Electric Vehicles smart charging for decarbonization

Master In Optimisation (Q. MERIGOT) Applied Game-Theory class (S. LASAULCE): TPs

2025-26, Dec. 18th - Jan. 8th

Olivier BEAUDE (*EDF R&D, OSIRIS department*)

For hurried readers: if you want to focus on the essentials, the ones needed to rapidly start coding (!!), look only at the parts in red. The rest is devoted to provide you with a global view of the context related to the numerical exercise proposed here.

N.B. The following **table of contents, and hyperlinks in the document can help you "navigating" efficiently** between the different parts.

Contents

1 Global setting

In this practical class, the **objective will be to consider a decentralized *charging mechanism* in order to get a "green" charging of a - pretty large - set of Electric Vehicles (EV)**: the fleet which is expected to be driving on the French roads in 2035-2050.

1.1 Practical context (a bit simplified for this exercise)

As we will try to smartly schedule the charge of French EVs, **it is first necessary to understand a few basics about the French electricity consumption. In particular, how are the (existing) electricity production assets managed in order to satisfy the demand?**

On this aspect, if more curious do not hesitate to look at:

- `https://www.rte-france.com/eco2mix` → a lot of information about real-time/historical metrics of the electricity system - focusing on France.
- Regarding - very! - specific situation faced last winter by French electricity system[1], the Ecowatt application/website could be also interesting (see `https://www.monecowatt.fr/`).

.

1.1.1 How to solve the supply-demand equilibrium in the electricity system? The "Unit Commitment" problem

In theory, the schematic principle to schedule electricity production is pretty simple, and can be expressed as follows:

$$\underset{(p_i(t))_{i,t}}{\text{minimize}} \sum_{i=1}^{I} \sum_{t=1}^{48} \alpha_i \times p_i(t)$$
$$\text{s.t.} \quad \begin{cases} \forall t = 1, \cdots, T, \ \sum_{i=1}^{I} p_i(t) = \ell(t) & \text{[French demand satisf.]} \\ \forall i = 1, \cdots, I, \ (p_i(t))_{t=1,\cdots,48} \in \mathcal{P}_i & \text{[Prod. unit i constraints]} \end{cases} \tag{1}$$

where:

- This problem is here written **for a given day, with half-hourly time-slots** (i.e. 48 time-slots in a day).
- $p_i(t)$ is the **production level** of unit i (nuclear, thermal, coal, renewable...). It is a power, measured typically in MW, up to 1.3GW if you consider last generation nuclear units!
- α_i is the **variable cost** to produce a "unit of electricity" (typically a MWh, or GWh) using unit i.

[1] With (i) Ukraine's crisis, and; (ii) Low nuclear assets availability.

- $\mathcal{P}_i$ is the set of **allowed (or "feasible") decisions for production unit**[2] i.
- $\ell(t)$ is the **demand (or "load"** as standardly named in this context) of the electricity system you are considering (**France for our class**).

In words, this optimization problem consists in:

- **Satisfying the electricity demand** (assumed to be fixed, "nonflexible").
- **By scheduling each production unit** i respecting its individual constraints (modeled here via set $\mathcal{P}_i$).
- **And at minimal cost**[3].

This optimization problem is called "Unit Commitment". See Sec. A for more information about specific difficulties, and research questions, on this problem.

For these practical classes, **remember that this "Unit Commitment" optimization problem satisfies electricity demand at each time-slot, and provides you with a production scheduling solution** that can look like the one in Fig. 1.

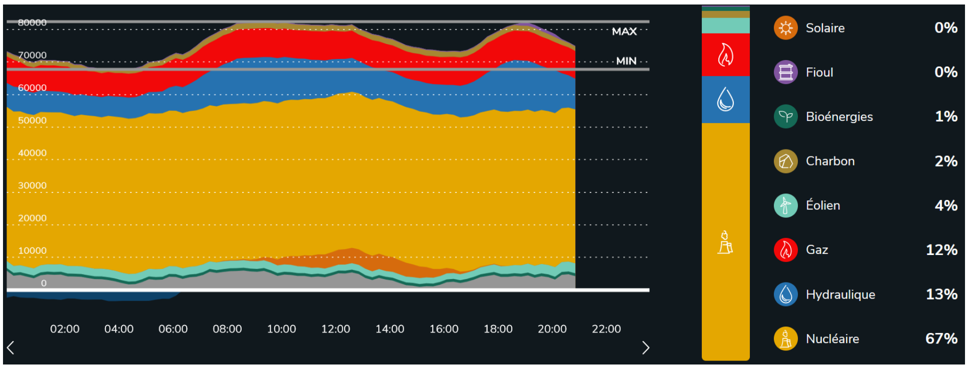

Figure 1: The solution of the Unit Commitment problem for 2021/1/12 (y-axis in MW). Source: *Eco2mix* (RTE).

1.1.2 A direct consequence of this "Unit Commitment": the CO2 content of the French electricity

When looking at Fig. 1 you can observe how, for each time-slot t, the different production units are "stacked" in order to satisfy the French demand: this is named the "merit-order". **A direct consequence of this scheduling is**

[2]First: do not produce more than the installed capacity (!): $\forall t = 1, \cdots, T$, $p_i(t) \leq P_i$.

[3]Here this cost is expressed with euros only, but could also integrate CO2 aspects (emissions) - that can be converted into monetary units, e.g. through a "carbon price".

that the CO2 content of the electricity produced <u>**at time-slot t**</u> **can be expressed as**:

$$\sum_{i=1}^{I} \mathrm{CO2}_i \times p_i(t)\,, \tag{2}$$

where $\mathrm{CO2}_i$ is the **CO2 emission rate associated to production unit** i - directly related to the carbon-content of the primary source of i. It is **expressed in gCO2eq./kWh**.

If you are interested by state-of-the-art values regarding these CO2 emission per type of production asset, do not hesitate to have a look at `https://bilans-ges.ademe.fr/` (you need to create an account, but it is worthwhile - and free!). A lot of details are provided to explain how these ones are calculated, and decomposed into different subcomponents (extraction and transport of primary energy resources, combustion part, losses, etc.).

For this class, <u>**combustion-only**</u> **CO2 emission rates will be considered, following values (in gCO2eq./kWh)** of Table 1.

Asset type	fuel	coal	gas	nuc.	wind	solar	hydro pump.	bioen.
CO2 emi. rate	777	986	429	0	0	0	0	494

Table 1: **Combustion-only CO2 emission rates** (gCO2eq./kWh) of the production assets used here. *A few very large values... and a lot of* 0. <u>Source</u>: Bilans GES ADEME.

As for the variable costs retained - necessary to build f^{CO2} with the proposed approach (in €/kWh) - they are provided in Table 2. These costs are taken from: `https://omnegy.com/la-mecanique-du-merit-order/` (except for bioenergy - arbitrarily? - set to 0€/kWh).

Asset type	fuel	coal	gas	nuc.	wind	solar	hydro pump.	bioen.
Variable cost	162	86	70	30	0	0	0	0

Table 2: **Variable costs** of the production assets used here. *A few decreasing values... and a lot of* 0. <u>Source</u>: `https://omnegy.com/la-mecanique-du-merit-order/`.

1.1.3 Simplified modelling of the CO2 emissions as a function of the electricity demand

As explained a little further, the objective of this class will be to "smartly" charge Electric Vehicles in order to diminish the content of the electricity used for this usage. **Calculating CO2 emission "scores" will necessitate to have a relation between the total French electricity demand and the CO2 emissions** associated to this demand level.

The way of **calculating this approximated curve is as follows**:

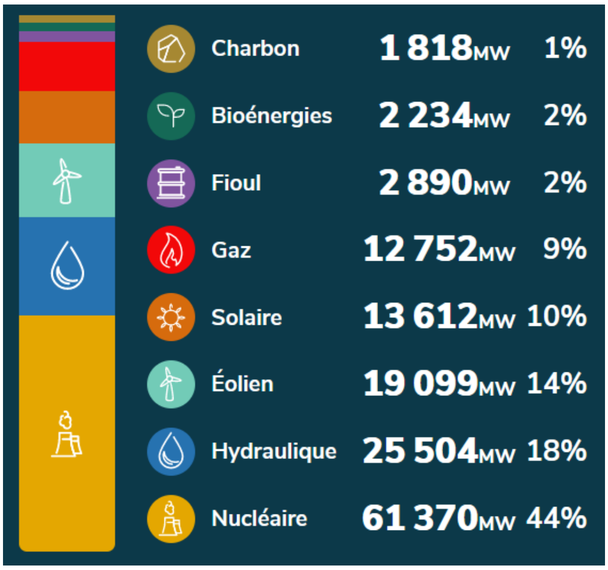

Figure 2: The **retained French electricity generation capacities (per type)** (2022 values). Source: *Eco2mix* (RTE). N.B.: "lost in translation?" "Charbon" is "coal"; rest pretty straightforward!

1. **Get capacity (maximal production level, in MW) of each type of production asset** (nuclear, thermal, etc.) from Fig. 2 (taken from `https://www.rte-france.com/eco2mix/les-chiffres-cles-de-lelectricite` - 2022 values). To take into account the intermittent nature of Variable Renewable Sources (solar and wind), "capacity factors" could (should?) be applied on this maximal power production level: e.g., multiplying by 0.3 (respectively 0.2) the wind (resp. solar) capacity value.

2. **Sort the different production units by increasing variable costs** (given in Table 2). For the units with a same variable cost (for example 0€/kWh) the unit with the smallest CO2 emission rate is taken first. From this point, production (type) units are ordered in set $\mathcal{I} = \{1, \cdots, I\}$.

3. **Start with the first unit ($i = 1$) in the order obtained in item 2., and set the CO2 emission rate to the one associated to this asset type** (see Table 1) for an electricity demand in $[0, P_1]$ (power interval, in MW) with P_1 the capacity of asset type $i = 1$ (as obtained in 1.).

4. **Then, consider the second unit in the order of 2.** ($i = 2$), **and set the CO2 emission rate as the sum of the preceding and the one of this second asset type**. This CO2 emission rate will be applied for a demand in interval $]P_1, P_2]$.

5. **Continue until all types of production assets have been considered.**

In the following, this function will be denoted by f^{CO2}, and can be written as follows:

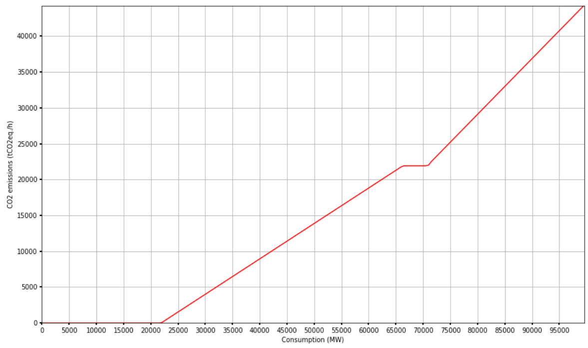

Figure 3: The **retained approximation of the CO2 emission curve**, as a function of the French demand of electricity. *For illustration; to be re-coded on your side, following methodology described previously (do not "extract"/"read" numerical values from this figure!).*

$$\forall \ell \in \left[0, \sum_{i=1}^{I} P_i\right], \; f^{\mathrm{CO2}}(\ell) = \sum_{i=0}^{I-1} (\mathrm{CO2}_{i+1} - \mathrm{CO2}_i) \times \left[\ell - \hat{P}_i\right]^+, \tag{3}$$

where:

- $[x]^+$ is the positive part of x, i.e. $\max(x, 0)$.
- $\hat{P}_i = \sum_{j=0}^{i} P_j$, for $i = 0, \cdots, I-1$, represents the **cumulative capacity of production units** from index 0 to i, having them ordered following the methodology described just above, and with the convention $\hat{P}_0 = P_0 = 0$MW.

Numerically, the obtained function should look like the one in Fig. 3.

Remarks:

1. The term $(\mathrm{CO2}_{i+1} - \mathrm{CO2}_i)$ in (3) is made to add the "marginal" contribution (i.e. CO2 emissions increase) due to the switch from using asset i to $i+1$, and given that, with the chosen formulation, $\mathrm{CO2}_i$ is "already" applied to the quantity produced by asset $i+1$.
2. The output of this function is in CO2eq./h (i.e. for an hour at this load level). To get the CO2 emissions associated to a given load level (in MW), this value has to be multiplied by the duration of the considered time-slot (here 0.5h).
3. Note that function f^{CO2} is **only an approximation** of the relation between French demand and resulting CO2 emissions. Indeed, because the production scheduling via problem (1) is not a functional relation this is more complex in practice... However it will be a **first step to do the numerical tests proposed in this class**. To test the relevance of this

functional representation, a piecewise-linear regression could be done on Eco2Mix data.

4. The **"cumulative" formulation** proposed in (3) is **not necessarily the most elegant one**. However it may be **directly useful for those that will write linear optimization problems in the following** (as explained in Appendix C).

5. A **typical average emission factor for electricity consumed in France** is of 56.9 gCO2eq./kWh (in 2021)[4].

1.2 A recent key evolution: the demand can become "flexible", for example when charging Electric Vehicles (EV)

Now consider the following modification in optimization problem (1)**: a part of the demand can be controlled** in order to get a "better solution" for this problem. **In our setting, we will assume that this part correspond to the Electric Vehicles (EV) charging process**. And that the **objective considered when scheduling EV consumption will be to get the lowest content of CO2 emissions associated to this electricity consumption**.

1.2.1 The basics of our EV flexible usage

The **constraints associated to the EV charging process** are the following:

- It **can only charge when connected** to the electricity system (in a charging station, via a plug). **In this class we will consider EV charging at home, during the night**. In turn, the period when the EVs will be available for charging is defined based on their arrival at home, and departure to work the next day. These information are provided in file */data/ev_scenarios.csv* - see App. B.1 to get a description of data.
- These **vehicles are first made to drive**. Then, the energy charged during the charging period must be sufficient such that the EV user can go to work the next day. This third parameter is also given in file */data/ev_scenarios.csv*; see App. B.1 for more detail.

Mathematically, the set of feasible decisions for EV j is then:

[4]Note that this value has significatively diminished in recent years: it was of 78.5 (resp. 60.9) gCO2eq./kWh in 2008 (resp. 2015). Also, interesting to have in mind that the 2021 value is composed of: (i) 13.8 gCO2eq./kWh for "upstream" process (building an asset, extracting and transporting primary energy resources); (ii) 38 gCO2eq./kWh for combustion-only; (iii) 5.15 gCO2eq./kWh for transport and distribution network losses. To be totally precise, we should compare here the obtained results on the "combustion-only" component. See `https://bilans-ges.ademe.fr/fr/basecarbone/donnees-consulter/liste-element/categorie/64` for more detail (in French).

$$\mathcal{L}_j = \Big\{ \left(\ell_j^{\mathrm{EV}}(t)\right)_t : \text{for } t \in \{a_j+1, ..., d_j-1\},\, 0 \leq \ell_j^{\mathrm{EV}}(t) \leq 7, \text{else } \ell_j^{\mathrm{EV}}(t) = 0, \sum_{t=a_j+1}^{d_j-1} \ell_j^{\mathrm{EV}}(t) = e_j/\delta \Big\} \tag{4}$$

with:

- $\delta = 0.5$h the considered **time-slot duration**.
- a_j (respectively d_j) the **arrival (respectively departure) time-slot** of EV j. Note that the formulation of the upper-bound constraint on the charging power of EV j implicitely assumes the following convention: an EV can neither charge during its arrival time-slot nor during its departure one[5].
- e_j EV j **charging need** (energy, in kWh).

Note that **this assumes** that:

- The **capacity of all EV chargers is the same**, with a maximal charging power of 7kW.
- **EV are not reinjecting electricity to the grid** (lower bound being 0kW). Including this additional flexibility corresponds to the "Vehicle To Grid" (V2G) functionality.
- **Formulating the energy need constraint as an equality forces EV not to charge more than the minimal amount needed. However, an inequality** (charge greater or equal to the need) **could be written instead... and lead to the same solutions hereafter**[6].

1.2.2 An EV trying alone to minimize the CO2 emissions associated to its charging operation

Consider first the **decision-making problem of a unique EV**:

Objective **Minimize the CO2 emissions** associated to its charging operation.

Constraints **While ensuring that its "charging need" being fullfilled** (based on the feasibility set $\mathcal{L}_j$ of (4)),

[5]This is a "robust" assumption implying that if a feasible solution is found by applying this convention, in practice the charging operator will have some margin, if EV arrives before the end of time-slot a_j and/or leaves after the beginning of time-slot d_j.

[6]Given the structure of current optimization problem an EV will not charge more than the minimal level e_j. Why?

which can be expressed as follows:

$$\begin{aligned} & \underset{\left(\ell_j^{\mathrm{EV}}(t)\right)_{t=1,\cdots,48}}{\text{minimize}} \quad \sum_{t=1}^{48} f^{\mathrm{CO2}}\left(\tilde{\ell}^{\mathrm{NF}}(t) + \sum_{j'\neq j} \ell_{j'}^{\mathrm{EV}}(t) + \ell_j^{\mathrm{EV}}(t)\right) \\ & \text{s.t.} \quad \left(\ell_j^{\mathrm{EV}}(t)\right)_{t=1,\cdots,48} \in \mathcal{L}_j \,, \end{aligned} \tag{5}$$

where $\tilde{\ell}^{\mathrm{NF}}(t)$ is the **"non-flexible" part of the load** (not controllable, considered as an input parameter), containing all the French electricity usages but the one of the EV (called "flexible" by opposition). In the following, **this optimization problem will be denoted by $\mathcal{P}_j$.**

Remarks:

1. Typically, and as proposed in the following, **this optimization problem is solved in advance, based on forecasts**, in particular for the non-flexible part of the consumption (**the tilda notation is used to indicate this distinction**). Assuming that current consumption data do not contain a significant part of EV consumption, data of column "forecast_day-1" in file */data/eCO2mix_RTE_Annuel-Definitif_2019_summer.csv* will be considered for this quantity. See App. B.2.

2. Given **charge will take place at home** in this exercise, and **during the night, it is not necessary to consider a full day in the definition of this problem** (i.e. 48 time-slots). The set of time-slots can be restricted to the period when EV is plugged-in at home, i.e. $\{a_j + 1, ..., d_j - 1\}$.

3. Optimization problem 5 is **parametrized by the charging profiles of all other EVs. To define and solve it, it is necessary to have some information about other EVs decisions**. A way of doing that will be considered through the Best-Response-Dynamics introduced justafter.

4. Note that, **more specifically, this optimization problem is parametrized by the aggregated consumption profile of all other EVs** $\sum_{j'\neq j} \ell_{j'}^{\mathrm{EV}}(t)$ (and even added to the profile of non-flexible usages, by $\tilde{\ell}^{\mathrm{NF}}(t)+\sum_{j'\neq j} \ell_{j'}^{\mathrm{EV}}(t)$). As a practical consequence, **EV j could be able to solve its own problem without having the detailed information over all its "neighbours" decisions; it is "privacy-preserving"** (in a certain limit).

5. Finally, note that **a unique EV taken alone has a negligible impact on the variation of CO2 emissions measured at the scale of France** (a maximal charging power of 7kW, compared to typical French consumption levels up to 100GW in winter as illustrated in Fig. 1). It will be discussed further how the proposed modelling could become more realistic.

2 The proposed decentralized mechanism to schedule EV charging

Having introduced the optimization problem to get the charging decision of a unique EV, it is now explained how to coordinate the charging of multiple EVs... up to the whole French EV fleet!

2.1 Algorithm proposed to coordinate EV charging decisions

In the following, **a decentralized algorithm will be proposed to address this objective**. It is based on the **Best-Response-Dynamics** (BRD), described by the following algorithm:

Input $\left(\ell_j^{\mathrm{EV},(0)}(t)\right)_{j,t}$, η, K for stopping criterion

Initial iteration $k = 0$

while $\sum_{j=1}^{J}\sum_{t=1}^{48}\left(\ell_j^{\mathrm{EV},(k)}(t) - \ell_j^{\mathrm{EV},(k-1)}(t)\right)^2 \geq \eta$ **and** $k \leq K$ **do**

Next iteration $k = k + 1$

for $j \in \{1, \cdots, J\}$ **do**

(I) Solve the smart charging optim. pb of EV j: $\mathcal{P}_j$ (def. in (5)) with

$\forall t,\ \ell_{j'}^{\mathrm{EV}}(t) = \ell_{j'}^{\mathrm{EV},(k+1)}(t)$ for $j' \leq j-1$, and $\ell_{j'}^{\mathrm{EV}}(t) = \ell_{j'}^{\mathrm{EV},(k)}(t)$ for $j' \geq j+1$

(II) Set $\ell_j^{\mathrm{EV},(k+1)}(t)$ to one of the argmin(s) of problem $\mathcal{P}_j$ (if there are multiple ones)

end for

end while

Figure 4: The proposed decentralized mechanism to coordinate EV charging decisions. *Once problem 5 is solved, it consists just in adding a while loop, and a loop over the set of EVs.*

Remarks:

1. In terms of implementation, once optimization problem $\mathcal{P}_i$ is coded, this mechanism is pretty straightforward to be obtained!
2. The parameters for the stopping criterion must verify the following:
   - η positive float, small compared to a "typical" value of $\sum_{t=1}^{48}\ell_j^{\mathrm{EV},(k)}(t)$ (which can be calibrated progressively during your tests).
   - K positive integer, not too big at the beginning to see how your algorithm behaves... without spending too much time in it.
3. Here the stopping criterion is based on a norm 2, another distance could be used instead.

4. It is assumed in this version of the BRD that the players (EVs) are updating their decisions sequentially (in the order of their indices). A version with simultaneous update is also possible. You could test this alternative if you want. Do you observe a different behaviour?

3 Objective of this practical work, questions... and the way I will be grading your work

Highlighted in yellow are the aspects to be treated a minima in your final report. For this **final report, 3 rules**:

1. **1 per group** (not individual work!).
2. **MAXIMUM 30 PAGES -** including code.
3. **Free format**: .pdf, notebook, etc.

Note that it will be **better to have this done based on real data** provided in this TP; nonetheless if processing these data seems too complex to you, you could provide these elements coded and tested on (randomly) simple generated data.

The **global objective of these TPs** will be to:

1. **Implement the proposed decentralized mechanism** of Fig. 4 (Best-Response-Dynamics, BRD). In sequence:
   (a) **Analyze and comment the formulation of the optimization problem of a given EV** (see Sec. 1.2.2). What do you think of the way the individual EV charging decision be taken into account in the global CO2 emissions minimization problem?
   (b) **Code and test the resolution of individual EV optimization problem** 5 - using the heuristic water-filling method described in Sec. D. What do you observe in terms of impact of an individual EV over French CO2 emissions?
   (c) **Implement the BRD "loop"**.

2. [More theoretical] Analyze the **behaviour of this mechanism**: Optimality of the solution found in 1.(b) with the heuristic method? Convergence of BRD in 1.(c)? Properties of the obtained solution? Are the obtained numerical results coherent with some results seen in the theoretical part of this class?

3. [More applicative] **Discuss the realism of the obtained results**: CO2 emissions "scores" - using the approximated function f^{CO2} to calculate them? Impact of the number of EVs you simulated? Of the season and year? How to scale-up individual EVs simulated as "meta-EVs" to represent the significant number (15 millions?) of vehicles envisioned in the near future?

4. Write and implement some **"reference" charging policies**:

- **"Plug-and-Charge"**: EV start charging as soon as they connect to the electricity system (at time-slot $t = a_j + 1$ for EV j), and until their energy need (e_j) being fullfilled;
- **"On-off peak fare"**: EV start charging at off-peak time, typically 10PM in France. Remark: this pricing system is the one currently used for Water-Heaters in France, a usage which is very close to the EV in terms of electricity consumption (both for energy and power levels);
- Other policies you could think relevant.

What is the **performance of these reference policies with respect to the decentralized mechanism?** Can you comment on the difficulties to apply - in practice - these policies compared to the decentralized scheme?

Appendices

A More about the Unit Commitment problem

The Unit Commitment problem has been the subject of a lot of research... which is continuing on some very **specific aspects**. Indeed, because:

- Its **size can be very big** (do you know the number of electricity production units available in France?).
- **Pretty complex constraints** can be "hidden" in set $\mathcal{P}_i$, for example "ramping constraints" (the production power cannot increase/decrease without limit between two consecutive time-slots[7]).
- The production units do not only participe to the proper satisfaction of electricity demand, but also to other services for the system[8]... which is not even written in (1)...

this problem can be very complex in practice!

B Data description

This section provides a description of the data provided to run a preliminar analysis in these TPs. Do not hesitate to enrich/create derived cases from this initial dataset.

Common format for all files:

- **type** is .csv;
- **column-separator** is ";";
- **decimal-separator** is "." (English standard).

B.1 Electric Vehicles arrival and departure at/from home

File : */data/ev_scenarios.csv*;

Content : column by column

- **[Integer; No unit; In set** $\{1, ..., T = 48\}$**].** *time_slot_arr* (respectively *time_slot_dep*): index of the time-slot at which an EV arrives (resp. departs) at (resp. from) home;

[7] Typically: $p_i(t+1) - p_i(t) \leq \mathrm{RC}_i^+$ to constrain "acceleration" of production level.

[8] For example, to "ancillary services", consisting in particular in ensuring that the - common in the European system - frequency level stays around its nominal value (50Hz) with a tolerance threshold of about... 0.050Hz; see `https://www.rte-france.com/riverains/la-frequence-electrique-un-indicateur-dequilibre-du-reseau` for more detail (in French).

- **[Float; kWh; In interval** $[0, 50]$**]**. *energy_need (kWh)*: (individual EV) energy to be added in the battery, for the next trip to be done. N.B. this assumes that the maximal energy need for commuting with an EV is 50kWh; in pratice this is largely sufficient, even if very cold weather... and very deep route!

B.2 eCO2mix data: for load forecast, and historical production data

File : */data/eCO2mix_RTE_Annuel-Definitif_2019_summer.csv.* 3 other files with the same format are provided to consider: (i) an alternative season (winter, in 2019); (ii) an alternative year (2020, with a Covid-effect?);

Content : column by column

- **[Str; No unit; In 2019 year]**. *date*: format yyyy-mm-dd HH:MM:SS. N.B. can be easily converted to datetime objects in Python using function datetime.strptime in package datetime.
  Example:
  # Import datetime package
  import datetime
  # Define date format
  date_format = "%Y-%m-%d %H:%M:%S"
  # Convert date in string into a datetime object
  date_as_datetime = datetime.datetime.strptime(date_as_str, date_format)
- **[Float; MW; Non-negative]** *forecast_day-1*: average French power demand over considered time-slot (of half an hour here);
- **[Float; MW; Non-negative]** *fuel / coal / gas / nuclear / wind / solar / hydro_pumping / bioenergy*: average power production per type of asset ("fuel", "coal", etc.);
- **[Float; gCO2eq./kWh; Non-negative]** *co2_rate*: CO2 rate of the total French electricity production at considered time-slot.

C Modelling a piecewise linear objective in an optimization "modeler"

A piecewise linear function that can be written as:

$$\forall \ell \in [0, C] \, , \, f(\ell) = \sum_{i=0}^{I-1} \alpha_i \times [\ell - \beta_i]^+ \tag{6}$$

with $\beta_0 \leq \beta_1 \leq \cdots \beta_{I-1}$ can be "coded" into a linear optimization framework:

- introducing variables $\ell_0^{\text{pos}}, \cdots, \ell_{I-1}^{\text{pos}}$;

- associated to the following constraints

$$\forall i \in \{0, \cdots, I-1\}, \quad \begin{cases} \ell_i^{\text{pos}} \geq 0 \\ \\ \ell_i^{\text{pos}} \geq \ell - \beta_i \end{cases} ; \tag{7}$$

- and considering as objective function

$$\sum_{i=0}^{I-1} \alpha_i \ell_i^{\text{pos}} . \tag{8}$$

Remark: this formulation only works if function f is convex, which implies that all α_i be nonnegative. Otherwise the solver will "have an interest" in setting $\ell_i^{\text{pos}} = +\infty$ for all i such that $\alpha_i < 0$.

Alternative formulation to be considered as soon as function f be not convex. To be very specific, in our case we have:

$$f(\ell_t) = f^{\text{CO2}}(\tilde{\ell}^{\text{NF}}(t) + \ell_t) \text{ with } f^{\text{CO2}}(\ell) = \alpha_i \ell \text{ for } \ell \in [P_{i-1}, P_i] \tag{9}$$

In this case binary variables have to be introduced to "code" the fact that $\tilde{\ell}^{\text{NF}}(t) + \ell_t \in \left[\hat{P}_{i-1}, \hat{P}_i\right]$. A possible way is the following:

1. set i_0 such that $\tilde{\ell}^{\text{NF}}(t) \in \left[\hat{P}_{i_0-1}, \hat{P}_{i_0}\right[$;
2. redefine interval bounds as follows:

$$\tilde{P}_{i_0-1} = 0, \text{ and } \forall i \in \{i_0, \cdots, I\}, \tilde{P}_i = \hat{P}_i - \tilde{\ell}^{\text{NF}}(t) , \tag{10}$$

3. introduce the following binary variables $y_{i_0}, \cdots, y_I \in \{0, 1\}$. In the following they will "code": $y_i = 1 \Leftrightarrow \tilde{\ell}^{\text{NF}}(t) + \ell_t \in \left[\hat{P}_{i-1}, \hat{P}_i\right[\Leftrightarrow \ell_t \in \left[\tilde{P}_{i-1}, \tilde{P}_i\right[$;
4. the idea is then to express ℓ_t as a convex combination of the extreme points of the intervals. Therefore continuous variables $\lambda_{i_0-1}, \cdots, \lambda_I \geq 0$ are introduced such that:

$$\begin{cases} \ell = \sum_{i=i_0}^{I} \lambda_i \tilde{P}_i \\ \\ \sum_{i=i_0}^{I} y_i = 1 \\ \\ \sum_{i=i_0}^{I} \lambda_i = 1 \\ \\ \forall i \in \{i_0, \cdots, I\}, y_i \leq \lambda_{i-1} + \lambda_i \end{cases} ; \tag{11}$$

Note that:

- last two constraints combined provide the following implication:

$$y_i = 1 \Rightarrow (\lambda_{i-1} + \lambda_i = 1, \;\text{ and }\; \forall j \in \{i_0 - 1, \cdots, I\} \setminus \{i-1, i\},\, \lambda_j = 0)\;; \tag{12}$$

- this formulation is supposed to be efficient, but possibly not the easiest one to be understood.

Last but not least, the objective function is obtained as:

$$\sum_{i=i_0-1}^{I} \lambda_i f^{\mathrm{CO2}}(\hat{P}_i)\,. \tag{13}$$

D A heuristic method providing the optimal solution of the 1-EV problem: Water-Filling

The charging optimization problem of EV i can also be solved using the following heuristic method, named "Water-Filling"[9].

Input $\left(\tilde{\ell}^{\mathrm{NF}}(t)\right)_t$, ϵ (small), K (pretty large)

Initialization $k = 0$: $\forall t$, $\ell_j^{\mathrm{EV},(0)}(t) = 0$

while $k \leq K$ **and** $\sum_{t=1}^{48} \ell_j^{\mathrm{EV},(k)}(t) < e_j/\delta$ **do**

Next iteration $k = k + 1$

(I) Get the set of "non-saturated" time-slots:

$$\mathcal{T}^{\mathrm{nonsat}} = \{t : \ell_j^{\mathrm{EV},(k)}(t) < 7\}$$

(II) In this set, get the subset of time-slots with minimal current total load:

$$\tilde{\mathcal{T}}^{\mathrm{nonsat}} = \underset{t \in \mathcal{T}^{\mathrm{nonsat}}}{\operatorname{argmin}} \;\; \tilde{\ell}^{\mathrm{NF}}(t) + \ell_j^{\mathrm{EV},(k)}(t)$$

(III) Add a small charging "quantity" (power) on all the obtained time-slots:

$$\forall t \in \tilde{\mathcal{T}}^{\mathrm{nonsat}},\, \ell_j^{\mathrm{EV},(k)}(t) = \ell_j^{\mathrm{EV},(k)}(t) + \epsilon$$

end while

Figure 5: The Water-Filling algorithm to solve the 1-EV optimization problem (5).

[9]EV charging flows on the nonflexible load curve, as would the water do on mountains and valleys...

This algorithm must provide the same solution as solving optimization problem (5). Why?

Remarks:

1. the maximal number of iterations K is to avoid an infinite while process, which is always a good practice in algorithmic numerical routines!

2. a - largely - more efficient version of this Water-Filling algorithm can be obtained by "tuning" the quantity filled ϵ at each iteration depending on the different non-flexible load levels, ordered in a good direction...

3. identically to the exact optimization problem of EV j (5), when using Water-Filling method for EV j, the temporal period considered can be restricted to $\{a_j + 1, \cdots, d_j - 1\}$ - instead of $\{1, \cdots, 48\}$.