}'_{a_i}(\tilde{z})}{\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})\left[1 - \mathcal{H}_{a_i}(\tilde{z})\right]}. \tag{40}$$

This system of differential equations in the two functions $w_{a_i}(\tilde{z})$ and $\mathcal{H}_{a_i}(\tilde{z})$ yield the optimal wage and vacancy policies for each MRPL, provided the necessary boundary conditions. Since the market MRPL distribution is not uniformly continuous (it exhibits discontinuities across productivity types), the boundary conditions are type-specific:

$$\lim_{\tilde{z}_{a_i} \to \underline{\tilde{z}}_{a_i}(z_1)} w_{a_i}(\tilde{z}_{a_i}) = \max\left\{ R^u_{a_i}, \frac{\mathbf{w}}{a} \right\}, \lim_{\tilde{z} \to \underline{\tilde{z}}_{a_i}(z_1)} \mathcal{H}_{a_i}(\tilde{z}_1) = 0,$$

$$\lim_{\tilde{z} \to \underline{\tilde{z}}_{a_i}(z_n)} w_{a_i}(\tilde{z}_{a_i}) = w_{a_i}(\bar{\tilde{z}}_{a_i}(z_{n-1})), \lim_{\tilde{z} \to \underline{\tilde{z}}_{a_i}(z_n)} \mathcal{H}_{a_i}(\tilde{z}) = \mathcal{H}_{a_i}(\bar{\tilde{z}}_{a_i}(z_{n-1})), \quad \forall n = 2, \ldots, \mathcal{Z}_i,$$

where $\underline{\tilde{z}}_{a_i}(z_n)$ is the infimum of the type-specific MRPL distribution for the $n-$th type. The first condition states that the lowest-MRPL firm offers the lowest piece rate possible, that is either the reservation wage or the minimum wage for the lowest-productivity type and the supremum of the type-specific wage offer distribution of the lower-productivity type for the others. Instead, the second condition states that the CDF of offered wages is equal to 0 at the lowest wage for each type.[51]

[50] Unlike in Engbom and Moser (2022), the MRPL is an equilibrium object in our model. It follows that the labor market block of our model is independent of the product market block for a given distribution of MRPL.

[51] In equilibrium, the labor market tightness adjusts to guarantee that $\lim_{\tilde{z} \to \bar{\tilde{z}}_{a_i}} \mathcal{H}_{a_i}(\tilde{z}) = 1$, where $\bar{\tilde{z}}_{a_i}$ is the highest MRPL level among active firms. Equivalently, the total mass of vacancies posted by firms need to equal

Let $x_{a_i}(\tilde{z}) \equiv \frac{2\nu s_{a_i}\lambda(\theta_{a_i})\mathcal{H}'_{a_i}(\tilde{z})}{\delta_{a_i}+s_{a_i}\lambda(\theta_{a_i})[1-\mathcal{H}_{a_i}(\tilde{z})]}$ be an inverse measure of search frictions faced by workers employed at a firm with MRPL $\tilde{z}$ in the job finding process. Solving the differential equation (40) allows computing the wage policy function analytically as follows:

$$w_{a_i}(\tilde{z}) = e^{-\int_{\underline{\tilde{z}}_{a_i}}^{\tilde{z}} x_{a_i}(z)dz} R^u_{a_i} + \int_{\underline{\tilde{z}}_{a_i}}^{\tilde{z}} \hat{z}\, e^{-\int_{\hat{z}}^{\tilde{z}} x(z)dz} x_{a_i}(\hat{z})\, d\hat{z}. \tag{41}$$

For the sake of intuition, notice that the assumption of stationary employment as labor supply function makes the firm's sequential search problem isomorphic to a simultaneous search problem, i.e., to a static allocation problem of workers to firms. According to this interpretation, the weighting factor attached to the reservation wage in Equation (41) equals the probability that a worker does not meet any firm with MRPL weakly lower than $\tilde{z}$. On the other hand, the integral of the weighting factor $e^{-\int_{\hat{z}}^{\tilde{z}} x_{a_i}(z)dz} x(\hat{z})$ attached to each MRPL level equals the probability that a worker meets at least one firm with MRPL weakly lower than $\tilde{z}$. Formally, $\int_{\underline{\tilde{z}}_{a_i}}^{\tilde{z}} e^{-\int_{\hat{z}}^{\tilde{z}} x_{a_i}(z)dz} x_{a_i}(\hat{z})\, d\hat{z} = 1 - e^{-\int_{\underline{\tilde{z}}_{a_i}}^{\tilde{z}} x_{a_i}(\hat{z})\, d\hat{z}}$. Hence, we can interpret the wage posting game as a first-price sealed-bid auction on workers with an unknown number of competitors pinned down by the extent of search frictions. It follows that $\frac{e^{-\int_{\hat{z}}^{\tilde{z}} x_{a_i}(z)dz} x_{a_i}(\hat{z})}{1-e^{-\int_{\underline{\tilde{z}}_{a_i}}^{\tilde{z}} x_{a_i}(\hat{z})\, d\hat{z}}}$ denotes the density of the highest-paying firm with lower MRPL than $\tilde{z}$ contacted by a worker, conditional on contacting any of them. Hence, the wage function (41) equals the expected outside option of a searching worker (16), as reported in the main text.

Of course, equation (41) holds only within productivity types (because of kinks in the market wage distribution). The market wage function is given by:

$$\begin{aligned} w_{a_i}(\tilde{z}_{a_i}(z_n)) = &\left[p_n(\tilde{z}_{a_i}(z_n)) \prod_{j=1}^{n-1} p_j(\bar{\tilde{z}}_{a_i}(z_j))\right] R^u_{a_i} + p_n(\tilde{z}_{a_i}(z_n)) \sum_{j=1}^{n-1} \prod_{v=j+1}^{n-1} p_v(\bar{\tilde{z}}_{a_i}(z_v)) \\ &\int_{\underline{\tilde{z}}_{a_i}(z_j)}^{\bar{\tilde{z}}_{a_i}(z_j)} p_j(\hat{z})\hat{z}x_{j,a_i}(\hat{z})d\hat{z} + \int_{\underline{\tilde{z}}_{a_i}(z_n)}^{\tilde{z}_{a_i}(z_n)} p_n(\hat{z})\hat{z}x_{n,a_i}(\hat{z})d\hat{z}, \end{aligned}$$

where $p_n(\tilde{z}_{a_i}(z_n)) \equiv e^{-\int_{\underline{\tilde{z}}_{a_i}(z_n)}^{\tilde{z}_{a_i}(z_n)} x_{n,a_i}(z)dz}$. Notice that the interpretation of the market wage function is exactly the same as that proposed in (16).

The markdown policy is given by the ratio between MRPL and wage: $\psi_{a_i}(\tilde{z}) = \frac{\tilde{z}}{w_{a_i}(\tilde{z})}$. The employment policy is determined by the labor supply curve (9): $\ell_{a_i}(\tilde{z}) = \frac{v_{a_i}(\tilde{z})}{V_{a_i}} \frac{u_{a_i}\lambda(\theta_{a_i})[\delta_{a_i}+s_{a_i}\lambda(\theta_{a_i})]}{\left[\delta_{a_i}+s_{a_i}\lambda(\theta_{a_i})(1-\mathcal{H}_{a_i}(\tilde{z}))\right]^2}$.

Second, for given residual distribution of marginal costs $\mathcal{C}(z) \sim \Phi^c_{k(a_i)}(z)$, we solve for the firm's product market policies. Maximizing (8) with respect to ℓ yields:

$$p_{k(a_i)}(\mathcal{C}(z), z) = \mu_{k(a_i)}(\mathcal{C}(z), z)\frac{\mathcal{C}(z)}{z}. \tag{42}$$

the aggregate V_{a_i}.

Optimal employment is determined by the product demand curve (10):

$$p_{k(a_i)}(\mathcal{C}(z),z) = \left(az\ell_{k(a_i)}(\mathcal{C}(z),z)\right)^{-\frac{1}{\sigma}} Y_{k(a_i)} \left(\ell_{k(a_i)}(\mathcal{C}(z),z)\right)^{\frac{1}{\sigma}-\frac{1}{\rho}} Y_i^{\frac{1}{\rho}-1}\alpha_i Y, \tag{43}$$

where $Y_{k(a_i)}(\ell_{k(a_i)}(\mathcal{C}(z),z)) = \left[\sum_{j\neq i}^{N_k} \bar{y}_{jk(a_i)}^{\frac{\sigma-1}{\sigma}} + \left(a\, z\, \ell_{k(a_i)}(\mathcal{C}(z),z)\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$ and $\bar{y}_{jk(a_i)}$ is the expected output of firm j, where the expectation is taken with respect to the residual distribution of MCL. Optimal employment then determines both the optimal market share and markup as follows: $sh_{k(a_i)}(\mathcal{C}(z),z) = \frac{(az\ell_{k(a_i)}(\mathcal{C}(z),z))^{\frac{\sigma-1}{\sigma}}}{Y_{k(a_i)}(\ell_{k(a_i)}(\mathcal{C}(z),z))^{\frac{\sigma-1}{\sigma}}}$, $\mu_{k(a_i)}(\mathcal{C}(z),z) = \frac{\sigma}{(\sigma-1)\left[1-\frac{\frac{\sigma}{\rho}-1}{\sigma-1}sh_{k(a_i)}(\mathcal{C}(z),z)\right]}$.

Finally, firm's policy functions obtain by setting $\Phi_{a_i}(\tilde{z},z) = \Phi^c_{k(a_i)}(\tilde{c},z)$ and equalizing MRPL and MCL: $\tilde{z}_{k(a_i)}(z) \equiv \frac{p_{k(a_i)}(z)}{\mu_{k(a_i)}(z)}z = \psi_{a_i}(z)w_{a_i}(z) \equiv \tilde{c}_{a_i}(z)$, where all the policy functions are set-valued.

B.8 Derivation of Equilibrium Markdown Function

In this Section we show how to derive the equilibrium markdown function reported in the main text (20) starting from the firm's marginal cost function.

$$\begin{aligned}
MCL(\ell) &= w(\ell) + \frac{\partial w(\ell)}{\partial \ell}\ell + \frac{\partial c\,(w(\ell),v(\ell))}{\partial v}\frac{\partial v(\ell)}{\partial \ell} + \frac{\partial c\,(w(\ell),v(\ell))}{\partial w}\frac{\partial w(\ell)}{\partial \ell} \\
&= w(\ell)\left(1+\epsilon_{w,\ell} + \frac{1}{w}\frac{\partial c\,(w(\ell),v(\ell))}{\partial v}\frac{\partial v(\ell)}{\partial \ell} + \frac{1}{w}\frac{\partial c\,(w(\ell),v(\ell))}{\partial w}\frac{\partial w(\ell)}{\partial \ell}\right) \\
&= w(\ell)\left(1+\frac{1+\partial c/\partial w \ell}{\epsilon_{\ell,w}}\right) = w(\ell)\left(1+\frac{1}{\nu\epsilon_{\ell,w}}\right),
\end{aligned}$$

where we made use of the firm's indifference condition between vacancies and wages (14)-(13), the total differential of the employment constraint, and the linearity of employment in vacancies:

$$\frac{\partial c/\partial v}{\partial \ell/\partial v} = \frac{\partial c}{\partial v}\frac{v}{\ell} = \left(1+\frac{1}{\ell}\frac{\partial c}{\partial w}\right)\frac{w}{\epsilon_{\ell,w}} \iff (1+\zeta)\frac{c}{w\ell}\epsilon_{\ell,w} = 1-\frac{\phi}{2}\frac{c}{w\ell}\epsilon_{\ell,w} \implies 1+\frac{\partial c}{\partial w}\frac{1}{\ell} = \frac{1}{\nu},$$
$$\ell = \ell(w,v) \implies \epsilon_{\ell,w}\epsilon_{w,\ell} + \epsilon_{v,\ell} = 1.$$

B.9 Aggregation

In this section we describe how to aggregate variables at different levels, according to the structure of our model.

Sectoral level. We start from the definition of sectoral revenues and wage bill: $P_{k(a_i)}Y_{k(a_i)} = \sum_{j=1}^{N_{k(a_i)}} p_{jk(a_i)}y_{jk(a_i)}$, $w_{k(a_i)}L_{k(a_i)} = \sum_{j=1}^{N_{k(a_i)}} w_{jk(a_i)}\ell_{jk(a_i)}$ Let $Z_k(a_i) = \frac{Y_k(a_i)}{aL_{k(a_i)}}$ denote the sectoral firm productivity. We define a sectoral net market power index, $\mathcal{M}_{k(a_i)}$, by assuming that the firm-level optimal price condition (15) holds at the sectoral level, as well: $P_{k(a_i)}Y_{k(a_i)} =$

$\sum_{j=1}^{N_{k(a_i)}} p_{jk(a_i)} y_{jk(a_i)}$, $\mathcal{M}_{k(a_i)} = \sum_{j=1}^{N_{k(a_i)}} \frac{w_{jk(a_i)}\ell_{jk(a_i)}}{w_{k(a_i)}L_{k(a_i)}} \mathcal{M}_{jk(a_i)}$, where $\mathcal{M}_{jk(a_i)} \equiv \mu_{jk(a_i)}\psi_{jk(a_i)}$ is the firm-level net market power index. Hence, the model-consistent aggregation of the net market power index is wage-bill-weighted. Along the same lines, we proceed by defining sectoral markup and markdown as their wage-bill-weighted averages: $\mu_{k(a_i)} \equiv \sum_{j=1}^{N_{k(a_i)}} \frac{w_{jk(a_i)}\ell_{jk(a_i)}}{w_{k(a_i)}L_{k(a_i)}} \mu_{jk(a_i)}$, $\psi_{k(a_i)} \equiv \sum_{j=1}^{N_{k(a_i)}} \frac{w_{jk(a_i)}\ell_{jk(a_i)}}{w_{k(a_i)}L_{k(a_i)}} \psi_{jk(a_i)}$. We notice that the sectoral net market power index equals the product between sectoral markup and markdown plus their covariance term: $\mathcal{M}_{k(a_i)} = \mu_{k(a_i)}\psi_{k(a_i)} + \mathbf{Cov}[\mu_{k(a_i)}, \psi_{k(a_i)}]$. Hence, some positive correlation between firm-level markups and markdowns makes the sectoral net market power index exceed the product between sectoral markups and markdowns.[52] Next, let the sectoral hiring cost and overhead cost equal $\mathcal{HC}_{k(a_i)} = \sum_{j=1}^{N_{k(a_i)}} ac_{jk(a_i)}$ and $\mathcal{FC}_{k(a_i)} = \sum_{j=1}^{\bar{N}_{k(a_i)}} a\kappa_{k(a_i)}$, respectively. The sectoral labor share equals: $LS_{k(a_i)} = \frac{aw_{k(a_i)}L_{k(a_i)}}{P_{k(a_i)}Y_{k(a_i)} - \mathcal{HC}_{k(a_i)} - \mathcal{FC}_{k(a_i)}}$

Industry level. Since workers' ability differ within industry, industry wage bill equals $W_iL_i = \int\int_0^{K_{a_i}} aw_{k(a_i)}L_{k(a_i)}dk(a_i)d\Omega_i(a)$. Repeating the same steps as at the sectoral level yields: $\mu_i = \int\int_0^{K_{a_i}} \frac{aw_{k(a_i)}L_{k(a_i)}}{W_iL_i}\mu_{k(a_i)}dk(a_i)d\Omega_i(a)$, $\psi_i = \int\int_0^{K_{a_i}} \frac{w_{k(a_i)}L_{k(a_i)}}{W_iL_i}\psi_{k(a_i)}dk(a_i)d\Omega_i(a)$, $\tilde{Z}_i = \frac{Y_i}{L_i}$, $LS_i = \frac{W_iL_i}{P_iY_i - \mathcal{C}_i^h - \mathcal{C}_i^f}$, where $\tilde{Z}_i$ is the industry-level (total) labor productivity.

Aggregate level. Repeating the same steps as at the industry level yields: $\mu = \sum_{i=1}^{I} \frac{W_iL_i}{WL}\mu_i$, $\psi = \sum_{i=1}^{I} \frac{W_iL_i}{WL}\psi_i$, $\tilde{Z} = \frac{Y}{L}$, $LS = \frac{WL}{PY - \mathcal{HC} - \mathcal{FC}}$, where $\tilde{Z}$ is the aggregate (total) labor productivity. Finally, aggregate value added (consumption) is defined as: $C = Y - \mathcal{HC} - \mathcal{FC}$.

B.10 Algorithm to solve the model

Consistently with our estimation strategy, we solve the discretized version of our model with a continuum of productivity types. Hence, we let residual wage (and MRPL) dispersion vanish and work with one-to-one firms' policy functions (rather than set-valued).

The model is solved by guess and verify for the collection of MRPL functions $\tilde{z}_{k(a_i)}(z)$. Although the same solution concept applies to any set of parameters, we will add some specific comments related to an increase in the MW parameter.

To initialize the solution routine, we make an initial guess on the collection of MRPL functions, $\tilde{z}^0_{k(a_i)}(z)$. If the MW is higher than some guessed $\tilde{z}$ values (with a positive number of firms), we adjust any such points upward until they all exceed the MW. This preliminary adjustment amounts to conjecturing that all the potential firms are active. In this respect, we leverage the theoretical insight that the presence of market power on both the labor and the product market guarantees that all firms make positive operating profits for *any* MW level. Since overhead costs do not affect firms' policy functions in any way, we can therefore find a candidate equilibrium for any set of active firms. To discipline the extensive margin of adjustment (firm exit) , we check ex-post whether the candidate equilibrium is sustainable, i.e., whether any

[52] In our simulations, the discrepancy is negligible, though.

firm is making negative profits at that equilibrium, and, if not, gradually removing firms from the market.

Upon initializing the solution routine as just described, we solve the model according to the $\tilde{z}^0(z)$ guess. This is performed in four steps:

1. Solve for the equilibrium in each labor market $a_i = 1, \ldots, IA$ as follows:

   – Compute the number of firms at each $\tilde{z}^0_{a_i}(z)$-value as a share of the number of active firms (which is set equal to the number of potential firms in the estimated model) to identify the discretized counterpart of the MRPL density $\phi_{a_i}(\tilde{z})$;

   – Guess the wage and wage offer distribution as a function of the guessed $\tilde{z}^0_{a_i}$ function, $w_{a_i}(\tilde{z}^0_{a_i})$ and $\mathcal{H}_{a_i}(\tilde{z}^0_{a_i})$. Apply a first-order Taylor expansion to the nonlinear system of differential equations (39)-(40) around the guessed wage and wage offer distribution functions to transform it into a linear system of differential equations. Then, use the Euler (finite difference) method to transform the linear system of differential equations into a linear system of difference equations with boundary conditions $w_{a_i}(\underline{\tilde{z}_{a_i}}) = R^u_{a_i}$ and $\mathcal{H}_{a_i}(\underline{\tilde{z}_{a_i}}) = 0$.[53];

   – Find the labor-market-specific equilibrium job finding rate by solving the linear system of difference equations: this can be performed efficiently by first identifying a lower bound and an upper bound to the equilibrium job finding rate corresponding to too much job creation, i.e., $\lim_{\tilde{z}_{a_i} \to \bar{\tilde{z}}_{a_i}} \mathcal{H}_{a_i}(\tilde{z}_{a_i}) > 1$, and too little job creation, i.e., $\lim_{\tilde{z}_{a_i} \to \bar{\tilde{z}}_{a_i}} \mathcal{H}_{a_i}(\tilde{z}_{a_i}) < 1$, respectively, and then applying a bisection algorithm within such bounds;

   – Compute the labor-market-specific policy functions:

   $$\{w_{a_i}(\tilde{z}^0_{a_i}), \mathcal{H}_{a_i}(\tilde{z}^0_{a_i}), v_{a_i}(\tilde{z}^0_{a_i}), \ell_{a_i}(\tilde{z}^0_{a_i}), \psi_{a_i}(\tilde{z}^0_{a_i})\}$$

   by making use of (40), (39), (38), (**??**), and (**??**), respectively.

2. Solve for the equilibrium in each product market $k = 1, \ldots, K$ as follows:

   – Compute sectoral, industry, and aggregate output by aggregating up firm-specific output policies $y_{k(a_i)}(\tilde{z}^0_{k(a_i)}(z)) = az\ell_{k(a_i)}(\tilde{z}^0_{k(a_i)}(z))$ according to (7), (6), and (5), respectively. To avoid that the number of sectors affects industry output, we remove love-of-variety effects from the discretized industry CES aggregator, that is, industry output reads:

   $$Y_i^{\text{discr}} = K_i^{-\frac{1}{\rho-1}} \left(\sum_{k=1}^{K_{a_L i}} Y_{k(a_L i)}^{\frac{\rho-1}{\rho}} + \sum_{k=1}^{K_{a_H i}} Y_{k(a_H i)}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}};$$

[53] We check that results are unchanged by applying the Euler method directly to the nonlinear system of differential equations.

– Compute firm-specific price policies by making use of the demand constraint (10), as well as sectoral, industry, and aggregate price indices according to (**??**)-(**??**), by aggregating up firm-level prices;

– Thanks to the oligopolistically-competitive product market structure and nested CES preferences, firm-level prices are sufficient for pinning down each firm's market share by

$$sh_{k(a_i)}(\tilde{z}^0_{k(a_i)}(z)) = \frac{p_{k(a_i)}\left(\tilde{z}^0_{k(a_i)}(z)\right)^{1-\sigma}}{N_{k(a_i)}\sum_{n=1}^{Z_{k(a_i)}} p_{k(a_i)}\left(\tilde{z}^0_{k(a_i)}(z_n)\right)^{1-\sigma}\gamma_{k(a_i)}(z_n)},$$

and markup policy by (18);

– Compute firm-specific profits as:

$$\pi_{k(a_i)}\left(\tilde{z}^0_{k(a_i)}(z_n)\right) = p_{k(a_i)}\left(\tilde{z}^0_{k(a_i)}(z_n)\right)y_{k(a_i)}\left(\tilde{z}^0_{k(a_i)}(z_n)\right) - ac_{k(a_i)}\left(\tilde{z}^0_{k(a_i)}(z_n)\right) - a\kappa_{k(a_i)}.$$

Compute the implied MRPLs, $\tilde{z}^{implied}_{k(a_i)}(z)$, by making use of the MRPL definition, i.e.,

$$\tilde{z}^{implied}_{k(a_i)}(z) \equiv \frac{p_{k(a_i)}\left(\tilde{z}^0_{k(a_i)}(z_n)\right)}{\mu_{k(a_i)}\left(\tilde{z}^0_{k(a_i)}(z_n)\right)}z.$$

3. Upon solving the model conditional on the $\tilde{z}^0_{k(a_i)}(z)$ guess, we proceed by verifying and potentially updating the guess. To do so, the solution algorithm goes through the following steps:

   – Let $\epsilon^0 \equiv \mathbb{E}\left[|\tilde{z}^0_{a_i}(z) - \tilde{z}^{implied}_{a_i}(z)|\right]$ denote the average convergence error, and set a sensitivity $\bar{\epsilon}$ ($= 10^{-4}$ in our simulations);[54]

   – If $\epsilon^0 > \bar{\epsilon}$, we update the initial $\tilde{z}^0$-guess via bisection, i.e. $\tilde{z}^1_{a_i}(z) = 0.5^*\tilde{z}^0_{a_i}(z) + 0.5^*\tilde{z}^{implied}_{a_i}(\tilde{z}^0_{a_i}(z))$. If the new guess $\tilde{z}^1$ features nonmonotonicity in some labor market, we reduce the weight on the implied MRPL until such nonmonotonicity disappears.[55] Repeat the steps 1-3 iteratively until finding a guess $\tilde{z}^n_{a_i}(z)$ at the $n-$th repetition such that $\epsilon^n < \bar{\epsilon}$;

   – Store the model solution for the $\tilde{z}^n_{a_i}(z)$ guess as candidate equilibrium.

4. Check that no firm makes negative profits in the candidate equilibrium. If it is the

[54]Our guess $\tilde{z}^0_{a_i}(z)$ is common across firms with the same productivity sourcing from the same labor market, i.e., $\tilde{z}^0_{k(a_i)}(z) = \tilde{z}^0_{a_i}(z)$. Due to integer constraints, sectors sourcing from the same labor market may not be exactly identical. In those cases, we compute the implied MRPL, $\tilde{z}^{implied}_{a_i}(z)$, as the sales-weighted average of implied MRPLs of firms with the same productivity.

[55]Because of the monotonicity restriction, this simple modified bisection algorithm is preferable to Jacobian-based methods, as well as more efficient.

case, the candidate equilibrium is sustainable and the model is solved. Otherwise, an extensive margin adjustment needs to be enacted:

– Following the equilibrium refinement device of Berry (1992), the firm making lowest (negative) profits is removed from the market. Since our model features identical sectors sourcing from the same labor market, we remove one of the worst loss-making firms in each identical sector at a time.[56] Then, the algorithm restarts from step 1.

[56] If the worst loss-making firms are more than 100 in each sector, we remove 10% of them to speed up the algorithm. We check that this shortcut has no bearing on the results.

B.11 Dynamic Firm's problem

The sequential firm's profit maximization problem reads:

$$\max_{w_t \geq \underline{w}/a, v_t \geq 0} \int_0^\infty e^{-rt}\, a \left[\left(p_{k(a_i)}(y_t) z - w_t \right) \ell_t - c_{a_i}(w_t, v_t) - \kappa_{a_i} \right] dt \tag{44}$$

$$\text{s.t.}\quad \dot{\ell}_t = -\left(\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i t})[1 - F_{a_i t}(w_t)] \right) \ell_t + \frac{v_t}{V_{a_i t}} \lambda(\theta_{a_i t})[u_{a_i t} + s_{a_i}(1 - u_{a_i t}) G_{a_i t}(w_t)], \tag{45}$$

$$p_{k(a_i)}(y_t) = y_t^{-\frac{1}{\sigma}}\, Y_{k_{a_i} t}(y_t)^{\frac{1}{\sigma} - \frac{1}{\rho}}\, Y_{it}^{\frac{1}{\rho} - 1} \alpha_i Y_t, \tag{46}$$

$$y_t = az\ell_t \tag{47}$$

We proceed by setting up the current value Hamiltonian and dropping time indices:

$$H(w, v; \ell) = \pi(w, v; \ell) + \xi \left(\frac{v}{V_{a_i}} \lambda(\theta_{a_i})[u_{a_i} + s_{a_i}(1 - u_{a_i}) G_{a_i}(w)] - \left(\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})[1 - F_{a_i}(w)] \right) \ell \right)$$

where $\pi(w, v; \ell) = \left(p_{k(a_i)}\left(y_{a_i}(\ell) \right) z - w \right) \ell - c_{a_i}(w, v)$ and ξ is the co-state variable.

The first-order conditions are:

$$\frac{\partial H}{\partial w} = 0 \iff \ell + \frac{\partial c_{a_i}(w, v)}{\partial w} = \xi \left(\frac{v}{V_{a_i}} \lambda(\theta_{a_i}) s_{a_i}(1 - u_{a_i}) g_{a_i}(w) + s_{a_i}\lambda(\theta_{a_i}) f_{a_i}(w) \ell \right) \tag{48}$$

$$\frac{\partial H}{\partial v} = 0 \iff \frac{\partial c_{a_i}(w, v)}{\partial v} = \xi \frac{\lambda(\theta_{a_i})[u_{a_i} + s_{a_i}(1 - u_{a_i}) G_{a_i}(w)]}{V_{a_i}} \tag{49}$$

$$\frac{\partial H}{\partial \ell} = r\xi - \dot{\xi} \iff \tilde{z}_{k(a_i)} - w - \xi \left(\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})[1 - F_{a_i}(w)] \right) = r\xi - \dot{\xi} \tag{50}$$

$$\frac{\partial H}{\partial \xi} = \dot{\ell} \iff \dot{\ell}_t = -\left(\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})[1 - F_{a_i}(w)] \right) \ell + \frac{v}{V_{a_i}} \lambda(\theta_{a_i})[u_{a_i} + s_{a_i}(1 - u_{a_i}) G_{a_i}(w)] \tag{51}$$

where $\tilde{z}_{k(a_i)} = \left(1 + \epsilon_{k(a_i)}^{-1}(\ell) \right) p_{k(a_i)} z$ denotes the MRPL. In stationary equilibrium, i.e., setting $\dot{\ell} = \dot{\xi} = 0$, the wage and vacancy policy functions read:

$$\tilde{z}_{k(a_i)} - w = \frac{r + \delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})[1 - F_{a_i}(w)]}{2 s_{a_i}\lambda(\theta_{a_i}) f_{a_i}(w)} \left(1 - \phi \frac{s_{a_i}\lambda(\theta_{a_i}) f_{a_i}(w)}{\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})[1 - F_{a_i}(w)]} \frac{c_{a_i}(w, v)}{\ell_{a_i}(w, v)} \right), \tag{52}$$

$$\tilde{z}_{k(a_i)} - w = \frac{r + \delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})[1 - F_{a_i}(w)]}{\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i})[1 - F_{a_i}(w)]} (1 + \zeta) \frac{c_{a_i}(w, v)}{\ell_{a_i}(w, v)} \tag{53}$$

Equations (52)-(53) boil down to (13)-(14) in the *timeless* limit, i.e., as $r \to 0$. By putting together the two FOCs, we can compute the counterpart of the equilibrium markdown function (20) as follows:

$$\psi_{a_i}(\tilde{z}) = 1 + \frac{r/(s_{a_i}\lambda(\theta_{a_i})) + \chi_{a_i}(\theta_{a_i}) + [1 - F_{a_i}(w_{a_i}(\tilde{z}))]}{2\nu f_{a_i}(w_{a_i}(\tilde{z})) w_{a_i}(\tilde{z})}. \tag{54}$$

Time discounting raises equilibrium markdowns, without changing any qualitative results.

C Efficiency Properties of Baseline Equilibrium

In this section we characterize the constrained efficient allocation of our baseline model. To do so, we solve the problem of a social planner that aims to maximize aggregate consumption in steady state: [57]

$$\mathcal{W} = \max_{\substack{\vec{v}^j_{a_i}(z_n)\geq 0,\\ \vec{q}_{1,a_i}(n,n')\in[0,1],\\ \vec{q}_{2,a_i}(j,j',n)\in[0,1]}} Y - \sum_{i=1}^{I} \int a M_{a_i} \left(\sum_{n}^{\mathcal{Z}_i} \int_0^1 c^j_{a_i}\left(\vec{q}_{1,a_i}, \vec{q}_{2,a_i}, V_{a_i}, v_{a_i}(z_n)\right) dj\ \gamma_{a_i}(z_n) \right) d\Omega_i(a)\Xi(i),$$

$$\text{s.t.}\quad Y = \prod_{i=1}^{I} Y_i^{\alpha_i}, \quad Y_i = \left(\int \int_0^{K_{a_i}} Y_{k(a_i)}^{\frac{\rho-1}{\rho}} dk_{a_i}\, d\Omega_i(a) \right)^{\frac{\rho}{\rho-1}}, \tag{55}$$

$$Y_{k(a_i)} = \left(N_{k(a_i)} \sum_{n=1}^{\mathcal{Z}_i} \int_0^1 y^j_{k(a_i)}(z_n)^{\frac{\sigma-1}{\sigma}} dj\ \gamma_{k(a_i)}(z_n) \right)^{\frac{\sigma}{\sigma-1}}, \tag{56}$$

$$y^j_{k(a_i)}(z_n) = a z_n \ell^j_{a_i}\left(\vec{q}_{a_i}(z_n, z_{-n}), V_{a_i}, v_{a_i}(z_n)\right), \tag{57}$$

$$\ell^j_{a_i}\left(\vec{q}_{a_i}, V_{a_i}, \vec{v}_{a_i}(z)\right) = \frac{v^j_{a_i}(z_n)}{V_{a_i}} \lambda_{a_i} u_{a_i} \frac{h^j_{a_i}(1,n) h^j_{a_i}(0,n) + s_{a_i}\lambda_{a_i}\left[(1-\bar{\mathcal{H}}^j_{2,a_i}(n))h^j_{a_i}(1,n) + \dots\right.}{h^j_{a_i}(1,n)\ h^j_{a_i}(0,n)\ h^j_{a_i}(q^j_2(n),n)} \tag{58}$$

$$\frac{\left.\dots(1-q^j_2(n))\left((1-\bar{\mathcal{H}}^j_{1,a_i}(n))h^j_{a_i}(0,n) - (1-\bar{\mathcal{H}}^j_{2,a_i}(n))h^j_{a_i}(1,n)\right)\right]}{}, \tag{59}$$

$$u_{a_i}(\lambda_{a_i}) = \frac{\delta_{a_i}}{\delta_{a_i} + \lambda_{a_i}}, \tag{60}$$

$$\lambda_{a_i}\left(V_{a_i}\right) = \mathcal{M}\left(1, \frac{V_{a_i}}{S_{a_i}(\lambda_{a_i}(V_{a_i}))}\right), \tag{61}$$

$$h^j_{a_i}(p,n) = \delta_{a_i} + s_{a_i}\lambda_{a_i}\left[1 - p\left(1 - \bar{\mathcal{H}}^j_{1,a_i}(n)\right) + (1-p)\left(1 - \bar{\mathcal{H}}^j_{2,a_i}(n)\right)\right], \tag{62}$$

$$\bar{\mathcal{H}}^j_{1,a_i}\left(\vec{q}_{1,a_i}, \vec{q}_{2,a_i}, V_{a_i}, \vec{v}_{a_i}(z_{n'})\right) = M_{a_i} \sum_{n'\neq n} q_{1,a_i}(n,n') \frac{\bar{v}_{a_i}(z_{n'})}{V_{a_i}} \gamma_{a_i}(z_{n'}) + M_{a_i} \frac{\bar{v}_{a_i}(z_n)}{V_{a_i}} \gamma_{a_i}(z_n) q^j_2(n), \tag{63}$$

$$\bar{\mathcal{H}}^j_{2,a_i}\left(\vec{q}_{1,a_i}, \vec{q}_{2,a_i}, V_{a_i}, \vec{v}_{a_i}(z_{n'})\right) = M_{a_i} \sum_{n'\neq n} q_{1,a_i}(n,n') \frac{\bar{v}_{a_i}(z_{n'})}{V_{a_i}} \gamma_{a_i}(z_{n'}), \tag{64}$$

$$q^j_2(n) = \int_0^1 q_{2,a_i}(j,j',n) \frac{v^{j'}_{a_i}(z_n)}{V_{a_i}(z_n)} dj', \tag{65}$$

$$q_{1,a_i}(n,n') + q_{a_i}(n',n) \leq 1,\ \forall n,n', \quad q_{2,a_i}(j,j',n) + q_{2,a_i}(j',j,n) \leq 1,\ \forall j,j',n, \tag{66}$$

where $V_{a_i} = M_{a_i} \sum_n \bar{v}_{a_i}(z_n)\gamma_{a_i}(z_n)$, $S_{a_i}\left(\lambda_{a_i}\right) = u_{a_i}(\lambda_{a_i}) + s_{a_i}\left(1 - u_{a_i}(\lambda_{a_i})\right)$, and $c^j_{a_i}(z_n) = \bar{c}_{a_i} h_{a_i}(0,z_n)^{\phi} \frac{v^j_{a_i}(z_n)^{1+\zeta}}{1+\zeta}$.

The social planner seeks to maximize aggregate consumption by leveraging two sets

[57] Hence, we abstract both from distributional concerns and elastic labor supply.

of control variables. First, the planner mandates each firm j with productivity z and skill requirement a operating in industry i to post a mass of vacancies $v^j_{a_i}(z)$. Second, the planner chooses the probability $q_{1,a_i}(n,n')$ with which a worker with skill a in industry i employed in a firm with productivity z_n transitions into a firm with productivity $z_{n'}$ if given the chance, and the probability $q_{2,a_i}(j,j',n)$ with which a worker with skill a in industry i employed in a firm with productivity z_n transitions into a firm with the same productivity if given the chance. These transition probabilities give rise to a a *shadow wage offer distribution* $\bar{\mathcal{H}}^j_{a_i}\left(\vec{q}_{1,a_i},\vec{q}_{2,a_i},V_{a_k},\vec{v}^{j'}_{a_i}(z_{n'})\right)$ as defined by (63).[58] Exactly as the equilibrium wage offer distribution, the shadow wage offer distribution plays an allocative role. Intuitively, firms' ranking in the $\mathcal{H}$ distribution determines the direction of worker reallocation. The problem setup allows for residual dispersion in policy functions if identical firms are dictated different reallocation patterns, i.e., if $q^j_2(n) \neq q^{j'}_2(n)$. The stationary employment constraint (58) equals the flow-consistent firm-level employment for a generic residual transition probability $q^j_2(n)$ among equally productive firms in the presence of finite productivity types (see (35) for its equilibrium counterpart). This expression has the property that $M_{a_i}\sum_n \bar{\ell}_{a_i}(z_n) = 1 - u_{a_i}(\lambda_{a_i})$, thus effectively representing a (frictional) labor resource constraint.

We now substitute for the tightness constraint (61) into the Beveridge curve (60) and plug the latter into the objective function. The Lagrangean associated to the social planner problem reads:

$$\mathcal{L}(\vec{v}^j_{a_i}(z),\vec{q}_{1,a_i},\vec{q}_{2,a_i}) = Y\left(\vec{\ell}^j_{a_i}\left(\vec{q}_{1,a_i},\vec{q}_{2,a_i},V_{a_i},\vec{v}^j_{a_i}(z)\right)\right) - \sum_{i=1}^{I}\int a\Bigg[M_{a_i}\Bigg(\sum_n^{\mathcal{Z}_i}\int_0^1 c^j_{a_i}\left(\vec{q}_{1,a_i},\vec{q}_{2,a_i},V_{a_i},\vec{v}^j_{a_i}(z_n)\right)dj$$
$$\gamma_{a_i}(z_n)\Bigg) - \sum_n\sum_{n'}\xi_{1,a_i}(n,n')[1-q_{1,a_i}(n,n')-q_{a_i}(n',n)] - \sum_n\int_0^1\int_0^1\xi_{2,a_i}(j,j',n)[1-q_{2,a_i}(j,j',n)-q_{a_i}(j',j,n)]$$
$$+\int_0^1 \xi^j_{v,a_i}(z_n)v^j(z_n)dj\Bigg]d\Omega_i(a)\Xi(i),$$

where $\xi^j_{v,a_i}(z_n) \geq 0$ is the Kuhn Tucker multiplier attached to the non-negativity constraint on vacancies, while $\xi_{1,a_i}(n,n')$ and $\xi_{2,a_i}(j,j',n)$ are the Lagrange multiplier attached to the adding-up constraints on the transition probabilities.

The efficient transition probability from two firms j and j' with the same productivity z_n solves:

$$\frac{\partial\mathcal{L}}{\partial q_{2,a_i}(j,j',n)} = \left[mp^j_{a_i}(z_n)\frac{\partial \ell^j_{a_i}(z_n)}{\partial q_{2,a_i}(j,j',n)} - \frac{\partial c^j_{a_i}}{\partial q_{2,a_i}(j,j',n)}\right] - \left[mp^{j'}_{a_i}(z_n)\frac{\partial \ell^{j'}_{a_i}(z_n)}{\partial q_{2,a_i}(j',j,n)} - \frac{\partial c^{j'}_{a_i}}{\partial q_{2,a_i}(j',j,n)}\right] \tag{67}$$
$$\geq 0,$$

$mp^j_{a_i}(z_n) \equiv \frac{\partial Y}{\partial \ell^j_{a_i}(z_n)}$ is the marginal product of labor of firm j with productivity z_n. Condition (67) is obviously satisfied with equality if $q^j_{2,a_i} = \frac{1}{2}\ \forall j$, so that all the firms with a certain productivity in a given labor market have the same vacancy yield. In principle, the planner

[58] Notice that (64) differs from (63) in that it ignores transitions between equally productive firms.

would be indifferent among any potentially nondegenerate distributions of q^j_{2,a_i} consistent with (67) holding as equality. Since firms are structurally identical, candidate distributions should have the property that $\frac{\partial}{\partial q^j_{2,a_i}}\left[mp^j_{a_i}(z_n)\frac{\partial \ell^j_{a_i}(z_n)}{\partial q_{2,a_i}(j,j',n)} - \frac{\partial c^j_{a_i}}{\partial q_{2,a_i}(j,j',n)}\right] = 0$. However, no such distributions generally exist. [59] Hence, we can specialize on the problem setup with no residual dispersion, thus dropping the j index henceforth. As a result, the stationary employment constraint (58) simplifies to its familiar expression:

$$\ell_{a_i}\left(\vec{q}_{a_i}, V_{a_i}, \vec{v}_{a_i}(z)\right) = \frac{v_{a_i}(z_n)}{V_{a_i}}\lambda_{a_i}u_{a_i}\frac{\delta_{a_i}+s_{a_i}\lambda_{a_i}}{h_{a_i}(1,n)h_{a_i}(0,n)}.$$

The efficient transition probability from a firm with productivity z_n to a firm with productivity $z_{n'}$ in labor market a_i solves:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial q_{1,a_i}(n,n')} &= \left[mp_{a_i}(z_n)\frac{\partial \ell_{a_i}(z_n)}{\partial q_{1,a_i}(n,n')} - \frac{\partial c_{a_i}}{\partial q_{1,a_i}(n,n')}\right]\gamma_{a_i}(z_n) - \left[mp_{a_i}(z_{n'})\frac{\partial \ell_{a_i}(z_{n'})}{\partial q_{1,a_i}(n',n)} - \frac{\partial c_{a_i}(z_{n'})}{\partial q_{1,a_i}(n',n)}\right]\gamma_{a_i}(z_{n'}) \\
&= \left[mp_{a_i}(z_n)\left(\frac{d\tilde{\ell}_{a_i}(z_n)}{d\bar{\mathcal{H}}_{1,a_i}(n)} + \frac{d\tilde{\ell}_{a_i}(z_n)}{d\bar{\mathcal{H}}_{2,a_i}(n)}\right) - mp_{a_i}(z_{n'})\left(\frac{d\tilde{\ell}_{a_i}(z_{n'})}{d\bar{\mathcal{H}}_{1,a_i}(n')} + \frac{d\tilde{\ell}_{a_i}(z_{n'})}{d\bar{\mathcal{H}}_{2,a_i}(n')}\right)\right] \\
&\quad + \frac{\phi}{1+\zeta}\bar{c}_{a_i}\left[\frac{v_{a_i}(z_{n'})^{\zeta}}{h_{a_i}(n')^{1-\phi}} - \frac{v_{a_i}(z_n)^{\zeta}}{h_{a_i}(n)^{1-\phi}}\right] \geq 0, \qquad (68)
\end{aligned}$$

where $\tilde{\ell} = \ell/v$ is the vacancy yield (independent of v) and the second line follows from simplifying for $\frac{\partial \bar{\mathcal{H}}_{s,a_i}(n)}{\partial q_{1,a_i}(n,n')} = \frac{\partial \bar{\mathcal{H}}_{s,a_i}(n)}{\partial q_{1,a_i}(n,n')} = \frac{v_{a_i}(z_{n'})}{V_{a_i}}\gamma_{a_i}(z_{n'})$, $s = 1,2$, and the functional form of hiring costs. Notice that (68) describes a knife-edge condition. Specifically, it holds as equality if hiring rates, marginal products, and vacancies are all equalized across all firms. However, if hiring rates and vacancies are equalized, all firms have the same employment according to (58). If firms have the same employment, marginal products are strictly increasing in productivity. Therefore, equation (68) needs to hold as inequality, thus describing a corner solution. Formally, $q_{1,a_i}(n,n') \in \{0,1\}\ \forall n \neq n'$. We proceed by guessing and verifying that the planner chooses to reallocate workers towards higher-productivity firms whenever an opportunity arises, that is, $q_{a_i}(n,n') = 1 \iff z_n < z_{n'}$. Hence, the shadow wage offer distribution equals the (complement to 1 of the) vacancy-weighted productivity distribution, i.e., $\bar{\mathcal{H}}_{a_i}\left(V_{a_i}, \vec{v}_{a_i}(z_{n'})\right) = \sum_{n'=n+1}^{\mathcal{Z}_i}\frac{\bar{v}_{a_i}(z_{n'})}{V_{a_i}}\gamma_{a_i}(z_{n'})$.

Efficient vacancy posting by a firm with productivity z operating in labor market a_i meets the following conditions:

[59] Assume firms with identical productivity z_n are assigned a signal $\varrho \sim \Phi_n$. Hence, the planner can condition the choice of the residual transition probability on ϱ, e.g., $q_{2,a_i}(\varrho,\varrho',n) = \mathcal{F}(\varrho/\varrho')$. If the planner chooses to do so, then $\frac{\partial q^{\varrho}_{2,a_i}}{\partial \varrho} \neq 0$. For this strategy to be efficient, it must be that $\frac{\partial}{\partial \varrho}\left[mp^{\varrho}_{a_i}(z_n)\frac{\partial \ell^{\varrho}_{a_i}(z_n)}{\partial q_{2,a_i}(\varrho,\varrho',n)} - \frac{\partial c^{\varrho}_{a_i}}{\partial q_{2,a_i}(\varrho,\varrho',n)}\right] = 0$. However, since efficient vacancies are increasing in vacancy yield and marginal product of labor, the derivative will be generally positive.

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial v_{a_i}(z_n)} = & \left[mp_{a_i}(z_n) \frac{\partial \ell_{a_i}(z_n)}{\partial v_{a_i}(z_n)} - \frac{\partial c_{a_i}(z_n)}{\partial v_{a_i}(z_n)} \right] M_{a_i} \gamma_{a_i}(z_n) \\
& + \sum_{n' \leq n}^{\mathcal{Z}_i} \left[mp_{a_i}(z_{n'}) \frac{\partial \ell_{a_i}(z_{n'})}{\partial \bar{\mathcal{H}}_{a_i}(n')} - \frac{\partial c_{a_i}(z_{n'})}{\partial \bar{\mathcal{H}}_{a_i}(n')} \right] \frac{\partial \bar{\mathcal{H}}_{a_i}(n')}{\partial v_{a_i}(z_n)} M_{a_i} \gamma_{a_i}(z_{n'}) \\
& + \sum_{n'=1}^{\mathcal{Z}_i} \left[mp_{a_i}(z_{n'}) \left(\frac{\partial \ell_{a_i}(z_{n'})}{\partial V_{a_i}} + \frac{\partial \ell_{a_i}(z_{n'})}{\partial \lambda_{a_i}} \frac{\partial \lambda_{a_i}}{\partial V_{a_i}} + \frac{\partial \ell_{a_i}(z_{n'})}{\partial \bar{\mathcal{H}}_{a_i}(n')} \frac{\partial \bar{\mathcal{H}}_{a_i}(n')}{\partial V_{a_i}} \right) \right. \\
& \left. - \frac{\partial c_{a_i}(z_{n'})}{\partial \bar{\mathcal{H}}_{a_i}(n')} \frac{\partial \bar{\mathcal{H}}_{a_i}(n')}{\partial V_{a_i}} \right] \frac{\partial V_{a_i}}{\partial v_{a_i}(z_n)} M_{a_i} \gamma_{a_i}(z_{n'}) \geq 0,
\end{aligned} \tag{69}
$$

$$
\xi_{v,a_i}(z_n) v_{a_i}(z_n) = 0, \tag{70}
$$

where (69) is the FOC with respect to vacancies, and (70) the respective complementary slackness condition.

The first line of (69) represents the *direct effect* of $v_{a_i}(z_n)$ on the firm's value added, that is, the excess marginal product of labor induced by the marginal vacancy over the marginal hiring cost.

The second line of (69) represents the *business-stealing effects* of $v_{a_i}(z)$ on the value added of lower-productivity firms operating in the same labor market. Specifically, the constrained social planner internalizes how vacancy posting by firms with higher productivity reduces the employment of firms with lower productivity, i.e., how the pace of worker reallocation affects the cross-sectional distribution of employment.[60]

The third and fourth line of (69) represent the *congestion effects* of $v_{a_i}(z_n)$ on the value added produced by workers operating in the same labor market. Specifically, the social planner internalizes that vacancy posting by some firm affects the meeting rate of all the firms via the induced change in aggregate vacancies and, in turn, labor market tightness. These negative (thin-market) externalities exerted on other firms are weighed against the positive (thick-market) externalities exerted on workers, that is, the marginal effect of $v_{a_i}(z_n)$ on the job finding rate – again mediated by the induced change in aggregate vacancies and labor market tightness.

Since both the shadow wage offer distribution and aggregate vacancies are linear in firm-level vacancies, equation (69) carries two important insights. First, conditional on a given rank in the productivity distribution, the business-stealing effects of vacancy posting by firm z_n in labor market a_i are independent of the mass of vacancies it posts. Second, the congestion effects of vacancy posting by firm z in labor market a_i are independent of the mass of vacancies it posts.

Let $E^{bs}_{a_i}(n)$ and $E^{c}_{a_i}$ denote the business-stealing effects and congestion effects of $v_{a_i}(z_n)$ on value added, respectively. It follows that vacancy posting of active firms in the efficient

[60] Intuitively, the higher the share of aggregate vacancies accounted for by firms with higher productivity, the lower the net poaching rate of firms with lower productivity.

allocation solves:

$$mp_{a_i}(z_n)\frac{\partial \ell_{a_i}(z_n)}{\partial v_{a_i}(z_n)} = \frac{\partial c_{a_i}(z_n)}{\partial v_{a_i}(z_n)} + E^{bs}_{a_i}(n) + E^{c}_{a_i}. \tag{71}$$

In words, the social planner equalizes the social marginal benefit of vacancy posting by firm z_n (left-hand side) to its social marginal cost (right-hand side). The latter is composed by the marginal vacancy posting cost and the external effects.

Notice that the social marginal benefit of vacancy posting equals the product between marginal product of labor and vacancy yield. Since the marginal product of labor is increasing in productivity, efficient vacancies are supermodular in productivity and vacancy yield. This property (coupled with the supermodularity of the output function in productivity and size) allows verifying our guess that the planner reallocates workers towards higher-productivity firms when it is given the chance, as an application of Topkis's theorem.

Relation with equilibrium allocation. In equilibrium, wages – rather than marginal products – play an allocative role via worker reallocation. Hence, for the efficient allocation to be sustainable in equilibrium, wages need to be increasing in productivity, i.e., $w'_{a_i}(z) > 0$. Notice that the social planner is indifferent between the composition of marginal costs between wages and markdowns.[61] However, comparing (71) with its equilibrium counterpart (14), it is apparent that both feature an "additive markdown", i.e., the difference between marginal (revenue) product and wage, defined as $\frac{\partial c_{a_i}(z)/\partial v_{a_i}(z)}{\partial \ell_{a_i}(z)/\partial v_{a_i}(z)}$. Hence, we proceed by defining the *shadow wage* as $w^{\star\star}_{a_i}(z) \equiv \frac{E^{bs}_{a_i}(n)+E^{c}_{a_i}}{\partial \ell_{a_i}(z)/\partial v_{a_i}(z)}$ and the *shadow markdown* as $\psi^{\star\star}_{a_i}(z) \equiv 1 + \frac{\partial c_{a_i}(z)/\partial v_{a_i}(z)}{w^{\star\star}_{a_i}(z)\partial \ell_{a_i}(z)/\partial v_{a_i}(z)}$. It follows that equilibrium markdowns are efficient if and only if wages are efficient. The efficient allocation would decentralize by dictating:

$$\mu_{a_i}(z_n) = 1, \tag{72}$$

$$\psi_{a_i}(z_n) = 1 + \frac{(1+\zeta)c_{a_i}(z_n)}{w_{a_i}(z_n)\ell_{a_i}(z_n)}, \tag{73}$$

$$w_{a_i}(z_n) = \frac{E^{bs}_{a_i}(n) + E^{c}_{a_i}}{\partial \ell_{a_i}(z_n)/\partial v_{a_i}(z_n)}. \tag{74}$$

Condition (72) makes sure that posted prices are efficient, i.e., equalizing the marginal product to the marginal *revenue* product of labor, condition (73) makes sure that markdowns are efficient for given wage policy, condition (74) makes sure that the wage policy induces efficient vacancy posting, i.e., equalizing social and private marginal cost.

How does the equilibrium wage function compare to its efficient counterpart (74)? First of all, we notice that residual wage (and marginal product) dispersion is inefficient. Yet, residual wage dispersion is needed to sustain the unique (mixed-strategy) Nash equilibrium in the wage-posting game with finite number of productivity types and continuous wage

[61] Since labor supply is inelastic, it is not influenced by the composition of income between wages and profits.

distributions. This allows establishing our first lemma:

Lemma A.2 (Efficiency requires infinite productivity types)
If the number of productivity types is finite, the wage-posting equilibrium with a continuous wage offer distribution is inefficient.

Hence, we proceed by comparing efficient and equilibrium allocation in the limit case of our economy as $\mathcal{Z}_i \to \infty$. For consistency with the social planner problem setup, we further assume that workers set their reservation wage statically under risk-neutrality, so that (**??**) boils down to the *static* reservation wage $\hat{R}^u_{a_i} : a\hat{R}^u_{a_i} + \Pi(a\hat{R}^u_{a_i}) = ab_{a_i}$. We carry out the comparison in two steps. First, we show that the equilibrium wage function internalizes the business-stealing effects of vacancy posting if the static reservation wage is zero, absent any additional source of inefficiency (i.e., provided that the marginal revenue product of labor equals its marginal product and congestion effects are zero). Second, we claim that equilibrium wages are generally inefficient for given marginal product function due to congestion effects.

Lemma A.3 (Wage posting internalizes business-stealing effects)
For given marginal product function $mp_{a_i}(z)$ and a zero static reservation wage ($\hat{R}^u_{a_i} = 0$), the equilibrium wage function internalizes the business-stealing effects of vacancy posting. Formally,

$$v_{a_i}(z) = v^\star_{a_i}(z) \implies mp_{a_i}(z)\frac{\partial \ell_{a_i}(z)}{\partial v_{a_i}(z)} = \frac{\partial c_{a_i}(z)}{\partial v_{a_i}(z)} + E^{bs}_{a_i}(\mathcal{H}_{a_i}(z)),$$

where $v^\star_{a_i}(z)$ is the equilibrium vacancy policy function and $\mathcal{H}_{a_i}(z)$ is the equilibrium wage offer distribution.

Proof. See C.0.1. ∎

In general, equilibrium vacancies and wages are inefficient for three reasons: static reservation wages are positive, markup rates are positive, and congestion effects are different from zero. [62] Indeed, evaluating (69) at the equilibrium solution yields:

$$\frac{\partial \mathcal{L}}{\partial v_{a_i}(z)}\Big|_{v_{a_i}(z)=v^\star_{a_i}(z)} = \frac{\mu_{a_i}(z)-1}{\mu_{a_i}(z)} mp_{a_i}(z)\frac{\partial \ell_{a_i}(z)}{\partial v_{a_i}(z)} + (1-\mathcal{P}_{a_i}(z))\frac{\partial \ell_{a_i}(z)}{\partial v_{a_i}(z)}\hat{R}^u_{a_i} - E^c_{a_i} \lesseqgtr 0,$$

where $\mathcal{P}_{a_i}(z) \equiv 1 - e^{-\int_{\underline{z}}^{z} x_{a_i}(\hat{z})d\hat{z}} \in (0,1]$ and $x_{a_i}(z) \equiv \frac{2\nu s_{a_i}\lambda(\theta_{a_i})\mathcal{H}'_{a_i}(z)}{\delta_{a_i}+s_{a_i}\lambda(\theta_{a_i})[1-\mathcal{H}_{a_i}(z)]}$. Notice that the sign of the congestion effects at the equilibrium solution is a priori ambiguous, though generally different from zero. If congestion effects are positive, i.e., $E^c_{a_i} > 0$, *ceteris paribus* equilibrium marginal costs (wages times markdowns) are lower than efficient. Since congestion effects show up as an additive wedge in the first-order condition (71) and wages are increasing in

[62] Equilibrium wages and markdowns would be the constrained efficient outcome of the wage-posting game if job creation were exogenous and firm size were pinned down by search frictions only (Moscarini and Postel-Vinay, 2013). Intuitively, firm size would be determined by workers' efficient turnover. However, insofar as firms optimize their employment size because of decreasing returns (in production or in revenues) and/or endogenous vacancy posting, equilibrium markdowns are generally inefficient.

productivity, lower-productivity firms are farther away from their efficient marginal cost in relative terms. As a result, low-productivity firms typically post an inefficient mass of vacancies, leading to labor misallocation as in Shimer and Smith (2001) and Acemoglu (2001).

We conclude this section by singling out the sources of inefficiency featured by the baseline equilibrium and how the introduction of a minimum wage may affect them. To do so, let $\Delta_{a_i}(z)$ denote the firm-specific *labor wedge*, that is, the ratio between social marginal benefit (left-hand side of (71)) and social marginal cost (right-hand side of (71)) of vacancy posting by a firm with productivity z_n operating in labor market a_i. In the efficient allocation, $\Delta_{a_i}(z) = 1\ \forall z,\ \forall a_i$. In the baseline equilibrium it equals:

$$\Delta_{a_i}(z) = \frac{\mu_{a_i}(z)}{1 + \frac{E^c_{a_i} - D_{a_i}(z)\hat{R}^u_{a_i}}{MCL_{a_i}(z)}}, \tag{75}$$

where $D_{a_i}(z) \equiv (1 - \mathcal{P}_{a_i}(z))\partial \ell_{a_i}(z)/\partial v_{a_i}(z)$, $\partial D_{a_i}(z)/\partial z \geq 0$ and $MCL_{a_i}(z) \equiv \psi_{a_i}(z) w_{a_i}(z)$. The baseline equilibrium is inefficient for two reasons. First, positive markup rates (due to imperfect substitutability across firm-level varieties) and distorted markdowns (due to congestion effects and positive static reservation wages) make the labor wedge generally differ from one in equilibrium. Positive markup rates push the labor wedge to be higher than one. Hence, if the net external effects in wage setting at the equilibrium solution are positive, equilibrium aggregate employment is inefficiently low. This means that value added would increase if all the firms posted more vacancies.

Second, heterogeneous markups (due to firms' granularity in their product market) and heterogeneous markdown distortions (due to heterogeneous impact of congestion effects and static reservation wages in the cross section) make the labor wedge generally differ across firms in equilibrium. This means that the economy features *misallocation* of labor across firms. Misallocation entails that the economy could produce the same amount of final output with lower aggregate hiring costs. This means that value added would increase by reallocating the equilibrium mass of equilibrium vacancies across firms. Hence, with a large number of productivity types, decentralizing the efficient allocation requires three policy instruments: (i) a size-dependent subsidy to neutralize markup distortions (Edmond et al., 2023), (ii) a linear vacancy posting tax (or subsidy) to neutralize markdown distortions due to congestion effects, and (iii) a firm-specific wage subsidy to neutralize positive static reservation wages.

However, the amount of information required to implement such first-best policies is arguably beyond the possibilities of policymakers. On the other hand, policymakers can directly control the static reservation wage through MW setting. Since $\partial D_{a_i}(z)/\partial z \geq 0$, the MW is expected to affect more the labor wedge of low-productivity firms. Since, for standard parametrizations, congestion effects are positive and markups are increasing in productivity, low-productivity firms typically post more vacancies than efficient. By constraining their optimal vacancy posting, a higher minimum wage is likely to reduce labor misallocation. In this sense, a minimum wage can be thought of as a second-best policy when policymakers do not have access to more targeted policy instruments.

C.0.1 Proof Lemma A.3

To establish the claim, we define a function $\mathcal{F}_{a_i}(v_{a_i}(z)) \equiv mp_{a_i}(z)\frac{\partial \ell_{a_i}(z)}{\partial v_{a_i}(z)} - \frac{\partial c_{a_i}(z)}{\partial v_{a_i}(z)} - E^{bs}_{a_i}\left(\mathcal{H}_{a_i}(z)\right)$, which equals zero at the efficient solution, i.e., $\mathcal{F}_{a_i}(v^{\star\star}_{a_i}(z)) = 0$. In equilibrium, optimal vacancy posting solves $mp_{a_i}(z)\frac{\partial \ell_{a_i}(z)}{\partial v_{a_i}(z)} - \frac{\partial c_{a_i}(z)}{\partial v_{a_i}(z)} - w_{a_i}(z)\frac{\partial \ell_{a_i}(z)}{\partial v_{a_i}(z)} = 0$. First of all, we notice that the business-stealing effects induced by the lowest-productivity firms are zero, i.e., $E^{bs}_{a_i}\left(\mathcal{H}_{a_i}(\underline{z}_{a_i})\right) = 0$. On the other hand, the lowest-productivity firms pay the reservation wage in equilibrium. Hence, as long as the reservation wage is positive, $\mathcal{F}_{a_i}(v^{\star}_{a_i}(\underline{z}_{a_i})) \neq 0$. Hence, equilibrium vacancy posting cannot be efficient.

We proceed by rearranging the expression for the business-stealing effects of vacancy posting as follows:

$$\begin{aligned}
E^{bs}(\mathcal{H}_{a_i}(z)) &\equiv -\frac{1}{M_{a_i}\gamma_{a_i}(z)}\int_{\underline{z}_{a_i}}^{z}\left[mp_{a_i}(\hat{z})\frac{\partial \ell_{a_i}(\hat{z})}{\partial \bar{\mathcal{H}}_{a_i}(\hat{z})} - \frac{\partial c_{a_i}(\hat{z})}{\partial \bar{\mathcal{H}}_{a_i}(\hat{z})}\right]\frac{\partial \bar{\mathcal{H}}_{a_i}(\hat{z})}{\partial v_{a_i}(z)} M_{a_i}\gamma_{a_i}(\hat{z})d\hat{z} \\
&= \int_{\underline{z}_{a_i}}^{z}\left[mp_{a_i}(\hat{z})\frac{2s_{a_i}\lambda_{a_i}\bar{\mathcal{H}}'_{a_i}(\hat{z})}{\delta_{a_i}+s_{a_i}\lambda_{a_i}\bar{\mathcal{H}}_{a_i}(\hat{z})}\frac{\ell_{a_i}(\hat{z})}{v_{a_i}(\hat{z})} + \phi\frac{s_{a_i}\lambda_{a_i}\bar{\mathcal{H}}'_{a_i}(\hat{z})}{\delta_{a_i}+s_{a_i}\lambda_{a_i}\bar{\mathcal{H}}_{a_i}(\hat{z})}\frac{c_{a_i}(\hat{z})}{v_{a_i}(\hat{z})}\right]d\hat{z} \\
&= \int_{\underline{z}_{a_i}}^{z}\left[\frac{1}{\nu}mp_{a_i}(\hat{z})\frac{\ell_{a_i}(\hat{z})}{v_{a_i}(\hat{z})} + \frac{\phi}{2\nu}\frac{c_{a_i}(\hat{z})}{v_{a_i}(\hat{z})}\right]x_{a_i}(\hat{z})d\hat{z} \\
&= \int_{\underline{z}_{a_i}}^{z}\left[\frac{1}{\nu}mp_{a_i}(\hat{z}) + \left(1-\frac{1}{\nu}\right)\frac{\partial c_{a_i}(\hat{z})/\partial v_{a_i}(\hat{z})}{\partial \ell_{a_i}(\hat{z})/\partial v_{a_i}(\hat{z})}\right]\frac{\partial \ell_{a_i}(\hat{z})}{\partial v_{a_i}(\hat{z})}x_{a_i}(\hat{z})d\hat{z} \\
&= \int_{\underline{z}_{a_i}}^{z}\left[\frac{1}{\nu}mp_{a_i}(\hat{z}) + \left(1-\frac{1}{\nu}\right)(mp_{a_i}(\hat{z}) - w_{a_i}(\hat{z}))\right]\frac{\partial \ell_{a_i}(\hat{z})}{\partial v_{a_i}(\hat{z})}x_{a_i}(\hat{z})d\hat{z},
\end{aligned}$$

where we made use of the following relationships and equilibrium conditions: $\frac{\partial \ell_{a_i}(\hat{z})}{\partial \bar{\mathcal{H}}_{a_i}(\hat{z})} = -\frac{2s_{a_i}\lambda_{a_i}}{\delta_{a_i}+s_{a_i}\lambda_{a_i}\bar{\mathcal{H}}_{a_i}(\hat{z})}\ell_{a_i}(\hat{z})$, $\frac{\partial \bar{\mathcal{H}}_{a_i}(\hat{z})}{\partial v_{a_i}(z)} = \frac{M_{a_i}}{V_{a_i}}\gamma_{a_i}(z)$, $\frac{\partial c_{a_i}(\hat{z})}{\partial \bar{\mathcal{H}}_{a_i}(\hat{z})} = \phi\frac{s_{a_i}\lambda_{a_i}}{\delta_{a_i}+s_{a_i}\lambda_{a_i}\bar{\mathcal{H}}_{a_i}(\hat{z})}c_{a_i}(\hat{z})$, $\frac{\partial c_{a_i}(\hat{z})}{\partial v_{a_i}(\hat{z})} = (1+\zeta)\frac{c_{a_i}(\hat{z})}{v_{a_i}(\hat{z})}$, $\bar{\mathcal{H}}'_{a_i}(\hat{z}) = \frac{M_{a_i}}{V_{a_i}}v_{a_i}(\hat{z})\gamma_{a_i}(\hat{z})$, and $\frac{\phi}{2(1+\zeta)\nu} = 1-\frac{1}{\nu}$.

Substituting for the equilibrium condition and the expression for business-stealing effects into $\mathcal{F}_{a_i}(v_{a_i}(z))$ yields:

$$\mathcal{F}_{a_i}(z) = w_{a_i}(mp_{a_i}(z))\frac{\partial \ell_{a_i}(z)}{\partial v_{a_i}(z)} - \int_{\underline{z}_{a_i}}^{z}\left[\frac{1}{\nu}mp_{a_i}(\hat{z}) + \left(1-\frac{1}{\nu}\right)(mp_{a_i}(\hat{z}) - w_{a_i}(\hat{z}))\right]\frac{\partial \ell_{a_i}(\hat{z})}{\partial v_{a_i}(\hat{z})}x_{a_i}(\hat{z})d\hat{z}.$$

Substituting for $\frac{\partial \ell_{a_i}(\hat{z})}{\partial v_{a_i}(\hat{z})}$ and collecting common terms, the function depends on the following three terms:

$$\begin{aligned}
\mathcal{F}_{a_i}(z) \propto \frac{w_{a_i}(mp_{a_i}(z))}{h_{a_i}(z)^2} &- \int_{\underline{z}_{a_i}}^{z}\frac{w_{a_i}(\hat{z})}{h_{a_i}(\hat{z})^2}\frac{x_{a_i}(\hat{z})}{\nu}d\hat{z} \\
&- \int_{\underline{z}_{a_i}}^{z}\frac{(mp_{a_i}(\hat{z}) - w_{a_i}(\hat{z}))}{h_{a_i}(\hat{z})^2}x_{a_i}(\hat{z})d\hat{z},
\end{aligned}$$

where $h_{a_i}(z) \equiv \delta_{a_i} + s_{a_i}\lambda_{a_i}(1-\mathcal{H}_{a_i}(z))$ is the hiring rate and $x_{a_i}(z) \equiv \frac{2\nu s_{a_i}\lambda_{a_i}\mathcal{H}'_{a_i}(z)}{h_{a_i}(z)}$.

We now focus on the second term and apply integration by parts:

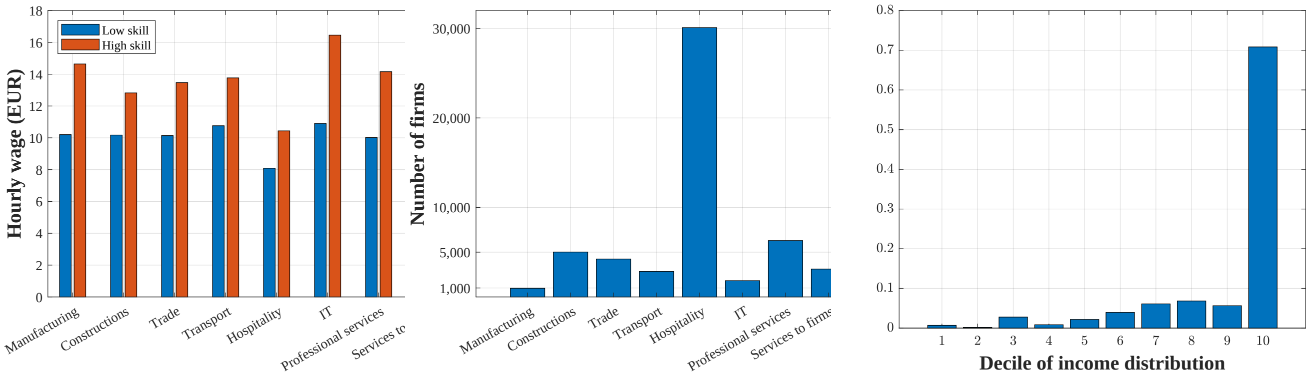
$$\int_{\underline{z}_{a_i}}^{z} \frac{w_{a_i}(\hat{z})}{h_{a_i}(\hat{z})^2} \frac{x_{a_i}(\hat{z})}{\nu} d\hat{z} = \int_{\underline{z}_{a_i}}^{z} w_{a_i}(\hat{z}) \frac{\partial h_{a_i}(\hat{z})^{-2}}{\partial \hat{z}} d\hat{z} = \frac{w_{a_i}(mp_{a_i}(z))}{h_{a_i}(z)^2} - \int_{\underline{z}_{a_i}}^{z} \frac{(mp_{a_i}(\hat{z}) - w_{a_i}(\hat{z}))}{h_{a_i}(\hat{z})^2} x_{a_i}(\hat{z}) d\hat{z},$$

where the last equality follows from the slope of the equilibrium wage function (40). Substituting back into the previous expression allows establishing the claim:

$$v_{a_i}(z) = v^{\star}_{a_i}(z) \implies \mathcal{F}_{a_i}(z) = 0.$$

D Structural Estimation

(a) Average wage, by labor mkt (b) Avg firms in sectors, by ind. (c) Profits share, by income



Source: Authors' calculations, INPS (2016-2018), Istat (2019), and SHIW (2016). *Note*: Sectors and industries are defined as 4-digit and 1-digit industry according to the ATECO classification.

D.1 Data description and sample selection

INPS data. We leverage social security data from INPS. Our dataset consists of the complete contribution histories of individuals who worked as employees in private-sector for at least one period in their life between 1990 and 2018. We use a random and representative 6.5% sample of this population. In this dataset, the unit of observation is an event that generated a contribution to the pension system. This allows us to derive complete job histories for the workers covered in the sample, as well as to distinguish their periods of employment and non-employment. Crucially for our analysis, the dataset also contains firm identifiers, which allow us to estimate the AKM equation. We select our sample by focusing on individuals aged 25-64. We do this to avoid capturing the latest phase of education in labor market careers. We do *not* exclude women nor part-time workers from our sample.[63] We end up with a sample of about 34 million yearly observations, covering 2.6 million distinct workers and 2.3 million distinct firms.

Istat data. Our data from Istat comes from two sources. First, we make use of publicly available data on the distribution of firms across 4-digit ATECO sectors, as well as on the employment share of firms with different employment size. Second, we leverage micro-data from the 2019 wave of the Structural Business Statistics to compute the HHI of 4-digit sectors and the semi-elasticity of firm-level labor share to market share. The dataset consists of the near-universe of Italian firms operating in the non-agricultural sector. We select our sample to cover all the 4-digit sectors in the economy excluding the financial sector. We consider as missing negative entries for "revenues", "value added", and "labor cost". We define labor share (LS) as the ratio between "labor cost" and "value added" at the firm-level. To guard

[63] We claim that this is potentially important for our analysis: given these groups of workers are overall paid less than average, they therefore constitute a relevant share of the population directly affected by a MW reform.

against outliers, we winsorize the labor share at 5%. We compute the firm-level market share (*sh*) in the 4-digit sector in which it operates as the ratio between firm-level revenues and sectoral revenues.

D.2 AKM estimation

For each worker, we first select the dominant (= highest-earnings) job spell within each year. We then compute average hourly wages by taking the ratio of yearly earnings and the imputed number of hours worked (obtained multiplying the number of days worked by 8 or 4, depending on whether the job is full or part-time). Finally, we estimate a classical AKM equation, that is a log-linear wage regression with worker and firm fixed effects:

$$\log w_{it} = \beta X_{i,t} + \alpha_i + \phi_{f(it)} + \epsilon_{i,t},$$

where $X_{i,t}$ represents a vector of time-varying controls, α_i's are the workers' fixed effects and ϕ_a's are the firms' fixed effects. We estimate the AKM equation without controls. This is consistent with our model, where the only source of heterogeneity is a permanent skill component. For the AKM estimation we use the longest possible time span in our data (1990-2018) to minimize the risk of capturing temporary shocks to workers' careers.

D.3 Computing estimation targets

We use our Istat micro-data to compute the HHI of each 4-digit sector k as: $HHI_k = \sum_{j=1}^{N_{j(k)}} sh_j^2$. We then aggregate sector-level HHIs by weighting them for the sectoral value added VA_k to compute our HHI target: $HHI = \frac{\sum_{k=1}^{K} VA_k HHI_k}{\sum_{k=1}^{K} VA_k}$. Next, we use the same Istat dataset to estimate the following regression model: $LS_{j(k)} = \beta \; sh_{j(k)} + \sum_{n=1}^{K} \delta_k \mathbb{1}_{n=k} + \epsilon_{j(k)}$, where the δ_ks are 4-digit-sector fixed effects. We normalize the estimated coefficient $\hat{\beta} = -0.274$ by the average labor share (0.610) to compute the semi-elasticity of the firm-level labor share to the market share, which we use as target in the structural estimation.

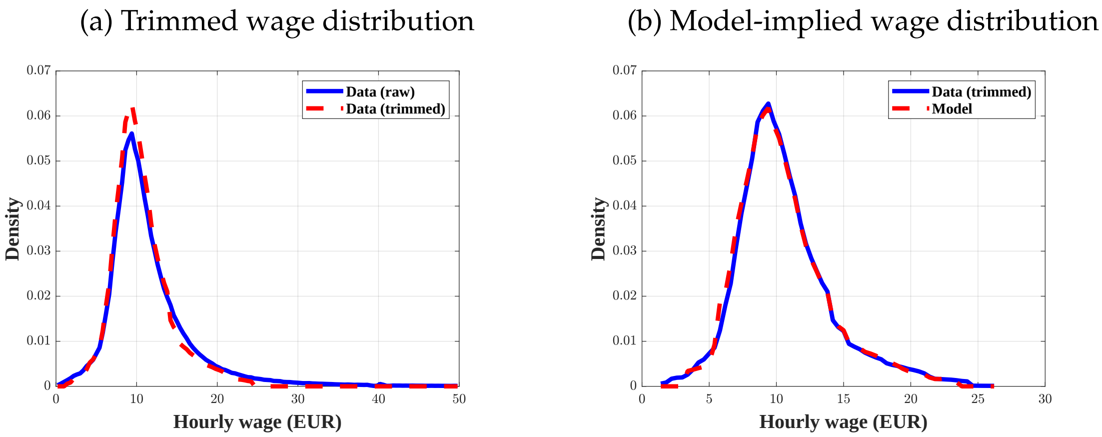
We partition our sample from INPS in two subsamples, each of which corresponds to half of the estimated AKM worker fixed effects distribution (low- and high-skilled workers). We then measure worker transition rates separately for each subgroup. In particular, we measure the EN rate, the job-to-job (J2J) rate, and the NE rate as the Poisson rates consistent with the monthly probability of moving from employment to non-employment, switching to another employer, and of moving from non-employment to employment, respectively. Finally, we focus on the time period between 2016 and 2018 and assign workers to a skill type (based on their estimated AKM worker fixed effect) and to the most frequent 1-digit industry they have been employed at. We use the resulting wage distributions at industry x skill level as targets in the structural estimation.

D.4 Estimating productivity from wages

In this section, we detail our strategy to estimate the productivity distribution of firms operating in a labor market from its wage distribution within the SMM routine.

Trimming wage distributions. First, we trim the lowest 1% and the highest $100tr$% of each wage distribution to guard against outliers, where tr is a parameter to be estimated. Figure (A.3a) reports the difference between the raw and the trimmed empirical wage distribution upon applying the optimal right trimming of $tr = 8.7\%$ delivered by our SMM estimator.

Figure A.3: Aggregate wage distribution - data vs model



Source: INPS data (2016-2018) and model. *Note*: Panel (a) compares the raw and the trimmed economy's employment wage distribution ($G(w)$); Panel (b) compares the trimmed and the model-implied economy's employment wage distribution. The economy's wage distribution in the data aggregates the 16 trimmed labor-market-specific wage distributions.

Cleaning wage distributions. For our model to replicate the trimmed empirical wage distributions, we pass them through a cleaning algorithm structured in two steps.

Admissibility condition. First, notice that the density of the MRPL distribution is structurally related to firms' wage and vacancy policies by the following relation:

$$\varphi_{a_i}(\tilde{z}) = \frac{\mathcal{H}'_{a_i}(\tilde{z})}{M_{a_i}} \frac{V_{a_i}}{v_{a_i}(\tilde{z})}. \tag{76}$$

Since $\mathcal{H}_{a_i}(\tilde{z})) = F_{a_i}(w_{a_i}(\tilde{z}))$, it follows that $\mathcal{H}'_{a_i}(\tilde{z}) = w'_{a_i}(\tilde{z})\, f_{a_i}(w_{a_i}(\tilde{z}))$. Following Bontemps et al. (2000), one can derive the following expression for $\mathcal{H}'_{a_i}(\tilde{z})$:

$$\mathcal{H}'_{a_i}(\tilde{z}) = \frac{2\nu s_{a_i}(1-u_{a_i})g^2_{a_i}(w)}{(1+2\nu)s_{a_i}(1-u_{a_i})g^2_{a_i}(w) - g'_{a_i}(w)\left[u_{a_i} + s_{a_i}(1-u_{a_i})G_{a_i}(w)\right]} \frac{(\delta_{a_i} + s_{a_i}\lambda(\theta_{a_i}))(1-u_{a_i})u_{a_i}g_{a_i}(w)}{\lambda(\theta_{a_i})\left[u_{a_i} + s_{a_i}(1-u_{a_i})G_{a_i}(w)\right]^2},$$

where the dependence of w on $\tilde{z}$ is left implicit. Turning back to Equation (76), we notice that $\varphi_{a_i}(\tilde{z}) \geq 0 \iff \mathcal{H}'_{a_i}(\tilde{z}) \geq 0$, provided that $v_{a_i}(\tilde{z}) > 0$. This implies that in order to have

$\varphi_{a_i}(\tilde{z}) \geq 0$, we need $\mathcal{H}'_{a_i}(\tilde{z}) \geq 0$, which is the case if and only if:

$$(1+2\nu)s_{a_i}(1-u_{a_i})g^2_{a_i}(w) \geq g'_{a_i}(w)\left(u_{a_i} + s_{a_i}(1-u_{a_i})G_{a_i}(w)\right).$$

Hence, for each labor market, we check the admissibility condition, and marginally shift mass from the (few) points of the empirical wage distributions for which this condition is not met. Essentially, the condition fails when the wage density grows *too quickly*, which in some cases happens in our data in the left part of the distributions. We work around this by applying the simple following algorithm. Let $G^D_{a_i}(w)$ be the discretized version of our wage distributions, that takes values on grid points $[w_1, w_2, ...w_N]$. We proceed as follows:

1. We identify the first grid point i^Y for which the admissibility condition holds, in the left part of the distribution;

2. Starting from w_1, we move mass from the lowest grid points to the $i^Y - 2$ grid point, until the condition for $i^Y - 1$ is met. If during the process the mass of w_1 runs out, then we move to w_2;

3. When the condition for $i^Y - 1$ is met, then we turn to move mass towards $i^Y - 3$;

4. We stop when the first grid point for which the admissibility condition holds is the second one with non-zero mass;

5. At the end of this, we check again the condition over the whole distribution. For all points for which this is not verified, we progressively add mass to the previous grid points, removing it from all other points of the distribution.

In fact, the adjustment takes place exclusively in the left part of the distributions, where rapidly growing density functions cannot be generated by the model. However, As shown in Figure (**??**), the distance between the left tails of the empirical (trimmed) wage distributions and the ones generated by the model is negligible. Hence, we claim that the failure of the admissibility conditions for some data points does not represent an issue for our analysis.

Replicate actual number of firms. Second, note that the number of firms is an important empirical moment to match, due to the granular product market structure of our model. To map the measure of firms populating our labor markets to their actual number, we propose the following normalization: we let the number of firms in the MRPL with lowest weighted density be equal to 1. We find this normalization the most sensible to introduce an integer constraint in the number of firms per MRPL level.[64] To make sure that our model replicates the actual number of firms, we implement the following algorithm:

[64] If the normalization yields a total number of firms in the model exceeding the actual number of firms with employees in Italy in 2019 (1,555,543), we perform a simple algorithm to transfer the mass corresponding to the MRPL grid point with the lowest number of firms to the closest, nonzero adjacent grid point in the same labor market.

1. Compute $M_{a_i}\varphi_{a_i}(\tilde{z})\ \forall a_i$ by inverting the (trimmed) empirical wage distributions according to the structure of the model (see next paragraph);

2. Let $\underline{\ell}$ and $\underline{\tilde{z}}$ denote the labor market and MRPL level corresponding to $\min\{M_{a_i}\varphi_{a_i}(\tilde{z})\}$. Apply the normalization: $N_{\underline{\ell}}(\underline{\tilde{z}}) = 1$.

3. Compute the number of firms in all the labor markets and MRPL values by rounding the expression $N_{a_i}(\tilde{z}) = \frac{M_{a_i}\varphi_{a_i}(\tilde{z})}{\min\{M_{a_i}\varphi_{a_i}(\tilde{z})\}}\forall a_i, \forall \tilde{z}$.

4. If the total number of firms in the model exceeds the actual number of firms with employees in Italy in 2019 ($1,555,543$), transfer the mass corresponding to the MRPL grid point with lowest number of firms to the closest, nonzero grid point to the left in the same labor market. To preserve the shape of the MRPL distribution, constrain the maximum mass to transfer to half the difference between the receiving grid point and the preceding nonzero grid point. If exceeding, split the mass over multiple grid points;

5. Iterate this procedure until the number of firms in the model is (weakly) lower than in the data. If it is lower, assess whether the error is higher in the last or second-to-last iteration step and choose the most precise one.

As shown in Figure (**??**), the distance between the right tails of the empirical (trimmed) wage distributions and the ones generated by the model is negligible. Hence, we claim that the integer constraint does not represent an issue for our analysis.

Inferring productivity from wages. We use the structure of our model to estimate the underlying productivity distributions to the observed wage distributions in two steps.

First, we leverage the equilibrium markdown function (20) and the mapping between wage offer distribution and wage employment distribution to infer the MRPL per wage level based on the shape of the employment wage distribution. Formally, $\tilde{z}_{a_i}(w; G_{a_i}(w)) = \psi_{a_i}(w)w = w + \frac{u_{a_i}+s_{a_i}(1-u_{a_i})G_{a_i}(w)}{2\nu s_{a_i}(1-u_{a_i})\Delta G_{a_i}(w)}\Delta w$. In this way, we recover the model-consistent MRPL distributions, $\tilde{z}_{a_i} \sim \Phi_{a_i}(\psi_{a_i}(w)w)$. We then impose the integer constraint to map the measure of firms into their actual number. We proceed by assigning as many firms as the average number of competing firms across 4-digit sectors in each industry to sectoral markets.[65]

Second, for given CES elasticities of demand and MRPL distributions in each product market, there exists a unique combination of prices, markups, CD industry weights, and physical productivities consistent with our model, which solves the following system of

[65]According to our model, product markets sourcing from the same labor market have the same productivity distribution. Hence, we allot firms to sectoral markets in order to minimize differences across them.

equations:

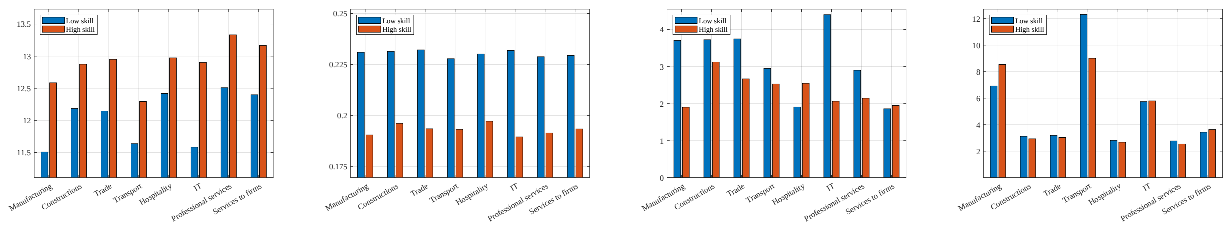
$$
\begin{cases}
z(\tilde{z}_{k(a_i)}) = \frac{\mu(\tilde{z}_{k(a_i)})}{p(\tilde{z}_{k(a_i)})}\tilde{z}_{k(a_i)}, \; p(\tilde{z}_{k(a_i)}) = y(\tilde{z}_{k(a_i)})^{-\frac{1}{\sigma}} Y_{k(a_i)}(\tilde{\boldsymbol{z}}_{k(a_i)})^{\frac{1}{\sigma}-\frac{1}{\rho}} Y_i(\tilde{\boldsymbol{z}}_i)^{\frac{1}{\rho}-1}\alpha_i(\tilde{\boldsymbol{z}})Y(\tilde{\boldsymbol{z}}), \\
sh(\tilde{z}_{k(a_i)}) = \frac{p(\tilde{z}_{k(a_i)})y(\tilde{z}_{k(a_i)})}{P_{k(a_i)}(\tilde{\boldsymbol{z}}_{k(a_i)})Y_{k(a_i)}(\tilde{\boldsymbol{z}}_{k(a_i)})}, \; \mu(\tilde{z}_{k(a_i)}) = \frac{\sigma}{(\sigma-1)\left[1-\frac{\sigma/\rho-1}{\sigma-1}sh(\tilde{z}_{k(a_i)})\right]}, \\
\alpha_i(\tilde{\boldsymbol{z}}) = \frac{P_i(\tilde{\boldsymbol{z}}_i)Y_i(\tilde{\boldsymbol{z}}_i)}{Y(\tilde{\boldsymbol{z}})},
\end{cases}
\tag{77}
$$

where variables in bold denote vectors at the level of aggregation corresponding to their subscript, and aggregators are defined in Equations (5)-(6). Solving the system of equations (77) allows us to back out the productivity distributions that rationalize the observed wage distributions through the lens of our model, $z_{a_i} \sim \Gamma_{a_i}\left(\frac{\mu_{k(a_i)}(\tilde{z})}{p_{k(a_i)}(\tilde{z})}\tilde{z}\right)$. Table (A.2a) shows that the estimated CD industry weights are consistent with the empirical revenue shares of the respective industry.

D.5 Estimation results

Figure A.4: Labor market parameters, by labor market

(a) $\log c_{a_i}$ (b) s_{a_i} (c) b_{a_i} (d) κ_{a_i}



Source: INPS matched employer-employee data (2016-2018), model.

Table A.1: Workers and firms distribution across labor markets

Industry	Manufacturing		Constructions		Trade		Transport		Hospitality		IT		Professional services		Services to firms	
Skill	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High
M_{a_i}	0.054	0.056	0.128	0.133	0.123	0.128	0.045	0.047	0.139	0.144	0.068	0.070	0.164	0.171	0.129	0.134
$\Xi(i)\omega_i(a)$	0.151	0.164	0.044	0.045	0.095	0.083	0.049	0.040	0.067	0.053	0.012	0.020	0.019	0.020	0.079	0.059

Table A.2: Industry-level statistics

(a) Industry weights and revenue shares

Industry	CD weight (model)	Revenue share (data)
Manufacturing	0.355	0.321
Constructions	0.085	0.133
Trade	0.179	0.319
Transport	0.090	0.052
Hospitality	0.092	0.028
IT	0.039	0.034
Professional services	0.042	0.049
Services to firms	0.119	0.065

(b) Avg labor share: 1^{st}-to-2^{nd} emp. tercile

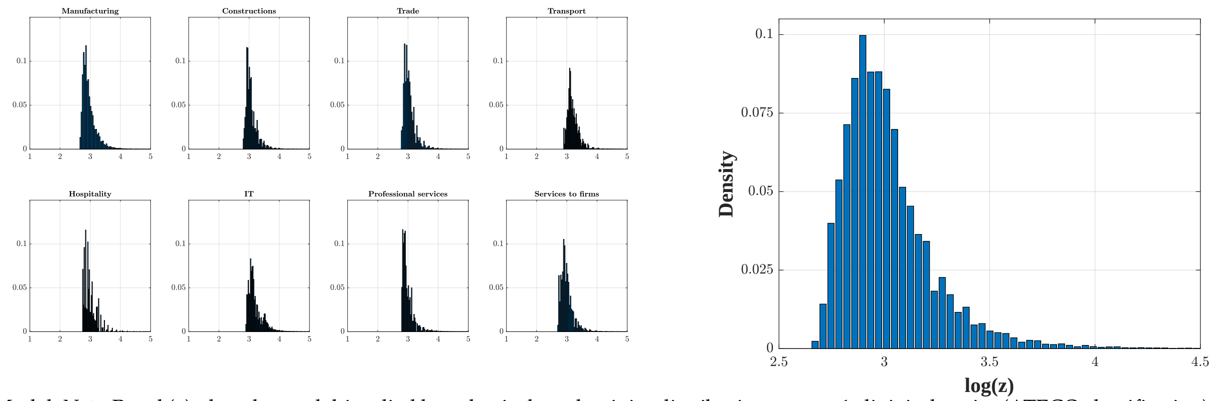
Industry	Model	Data
Manufacturing	0.928	0.838
Constructions	0.941	0.910
Trade	0.942	0.888
Transport	0.916	0.855
Hospitality	0.905	0.829
IT	0.948	0.914
Professional services	0.931	0.907
Services to firms	0.868	0.879

Source: Model and Istat. *Note*: The left table reports the model-implied Cobb-Douglas industry weights and the empirical industry revenue shares. The empirical values (second column) are computed on Istat data (2019). The right table reports the ratio between the average labor share in the first and the second tercile of firms ranked by employment for each 1-digit industry. The empirical values (second column) are computed on the Structural Business Statistics dataset of Istat (2019).

Figure A.5: Physical productivity distribution

(a) By industry

(b) Aggregate

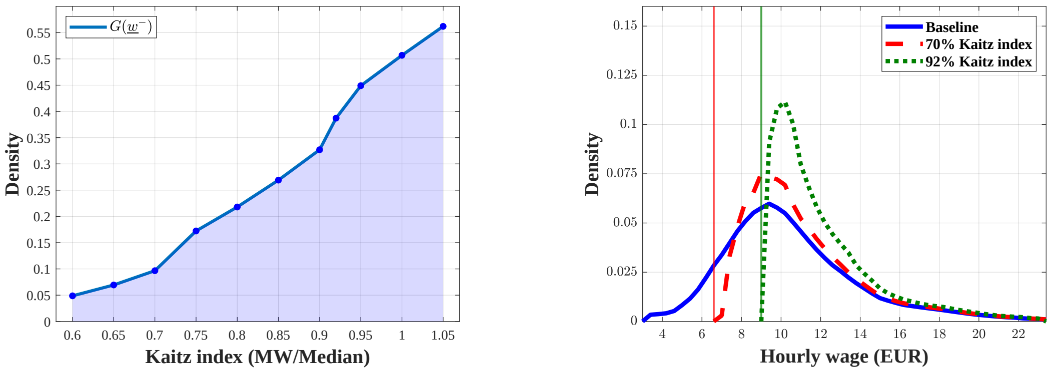


Source: Model. *Note*: Panel (a) plots the model-implied log-physical productivity distributions across 1-digit industries (ATECO classification); Panel (b) plots the model-implied aggregate log-physical productivity distribution.

E Equilibrium Effects of the Minimum Wage

Figure A.6: MW bite and wage distribution by reform

(a) Share directly affected workers by reform (b) Wage distribution response



Source: Model. *Note*: Panel (a) plots the share of directly affected workers (i.e., workers earning strictly less than the mandated MW in the baseline equilibrium) against the Kaitz index of the MW reform. Panel (b) reports the density of the aggregate employment wage distribution $G(w)$ in the baseline equilibrium (solid blue line), after a 70%-Kaitz-index MW reform (red dashed line), and after a 92%-Kaitz-index MW reform (green dotted line).

Table A.3 reports the effects on the main aggregate variables of the two policy experiments surveyed in the main text: the welfare-maximizing MW equal to the 10^{th} percentile of the original wage distribution (70% Kaitz index), and the consumption-maximizing MW equal to the 40^{th} percentile of the initial distribution (92% Kaitz index).

Table A.3: Policy experiments

Variable	Baseline	70% Kaitz index	92% Kaitz index
Panel a. Aggregate statistics			
Value Added	1.000	1.014	1.031
Gross output	1.000	1.000	1.002
Labor share	0.733	0.745	0.753
Aggregate welfare	1.000	1.005	0.987
Unemployment rate	0.135	0.155	0.203
Labor productivity	1.000	1.024	1.087
Average wage	1.000	1.055	1.149
Average firm size	8.737	10.32	15.00
Panel b. Market power statistics			
Aggregate markup	1.139	1.140	1.143
Aggregate markdown	1.471	1.427	1.385
Panel c. Labor market transitions			
Job finding rate	0.161	0.136	0.098
Job separation rate	0.025	0.025	0.025

Source: Model. *Note*: The variables Value added, Gross output, Aggregate Welfare, Labor productivity, and Average wage are normalized to 1 in the baseline equilibrium.

We now derive an aggregate index of market power adjusted for hiring costs. We start by

Table A.4: Behavior vs. selection: decomposition of main aggregate effects

Variable	Percent change	Behavioral share	Compositional share
Average wage	14.85 %	43.59 %	56.41 %
Average labor share	2.33 %	114.66 %	-14.66 %
Aggregate markup rate	3.21 %	55.66 %	44.34 %
Aggregate markdown rate	-18.19 %	109.32 %	-9.32 %

Source: Model. *Note*: The table reports percentage changes between the baseline equilibrium (no MW) and the consumption-maximizing MW (92% Kaitz index). Each aggregate variable X corresponds to a weighted average of the firm-specific variable $x(i)$ $X = \int x(j)\,\hat{w}(j)dj$, where i denotes a specific firm and $\hat{w}(j)$ is the firm-specific weight according to the aggregation results reported in Appendix B.9. The change in the variable X can be decomposed as follows: $\Delta X = \int \Delta x(j)\,\hat{w}'(j)dj + \int x(j)\,\Delta\hat{w}(j)dj$, where $\hat{w}'(j)$ denotes the weight after the change. The first term represents the behavioral effect, the second represents the compositional effect.

rearranging the equilibrium labor share in value added at the firm-level as follows:

$$LS_{a_i}(z) \equiv \frac{aw_{a_i}(z)\ell_{a_i}(z)}{p_{k(a_i)}(z)y_{a_i}(z) - ac_{a_i}(z)} = \frac{aw_{a_i}(z)\ell_{a_i}(z)}{\mu_{k(a_i)}(z)\psi_{a_i}(z)aw_{a_i}\ell_{a_i}(z) - ac_{a_i}(z)}$$
$$= \frac{aw_{a_i}(z)\ell_{a_i}(z)}{\mu_{k(a_i)}(z)\psi_{a_i}(z)aw_{a_i}\ell_{a_i}(z) - \frac{\psi_{a_i}(z)-1}{1+\zeta}aw_{a_i}\ell_{a_i}(z)} = \frac{1}{\mu_{k(a_i)}(z)\psi_{a_i}(z) - \frac{\psi_{a_i}(z)-1}{1+\zeta}},$$

where in the first step we substitute for the optimal price (15) and in the second step for the optimal markdown in terms of hiring costs (20). From this expression, we see that labor market power affects the labor share both directly and through hiring costs.[66] Hence, we define an adjusted market power index at the firm-level as: $\mathcal{M}^{adj}_{a_i}(z) \equiv \mu_{k(a_i)}(z)\psi_{a_i}(z) - \frac{\psi_{a_i}(z)-1}{1+\zeta}$. We then apply a first-order approximation to define aggregate adjusted market power as: $\mathcal{M}^{adj} \equiv \mu\psi - \frac{\psi-1}{1+\zeta}$, where aggregate variables are defined in Appendix B.9. Finally, we decompose the aggregate adjusted market power response to the MW into the labor and product market power responses as follows:

$$\Delta\mathcal{M}^{adj}(\underline{w}') = \underbrace{\left(\mu(\underline{w}) - \frac{1}{1+\zeta}\right)\Delta\psi(\underline{w}')}_{\text{labor market power response}} + \underbrace{\psi(\underline{w})\Delta\mu(\underline{w}')}_{\text{product market power response}}. \tag{78}$$

E.1 Robustness: Nominal Minimum Wage

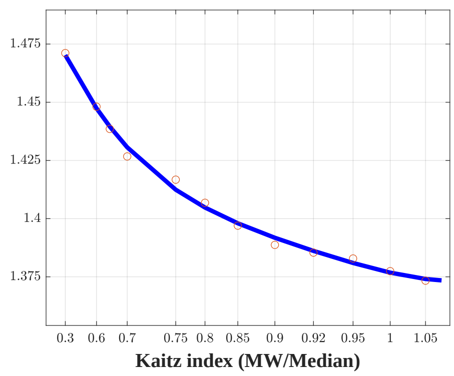
In the main text, we analyze the response of our economy to *real* MWs, i.e. set in terms of final good. However, in the reality, mandated MWs are *nominal*, i.e. set in monetary terms. As a robustness check, we repeat our policy experiments in an economy where money, rather than the final good, is the numeraire. To do so, we keep money supply equal to the nominal output of the baseline economy, i.e., $M^s = PY$, and let the aggregate price level adjust in response to a monetary MW.

In Figure (A.14) we compare the aggregate response of the economy where the MW is set

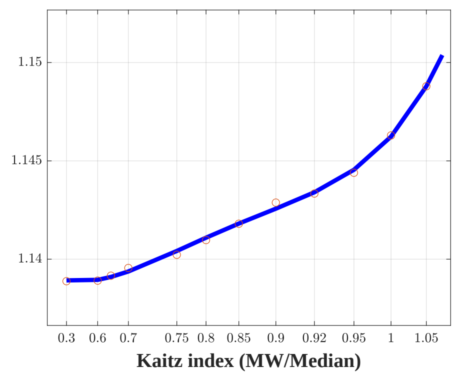
[66]Similarly, we can rearrange equilibrium profits at the firm-level as: $\pi_{a_i}(z) = \left[\mu_{k(a_i)}(z)\psi_{a_i}(z) - \frac{\psi_{a_i}(z)-1}{1+\zeta} - 1\right] aw_{a_i}\ell_{a_i}(z)$.

Figure A.7: Market power indices

(a) Aggregate markdown response



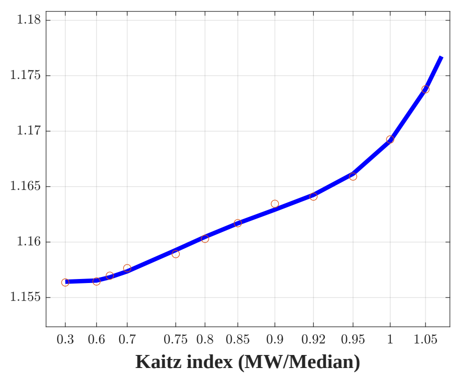
(b) Aggregate markup response



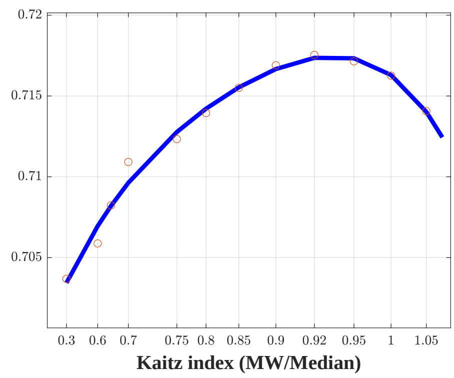
Source: Model. *Note*: Aggregate markdown and markup are cost-weighted averages of the respective firm-level variables, where weights are given by the wage bill. See Appendix (B.9) for the model-consistent aggregation. The x-axis is scaled so as to reflect the share of directly affected workers. The red circular markers represent the simulated data, the blue solid lines their 4^{th}-order polynomial fit.

Figure A.8: Markup and labor share in manufacturing

(a) Manufacturing markup response



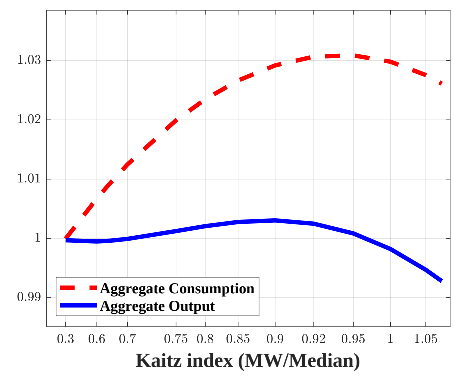
(b) Manufacturing labor share response



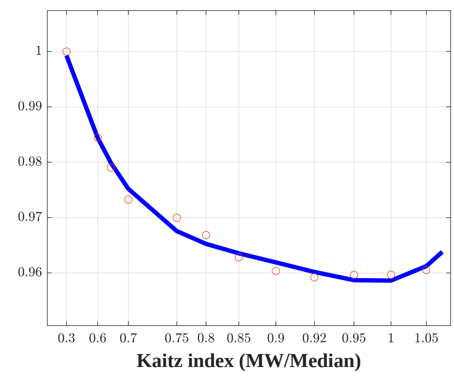
Source: Model. *Note*: Manufacturing markup is the cost-weighted average of the firm-level markup in the manufacturing industry, where weights are given by the wage bill. The x-axis is scaled so as to reflect the share of directly affected workers. The red circular markers represent the simulated data, the blue solid lines their 4^{th}-order polynomial fit.

Figure A.9: Output and profits

(a) Output vs consumption response

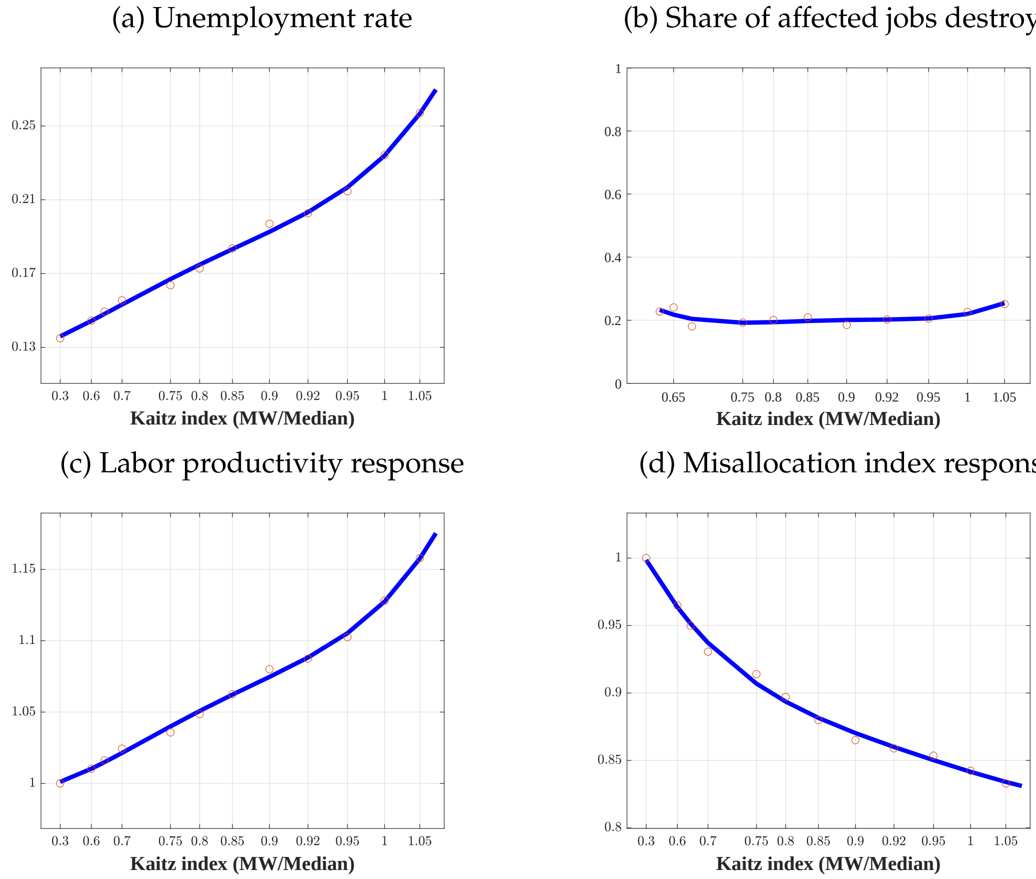


(b) Aggregate profits response



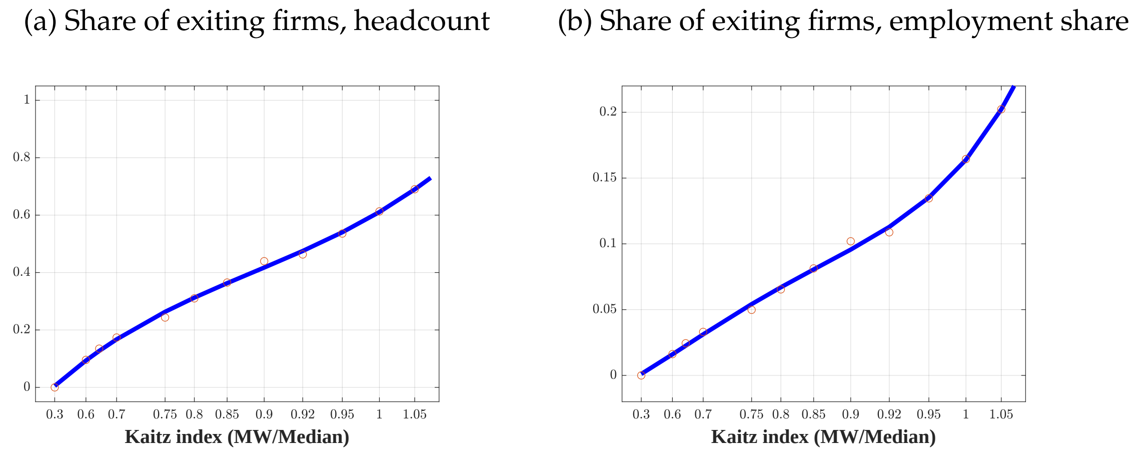
Source: Model. *Note*: Output is final good production (Y). Aggregate profits equal the sum of operating profits net of overhead costs. The x-axis is scaled so as to reflect the share of directly affected workers. The red circular markers represent the simulated data, the blue solid lines their 4^{th}-order polynomial fit.

Figure A.10: Employment and productivity effects of the MW



Source: Model. *Note*: The share of affected jobs destroyed is computed as the ratio of workers earning less than the MW in the baseline equilibrium and the difference in the unemployment rate between the simulated MW reform and the baseline. Labor productivity is defined as $\frac{Y}{L}$. The misallocation index equals total hiring cost per unit of final good relative to the baseline economy. Hiring cost per unit of final good are computed as $\frac{\mathcal{C}_h}{Y}$, where $\mathcal{C}_h$ denotes total hiring costs. The x-axis is scaled so as to reflect the share of directly affected workers. The red circular markers represent the simulated data, the blue solid lines their 4^{th}-order polynomial fit.

Figure A.11: Firm exit effects of the MW

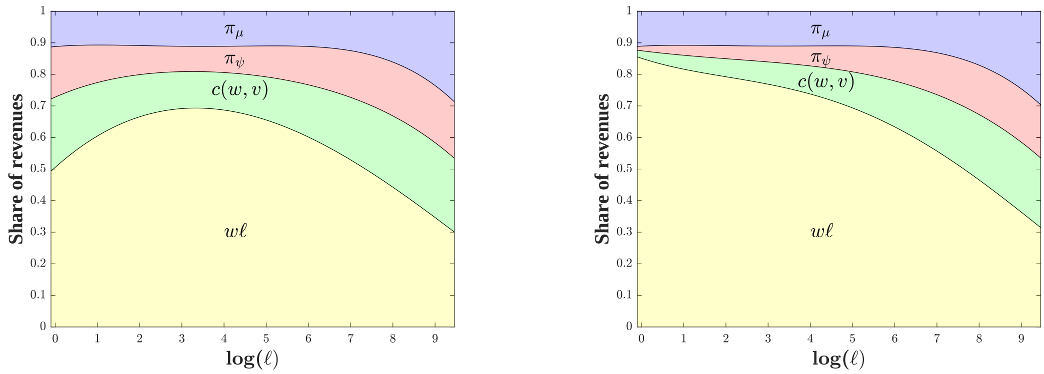


Source: Model. *Note*: The *share of exiting firms firms, headcount* (Panel A) is the ratio between the number of exiting firms after a MW reform relative to the baseline economy; The *share of exiting firms firms, employment share* (Panel A) is the ratio between the baseline employment of firms exiting after a MW reform relative to total employment in the baseline economy. The x-axis is scaled so as to reflect the share of directly affected workers. The red circular markers represent the simulated data, the blue solid lines their 4^{th}-order polynomial fit.

in nominal terms, money is the numeraire, and money supply is fixed, to our baseline. As apparent, setting the MW in real terms or in nominal terms does not make any significant difference.

Figure A.12: Revenue shares response by firm size

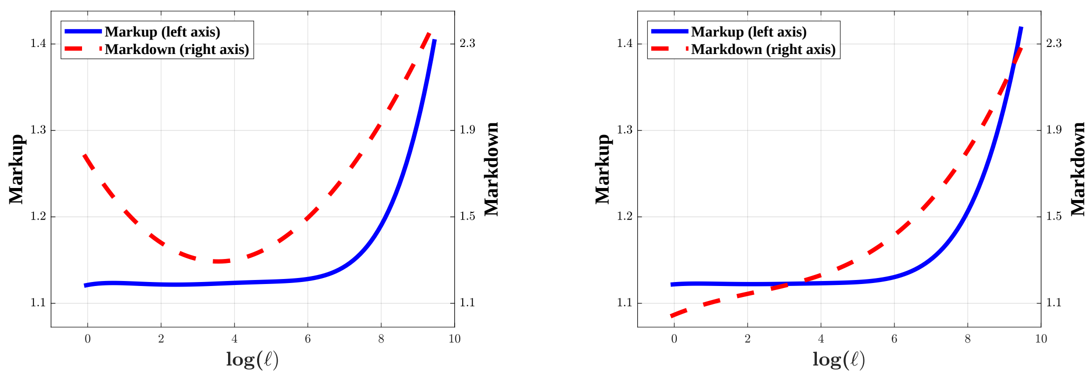
(a) Baseline (b) 100% Kaitz index



Source: Model.

Figure A.13: Market power response by firm size

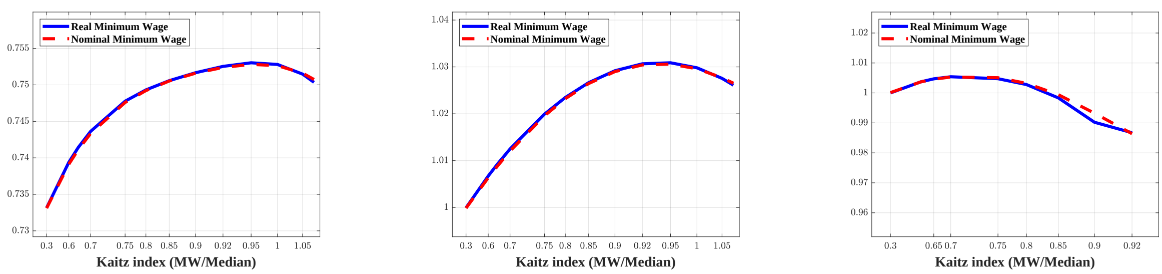
(a) Baseline (b) 100% Kaitz index



Source: Model.

Figure A.14: Aggregate response: Real vs nominal Minimum Wage

(a) Aggregate labor share (b) Aggregate value added (c) Aggregate welfare



Source: Model. *Note*: The solid blue line represents the response of our baseline economy to a real minimum wage, while the dashed red line represents the response of our baseline economy to a nominal minimum wage. The x-axis is scaled so as to reflect the share of directly affected workers.

F Welfare Analysis

Following Floden (2001), utilitarian welfare changes from MW reforms can be decomposed into level, uncertainty, and distributional effects. Let ω_U be the consumption-equivalent welfare change from raising the MW from its baseline level ($\underline{\mathbf{w}}$) to $\underline{\mathbf{w}}'$, that is,

$$\omega_U = \left(\frac{\mathcal{W}(\underline{\mathbf{w}}')}{\mathcal{W}(\underline{\mathbf{w}})} \right)^{\frac{1}{1-\vartheta}} - 1,$$

where $\mathcal{W}$ denotes aggregate welfare, as defined in (**??**).

Floden (2001) shows that utilitarian welfare admits the following multiplicative decomposition:

$$\omega_U = (1 + \omega_{lev})(1 + \omega_{unc})(1 + \omega_{distr}) - 1. \tag{79}$$

We now derive the three components one at a time. In our context, the *level effect* simply boils down to the aggregate consumption response:

$$\omega_{lev} = \frac{C(\underline{\mathbf{w}}')}{C(\underline{\mathbf{w}})} - 1. \tag{80}$$

Express the value of unemployment and employment at wage w in sequence form as:

$$\begin{aligned} U_{a_i} &= \mathbb{E}_t \left[\mathcal{U}_{a_i}(b_{a_i}, \{c_s\}_{s=t+1}^{\infty}) \right] \\ W_{a_i}(w) &= \mathbb{E}_t \left[\mathcal{U}_{a_i}(w, \{c_s\}_{s=t+1}^{\infty}) \right], \end{aligned}$$

where $\mathcal{U}$ is the lifetime utility function associated with our CRRA instantaneous utility function. Hence, we can compute the certainty-equivalent consumption, C^e, for each expected consumption stream starting from any labor market state as:

$$\begin{aligned} C^e(U_{a_i}) &= \mathcal{U}_{a_i}^{-1} \left(\mathbb{E}_t \left[\mathcal{U}_{a_i}(b_{a_i}, \{c_s\}_{s=t+1}^{\infty}) \right] \right) \\ C^e(W_{a_i}(w)) &= \mathcal{U}_{a_i}^{-1} \left(\mathbb{E}_t \left[\mathcal{U}_{a_i}(w, \{c_s\}_{s=t+1}^{\infty}) \right] \right). \end{aligned}$$

Let $C^e(\underline{\mathbf{w}})$ denote the aggregate certainty-equivalent consumption, that is,

$$C^e(\underline{\mathbf{w}}) = \sum_{i=1}^{I} \int \left[u_{a_i}(\underline{w}) C^e\left(U_{a_i}(\underline{w})\right) + \left(1 - u_{a_i}(\underline{w})\right) \int C^e\left(W_{a_i}(aw; \underline{w})\right) \, dG_{a_i}(w; \underline{w}) \right] d\Omega_i(a) \, d(i).$$

In turn, let C^d be the aggregate consumption-equivalent utility of certainty-equivalent consumption, that is,

$$C^d(\underline{\mathbf{w}}) = \left(\sum_{i=1}^{I} \int \left[u_{a_i}(\underline{w}) C^e\left(U_{a_i}(\underline{w})\right)^{1-\vartheta} + \left(1 - u_{a_i}(\underline{w})\right) \int C^e\left(W_{a_i}(aw; \underline{w})\right)^{1-\vartheta} \, dG_{a_i}(w; \underline{w}) \right] d\Omega_i(a) \, d(i) \right)^{\frac{1}{1-\vartheta}}.$$

The *uncertainty effect* is the change in the relative aggregate certainty-equivalent consumption with respect to aggregate consumption:

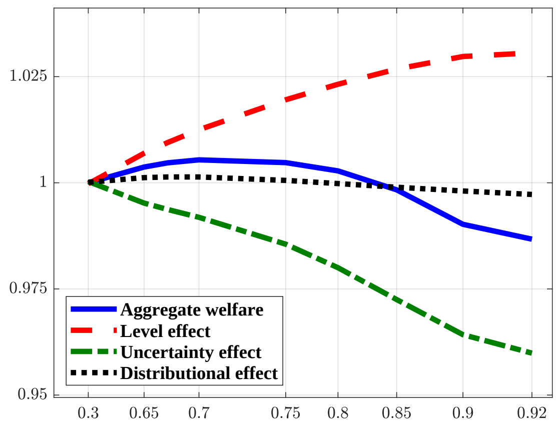
$$\omega_{unc} = \frac{C^e(\underline{\mathbf{w}}')/C(\underline{\mathbf{w}}')}{C^e(\underline{\mathbf{w}})/C(\underline{\mathbf{w}})} - 1. \tag{81}$$

Finally, the *distributional effect* is the change in the aggregate consumption-equivalent utility of certainty-equivalent consumption with respect to certainty-equivalent consumption:

$$\omega_{distr} = \frac{C^d(\underline{\mathbf{w}}')/C^e(\underline{\mathbf{w}}')}{C^d(\underline{\mathbf{w}})/C^e(\underline{\mathbf{w}})} - 1. \tag{82}$$

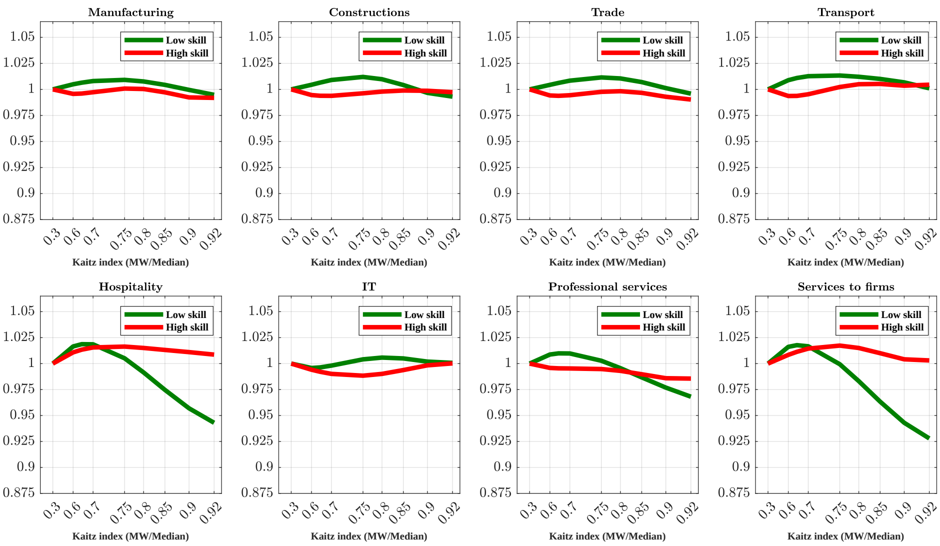
See Dávila and Schaab (2022) for further details on welfare assessments with heterogeneous agents.

Figure A.15: Welfare effects, decomposition



Source: Model. *Note*: The figure decomposes the response of aggregate utilitarian welfare into level, uncertainty, and distributional effects as defined in Floden (2001).

Figure A.16: Breaking down the distributional impact of the MW



Source: Model. *Note*: The graph reports the utilitarian welfare change for workers operating in each industry. The green lines represent low-skilled workers, the red lines high-skilled workers.

G The Role of Endogenous Markups

In the alternative economy we assume monopolistic competition on the product markets, that is, we set $\rho = \sigma$. This implies that markups are exogenous and constant across all firms. We proceed by re-estimating our baseline model according to the same strategy outlined in Section (4). For the counterfactual economy to be as close as possible to our baseline, we use the linear (and equally weighted) combination of weighted average HHI in 4-digit sectors and the semi-elasticity of the labor share with respect to the market share as target to estimate the hiring elasticity of hiring costs, ϕ.

Table A.5: Internally estimated parameters: benchmark vs monopolistic competition

Parameter	Description	Value (Benchmark)	Value (Monopolistic comp.)
a_H	Skill parameter, high skills	1.320	1.337
ζ	Vacancy convexity hiring costs	0.772	0.796
ϕ	Hiring rate elasticity hiring costs	1.715	1.745
σ	Elast. of subst. within sectors	9.171	8.193
ρ	Elast. of subst. across sectors	3.049	8.193
χ	Meeting efficiency	1.465	1.502
tr	Right trimming wage distributions	0.087	0.050
Θ	SMM loss function	0.017	0.015

Table (A.5) compares the differences in the estimated parameters across the two economies. As intuitive, the estimated elasticity of substitution is lower than in the baseline model to replicate the same aggregate markup target. Since we target the aggregate markup, the *average* elasticity of demand across firms remain constant, though.[67] Both structural models are successful at replicating the estimation targets accurately. Hence, absent a MW, the economy with exogenous markups is observationally equivalent to our baseline with endogenous markups. It follows that differential responses to the MW across the two economies can be attributed both to the endogenous markup response and the differential passthrough of the MW across the two demand structures.

[67]While the other parameters are similar across the two economies, the share of right trimming is significantly lower. If we fixed the share of right trimming to equal that of the baseline model, the model with exogenous markups would be unable to replicate the estimation targets (the SMM loss function would equal 0.140 instead of 0.015). We argue that the change in the right tail of the wage distribution induced by differential trimming (by less than $4pp$) introduces less bias than missing the other estimation targets.

H Empirical Validation

H.1 Institutional background

The collective bargaining system in Italy consists of a large number of contracts negotiated between trade unions and employers' associations (see Boeri (2012) for the economic implications of this system). Beyond regulating other aspects of labor contracts (maximum number of hours of work, number of days off, rules for promotions and training), these agreements set wage floors that are sector and skill-specific, and typically have a duration of 3 years (2 years prior to 2009). Importantly, these contracts have a virtually universal coverage – i.e., their validity extends *erga omnes* – and are generally used by labor courts and labor inspectors as a reference for a "fair wage". As a consequence, non-compliance to contractual wages in Italy is extremely rare (Adamopoulou and Villanueva, 2022). Moreover, bargaining at the firm level is also very unusual, with the exception of a few large firms. Therefore, agreed wages represent a very important component of worker pay in the Italian economy.[68] For a more detailed description of the institutional framework see D'Amuri and Nizzi (2018).

H.2 Data

Balance sheet data. We leverage administrative data on the balance sheets for the universe of incorporated Italian firms covering the period 2005-2020 from CERVED. For each firm-year we match information on the number of employees and on the total wage bill coming from INPS (the Italian Social Security institute), so that we can derive a measure of average wage. Finally, the industry classification is used to match the relevant level of contractual wage for each year.

Contractual wages data. The Italian collective bargaining system is highly centralized. Collective agreements are signed at the nationwide industry level, with no room for further adjustments at the local level. Contracts envisage wage floors for different worker categories. In this paper, we use data on average contractual wages at the detailed industry level (3-digit) that are made publicly available by Istat for the period 2005-2020 at the monthly frequency, computed as a weighted average of the levels set by the detailed collective contracts within each industry.

H.3 Empirical Results

We use our micro-data to measure concentration by computing the revenue-based HHI for each 4-digit sector, and to compute the ratio of the sectoral value added and employment as a

[68] Indeed, D'Amuri and Nizzi (2018) document that in the period 2005-2016 contractual wages defined at national level accounted for about 88% of overall total gross earnings. Moreover, a number of studies demonstrates the important role of collective bargaining for downward wage rigidity (Devicienti et al., 2007), wage inequality (Erikson and Ichino, 1994; Manacorda, 2004; Devicienti et al., 2019; Leonardi et al., 2019) and regional differences in employment (Boeri et al., 2021) in Italy.

proxy of sectoral productivity. As shown in Table (A.6a), higher industry-specific wage floors increase both concentration and sectoral productivity, consistent with worker reallocation towards more productive firms. On average, a 1-percent increase in the wage floor causes an increase in concentration of about 0.2pp and a rise in measured labor productivity slightly above 1 percent.

Then, we estimate the firm-level labor share response to a rise in the contractual wage floor in a given year allowing for heterogeneous effects by sectoral concentration:

$$LS_{i,t} = \beta_1 \log MW_{s_3(i),t} + \beta_2 \log MW_{s_3(i),t} \cdot HHI_{s_4(i)} + \beta_3 HHI_{s_4(i)} + \gamma_{s_2(i)} \cdot \phi_t + \alpha_i + \epsilon_{i,t}.$$

The coefficients of interest are β_1, which captures the effect of the wage floor in highly competitive markets ($HHI_{s_4(i)} \approx 0$), and β_2, which captures the gradient of the effect with respect to market concentration. Table A.6b reports the regression results. As expected, the average firm-level labor share response to the MW is positive. However, we uncover a significant and negative partial effect of baseline product market concentration on the firm-level labor share response. This implies that the higher product market concentration is, the lower the firm-level labor share response to the MW, lending support to the hypothesis that product market competition is an important determinant of the labor share response.

Table A.6: Response to contractual wage floor

(a) Sector-level

	(1) HHI (4-digit)	(2) Log labor productivity
Log wage floor	0.174	1.328**
	(0.099)	(0.485)
Time FE	Yes	Yes
Sector FE (4-digit)	Yes	Yes
N	7,381	7,381
R^2	0.871	0.893

(b) Firm-level Labor Share

	(1) Labor share	(2) Labor share
Log wage floor	0.526**	0.522***
	(0.177)	(0.143)
Log wage floor × HHI (4-digit)		-1.086*
		(0.509)
Time × Industry (2-digit) FE	Yes	Yes
Firm FE	Yes	Yes
N	6.12e+06	6.12e+06
R^2	0.733	0.734

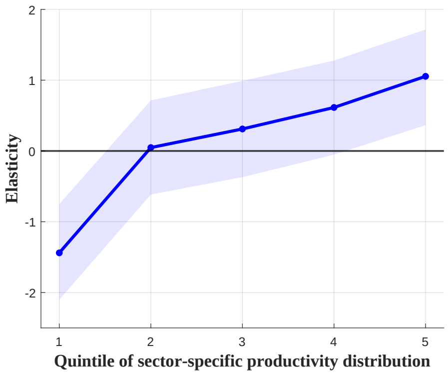
Source: CERVED (2005-2020), INPS, and Istat data. *Note*: Panel (a) reports linear regressions of the HHI of concentration and log average labor productivity at 4-digit level. Formally, $Y_{s_4,t} = \beta_1 \log MW_{s_3,t} + \phi_t + \alpha_{s_4} + \epsilon_{s_4,t}$. Standard errors clustered at the sector-level in parentheses. Panel (b) reports linear regressions of labor share, defined as the ratio between total labor costs and value added. Regression models in column (2) also control for the 4-digit sector-specific HHI. Standard errors clustered at the firm-level in parentheses.

H.4 Determinants of the labor share response

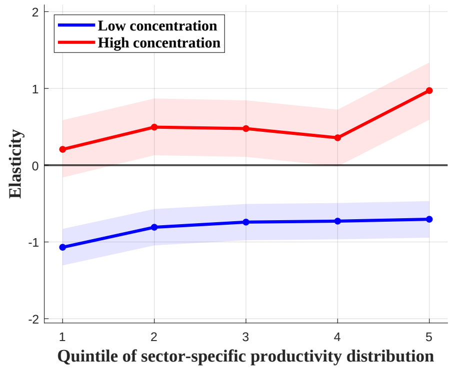
Several adjustment mechanisms at the firm level may bring about changes in the measured labor share. The data at our disposal allow us to dig deeper into the nature of these adjustments. Therefore, in Table A.7 we repeat the same regression of Column 2 of Table A.6b, for different dependent variables: log average wage, log size, log value added and log profits. Our estimates show that the average effect on wages does not depend on the level of concentration. Instead, firm size and value added respond differently depending on the HHI. In particular, firms in more concentrated sectors grow by less (or even shrink) following the wage increase. The dynamics of the value added qualitatively follows the one of firm size, but

Figure A.17: Heterogeneous response by productivity

(a) Log value added



(b) Markup, by concentration



Source: CERVED (2005-2020), INPS and Istat data. *Note*: Panel (a) plots the estimated coefficients of the interaction term between the binned level of sector-specific labor productivity and the natural logarithm of wage floor. The underlying regression is equation (21). Panel (b) reports the average semi-elasticity of firm-level markup to the MW by productivity bins, computed from the regression model (22). Markups are estimated on revenue data as in De Loecker and Warzynski (2012) and Ciapanna et al. (2022). For both panels, standard errors are clustered at the firm-level. Shaded areas represent 90% confidence intervals.

it is much less pronounced. As a result, profits in high-concentration sectors *increase* with the wage floor, in stark contrast with what happens in low-concentration sectors. Taken together, these findings show that the differential reaction of the labor share is strictly associated with opposite dynamics of the profit shares at the firm level.

Table A.7: The Effect of Minimum Wages on the Determinants of the Labor Share

	(1) Log avg wage	(2) Log size	(3) Log value added	(4) Log profits
Log wage floor	0.369***	0.952*	0.293	-1.158
	(0.068)	(0.392)	(0.396)	(0.779)
Log wage floor × HHI (4-digit)	-0.030	-1.414	-0.634	1.487
	(0.192)	(1.412)	(0.532)	(1.318)
Time × Industry (2-digit) FE	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
N	6,155,191	6,158,901	5,774,132	4,915,300
R^2	0.932	0.988	0.978	0.952

Clustered standard errors at the firm-level in parentheses.

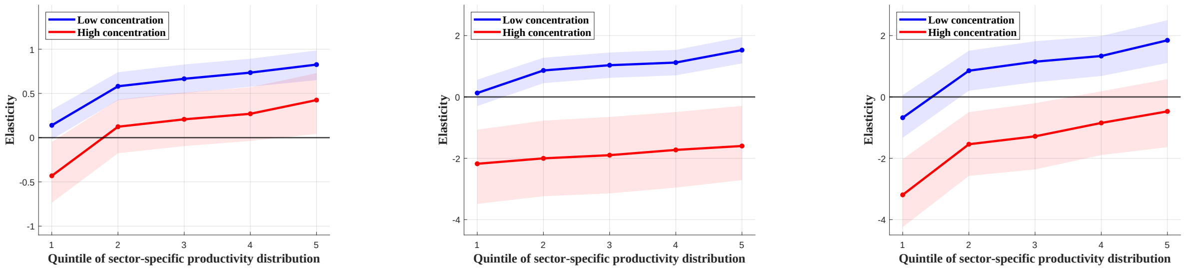
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Source: CERVED (2005-2020), INPS and Istat data. *Note*: Linear regressions of log average wage, log firm size, log value added and log profits (EBITDA). Regression models in all columns also control for the industry-specific level of HHI.

We proceed by breaking down the cross-sectional labor share response into its components (Figure A.18). We uncover a positive gradient by firm productivity in the response of both firm size and value added. However, the change in value added is larger than the one of firm size for low-productivity firms, causing a drop in their profits (Figure (A.9b)). Instead, more productive firms experience a less than proportional variation in value added relative to the change in firm size. As a consequence, profits tend to *rise* in the upper part of the productivity distribution.

Figure A.18: Heterogeneous Response of Labor Share Determinants

(a) Average wage (b) Employment (c) Value added



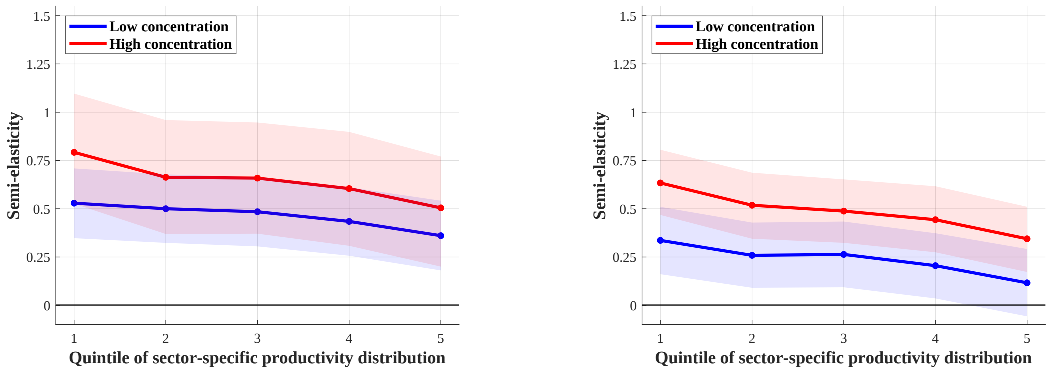
Source: CERVED (2005-2020), INPS and Istat data. *Note*: linear regressions of log average wage, log firm size, and log value added. All panels plot the estimated coefficients of interaction terms between the quintile of the sector-specific productivity distributions and the natural logarithm of wage floor. Standard errors are clustered at the firm-level. Shaded areas represent 90% confidence intervals.

H.5 Product or Labor Market Concentration?

In Section (8) we have established empirically that sectoral concentration is an important predictor of the labor share response to the MW. According to our theory, sectoral concentration matters for product market competition, in that more concentration implies more market power. Mapping product markets into detailed sectors (4-digit, in our case) is a rather standard practice in the literature. However, one may still worry that the degree of sectoral concentration also affects *labor* market competition. To address this concern, we construct proxies of labor market competition and test whether and how adding them to our regression models affects the results. Following the existing literature (Berger et al., 2022; Azar et al., 2024), we work with two definitions of labor market, embracing its geographical dimension: province (administrative geographical units) and province-industry (2-digit).As shown in Table A.8, the estimates of our coefficients of interest are remarkably robust to the inclusion of these additional fixed effects (Columns 2 and 3). Moreover, Figure (A.19) shows that the labor share response is higher in *highly* concentrated labor markets than in low labor concentrated markets across the entire productivity distribution. In sum, raising the MW increases the firm-level labor share the more labor markets are concentrated to start with. The opposite holds for product market concentration.

Figure A.19: Firm-level response by labor market concentration and productivity

(a) Local concentration   (b) Local concentration x 1-digit industry



Source: CERVED (2005-2020), INPS and Istat data. *Note*: The graphs report the average semi-elasticity of firm-level labor share to the MW by productivity bins, computed from the regression model (22). Panel a) uses the HHI of local labor markets (province-level) as a proxy for labor market concentration. Panel b) uses the HHI of local labor markets (province-level) per 1-digit industry as a proxy for labor market concentration. Standard errors are clustered at the firm-level. Confidence intervals are 95%.

Table A.8: Robustness Checks: Controlling for labor market competition

	(1) Labor share	(2) Labor share	(3) Labor share	(4) Labor share	(5) Labor share
Log wage floor	0.515***	0.511***	0.406***	0.472***	0.374***
	(0.132)	(0.119)	(0.076)	(0.084)	(0.068)
Log wage floor × HHI revenues (4-digit)	-1.105*	-1.024*	-0.705**	-1.025*	-0.702**
	(0.473)	(0.408)	(0.260)	(0.405)	(0.257)
Log wage floor × HHI wage bill (prov)				3.060	
				(5.895)	
Log wage floor × HHI wage bill (prov × 1-digit)					0.336
					(0.848)
Firm FE	Yes	Yes	Yes	Yes	Yes
Time × Industry (2-digit) FE	Yes	Yes	Yes	Yes	Yes
Time × Province FE	No	Yes	No	Yes	No
Time × Industry (1-digit) × Province FE	No	No	Yes	No	Yes
N	5,613,465	5,613,462	5,612,641	5,540,053	5,539,139
R^2	0.744	0.746	0.753	0.746	0.754

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Source: CERVED (2005-2020), INPS and Istat data. *Note*: Linear regressions of labor share, defined as the ratio between total labor costs and value added. Regression models also control for the 4-digit sector-specific HHI. Clustered standard errors at the firm-level in parentheses.