MGT-430 Quantitative systems modeling techniques

Project

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Problem:

With the globalization, the production of multinational compagnies increases. It is accompanied by an ever-greater digitalization of these big compagnies and induces a fierce concurrence between them.

In this context, optimizing is becoming mandatory in all kind of sectors.

In this project, we focus on optimizing the packaging cost of a large amount of raw material and finding a possible way of splitting the boxes containing the raw material between trucks.

Possible practical application:

Fast-moving consumer goods[1] (FMCG) is a type of goods with specific characteristics like:

- High production volumes
- Low profit
- Extensive distribution

A good supply chain is thus mandatory for the companies to stay competitive. Of course, the packaging and transportation are non-negligible parts of the supply chain since FMCG production volume is high and its distribution is wide.

So, the vast majority of FMCG productors like Coca Cola, Nescafe, Ariel, Pampers and so on probably face the problems inherent in this project.

[1] « Fast-Moving Consumer Goods », in *Wikipedia*, 6 April 2020, https://en.wikipedia.org/w/index.php?title=Fast-moving_consumer_goods&oldid=949498009.

Exercise:

- We would like to ship raw material, contained in boxes, to a factory by using trucks.
- We know the ordered raw material weight (in kg) and its density (in kg per L):

Ordered raw material weight [kg]	**Density [kg/L]**
20’250	2

- There are 5 box types. For each box type ‘i’, we know its volume V_i (in L), its cost C_i (in CHF) and the number of available boxes N_i:

Type of box	**V_i [L]**	**C_i [CHF]**	**N_i**
a	10	7	20
b	50	33	20
c	100	65	20
d	200	125	40
e	275	165	40

- There are 3 available trucks. For each truck ‘t’, we know its maximum carrying weight W_t (in kg) and maximum carrying volume V_t (in L):

Truck	**W_t [kg]**	**V_t [L]**
1	5’000	6’000
2	7’000	8’000
3	9’000	10’000

- During the delivery, each box needs to be exactly half full (to always have a feasible solution, you can assume that the ordered raw material weight is always a multiple of 10 → since the density equals 2, the raw material volume is always a multiple of 5 $= {}^{V_a}/_{2}$).

<u>Goal:</u>

Minimize the boxes combination cost and give a possible repartition between trucks.

Model:

- **Variables:**

  X_{it} = number of boxes of type ‘i’ in truck ‘t’.

- **Goal:**

  Minimize the box combination cost: $\sum_{i\in\{a,b,c,d,e\}} C_i \times \sum_{t\in\{1,2,3\}} X_{it}$

- **Constraints:**
  - Integer constraint: X_{it} is integer for all $(i,t) \in \{a,b,c,d,e\} \times \{1,2,3\}$
  - Positivity constraint: $X_{it} \geq 0$ for all $(i,t) \in \{a,b,c,d,e\} \times \{1,2,3\}$
  - Max number of box constraint: $\sum_{t\in\{1,2,3\}} X_{it} \leq N_i$ for all $i \in \{a,b,c,d,e\}$
  - Max volume for each truck constraint: $\sum_{i\in\{a,b,c,d,e\}} V_i X_{it} \leq V_t$ for all $t \in \{1,2,3\}$
  - Max weight for each truck constraint: Raw material weight in truck ‘t’ [kg] = $\frac{\sum_{i\in\{a,b,c,d,e\}} V_i X_{it} [L]}{2} * Raw\ material\ density\ [kg/L] \leq W_t$ for all $t \in \{1,2,3\}$

❖ *Note:* $\frac{ordered\ raw\ material\ weight\ [kg]}{raw\ material\ density\ [kg/L]} = ordered\ raw\ material\ volume\ [L]$

  - Raw material command constraint:

    $\sum_{i\in\{a,b,c,d,e\}} V_i \times \sum_{t\in\{1,2,3\}} X_{it} \geq$ ordered raw material volume
  - Half full boxes constraint:

    Ordered raw material volume = $\frac{\sum_{i\in\{a,b,c,d,e\}} V_i \times \sum_{t\in\{1,2,3\}} X_{it}}{2}$

Excel solver solution:

See ‘PILOTTO-Loris.xlsx’ Excel file.

Optimal solution:

- Boxes combination minimal cost: 12’413 CHF.

- Repartition of boxes in trucks:

	Truck 1	Truck 2	Truck 3	Total number of boxes
Nbr of box a	0	0	0	0
Nbr of box b	1	0	0	1
Nbr of box c	11	0	1	12
Nbr of box d	19	21	0	40
Nbr of box e	0	10	30	40

Obviously, this boxes repartition is not unique since a lot of permutations are possible.

We can still notice that boxes with a high volume are the most used. It is explained by the fact that they have lower ${}^{cost}/_{volum}$ ratio than boxes with a small volume so they are more ‘interesting’ for our minimization cost objective.

- Boxes volume in each truck:

Volume in truck 1 [L]	4950
Volume in truck 2 [L]	6950
Volume in truck 3 [L]	8350

- Boxes weight in each truck:

Weight in truck 1 [kg]	4950
Weight in truck 2 [kg]	6950
Weight in truck 3 [kg]	8350

We can see that in each truck, the volume and weight value are the same. It is because we have the constraint ‘Half full boxes constraint’ so: $boxes\ weight\ in\ each\ truck\ = \frac{boxes\ volume\ in\ each\ truck\ [L]}{2} * raw\ material\ density\ [kg/L] = boxes\ volume\ in\ each\ truck$, since ‘raw material density [kg/L]’ = 2.

We can also notice that the ‘Max weight for each truck constraint’ is the one close to its limit (among the constraints related to trucks) since $W_t = \{5'000, 7'000, 9'000\}$ and $V_t = \{6'000, 8'000, 10'000\}$.

Sensitivity analysis:

- Increasing the ordered raw material to 21'000 kg:
  - Boxes combination minimal cost increase (as expected) to 12'900 CHF
  - Weights in trucks 1, 2 & 3 are respectively of 5'000, 7'000 & 9'000 kg (*i.e.* the max values) while there still are available boxes.
    ⇨ The limitation in this command is not the boxes but the trucks maximal weight.
    ⇨ 21'000 kg is the maximum delivery capacity.

- Reduce the cost of box b to $C_b = 30$:
  ⇨ The $^{price}/_{volume}$ ratio of box b is now the most advantageous. It changes the total number of each box type in the optimal solution:

	Truck 1	Truck 2	Truck 3	Total number of boxes
Nbr of box a	0	5	0	5
Nbr of box b	20	0	0	20
Nbr of box c	2	0	0	2
Nbr of box d	19	21	0	40
Nbr of box e	0	10	30	40

Box type c & a are chosen only once all more advantageous box type (b, d & e) are taken. Here, the last 250 kg of raw material are first put in two boxes of type c (*i.e.* 200 kg in box type c) and finally, the remaining 50 kg are put in 5 boxes of type a (since $5*C_a < 1*C_c$). The boxes combination minimal cost also decreases since we reduced the cost of one used type of box.

Challenging extensions:

- We could relax one of the constraints. For example, for a more realistic problem, 'each box needs to be exactly half full' could be replaced by 'each box needs to be at least half full'. Two constraints would have to be replaced:
  1) Half full boxes constraint:

$$\textit{ordered raw material volume} \geq \frac{\sum_{i\in\{a,b,c,d,e\}} V_i \times \sum_{t\in\{1,2,3\}} X_{it}}{2}$$

  2) Max weight for each truck constraint: $\frac{\textit{Ordered raw material weight } [kg]}{\sum_{i\in\{a,b,c,d,e\}} V_i \times \sum_{t\in\{1,2,3\}} X_{it} \; [L]} \times$

$$\sum_{i\in\{a,b,c,d,e\}} V_i X_{it} \leq W_t \text{ for all } t \in \{1,2,3\}$$

  Note: The second constraint becomes nonlinear. This problem is still solvable with Excel solver, but a nonlinear solving method (like 'GRG non-linear') needs to be selected: See 'PILOTTO-Loris-nonlinear-extension.xlsx' file

- Here we consider a company with a single production site and no delivery price. We could add variables for the truck delivery prices and consider several production sites by allocating respective values (prices/box types/trucks/…) and variables to each separated production site.

REFERENCE: « Fast-Moving Consumer Goods ». In *Wikipedia*, 6 April 2020. https://en.wikipedia.org/w/index.php?title=Fast-moving_consumer_goods&oldid=949498009.