REPORT

Subject: **AADEC**

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Lab Nr 1 Date 23.02.2024 Topic: "Spectral Analysis of Deterministic Signals" Variant 6	Wojciech Kasolik Informatyka II stopień, stacjonarne, I semestr, gr. 1a

1. Problem Statement:

The DFT and IDFT basically solve two sets of linear equations, that are linked as forward and inverse problem.

This is revealed with the important property of the Fourier matrix

$$\mathbf{W}^{-1} = \frac{\mathbf{W}^{\mathrm{H}}}{N} = \frac{\mathbf{W}^{*}}{N},$$

the latter holds since the matrix is symmetric.

Thus, we see that by our convention, the DFT is the inverse problem (signal analysis) and the IDFT is the forward problem (signal synthesis)

$$\text{DFT: } \mathbf{x}_{\mu} = \mathbf{W}^{*}\mathbf{x}_{k} \rightarrow \mathbf{x}_{\mu} = N\mathbf{W}^{-1}\mathbf{x}_{k}$$

$$\text{IDFT: } \mathbf{x}_{k} = \frac{1}{N}\mathbf{W}\mathbf{x}_{\mu}.$$

The occurrence of the N, $1/N$ factor is due to the prevailing convention in signal processing literature.

If the matrix is normalised as $\frac{\mathbf{W}}{\sqrt{N}}$, a so called unitary matrix results, for which the important property

$$(\frac{\mathbf{W}}{\sqrt{N}})^{\mathrm{H}}(\frac{\mathbf{W}}{\sqrt{N}}) = \mathbf{I} = (\frac{\mathbf{W}}{\sqrt{N}})^{-1}(\frac{\mathbf{W}}{\sqrt{N}})$$

holds, i.e. the complex-conjugate, transpose is equal to the inverse $(\frac{\mathbf{W}}{\sqrt{N}})^{\mathrm{H}} = (\frac{\mathbf{W}}{\sqrt{N}})^{-1}$ and due to the matrix symmetry also $(\frac{\mathbf{W}}{\sqrt{N}})^{*} = (\frac{\mathbf{W}}{\sqrt{N}})^{-1}$ is valid.

This tells that the matrix $\frac{\mathbf{W}}{\sqrt{N}}$ is **orthonormal**, i.e. the matrix spans a orthonormal vector basis (the best what we can get in linear algebra world to work with) of N normalized DFT eigensignals.

So, DFT and IDFT is transforming vectors into other vectors using the vector basis of the Fourier matrix.

2. Input Data:

6.

$$\mathbf{x}_{\mu} = [7, 2, 4, 3, 4, 5, 0, 0, 0, 0]^{\mathrm{T}} \tag{22}$$

3. Remote repository link:

https://github.com/Lorn-Hukka/AADEC/tree/main/lab01

4. Execution / Source Code:

```python
# Wojciech Kasolik LAB 01

import numpy as np
import matplotlib.pyplot as plt


Xu = [7, 2, 4, 3, 4, 5, 0, 0, 0, 0]
N = len(Xu)
```
✓ 0.0s

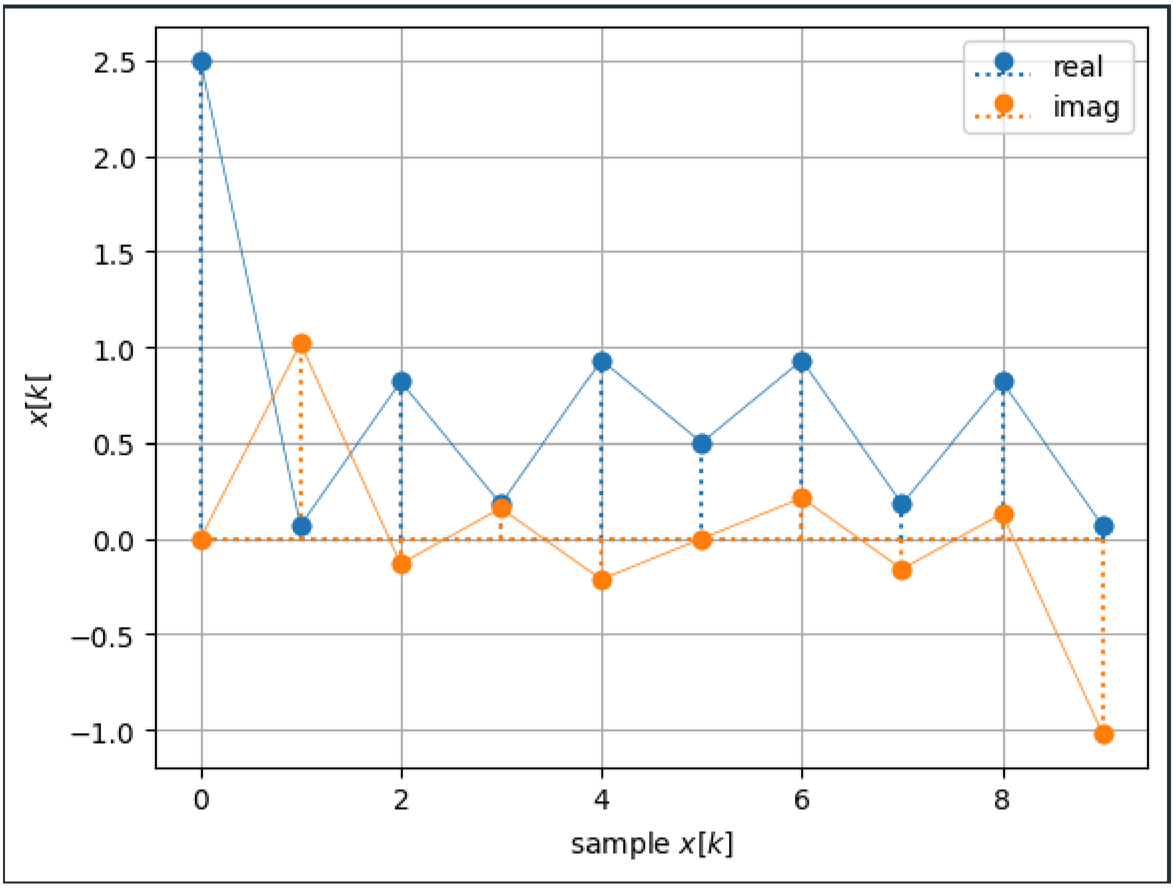
```python
k = np.arange(N)
mu = np.arange(N)
K = np.outer(k, mu)
W = np.exp(+1j*2*np.pi/N*K)
```
✓ 0.0s

```python
np.set_printoptions(precision=2, suppress=True)
display(K)
display(W)
```
✓ 0.0s

```python
signal = 1/N * np.matmul(W, Xu)
display(signal)
```
✓ 0.0s

```python
plt.stem(k, np.real(signal), label='real', markerfmt='C0o', basefmt='C0:', linefmt='C0:')
plt.stem(k, np.imag(signal), label='imag', markerfmt='C1o', basefmt='C1:', linefmt='C1:')

plt.plot(k, np.real(signal), 'C0o-', lw=0.5)
plt.plot(k, np.imag(signal), 'C1o-', lw=0.5)
plt.xlabel(r'sample $x[k]$')
plt.ylabel(r'$x[k[$')
plt.legend()
plt.grid(True)
```



5. Conclusions:

The transformation of the time-domain signal (Xu) into its frequency-domain representation facilitates the examination of the signal's spectral characteristics. The resulting array of DFT coefficients encapsulates the signal's frequency components.